**Algebraic Reasoning Game** 



Carole Greenes
Tanner Wolfram

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Alge-Grid: What's the a?

Factor Max

Make It Proper

Pattern Grid-unLocks

Play It Positively or Negatively!

Shape-Up

Practice Research Innovation in Mathematics Education (PRIME) Group

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#### **Author Bios**



Carole Greenes, Ed.D. is Professor Emerita, Mathematics Education at Arizona State University. While at ASU, she served as Associate Vice President for STEM Education, Dean of the School of Educational Innovation and Teacher Preparation, Director of the Practice Research and Innovation in Mathematics Education (PRIME) Center, Director of the Vertically Integrated Projects program that provides research experiences for undergraduate students, and Professor of Mathematics Education in the Ira A. Fulton Schools of Engineering, the College of Liberal Arts and Sciences, and the Mary Lou Fulton Teachers College. Currently, she directs the PRIME Group that develops books of challenge problems and games for students, grades K - 12, and authors Carole's Corner for the Center for Mathematics and Teaching, Inc. in California. Carole is author of more than 350 books for students, PreK-12 and college, and teachers; 81 articles; six mathematical musicals; and two histories of mathematics in story and song. She served as editor of the Arizona

Association of Teachers of Mathematics journal, *OnCore*, and author of the online monthly free *MATHgazine Senior* (grades 8-12), *MATHgazine Junior* (grades 5-8), *MATHgazine Elementary* (grades 3-5) and *MATHgazine Primary* (grades K-2). In 2003, Greenes was inducted into the Massachusetts Mathematics Educators' Hall of Fame. In 2011, she received the NCSM Ross Taylor/Glenn Gilbert National Leadership Award in Mathematics Education. In 2016, she received the Copper Apple Award for Leadership in Mathematics in Arizona, and in 2018 she received the National Council of Teachers of Mathematics Lifetime Achievement Award. Her 2021 and 2022 books/games include: *Alge-Grid: What'the a?*, *Pattern Grid-unLocks, Play It Positively or Negatively?! Factor Max!*, *Make It Proper!*, and *Shape-Up*. She is author of *Carole's Corner* and *Carole's Commentary* for the Center for Mathematics and Teaching.



Tanner Wolfram is a graduate, Summa cum Laude, of Barrett, The Honors College at Arizona State University with a major in Physics and minors in both Spanish and Chinese. Tanner is coauthor of *Make It Proper, Solve It Positively and Negatively!*, Pattern Grid-unLocks, Factor Max, Alge-Grid: What's the a?, Make It Proper, and Shape-Up puzzle books distributed by the Center for Mathematics and Teaching, and senior author of the Facasumi Puzzle Book for the Arizona Association of Teachers of Mathematics. From Spring 2016 to Fall 2020, Tanner served as Senior Project Assistant in the Practice, Research, and Innovation in Mathematics Education (PRIME) Center at ASU, and is now co-Director of the PRIME Group. During his time with the PRIME Center, Tanner assisted with the NSF-funded App Maker Pro (AMP) project, contributed to and edited eight

MATHadazzle Puzzle Books, co-authored six articles, and co-edited free monthly online MATHgazines (Senior, Junior, Elementary, and Primary).

#### Reasoning Algebraically and Logically

#### What is READY-X?

READY-X is an algebraic reasoning game that requires solution of equations and systems of equations.

**Gameboard:** The board is a 3-by-4 grid with 3 rows, 4 columns, and 12 spaces.

Each space contains one of the letters: R, E, A, D, Y, or X, with some letters repeated in the grid. Within each grid, same letters represent same numbers. Letter values differ with different grids.

Letter values may be 1, 2, 3, 4, 5, 6, 7, 8, 9 or 10.

At the end of each row and at the bottom of each column is the sum of the numbers in that row or column. Therefore, all rows and columns are addition equations.

That is a total of 7 equations: 4 equations with three variables and 3 equations with four variables.

To complete the grid and determine the value of each letter, R, E, A, D, Y, and X, equations and systems of equations must be solved.

#### **Example:**

One Row of a Grid shows: Y + Y + 2 = 12. Then 2Y + 2 = 12. Then 2Y = 10, and Y = 5

One Column of the same Grid shows: R + R + Y + R = 29. Replace Y with 5. Then 3R + 5 = 29. Then 3R = 24, and R = 8.

#### The 40 problems are arranged by difficulty

Section 1: 10 Warm Up

**Section 2: 20 Exercise Your Skills** 

**Section 3: 10 Demonstrate Your X-pertise** 

#### **Possible Solutions** are presented at the end of the book.

Note that solutions are labeled "Possible" because, for most problems, there is more than one solution method. You may want to compare solution methods with others.

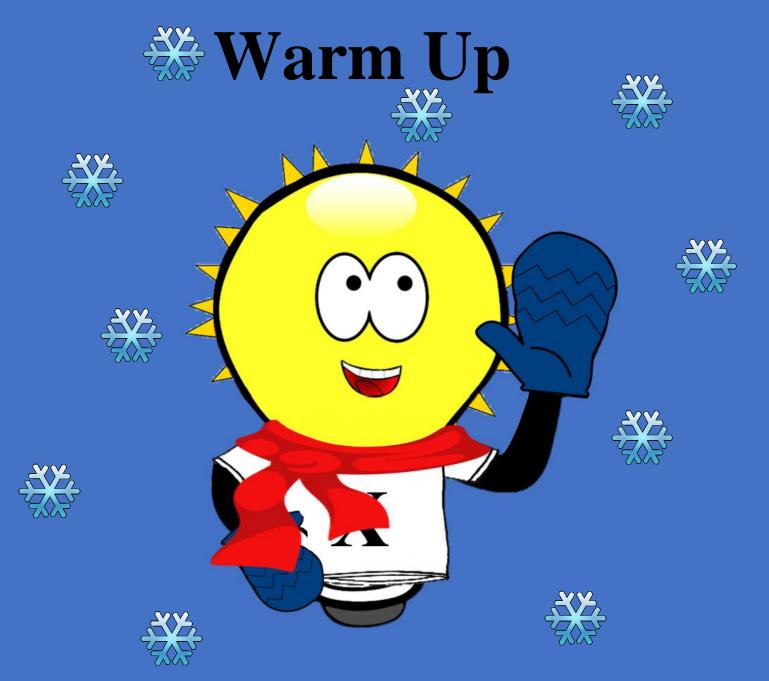
#### Ready to become a READY-Xpert? Give it a try!





# Section 1:





Solve for the values of R, E, A, D, Y, X.

#### **Columns** 1 2 3 $\mathbf{X}$ R $\mathbf{E}$ **17** 1 $\mathbf{X}$ $\mathbf{D}$ $\mathbf{D}$ **13** 2 Rows Y $\mathbf{X}$ A **17** 3 $\mathbf{X}$ $\mathbf{D}$ E **14** 4 **22 12 27**

$$\mathbf{R} = \underline{\hspace{1cm}} \mathbf{E} = \underline{\hspace{1cm}} \mathbf{A} = \underline{\hspace{1cm}} \mathbf{D} = \underline{\hspace{1cm}} \mathbf{Y} = \underline{\hspace{1cm}} \mathbf{X} = \underline{\hspace{1cm}}$$

			<b>Columns</b>		
		1	2	3	
	1	Y	Y	R	12
Dowe	2	X	X	R	28
Rows	3	D	D	D	12
	4	A	E	R	20
	,	23	21	28	_

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns			
		1	2	3		
	1	X	X	D	17	
Rows	2	A	Y	D	16	
	3	Y	X	D	14	
	4	E	R	D	17	
	'	14	14	36	_	

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns			
		1	2	3		
	1	R	Y	R	13	
Rows	2	E	Y	E	19	
	3	A	Y	A	29	
	4	D	Y	X	17	
	'	18	36	24	_	

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns		
		1	2	3	
	1	A	Y	X	21
Rows	2	A	D	X	20
Kows	3	A	E	R	22
	4	A	A	R	21
		32	22	30	_

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns		
		1	2	3	
	1	R	D	E	15
D	2	A	A	A	3
Rows	3	Y	D	Y	18
	4	X	D	A	15
	l	27	13	11	_

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns		
		1	2	3	
Rows	1	R	D	E	22
	2	R	X	A	15
	3	R	E	Y	15
	4	R	Y	Y	11
	,	20	29	14	_

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns		
		1	2	3	
Rows	1	A	E	A	25
	2	R	E	R	17
	3	D	E	D	19
	4	Y	E	X	20
	·	23	28	30	_

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns			
		1	2	3		
Rows	1	X	R	D	24	
	2	Y	R	E	13	
	3	X	X	X	21	
	4	A	X	R	21	
	·	22	30	27	<del>_</del>	

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns		
		1	2	3	
	1	Y	A	X	9
Down	2	D	R	X	14
Rows	3	E	A	X	15
	4	R	R	R	21
		24	22	13	_1

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

# Section 2: Exercise Your Skills



			Columns		
		1	2	3	
Rows	1	E	E	E	30
	2	D	X	E	20
	3	D	R	A	11
	4	X	R	Y	18
		23	21	35	_

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns		
		1	2	3	
	1	A	E	Y	12
Rows	2	Y	A	A	11
	3	A	D	A	9
	4	E	R	X	18
		16	17	17	_

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns		
		1	2	3	
Rows	1	X	A	R	14
	2	A	E	R	21
	3	Y	X	A	10
	4	R	D	R	15
		16	19	25	_

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			<b>Columns</b>		
		1	2	3	
Rows	1	X	E	R	14
	2	X	A	D	6
	3	R	R	A	11
	4	R	X	Y	15
		10	17	19	_1

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns		
		1	2	3	
	1	X	A	R	16
Dows	2	A	R	R	18
Rows	3	Y	D	X	16
	4	E	A	X	18
		27	25	16	

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns		
		1	2	3	
	1	R	X	R	23
D	2	D	Y	Y	9
Rows	3	R	X	D	21
	4	D	E	A	14
		24	23	20	_

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns		
		1	2	3	
Rows	1	R	E	A	18
	2	Y	D	Y	17
	3	D	A	D	16
	4	R	R	X	20
		24	25	22	_

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns		
		1	2	3	
	1	D	A	E	19
Dows	2	X	A	A	24
Rows	3	A	X	E	20
	4	R	X	Y	16
		27	34	18	_

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns		
		1	2	3	
Rows	1	Y	E	R	12
	2	D	E	A	20
	3	D	Y	X	15
	4	Y	D	D	21
		22	15	31	_

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns				
		1	2	3			
	1	E	Y	X	8		
Rows	2	D	E	D	16		
	3	Y	E	A	7		
	4	R	Y	D	15		
		19	10	17			

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

		Columns				
		1	2	3		
Rows	1	Y	X	R	14	
	2	R	E	R	19	
	3	R	A	X	11	
	4	Y	D	X	12	
		26	12	18		

$$\mathbf{R} = \underline{\hspace{1cm}} \mathbf{E} = \underline{\hspace{1cm}} \mathbf{A} = \underline{\hspace{1cm}} \mathbf{D} = \underline{\hspace{1cm}} \mathbf{Y} = \underline{\hspace{1cm}} \mathbf{X} = \underline{\hspace{1cm}}$$

			Columns		
		1	2	3	
	1	E	A	A	20
Dowe	2	R	E	X	10
Rows	3	D	E	X	13
	4	Y	A	Y	11
		16	22	16	_

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns		
		1	2	3	
Rows	1	X	A	D	22
	2	D	A	D	18
	3	R	Y	E	18
	4	D	Y	A	20
	,	25	32	21	_

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns		
		1	2	3	
	1	X	D	R	11
Dows	2	R	D	D	9
Rows	3	X	E	Y	<b>17</b>
	4	R	E	A	19
		20	18	18	_

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns		
		1	2	3	
Rows	1	A	D	D	27
	2	X	X	D	28
	3	D	Y	E	22
	4	X	R	R	21
		35	33	30	

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns		
		1	2	3	
Rows	1	A	Y	A	29
	2	R	X	R	17
	3	D	Y	E	13
	4	Y	X	R	21
	·	25	32	23	_

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			<b>Columns</b>		
		1	2	3	
Rows	1	E	A	R	9
	2	R	X	D	22
	3	E	D	Y	14
	4	R	Y	E	12
		16	22	19	_

$$\mathbf{R} = \underline{\hspace{1cm}} \mathbf{E} = \underline{\hspace{1cm}} \mathbf{A} = \underline{\hspace{1cm}} \mathbf{D} = \underline{\hspace{1cm}} \mathbf{Y} = \underline{\hspace{1cm}} \mathbf{X} = \underline{\hspace{1cm}}$$

			Columns		
		1	2	3	
Rows	1	A	D	X	21
	2	E	A	Y	19
	3	E	E	X	19
	4	Y	R	Y	7
		25	21	20	_

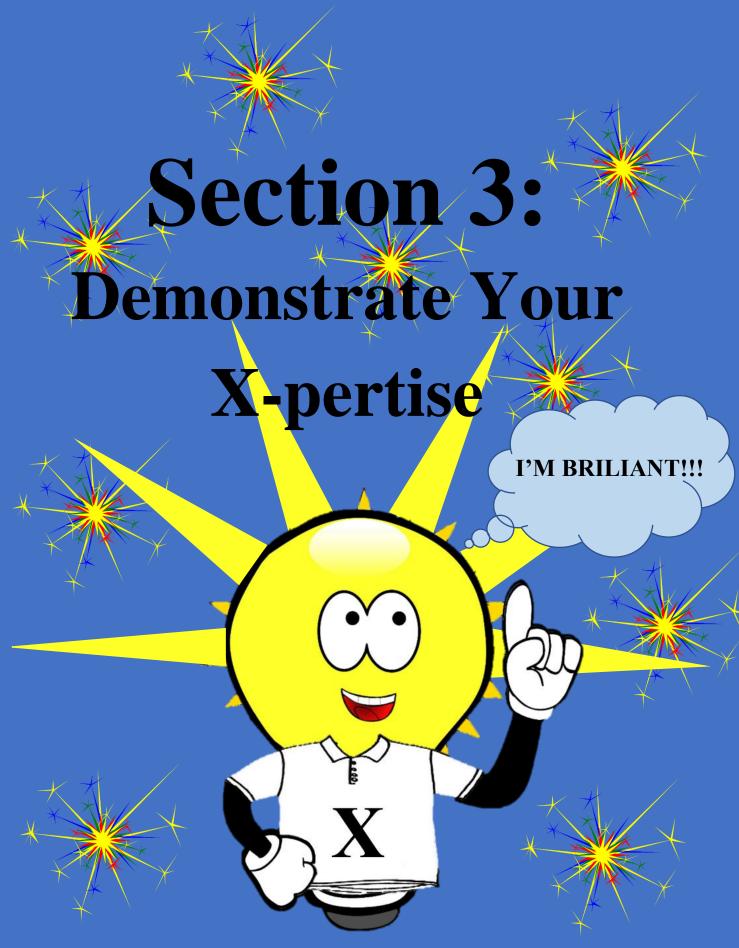
$$\mathbf{R} = \underline{\hspace{1cm}} \mathbf{E} = \underline{\hspace{1cm}} \mathbf{A} = \underline{\hspace{1cm}} \mathbf{D} = \underline{\hspace{1cm}} \mathbf{Y} = \underline{\hspace{1cm}} \mathbf{X} = \underline{\hspace{1cm}}$$

			Columns		
		1	2	3	
Rows	1	R	E	E	26
	2	D	E	R	21
	3	D	Y	A	15
	4	R	X	X	14
		22	33	21	

$$\mathbf{R} = \underline{\hspace{1cm}} \mathbf{E} = \underline{\hspace{1cm}} \mathbf{A} = \underline{\hspace{1cm}} \mathbf{D} = \underline{\hspace{1cm}} \mathbf{Y} = \underline{\hspace{1cm}} \mathbf{X} = \underline{\hspace{1cm}}$$

			Columns		
		1	2	3	
Rows	1	E	X	E	20
	2	R	A	A	7
	3	E	R	D	11
	4	R	Y	R	11
	l	20	16	13	_

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$



			Columns		
		1	2	3	
	1	A	Y	E	21
Rows	2	E	E	R	18
Rows	3	D	A	X	23
	4	R	X	A	19
		28	27	26	_

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns		
		1	2	3	
Rows	1	A	X	R	17
	2	A	E	X	11
	3	R	X	D	18
	4	D	Y	Y	23
	,	28	15	26	1

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns				
		1	2	3			
	1	X	A	Y	15		
Down	2	R	Y	A	14		
Rows	3	A	D	D	19		
	4	Y	E	X	9		
		16	21	20			

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns				
		1	2	3			
Rows	1	R	A	D	11		
	2	X	Y	R	11		
	3	D	E	R	9		
	4	E	D	E	10		
		16	15	10	_		

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns				
		1	2	3			
Rows	1	R	Y	R	18		
	2	E	Y	E	24		
	3	X	Y	X	22		
	4	D	D	A	14		
		20	33	25	_		

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns		
		1	2	3	
Rows	1	D	R	E	20
	2	E	Y	E	21
	3	R	A	X	13
	4	A	X	Y	12
		21	16	29	_

$$\mathbf{R} = \underline{\hspace{1cm}} \mathbf{E} = \underline{\hspace{1cm}} \mathbf{A} = \underline{\hspace{1cm}} \mathbf{D} = \underline{\hspace{1cm}} \mathbf{Y} = \underline{\hspace{1cm}} \mathbf{X} = \underline{\hspace{1cm}}$$

			Columns		
		1	2	3	
Rows	1	R	D	A	21
	2	D	E	X	19
	3	X	R	D	15
	4	Y	A	E	23
		20	31	27	_

$$\mathbf{R} = \underline{\hspace{1cm}} \mathbf{E} = \underline{\hspace{1cm}} \mathbf{A} = \underline{\hspace{1cm}} \mathbf{D} = \underline{\hspace{1cm}} \mathbf{Y} = \underline{\hspace{1cm}} \mathbf{X} = \underline{\hspace{1cm}}$$

			Columns		
		1	2	3	
Rows	1	A	A	D	10
	2	A	X	X	10
	3	X	R	A	9
	4	Y	E	X	22
		16	19	16	_

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns		
		1	2	3	
Rows	1	D	R	A	22
	2	D	Y	Y	11
	3	R	X	Y	20
	4	A	E	X	23
		25	28	23	_

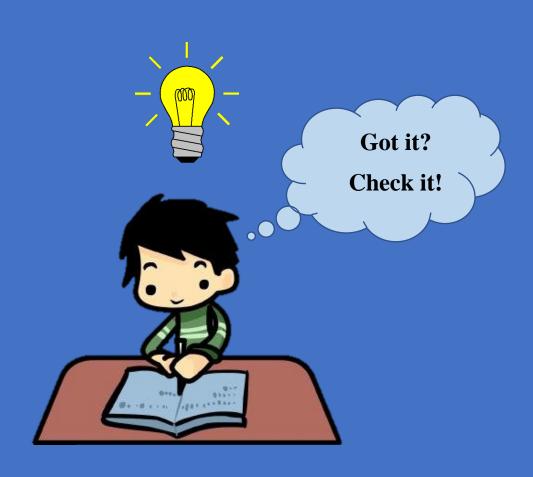
$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

			Columns		
		1	2	3	
Rows	1	X	X	A	14
	2	R	R	D	11
	3	E	E	Y	20
	4	Y	E	A	17
		19	20	23	_

$$\mathbf{R} = \underline{\qquad} \mathbf{E} = \underline{\qquad} \mathbf{A} = \underline{\qquad} \mathbf{D} = \underline{\qquad} \mathbf{Y} = \underline{\qquad} \mathbf{X} = \underline{\qquad}$$

## Solutions

## Section 1: Warm Up



$$R = 8$$
  $E = 6$   $A = 10$   $D = 5$   $Y = 4$   $X = 3$ 

#### **Possible Solution**

Column 2: X + X + X + X = 12. Then 4X = 12, and X = 3.

Row 2: D + X + D = 13. Replace X with 3. Then D + 3 + D = 13. Then 2D + 3 = 13. Then 2D = 10, and D = 5.

Row 4: D + X + E = 14. Replace D with 5 and X with 3. Then 5 + 3 + E = 14, and E = 6.

Column 3: E + D + A + E = 27. Replace each E with 6 and D with 5. Then 6 + 5 + A + 6 = 27, and A = 10.

Row 3: Y + X + A = 17. Replace X with 3 and A with 10. Then Y + 3 + 10 = 17, and Y = 4.

Row 1: R + X + E = 17. Replace X with 3 and E with 6. Then R + 3 + 6 = 17, and R = 8.

$$R = 8$$
  $E = 5$   $A = 7$   $D = 4$   $Y = 2$   $X = 10$ 

#### **Possible Solution**

Row 3: D + D + D = 12. Then 3D = 12, and D = 4.

Column 3: R + R + D + R = 28. Replace D with 4. Then R + R + 4 + R = 28. Then 3R + 4 = 28. Then 3R = 24, and R = 8.

Row 1: Y + Y + R = 12. Replace R with 8. Then Y + Y + 8 = 12. Then 2Y + 8 = 12. Then 2Y = 4, and Y = 2.

Row 2: X + X + R = 28. Replace R with 8. Then X + X + 8 = 28. Then 2X + 8 = 28. Then 2X = 20, and X = 10.

Column 1: Y + X + D + A = 23. Replace Y with 2, X with 10, and D with 4. Then 2 + 10 + 4 + A = 23, and A = 7.

Row 4: A + E + R = 20. Replace A with 7 and R with 8. Then 7 + E + 8 = 20, and E = 5.

$$R = 5$$
  $E = 3$   $A = 6$   $D = 9$   $Y = 1$   $X = 4$ 

#### **Possible Solution**

Column 3: D + D + D + D = 36. Then 4D = 36, and D = 9.

Row 1: X + X + D = 17. Replace D with 9. Then X + X + 9 = 17. Then 2X + 9 = 17. Then 2X = 8, and X = 4.

Row 3: Y + X + D = 14. Replace X with 4 and D with 9. Then Y + 4 + 9 = 14, and Y = 1.

Row 2: A + Y + D = 16. Replace Y with 1 and D with 9. Then A + 1 + 9 = 16, and A = 6.

Column 2: X + Y + X + R = 14. Replace each X with 4 and Y with 1. Then 4 + 1 + 4 + R = 14, and R = 5.

Row 4: E + R + D = 17. Replace R with 5 and D with 9. Then E + 5 + 9 = 17, and E = 3.

$$R = 2$$
  $E = 5$   $A = 10$   $D = 1$   $Y = 9$   $X = 7$ 

#### **Possible Solution**

Column 2: Y + Y + Y + Y = 36. Then 4Y = 36, and Y = 9.

- Row 1: R + Y + R = 13. Replace Y with 9. Then R + 9 + R = 13. Then 2R + 9 = 13. Then 2R = 4, and R = 2.
- Row 2: E + Y + E = 19. Replace Y with 9. Then E + 9 + E = 19. Then 2E + 9 = 19. Then 2E = 10, and E = 5.
- Row 3: A + Y + A = 29. Replace the Y with 9. Then A + 9 + A = 29. Then 2A + 9 = 29. Then 2A = 20, and A = 10.
- Column 1: R + E + A + D = 18. Replace R with 2, E with 5, and A with 10. Then 2 + 5 + 10 + D = 18. Then D = 1.
- Row 4: D + Y + X = 17. Replace D with 1 and Y with 9. Then 1 + 9 + X = 17, and X = 7.

$$R = 5$$
  $E = 9$   $A = 8$   $D = 2$   $Y = 3$   $X = 10$ 

#### **Possible Solution**

Column 1: A + A + A + A = 32. Then 4A = 32, and A = 8.

Row 4: A + A + R = 21. Replace each A with 8. Then 8 + 8 + R = 21, and R = 5.

Row 3: A + E + R = 22. Replace A with 8 and R with 5. Then 8 + E + 5 = 22, and E = 9.

Column 3: X + X + R + R = 30. Replace each R with 5. Then X + X + 5 + 5 = 30. Then 2X + 10 = 30. Then 2X = 20, and X = 10.

Row 1: A + Y + X = 21. Replace A with 8 and X with 10. Then 8 + Y + 10 = 21, and Y = 3.

Row 2: A + D + X = 20. Replace A with 8 and X with 10. Then 8 + D + 10 = 20, and D = 2.

$$R = 9$$
  $E = 2$   $A = 1$   $D = 4$   $Y = 7$   $X = 10$ 

#### **Possible Solution**

Row 2: A + A + A = 3. Then 3A = 3, and A = 1.

Column 2: D + A + D + D = 13. Replace A with 1. Then 3D + 1 = 13. Then 3D = 12, and D = 4.

Row 4: X + D + A = 15. Replace D with 4 and A with 1. Then X + 4 + 1 = 15, and X = 10.

Row 3: Y + D + Y = 18. Replace D with 4. Then 2Y + 4 = 18. Then 2Y = 14, and Y = 7.

Column 3: E + A + Y + A = 11. Replace each A with 1 and Y with 7.

Then E + 1 + 7 + 1 = 11, and E = 2.

Row 1: R + D + E = 15. Replace D with 4 and E with 2.

Then R + 4 + 2 = 15, and R = 9.

$$R = 5$$
  $E = 7$   $A = 1$   $D = 10$   $Y = 3$   $X = 9$ 

#### **Possible Solution**

Column 1: R + R + R + R = 20. Then 4R = 20, and R = 5.

Row 4: R + Y + Y = 11. Replace R with 5. Then 5 + Y + Y = 11.

Then 5 + 2Y = 11. Then 2Y = 6, and Y = 3.

Row 3: R + E + Y = 15. Replace R with 5 and Y with 3. Then 5 + E + 3 = 15, and E = 7.

Row 1: R + D + E = 22. Replace R with 5 and E with 7. Then 5 + D + 7 = 22, and D = 10.

Column 2: D + X + E + Y = 29. Replace D with 10, E with 7, and Y with 3.

Then 10 + X + 7 + 3 = 29, and X = 9.

Row 2: R + X + A = 15. Replace R with 5 and X with 9. Then 5 + 9 + A = 15, and A = 1.

$$R = 5$$
  $E = 7$   $A = 9$   $D = 6$   $Y = 3$   $X = 10$ 

#### **Possible Solution**

Column 2: E + E + E + E = 28. Then 4E = 28, and E = 7.

Row 1: 
$$A + E + A = 25$$
. Replace E with 7. Then  $A + 7 + A = 25$ .  
Then  $2A + 7 = 25$ . Then  $2A = 18$ , and  $A = 9$ .

Row 2: 
$$R + E + R = 17$$
. Replace E with 7. Then  $R + 7 + R = 17$ . Then  $2R + 7 = 17$ . Then  $2R = 10$ , and  $R = 5$ .

Row 3: 
$$D + E + D = 19$$
. Replace E with 7. Then  $D + 7 + D = 19$ . Then  $2D + 7 = 19$ . Then  $2D = 12$ , and  $D = 6$ 

Column 3: 
$$A + R + D + X = 30$$
. Replace A with 9, R with 5, and D with 6.  
Then  $9 + 5 + 6 + X = 30$ , and  $X = 10$ .

Row 4: 
$$Y + E + X = 20$$
. Replace E with 7 and X with 10.  
Then  $Y + 7 + 10 = 20$ , and  $Y = 3$ .

$$R = 8$$
  $E = 3$   $A = 6$   $D = 9$   $Y = 2$   $X = 7$ 

#### **Possible Solution**

Row 3: X + X + X = 21. Then 3X = 21, and X = 7.

Column 2: R + R + X + X = 30. Replace each X with 7. Then R + R + 7 + 7 = 30. Then 2R + 14 = 30. Then 2R = 16, and R = 8.

Row 1: X + R + D = 24. Replace X with 7 and R with 8. Then 7 + 8 + D = 24, and D = 9.

Row 4: A + X + R = 21. Replace X with 7 and R with 8. Then A + 7 + 8 = 21, and A = 6.

Column 3: D + E + X + R = 27. Replace D with 9, X with 7, and R with 8. Then 9 + E + 7 + 8 = 27, and E = 3.

Row 2: Y + R + E = 13. Replace R with 8 and E with 3. Then Y + 8 + 3 = 13, and Y = 2.

$$R = 7$$
  $E = 9$   $A = 4$   $D = 5$   $Y = 3$   $X = 2$ 

#### **Possible Solution**

Row 4: R + R + R = 21. Then 3R = 21, and R = 7.

Column 3: X + X + X + R = 13. Replace R with 7. Then X + X + X + 7 = 13. Then 3X + 7 = 13. Then 3X = 6, and X = 2.

Column 2: A + R + A + R = 22. Replace each R with 7. Then A + 7 + A + 7 = 22. Then 2A + 14 = 22. Then 2A = 8, and A = 4.

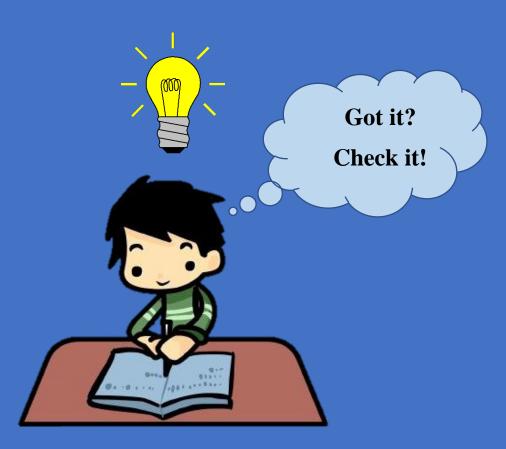
Row 1: Y + A + X = 9. Replace A with 4 and X with 2. Then Y + 4 + 2 = 9, and Y = 3.

Row 2: D + R + X = 14. Replace R with 7 and X with 2. Then D + 7 + 2 = 14, and D = 5.

Row 3: E + A + X = 15. Replace A with 4 and X with 2. Then E + 4 + 2 = 15, and E = 9.

## Solutions

# Section 2: Exercise Your Skills



$$R = 2$$
  $E = 10$   $A = 6$   $D = 3$   $Y = 9$   $X = 7$ 

#### **Possible Solution**

Row 1: E + E + E = 30. Then 3E = 30, and E = 10.

All of Row 2 is in Column 1. Line up like variables and subtract.

$$D + D + X + E = 23$$
-  $D + X + E = 20$ 
 $D = 3$ 

Row 2: D + X + E = 20. Replace D with 3 and E with 10. Then 3 + X + 10 = 20, and X = 7.

Column 2: E + X + R + R = 21. Replace E with 10 and X with 7.

Then 10 + 7 + R + R = 21. Then 2R + 17 = 21. Then 2R = 4, and R = 2.

Row 3: D + R + A = 11. Replace D with 3 and R with 2. Then 3 + 2 + A = 11, and A = 6.

Row 4: X + R + Y = 18. Replace X with 7 and R with 2. Then 7 + 2 + Y = 18, and Y = 9.

$$R = 7$$
  $E = 5$   $A = 4$   $D = 1$   $Y = 3$   $X = 6$ 

#### **Possible Solution**

All of Row 1 is in Column 1. Line up like variables and subtract.

$$A + A + E + Y = 16$$
-  $A + E + Y = 12$ 
 $A = 4$ 

Row 2: Y + A + A = 11. Replace each A with 4. Then Y + 4 + 4 = 11, and Y = 3.

Row 1: A + E + Y = 12. Replace A with 4 and Y with 3. Then 4 + E + 3 = 12, and E = 5.

Row 3: A + D + A = 9. Replace each A with 4. Then 4 + D + 4 = 9, and D = 1.

Column 3: Y + A + A + X = 17. Replace Y with 3 and each A with 4.

Then 3 + 4 + 4 + X = 17, and X = 6.

Row 4: E + R + X = 18. Replace E with 5 and X with 6. Then 5 + R + 6 = 18, and R = 7.

$$R = 6$$
  $E = 8$   $A = 7$   $D = 3$   $Y = 2$   $X = 1$ 

#### **Possible Solution**

All of Row 3 is in Column 1. Line up like variables and subtract.

$$R + Y + X + A = 16$$

$$- Y + X + A = 10$$

$$R = 6$$

Row 4: R + D + R = 15. Replace each R with 6. Then 6 + D + 6 = 15, and D = 3.

Column 3: R + R + A + R = 25. Replace each R with 6. Then 6 + 6 + A + 6 = 25, and A = 7.

Row 2: A + E + R = 21. Replace A with 7 and R with 6. Then 7 + E + 6 = 21, and E = 8.

Row 1: X + A + R = 14. Replace A with 7 and R with 6. Then X + 7 + 6 = 14, and X = 1.

Row 3: Y + X + A = 10. Replace X with 1 and A with 7. Then Y + 1 + 7 = 10, and Y = 2.

$$R = 4$$
  $E = 9$   $A = 3$   $D = 2$   $Y = 10$   $X = 1$ 

#### **Possible Solution**

All of Row 1 is in Column 2. Line up like variables and subtract.

$$A + X + E + R = 17$$
-  $X + E + R = 14$ 
 $A = 3$ 

Row 3: R + R + A = 11. Replace A with 3. Then 2R + 3 = 11. Then 2R = 8, and R = 4.

Column 1: X + X + R + R = 10. Replace each R with 4. Then X + X + 4 + 4 = 10. Then 2X + 8 = 10. Then 2X = 2, and X = 1.

Row 1: X + E + R = 14. Replace X with 1 and R with 4. Then 1 + E + 4 = 14, and E = 9.

Row 2: X + A + D = 6. Replace X with 1 and A with 3. Then 1 + 3 + D = 6, and D = 2.

Row 4: R + X + Y = 15. Replace R with 4 and X with 1. Then 4 + 1 + Y = 15, and Y = 10.

$$R = 5$$
  $E = 7$   $A = 8$   $D = 4$   $Y = 9$   $X = 3$ 

#### **Possible Solution**

Column 3: R + R + X + X = 16. Then 2R + 2X = 16, and R + X = 8.

Row 1: X + A + R = 16. Replace (R + X) with 8. Then 8 + A = 16, and A = 8.

Row 2: A + R + R = 18. Replace A with 8. Then 8 + R + R = 18. Then 2R + 8 = 18. Then 2R = 10, and R = 5.

Row 1: X + A + R = 16. Replace A with 8 and R with 5. Then X + 8 + 5 = 16, and X = 3.

Row 4: E + A + X = 18. Replace A with 8 and X with 3. Then E + 8 + 3 = 18, and E = 7.

Column 2: A + R + D + A = 25. Replace each A with 8 and R with 5. Then 8 + 5 + D + 8 = 25, and D = 4.

Row 3: Y + D + X = 16. Replace D with 4 and X with 3. Then Y + 4 + 3 = 16, and Y = 9.

$$R = 7$$
  $E = 3$   $A = 6$   $D = 5$   $Y = 2$   $X = 9$ 

#### **Possible Solution**

Column 1: R + D + R + D = 24. Then 2R + 2D = 24. Then R + D = 12.

Row 3: R + X + D = 21. Replace (R + D) with 12. Then 12 + X = 21, and X = 9.

Row 1: R + X + R = 23. Replace X with 9. Then 2R + 9 = 23. Then 2R = 14, and R = 7.

From Column 1: R + D = 12. Replace R with 7. Then 7 + D = 12, and D = 5.

Row 2: D + Y + Y = 9. Replace D with 5. Then 5 + 2Y = 9. Then 2Y = 4, and Y = 2.

Column 2: X + Y + X + E = 23. Replace each X with 9 and Y with 2.

Then 9 + 2 + 9 + E = 23, and E = 3.

Row 4: D + E + A = 14. Replace D with 5 and E with 3. Then 5 + 3 + A = 14, and A = 6.

$$R = 6$$
  $E = 10$   $A = 2$   $D = 7$   $Y = 5$   $X = 8$ 

#### **Possible Solution**

All of Row 1 is in Column 2. Line up like variables and subtract.

$$D + R + E + A = 25$$
-  $R + E + A = 18$ 
 $D = 7$ 

Row 3: D + A + D = 16. Replace each D with 7. Then 7 + A + 7 = 16, and A = 2.

Row 2: 
$$Y + D + Y = 17$$
. Replace D with 7. Then  $Y + 7 + Y = 17$ . Then  $2Y + 7 = 17$ . Then  $2Y = 10$ , and  $Y = 5$ .

Column 1: 
$$R + Y + D + R = 24$$
. Replace Y with 5 and D with 7. Then  $R + 5 + 7 + R = 24$ .  
Then  $2R + 12 = 24$ . Then  $2R = 12$ , and  $R = 6$ .

Row 4: R + R + X = 20. Replace each R with 6. Then 6 + 6 + X = 20, and X = 8.

Row 1: R + E + A = 18. Replace R with 6 and A with 2. Then 6 + E + 2 = 18, and E = 10.

$$R = 1$$
  $E = 3$   $A = 7$   $D = 9$   $Y = 5$   $X = 10$ 

#### **Possible Solution**

Column 2: A + A + X + X = 34. Then 2A + 2X = 34. Then A + X = 17.

Row 2: X + A + A = 24. Replace (A + X) with 17. Then A + 17 = 24, and A = 7.

From Column 2: A + X = 17. Replace A with 7. Then 7 + X = 17, and X = 10.

Row 3: A + X + E = 20. Replace A with 7 and X with 10. Then 7 + 10 + E = 20, and E = 3.

Row 1: D + A + E = 19. Replace A with 7 and E with 3. Then D + 7 + 3 = 19, and D = 9.

Column 1: D + X + A + R = 27. Replace D with 9, X with 10, and A with 7. Then 9 + 10 + 7 + R = 27, and R = 1.

Row 4: R + X + Y = 16. Replace R with 1 and X with 10. Then 1 + 10 + Y = 16, and Y = 5.

$$R = 9$$
  $E = 2$   $A = 8$   $D = 10$   $Y = 1$   $X = 4$ 

#### **Possible Solution**

All of Row 4 is in Column 1. Line up like variables and subtract.

$$Y + Y + D + D = 22$$
-  $Y + D + D = 21$ 
 $Y = 1$ 

Row 4: Y + D + D = 21. Replace Y with 1. Then 1 + 2D = 21. Then 2D = 20, and D = 10.

Row 3: D + Y + X = 15. Replace D with 10 and Y with 1. Then 10 + 1 + X = 15, and X = 4.

Column 2: E + E + Y + D = 15. Replace Y with 1 and D with 10. Then E + E + 1 + 10 = 15. Then 2E + 11 = 15. Then 2E = 4, and E = 2.

Row 2: D + E + A = 20. Replace D with 10 and E with 2. Then 10 + 2 + A = 20, and A = 8.

Row 1: Y + E + R = 12. Replace Y with 1 and E with 2 and. Then 1 + 2 + R = 12, and R = 9.

$$R = 8$$
  $E = 4$   $A = 2$   $D = 6$   $Y = 1$   $X = 3$ 

#### **Possible Solution**

All of Row 4 is in Column 1. Line up like variables and subtract.

$$E + D + Y + R = 19$$
-  $D + Y + R = 15$ 
 $E = 4$ 

Row 2: D + E + D = 16. Replace the E with 4. Then 2D + 4 = 16. Then 2D = 12, and D = 6.

Column 2: Y + E + E + Y = 10. Replace each E with 4.

Then 2Y + 8 = 10. Then 2Y = 2, and Y = 1.

Row 1: E + Y + X = 8. Replace E with 4 and Y with 1. Then 4 + 1 + X = 8, and X = 3.

Row 3: Y + E + A = 7. Replace Y with 1 and E with 4. Then 1 + 4 + A = 7, and A = 2.

Row 4: R + Y + D = 15. Replace Y with 1 and D with 6.

Then R + 1 + 6 = 15, and R = 8.

$$R = 8$$
  $E = 3$   $A = 2$   $D = 6$   $Y = 5$   $X = 1$ 

#### **Possible Solution**

Column 1: Y + R + R + Y = 26. Then 2Y + 2R = 26, and Y + R = 13

Row 1: Y + X + R = 14. Replace (Y + R) with 13. Then 13 + X = 14, and X = 1.

Column 3: R + R + X + X = 18. Replace each X with 1. Then R + R + 1 + 1 = 18. Then 2R + 2 = 18. Then 2R = 16, and R = 8.

From Column 1: Y + R = 13. Replace R with 8. Then Y + 8 = 13, and Y = 5.

Row 3: R + A + X = 11. Replace R with 8 and X with 1. Then 8 + A + 1 = 11, and A = 2.

Row 2: R + E + R = 19. Replace each R with 8. Then 8 + E + 8 = 19, and E = 3.

Row 4: Y + D + X = 12. Replace Y with 5 and X with 1. Then 5 + D + 1 = 12, and D = 6.

$$R = 5$$
  $E = 2$   $A = 9$   $D = 8$   $Y = 1$   $X = 3$ 

#### **Possible Solution**

All of Row 1 is in Column 2. Line up like variables and subtract.

$$E + E + A + A = 22$$
-  $E + A + A = 20$ 
 $E = 2$ 

Row 1: E + A + A = 20. Replace E with 2. Then 2 + A + A = 20. Then 2 + 2A = 20. Then 2A = 18, and A = 9.

Row 4: Y + A + Y = 11. Replace A with 9. Then Y + 9 + Y = 11. Then 2Y + 9 = 11. Then 2Y = 2, and Y = 1.

Column 3: A + X + X + Y = 16. Replace A with 9 and Y with 1. Then 9 + X + X + 1 = 16. Then 2X + 10 = 16. Then 2X = 6, and X = 3.

Row 2: R + E + X = 10. Replace E with 2 and X with 3. Then R + 2 + 3 = 10, and R = 5.

Row 3: D + E + X = 13. Replace E with 2 and X with 3. Then D + 2 + 3 = 13, and D = 8.

$$R = 9$$
  $E = 3$   $A = 10$   $D = 4$   $Y = 6$   $X = 8$ 

#### **Possible Solution**

Column 2: A + A + Y + Y = 32. Then 2A + 2Y = 32. Then A + Y = 16.

Row 4: D + Y + A = 20. Replace (A + Y) with 16. Then D + 16 = 20, and D = 4.

Row 2: D + A + D = 18. Replace each D with 4. Then 4 + A + 4 = 18, and A = 10.

From Column 2: A + Y = 16. Replace A with 10, then 10 + Y = 16, and Y = 6.

Row 1: X + A + D = 22. Replace A with 10 and D with 4. Then X + 10 + 4 = 22, and X = 8.

Column 1: X + D + R + D = 25. Replace X with 8 and each D with 4.

Then 8 + 4 + R + 4 = 25, and R = 9.

Row 3: R + Y + E = 18. Replace R with 9 and Y with 6. Then 9 + 6 + E = 18, and E = 3.

$$R = 7$$
  $E = 8$   $A = 4$   $D = 1$   $Y = 6$   $X = 3$ 

#### **Possible Solution**

Column 1: X + R + X + R = 20. Then 2X + 2R = 20, and X + R = 10.

Row 1: X + D + R = 11. Replace (X + R) with 10. Then D + 10 = 11, and D = 1.

Row 2: R + D + D = 9. Replace each D with 1. Then R + 1 + 1 = 9, and R = 7.

Row 1: X + D + R = 11. Replace D with 1 and R with 7. Then X + 1 + 7 = 11, and X = 3.

Column 2: D + D + E + E = 18. Replace each D with 1. Then 1 + 1 + E + E = 18, Then 2 + 2E = 18, Then 2E = 16, and E = 8.

Row 3: X + E + Y = 17. Replace X with 3 and E with 8. Then 3 + 8 + Y = 17, and Y = 6.

Row 4: R + E + A = 19. Replace R with 7 and E with 8. Then 7 + 8 + A = 19, and A = 4.

$$R = 6$$
  $E = 4$   $A = 7$   $D = 10$   $Y = 8$   $X = 9$ 

#### **Possible Solution**

All of Row 2 is in Column 1. Line up like variables and subtract.

$$A + X + X + D = 35$$
-  $X + X + D = 28$ 
 $A = 7$ 

Row 1: A + D + D = 27. Replace A with 7. Then 7 + D + D = 27. Then 2D + 7 = 27. Then 2D = 20, and D = 10.

Row 2: X + X + D = 28. Replace D with 10. Then 2X + 10 = 28. Then 2X = 18, and X = 9.

Row 4: X + R + R = 21. Replace X with 9. Then 9 + R + R = 21. Then 9 + 2R = 21. Then 2R = 12, and R = 6.

Column 3: D + D + E + R = 30. Replace each D with 10 and R with 6. Then 10 + 10 + E + 6 = 30, and E = 4.

Row 3: D + Y + E = 22. Replace D with 10 and E with 4. Then 10 + Y + 4 = 22, and Y = 8.

$$R = 5$$
  $E = 3$   $A = 10$   $D = 1$   $Y = 9$   $X = 7$ 

#### **Possible Solution**

Column 2: Y + X + Y + X = 32. Then 2X + 2Y = 32. Then Y + X = 16.

Row 4: Y + X + R = 21. Replace (Y + X) with 16. Then 16 + R = 21, and R = 5.

Row 2: R + X + R = 17. Replace each R with 5. Then 5 + X + 5 = 17, and X = 7.

From Column 2: Y + X = 16. Replace X with 7. Then Y + 7 = 16, and Y = 9.

Row 1: A + Y + A = 29. Replace Y with 9. Then 2A + 9 = 29. Then 2A = 20, and A = 10.

Column 3: A + R + E + R = 23. Replace A with 10, and each R with 5.

Then 10 + 5 + E + 5 = 23, and E = 3.

Row 3: D + Y + E = 13. Replace Y with 9 and E with 3. Then D + 9 + 3 = 13, and D = 1.

$$R = 5$$
  $E = 3$   $A = 1$   $D = 7$   $Y = 4$   $X = 10$ 

#### **Possible Solution**

All of Row 3 is in Column 3. Line up like variables and subtract.

$$R + D + Y + E = 19$$
-  $D + Y + E = 14$ 
 $R = 5$ 

Column 1: E + R + E + R = 16. Replace each R with 5.

Then E + 5 + E + 5 = 16. Then 2E + 10 = 16. Then 2E = 6, and E = 3.

Row 4: R + Y + E = 12. Replace R with 5 and E with 3. Then 5 + Y + 3 = 12, and Y = 4.

Row 1: E + A + R = 9. Replace E with 3 and R with 5. Then 3 + A + 5 = 9, and A = 1.

Row 3: E + D + Y = 14. Replace E with 3 and Y with 4. Then 3 + D + 4 = 14, and D = 7.

Row 2: R + X + D = 22. Replace R with 5 and D with 7. Then 5 + X + 7 = 22, and X = 10.

$$R = 1$$
  $E = 6$   $A = 10$   $D = 4$   $Y = 3$   $X = 7$ 

#### **Possible Solution**

All of Row 2 is in Column 1. Line up like variables and subtract.

$$E + E + A + Y = 25$$
-  $E + A + Y = 19$ 
 $E = 6$ 

Row 3: E + E + X = 19. Replace each E with 6. Then 6 + 6 + X = 19, and X = 7.

Column 3: X + Y + X + Y = 20. Replace each X with 7. Then 7 + Y + 7 + Y = 20. Then 2Y + 14 = 20. Then 2Y = 6, and Y = 3.

Row 4: Y + R + Y = 7. Replace each Y with 3. Then 3 + R + 3 = 7, and R = 1.

Row 2: E + A + Y = 19. Replace E with 6 and Y with 3. Then 6 + A + 3 = 19, and A = 10.

Row 1: A + D + X = 21. Replace A with 10 and X with 7. Then 10 + D + 7 = 21, and D = 4.

$$R = 6$$
  $E = 10$   $A = 1$   $D = 5$   $Y = 9$   $X = 4$ 

#### **Possible Solution**

Column 1: R + D + D + R = 22. Then 2R + 2D = 22. Then R + D = 11.

Row 2: D + E + R = 21. Replace (R + D) with 11. Then E + 11 = 21, and E = 10.

Row 1: R + E + E = 26. Replace each E with 10. Then R + 10 + 10 = 26, and R = 6.

From Column 1: R + D = 11. Replace R with 6. Then 6 + D = 11, and D = 5.

Row 4: R + X + X = 14. Replace R with 6. Then 6 + X + X = 14. Then 2X + 6 = 14. Then 2X = 8, and X = 4.

Column 2: E + E + Y + X = 33. Replace each E with 10 and X with 4. Then 10 + 10 + Y + 4 = 33, and Y = 9.

Row 3: D + Y + A = 15. Replace D with 5 and Y with 9. Then 5 + 9 + A = 15, and A = 1.

$$R = 3$$
  $E = 7$   $A = 2$   $D = 1$   $Y = 5$   $X = 6$ 

#### **Possible Solution**

All of Row 3 is in Column 3. Line up like variables and subtract.

$$A + E + D + R = 13$$
-  $E + D + R = 11$ 
 $A = 2$ 

Row 2: R + A + A = 7. Replace each A with 2. Then R + 2 + 2 = 7, and R = 3.

Row 4: R + Y + R = 11. Replace each R with 3. Then 3 + Y + 3 = 11, and Y = 5.

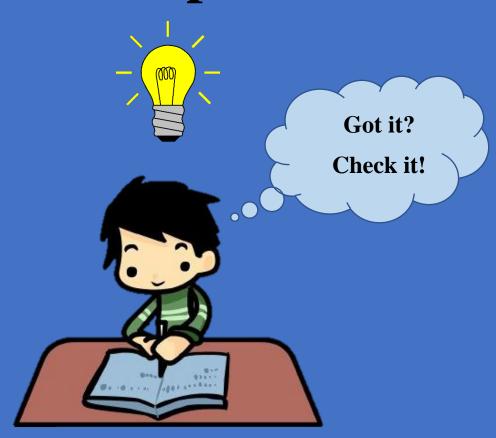
Column 1: E + R + E + R = 20. Replace each R with 3. Then E + 3 + E + 3 = 20. Then 2E + 6 = 20. Then 2E = 14, and E = 7.

Row 1: E + X + E = 20. Replace each E with 7. Then 7 + X + 7 = 20, and X = 6.

Row 3: E + R + D = 11. Replace E with 7 and R with 3. Then 7 + 3 + D = 11, and D = 1.

# Solutions

# Section 3: Demonstrate Your X-pertise



$$R = 4$$
  $E = 7$   $A = 9$   $D = 8$   $Y = 5$   $X = 6$ 

#### **Possible Solution**

All of Row 4 is in Column 3. Line up like variables and subtract.

$$E + R + X + A = 26$$
-  $R + X + A = 19$ 
 $E = 7$ 

Row 2: E + E + R = 18. Replace each E with 7. Then 7 + 7 + R = 18, and R = 4.

Column 1: A + E + D + R = 28. Replace E with 7 and R with 4.

Then A + 7 + D + 4 = 28. Then A + D = 17.

Row 3: D + A + X = 23. Replace (A + D) with 17. Then 17 + X = 23, and X = 6.

Row 4: R + X + A = 19. Replace R with 4 and X with 6. Then 4 + 6 + A = 19, and A = 9.

From Column 1: A + D = 17. Replace A with 9. Then 9 + D = 17, and D = 8.

Row 1: A + Y + E = 21. Replace A with 9 and E with 7. Then 9 + Y + 7 = 21, and Y = 5.

$$R = 9$$
  $E = 3$   $A = 6$   $D = 7$   $Y = 8$   $X = 2$ 

#### **Possible Solution**

All of Row 3 is in Column 3. Line up like variables and subtract.

$$Y + R + X + D = 26$$
-  $R + X + D = 18$ 
 $Y = 8$ 

Row 4: D + Y + Y = 23. Replace each Y with 8. Then D + 8 + 8 = 23, and D = 7.

Row 3: R + X + D = 18. Replace D with 7. Then R + X + 7 = 18, and R + X = 11.

Row 1: A + X + R = 17. Replace (R + X) with 11. Then A + 11 = 17, and A = 6.

Column 1: A + A + R + D = 28. Replace each A with 6 and D with 7.

Then 6 + 6 + R + 7 = 28, and R = 9.

From Row 3: R + X = 11. Replace R with 9. Then 9 + X = 11, and X = 2.

Row 2: A + E + X = 11. Replace A with 6 and X with 2. Then 6 + E + 2 = 11, and E = 3.

$$R = 1$$
  $E = 3$   $A = 9$   $D = 5$   $Y = 4$   $X = 2$ 

#### **Possible Solution**

All of Row 1 is in Column 3. Line up like variables and subtract.

$$D + X + A + Y = 20$$
-  $X + A + Y = 15$ 
 $D = 5$ 

Row 3: A + D + D = 19. Replace each D with 5. Then A + 5 + 5 = 19, and A = 9.

Column 3: Y + A + D + X = 20. Replace A with 9 and D with 5.

Then 
$$Y + 9 + 5 + X = 20$$
. Then  $Y + X + 14 = 20$ . Then  $Y + X = 6$ 

Column 1: X + R + A + Y = 16. Replace (Y + X) with 6, and A with 9.

Then 6 + R + 9 = 16, and R = 1.

Row 2: R + Y + A = 14. Replace R with 1 and A with 9.

Then 
$$1 + Y + 9 = 14$$
, and  $Y = 4$ .

From Column 3: Y + X = 6. Replace Y with 4. Then 4 + X = 6, and X = 2.

Row 4: Y + E + X = 9. Replace Y with 4 and X with 2.

Then 
$$4 + E + 2 = 9$$
, and  $E = 3$ .

$$R = 1$$
  $E = 2$   $A = 4$   $D = 6$   $Y = 3$   $X = 7$ 

#### **Possible Solution**

All of Row 3 is in Column 3. Line up like variables and subtract.

$$R + D + E + R = 10$$
-  $D + E + R = 9$ 
 $R - 1$ 

Column 3: D + R + R + E = 10. Replace each R with 1.

Then D + 1 + 1 + E = 10. Then D + E = 8.

Row 4: E + D + E = 10. Replace (D + E) with 8. Then E + 8 = 10, and E = 2.

From Column 3: D + E = 8. Replace E with 2. Then D + 2 = 8, and D = 6.

Row 1: R + A + D = 11. Replace R with 1 and D with 6. Then 1 + A + 6 = 11, and A = 4.

Column 2: A + Y + E + D = 15. Replace A with 4, E with 2, and D with 6.

Then 4 + Y + 2 + 6 = 15, and Y = 3.

Row 2: X + Y + R = 11. Replace Y with 3, and R with 1. Then X + 3 + 1 = 11, and X = 7.

$$R = 4$$
  $E = 7$   $A = 8$   $D = 3$   $Y = 10$   $X = 6$ 

#### **Possible Solution**

Columns 3 and 1 have three variables in common. Line up like variables and subtract.

$$X + R + E + A = 25$$
-  $X + R + E + D = 20$ 
 $A - D = 5 \text{ or } A = D + 5$ 

Row 4: 
$$D + D + A = 14$$
. Replace A with  $(D + 5)$ . Then  $D + D + D + 5 = 14$ .  
Then  $3D + 5 = 14$ . Then  $3D = 9$ , and  $D = 3$ .

From subtraction of Column 1 from Column 3: A = D + 5. Replace D with 3. Then A = 3 + 5, and A = 8.

Column 2: 
$$Y + Y + Y + D = 33$$
. Replace D with 3. Then  $Y + Y + Y + 3 = 33$ .  
Then  $3Y + 3 = 33$ . Then  $3Y = 30$ , and  $Y = 10$ .

Row 1: R + Y + R = 18. Replace Y with 10. Then R + 10 + R = 18. Then 2R + 10 = 18. Then 2R = 8, and R = 4.

Row 2: 
$$E + Y + E = 24$$
. Replace Y with 10. Then  $E + 10 + E = 24$ . Then  $2E + 10 = 24$ .  
Then  $2E = 14$ , and  $E = 7$ .

Row 3: X + Y + X = 22. Replace Y with 10. Then X + 10 + X = 22. Then 2X + 10 = 22. Then 2X = 12, and X = 6.

$$R = 4$$
  $E = 9$   $A = 1$   $D = 7$   $Y = 3$   $X = 8$ 

#### **Possible Solution**

All of Row 4 is in Column 2. Line up like variables and subtract.

$$R + A + X + Y = 16$$
-  $A + X + Y = 12$ 
 $R = 4$ 

All of Row 1 is in Column 1. Line up like variables and subtract.

$$A + D + R + E = 21$$
-  $D + R + E = 20$ 
 $A = 1$ 

Row 3: R + A + X = 13. Replace R with 4 and A with 1. Then 4 + 1 + X = 13, and X = 8.

Row 4: A + X + Y = 12. Replace A with 1 and X with 8. Then 1 + 8 + Y = 12, and Y = 3.

Row 2: E + Y + E = 21. Replace Y with 3. Then E + 3 + E = 21. Then 2E + 3 = 21. Then 2E = 18, and E = 9.

Row 1: D + R + E = 20. Replace R with 4 and E with 9. Then D + 4 + 9 = 20, and D = 7.

$$R = 6$$
  $E = 10$   $A = 8$   $D = 7$   $Y = 5$   $X = 2$ 

#### **Possible Solution**

All of Row 2 is in Column 3. Line up like variables and subtract.

$$A + X + D + E = 27$$
-  $X + D + E = 19$ 
 $A = 8$ 

All of Row 1 is in Column 2. Line up like variables and subtract.

$$E + D + R + A = 31$$
-  $D + R + A = 21$ 
 $E = 10$ 

Row 4: Y + A + E = 23. Replace A with 8 and E with 10.

Then Y + 8 + 10 = 23, and Y = 5.

Row 1: R + D + A = 21. Replace A with 8. Then R + D = 13.

Column 1: R + D + X + Y = 20. Replace (R + D) with 13 and Y with 5. Then 13 + X + 5 = 20, and X = 2.

Column 3: A + X + D + E = 27. Replace A with 8, X with 2, and E with 10. Then 8 + 2 + D + 10 = 27, and D = 7.

From Row 1: R + D = 13. Replace D with 7. Then R + 7 = 13, and R = 6.

$$R = 3$$
  $E = 10$   $A = 2$   $D = 6$   $Y = 8$   $X = 4$ 

#### **Possible Solution**

All of Row 2 is in Column 3. Line up like variables and subtract.

$$D + X + A + X = 16$$
-  $X + A + X = 10$ 
 $D = 6$ 

Row 1: A + A + D = 10. Replace D with 6. Then 2A + 6 = 10. Then 2A = 4, and A = 2.

Column 3: D + X + A + X = 16. Replace D with 6 and A with 2.

Then 6 + X + 2 + X = 16. Then 2X + 8 = 16. Then 2X = 8, and X = 4.

Row 3: X + R + A = 9. Replace X with 4 and A with 2. Then 4 + R + 2 = 9, and R = 3.

Column 1: A + A + X + Y = 16. Replace each A with 2 and X with 4.

Then 2 + 2 + 4 + Y = 16, and Y = 8.

Row 4: Y + E + X = 22. Replace Y with 8 and X with 4. Then 8 + E + 4 = 22, and E = 10.

$$R = 10$$
  $E = 8$   $A = 9$   $D = 3$   $Y = 4$   $X = 6$ 

#### **Possible Solution**

All of Row 1 is in Column 1. Line up like variables and subtract.

$$D + D + R + A = 25$$
-  $D + R + A = 22$ 
 $D = 3$ 

Row 2: D + Y + Y = 11. Replace D with 3. Then 3 + 2Y = 11. Then 2Y = 8, and Y = 4.

Column 3: A + Y + Y + X = 23. Replace each Y with 4. Then A + 4 + 4 + X = 23. Then A + X + 8 = 23. Then A + X = 15.

Row 4: A + E + X = 23. Replace (A + X) with 15. Then E + 15 = 23, and E = 8.

Row 1: D + R + A = 22. Replace D with 3. Then 3 + R + A = 22. Then R + A = 19.

Column 2: R + Y + X + E = 28. Replace Y with 4 and E with 8. Then R + 4 + X + 8 = 28. Then R + X = 16

From Row 1: R + A = 19. From Column 2: R + X = 16. Line up like variables and subtract.

$$R + A = 19$$
-  $R + X = 16$ 
A - X = 3 or A = X + 3

Row 4: A + E + X = 23. Replace A with (X + 3) and E with 8.

Then X + 3 + 8 + X = 23. Then 2X + 11 = 23. Then 2X = 12, and X = 6

From Column 2: R + X = 16. Replace X with 6. Then R + 6 = 16, and R = 10.

From Row 1: R + A = 19. Replace R with 10. Then 10 + A = 19, and A = 9.

$$R = 1$$
  $E = 7$   $A = 4$   $D = 9$   $Y = 6$   $X = 5$ 

#### **Possible Solution**

Columns 1 and 2 have three variables in common. Line up like variables and subtract.

$$X + R + E + E = 20$$
-  $X + R + E + Y = 19$ 
 $E - Y = 1 \text{ or } Y = E - 1$ 

Row 3: E + E + Y = 20. Replace Y with (E - 1). Then E + E + E - 1 = 20. Then 3E = 21, and E = 7.

Row 3: E + E + Y = 20. Replace each E with 7. Then 7 + 7 + Y = 20, and Y = 6.

Row 4: Y + E + A = 17. Replace Y with 6 and E with 7. Then 6 + 7 + A = 17, and A = 4.

Row 1: X + X + A = 14. Replace A with 4. Then X + X + 4 = 14. Then 2X = 10, and X = 5.

Column 3: A + D + Y + A = 23. Replace each A with 4 and Y with 6. Then 4 + D + 6 + 4 = 23, and D = 9.

Row 2: R + R + D = 11. Replace D with 9. Then R + R + 9 = 11. Then 2R + 9 = 11. Then 2R = 2, and R = 1.

