7-8
TEACHER PACKET

MATHLINKS: GRADE 7
TEACHER PACKET 8
EXPLORING EXPRESSIONS AND EQUATIONS

Table of Contents

ANNOTATIONS in this teacher packet provide additional program information for prospective users. Please go to http://mathandteaching.org/register/ and register to view all content components.

There are 16 student packets in the Grade 7 Program. This excerpt from the Teacher Guide accompanies Student Packet 8. For easy reference, the color of the cover of student packet will match the teacher packet.

GENERAL INFORMATION

PACING PLAN SUGGESTIONS

TRADITIONAL MATH SCHEDULE				
Days-Modified	Days-Basic	Days-Enriched	Lesson	Review/Practice
3	3	3	[8.1] Pages 0, 1-9	Pages 26-28
3	3	3	[8.2] Pages 0, 10-17	Pages 29-31
3	3	3	[8.3] Pages 0, 18-25	Pages 32-37
2	2	2	Catch up, Tasks, Assessment	

BLOCK SCHEDULE				
Days-Modified	Days-Basic	Days-Enriched	Lesson	Review/Practice
2	2	2	[8.1] Pages 0, 1-9	Pages 26-28
2	2	2	[8.2] Pages 0, 10-17	Pages 29-31
2	2	2	[8.3] Pages 0, 18-25	Pages 32-37
1	1	1	Catch up, Tasks, Assessment	

- Lesson pages are not intended to be used only as class work be used only as homework. How they are used is up to the te

- The number of days estimated for each lesson will vary depe and student proficiency.

Every student packet includes three concept lessons and a review section. Packets generally take 1-3 weeks.

- Although they are listed at the end of the tables, use catch up days when needed.

- Tasks may be assigned at any time after students have completed the prerequisite content work.

- Multiple assessment measures are encouraged, including (but not limited to) quizzes, tasks, assessment challenges, strategically selected student pages, skill builders, selected response page, knowledge check, etc.

- Consider requiring a math journal, to be collected and checked periodically, or collecting an “exit slip” at the end of selected class periods. Journals and exits slips may include short skills review, explanations of concepts, or anything else the instructor may want to assess.

- As part of a modified program, consider omitting the following, depending upon time constraints: Student Packet 8: Page 9, and the more difficult problems on Pages 15-17, 22-24, 26-33.

COMMON CORE STATE STANDARDS – MATHEMATICS

STANDARDS FOR MATHEMATICAL CONTENT

6.NS.B*	**Compute fluently with multi-digit numbers and find common factors and multiples.**
6.EE 3*	Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*
6.EE 4*	Identify when two expressions are equivalent (i.e. when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*
6.EE.B*	**Reason about and solve one-variable equations and inequalities.**[1]
6.EE 6*	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
7.NS.A	**Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.**[1]
7.NS.3	Solve real-world and mathematical problems involving the four operations with rational numbers.
7.EE.A	**Use properties of operations to generate equivalent expressions.**[1]
7.EE 1	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
7.EE 2	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*
7.EE.B	**Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**[1]
7.EE.3	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

*Content essential for success in 7th grade
[1] A major cluster for the grade level.

STANDARDS FOR MATHEMATICAL PRACTICE

MP1	Make sense of problems and persevere in solving them.
MP2	Reason abstractly and quantitatively.
MP3	Construct viable arguments and critique the reasoning of others.
MP4	Model with mathematics.
MP7	Look for and make use of structure.

PACKET PLANNING INFORMATION

Assessments*, Reproducibles, and Tasks****

Quiz 8A, 8B
Proficiency Challenge 8
Test 8 (see Assessment Tab, page iv)

Reproducible 25:	Extra Border Grids (12×12 and 8×8) (1/student, optional) [8.1]
Reproducible 26:	Analyzing the Border Problem (1/student) [8.1]
Reproducible 27:	Hundreds Chart (1/student, optional) [8.2]
Reproducible 28:	Polygon Puzzle Pieces (1/student + 1/group) [8.3]

Task, Page 12:	Border Tile Extensions [8.1]
Task, Page 13:	Hundred Chart Explorations [8.2]
Task, Page 14:	Polygon Area Puzzle Challenge [8.3]

*Located in the assessment envelope and on the secure website
**Located in the back of the Teacher Guide

Materials

- Colored pencils, crayons, or highlighters [8.1]
- Graph paper [8.1]

***MathLinks:* Grade 7 Resource Guide (Part 1)**

Key vocabulary in the Word Bank:

- deductive reasoning
- distributive property
- equation
- equivalent expressions
- evaluate
- expression
- greatest common factor
- inductive reasoning
- solve an equation
- variable

Explanations and examples:

- Mathematical Symbols and Language
- Variables and Expressions

Prepare Ahead

Go to www.mathandteaching.org for additional resources.

Lesson 8.3:
- Reproducible 28 will be used as a puzzle during the first part of the lesson (1 per group). Ask students to cut one puzzle per group and put pieces in envelopes or plastic bags.

Technology Resources

This applet is used to explore the number of chairs needed when tables are rearranged in a restaurant.
http://illuminations.nctm.org/Activity.aspx?id=3542

Use this applet to determine how many handshakes occur when *n* people meet.
http://illuminations.nctm.org/Activity.aspx?id=4150

A good source for simple algebra games students can play for free can be found at the link below.
http://www.sheppardsoftware.com/math.htm

Options for a Substitute

Any time:	Pages 26-29
After 3.1:	Pages 8-9, 30-31
After 3.2:	Pages 32
After 3.3:	Pages 25, 33-37

This page summarizes planning information to get you started.

TEACHER CONTENT INFORMATION

MATH NOTES

MN1: Conjecture vs. Proof [8.2]

A "number trick" is a popular classroom activity in which students choose a starting number and perform a sequence of operations that eventually lead back to the original starting number. Here is an example of a number trick.

Step	Directions	Using specific starting values		Using a variable
1	Choose a number	3	0.5	n
2	Multiply by 4	12	2	$4n$
3	Add 6	18	8	$4n + 6$
4	Subtract the original number	15	7.5	$3n + 6$
5	Divide by 3	5	2.5	$n + 2$
6	Subtract 2	3	0.5	n

Based on the results for specific starting numbers, students use inductive reasoning to conjecture that for all starting values, the result is the original number.

By following the steps of this trick using n to represent any number, students prove that for all starting values, the result is the original number.

When students in a classroom select different numbers to test a number trick such as this one, they may convince themselves that the trick will always work. Their generalization is a conjecture because it has not been proven to be true nor shown to be false. The sort of question, "Will this trick work for all numbers?" is a very important one in mathematics. Certainly it is impossible to try all numbers. The use of symbolic algebra for the purpose of generalization is an efficient way to prove this conjecture and it provides a convincing way to show the usefulness of algebra.

Most Math Notes were written by our mathematicians. They provide additional content information for teachers, often beyond what students will learn.

MATH NOTES (Continued)

MN2: Inductive and Deductive Reasoning [8.2]

Consider this problem: What is the sum of the first n odd numbers?

To investigate this problem, we might begin by creating a table of sums of odd numbers.

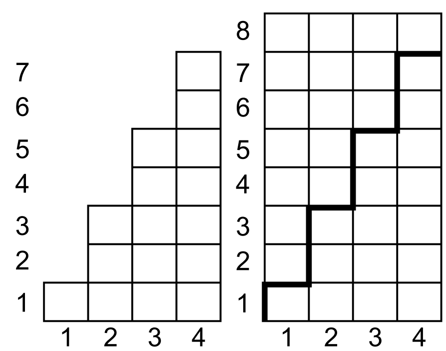
n	nth odd number	sum of first n odd numbers
1	1	1
2	3	4 = 1 + 3
3	5	9 = 1 + 3 + 5
4	7	16 = 1 + 3 + 5 + 7

A pattern is developing. Based on the table, we might guess that the sum of the first n odd numbers is n^2. After completing the table to, say, $n = 100$, perhaps with the aid of a computer, we have substantial evidence that our guess is correct. Arguing on the basis of the pattern that has developed, we conclude that the sum of the first n odd integers is n^2. This conclusion is based on inductive reasoning. It is arrived at through consideration of experimental evidence or special cases. Though the evidence might be overwhelming and we might be confident that the result is correct, we cannot be absolutely sure until we find a proof. At this point, the conclusion is a conjecture.

Toward finding a proof, we proceed as follows. Represent the sum of the first n odd numbers 1, 3, 5, 7, … as an area under a stair step pattern with total width n and total height $2n – 1$, as in the figure below. The area of the figure is the sum of the first n odd numbers. If we duplicate the figure, rotate it half a turn, and place it above the original stairway, we see that the figures fit together to form a rectangle of width n and total height $(2n – 1) + 1 = 2n$. Thus twice the area is $n(2n) = 2n^2$.

Hence the area, which is the sum of the first n odd numbers, is n^2.

We have arrived at this conclusion by deductive reasoning, using the given information, previously established facts, and accepted rules of logic. We can be absolutely sure that the conclusion is true.



TEACHING NOTES

TN1: Select Standards for Mathematical Practice Examples [8.1, 8.2, 8.3]

Here are a few examples of how the Standards for Mathematical Practice are applied in these lessons.

Teaching Notes were written by math educators. TN1 links packet contents to Standards for Mathematical Practice. TNs 2-4 focus on strategies for special populations. TN5 gives suggestions to motivate students by creating an "itch" for learning. Other teaching information begins in TN6.

MP1 Make sense of problems and perseve
tile patterns. Encourage students to e
solution is found. [8.3] Encourage stu
many relationships among shapes.

MP2 Reason abstractly and quantitatively.
various border grids, and then genera
hundreds chart, make conjectures, an
[8.3] Students create equations to describe areas of polygon shapes.

MP3 Construct viable arguments and critique the reasoning of others. [8.1] Students create expressions for borders and justify their reasoning with class discussions. Practice 1 and Practice 2 include problems where students critique the reasoning of others. [8.3] Students explain relationships among polygon pieces orally and in writing.

MP4 Model with mathematic. [8.3] Paintings on the Wall requires students to create diagrams and expressions to solve a real-life problem.

MP7 Look for and make use of structure. [8.1] Students use the structure from each of the 12×12 border grid expressions to generate expressions for 8×8, 5×5, and $n \times n$ grids. [8.2] Students generalize patterns on the hundreds chart based on the number patterns within the chart.

TN2: Strategies for English Learners [8.1, 8.2, 8.3]

Building Background

(Emphasize key vocabulary.) [8.1] Be sure students understand the difference between "border area" and "perimeter." Write vocabulary such as "distributive property" and "factor" on the board as it arises. [8.2] Be sure that students understand that the phrase "2×2 square" does not refer to a multiplication problem. Use visuals to explain the meaning of row, column, and diagonal.

Comprehensible Input

(Explain academic tasks clearly.) [8.1, 8.2, 8.3] Connect word descriptions to visuals and numbers so that English learners increase vocabulary for describing patterns.

Instructional Strategies

(Make concepts clear with visuals. Use hands-on activities.) [8.1, 8.2, 8.3] All three lessons rely heavily on visuals or manipulatives. Provide ample opportunities for students to draw and build as they work. Create extra reproducible pages as needed.

TEACHING NOTES (Continued)

TN3: Strategies for Special Learners [8.1, 8.2, 8.3]

Increase communication and participation

(Encourage students to demonstrate what they have learned to their peers.) [8.1, 8.2, 8.3] These lessons provide ample opportunities for partner work. As students explain their thinking to each other, they clarify thinking and increase their confidence.

(Allow alternative methods to express mathematical ideas.) [8.1, 8.2, 8.3] Encourage students who have difficulty expressing themselves to “show” others rather than “tell.” Help students during this process to pair the correct academic vocabulary to what they are demonstrating with a visual or manipulative.

Differentiate instructional strategies

(Include color to emphasize concepts.) [8.1, 8.2] Give students copies of R25 and R27, and highlighters or colored pencils to help them visualize patterns.

TN4: Strategies for Enrichment [8.1, 8.2, 8.3]

[8.1] After completing each border problem, ask students to make generalizations about the unshaded region of the diagrams. Ask students to create their own border pattern and represent it visually, numerically, symbolically, and verbally.

[8.2, 8.3] Mathematically curious students will find many interesting patterns in these open investigations.

TN5: Creating an Itch [8.1]

[8.1] Create a need to understand the meaning of equivalent expressions. Write the expressions $x^2 - 1$ and $1 - x^2$ on the board. Harry says these are equivalent because the expressions are true if $x = 1$ or $x = -1$. Do you agree with Harry? Allow students to express opinions, but there is no need to resolve the issue now. As the lesson unfolds, help students to understand that equivalent algebraic expressions must be true for all substituted values. While the numerical expressions $1 - 1^2$ and $1^2 - 1$ are equivalent, the algebraic expressions are not equivalent, because there are values of the variable x (such as $x = 0$, or $x = 2$) for which, when substituted, the expressions are not equal. Revisit the problem when students tackle Tere’s problem (students explain why $2n$ and n^2 are not equivalent).

TN6: Fostering Independent Thinking [8.1, 8.2, 8.3]

Lessons in this packet require a lot of exploration. Students may want immediate reassurance from an expert that their patterns, expressions, and rules are correct. However, for students to become independent, confident thinkers, they must be given adequate time to think through a problem, and sufficient opportunities to resolve misconceptions and unclear concepts through reflective thought and discussion with peers.

To avoid contributing to a student’s “learned helplessness,” teachers are encouraged to refrain from providing feedback and answers too quickly. Students will improve their thinking skills and gain confidence as they work through problems themselves.

TEACHING NOTES (Continued)

TN7: Confusing Perimeter and Border [8.1]

Students often confuse the concepts of area and perimeter. Area is the number of square units that a shape covers, while perimeter is the distance (length) around the shape.

Consider Figure 1 below. Each side of a small square is one unit of length, so the area of one small square is one square unit. The perimeter of the entire figure is 16 units and its area is 16 square units. Consider the following exchange between teacher and student.

Teacher: "What is the perimeter of this square?"
Student: "16."

If the student omits units, or the teacher does not ask a follow-up question, student understanding of the difference between length and area remains unknown. This student may be thinking about area when answering the perimeter question.

This lesson requires students to write expressions for the number of shaded border squares that surround grids, like Figures 2 and 3 below. Students may confuse the idea of finding the number of border squares (calculating area) with finding perimeter (calculating length).

In Figure 2 below, the perimeter of the entire figure is 16 units, but the area of the border is 12 square units. Be sure that when students count border squares they understand that they are calculating areas, NOT perimeters.

Figure 1

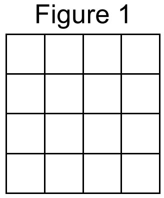


Figure 2

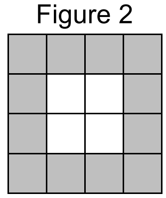
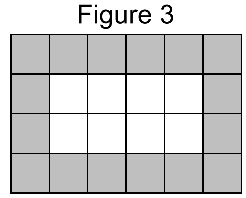


Figure 3



While Math Notes and Teaching Notes were written to support instruction, we strongly recommend professional development to help teachers learn content and strategies for effective implementation of a Common Core program using MathLinks resources.

EQUIVALENT EXPRESSIONS

Summary	Goals
Students write numerical expressions to represent geometric patterns, describe patterns in words, and generalize using variable expressions. Students apply properties to generate equivalent expressions that include integers. *6.NS.B, 7.EE.A, 7.EE.B*	• Write, evaluate, and simplify expressions. • Describe geometric patterns numerically, pictorially, symbolically, and verbally. • Interpret expressions in terms of their geometric context. • View algebra as a useful mathematical tool.

PREVIEW / WARMUP

Every lesson begins with a black bar title, summary and goals, which are also listed in the student packet.

Wh

ce the goals and standards of the lesson. Discuss
nt vocabulary as relevant.

Create an “itch” here. See Teaching Note 5.

Pag
Wo

ts rewrite each arithmetic problem as an expression horizontally. This form is
ed when writing expressions throughout this lesson.

Pag
Equ
Exp

ts demonstrate whether or not the pairs of numerical expressions in each row
ivalent.

What does it mean for numerical expressions to be equivalent? They represent the same value. That is, they name the same number.

INTRODUCE 1

Whole Class / Individuals / Partners

Page 2
The Border Problem
(for display only)

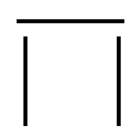
This exploration calls for teacher facilitation of student thinking, rather than direct instruction. Students can easily count the border tiles (44 tiles in a 12 × 12 square). We want students to use creativity as they write expressions for the number of border tiles, show they are equivalent, and generalize their patterns.

- Display a 12 × 12 Border Challenge. Ask students to think about how they might figure out the number of shaded border squares without actually counting all of them one by one. Allow individual “think time” before sharing ideas.

We could easily count to find the number of border squares (44). Using words or a numerical expression, how can we describe the border? If students are stuck, read statement (a) below to clarify the task. If they are still stuck, read statement (b).

(a) ***Aja looked at the 12 × 12 square and counted all the border squares one by one (1, 2, 3, 4, …) all the way to 44. This confirms that there are 44 squares on the border, but our task is to write expressions for “shortcuts.”***

(b) ***Dan said he had a quicker way and drew the picture to the right. “There is a group of 12 squares along the top row, another 12 along the bottom row, and two groups of 10 squares that are left along each side column.” How did Dan see that?*** Students can



) or 2(12 + 10).

The Student Packet IS NOT intended to be a stand-alone workbook! It is structured workspace to support all learners. Lesson plans in the teacher guide provide guidance for classroom activities, interactions, and questioning.

INTRODUCE 1 (Continued)

- If a numerical expression is offered, transcribe it, ask the student to describe it verbally, and connect the expression to the grid. If a verbal description is offered, ask the student (or the class) to translate it into a numerical expression, and connect the expression to the grid.

- Continue the process until multiple expressions have been shared, or offer more work time for students to come up with more expressions and then discuss them.

See SUMMARIZE 2 in this lesson for examples of valid expressions for the border problem.

EXPLORE 1 / SUMMARIZE 1

Individuals / Partners

Pages 2-3
The Border Problem

Reproducible 25
Extra Border Grids (optional)

Materials
- Colored pencils, crayons, or highlighters,
- extra graph paper (optional)

- Students use the ideas discussed for the 12 × 12 grid to generate more pictures, numerical expressions, and word statements for an 8 × 8 grid.

- After some work time, ask students to contribute expressions and explanations for the 8 × 8 grid.

- Compare expressions for the 8 × 8 and 12 × 12 grids. Identify expressions that represent similar structural thinking.

  For the 12 × 12 grid, we said that we can add 4 corner squares to 4 groups of 10 along the 4 edges [4 + 4(10)]. What is a description for the same pattern on the 8 × 8 grid? Add 4 corner squares to 4 groups of 6 along the 4 edges [4 + 4(6)].

  What is the same about these two descriptions? They both have 4 corner squares and 4 groups along the edges.

  What is different? The number of squares along the edges.

  How is the number of squares along the edges excluding the corners related to the number of squares along the edges including the corners? In both cases, the number of squares excluding the corners is two less on each side.

- Ask students to try to generate expressions for a 5 × 5 grid. No grid is provided to encourage abstract thinking. However, students may draw one if needed.

- Help students to make connections among 12 × 12, 8 × 8, and 5 × 5 expressions with similar structure. This is an important step toward generalizing.

  One 12 × 12 grid expression was 2(12) + 2(10). Its 8 × 8 grid companion was 2(8) + 2(6). What is the 5 × 5 companion to these expressions? 2(5) + 2(3). ***How do these expressions relate to one another?*** They all have the same structure, 2(__) + 2(__).The first blank is equal to the side length of the original grid. The second blank is 2 less than the side length of the original grid.

Every lesson follows a general pattern (warmup, introduce, explore, summarize, practice, extend, closure). Grouping strategies, student packet page(s), and materials needed for each phase of the lesson are in the left column.

EXTEND 1

Whole Class / Partners / Individuals Page 4 Expressions and the Distributive Property	• Discuss equivalent numerical and variable expressions. Students list three of the expressions for 8 × 8 grids and simplify them, getting results of 28 each time to verify that they are equivalent. Students simplify the given variable expression and rewrite it using the distributive property. They substitute in the given values for each expression. ***Do you think these expressions are equivalent?*** They seem to be equivalent, based on evaluating the expressions for $x = -1$ and $y = 4$. But we could never check *all* possible values of x and y. We need other methods to verify that two expressions are equivalent. In part (d), the distributive property is used to justify that they really are equivalent. (Problem 3) ***What is an expression for the 12 × 12 grid that can be rewritten using the distributive property?*** Answers may vary. One example shown previously is 2(12) + 2(10) = 2(12 + 10).

PRACTICE 1

Partners / Individuals Page 5 Practice 1	• This page is appropriate for class work or homework. Revisit the "itch" here. See Teaching Note 5. ***What is Tere's error?*** While the numerical expressions are equal for $n = 2$ because $2(2) = 2^2$, there are other values of m (such as $n = 3$ or $n = 4$) for which they are not equal.

INTRODUCE 2

Whole Class / Individuals Page 6 Revisiting the Border Problem Reproducible 26 Analyzing the Border Problem (optional)	To generalize the expressions for the Border Problem, use Reproducible 26 for an open investigation or pages 6-7 for structured workspace. Consider using Reproducible 26 with the entire class first. Then partners or individuals can complete pages 6-7. If time is limited, one or the other may be omitted. • Help students interpret Jaime's sketch, write expressions to match the visual representation and generalize it, and show that expressions are equivalent. ***What do the lines in Jaime's sketch mean?*** We count all tiles on the top and the bottom. We count all tiles on the sides except for the corners. ***State each of the expressions in words.*** Expression 1: top row plus bottom row plus remaining left side plus remaining right side. Expression 2: two full rows plus two remaining columns. ***How do we show that numerical expressions are equivalent?*** Evaluate each expression. If they represent the same value, then they are equivalent. ***How would you generalize these expressions?*** Expression 1: $n + n + (n – 2) + (n – 2)$ Expression 2: $2n + 2(n – 2)$ ***How do we show that algebraic expressions are equivalent?*** If they can be written in the same form (e.g., simplify them). They are both equivalent to $4n – 4$.

Options for delivery of a lesson are often included in the Teacher Packet. Online professional development support for many lessons is available at www.mathandteaching.org through a secure teacher login

EXPLORE 2

Partners/ Individuals

Page 7
Revisiting the Border Problem

- Encourage students to work together to interpret Denny and LaTonya's sketches. They are expected to write expressions that connect to the visual representations. For example, one valid expression for Denny's drawing might be 12 + 11 + 2(10). One valid expression for LaTonya's expression might be 4(10) + 4.

SUMMARIZE 2

Whole Class

Pages 6-7
Revisiting the Border Problem

Reproducible 26
Analyzing the Border Problem (optional)

- Discuss pages 6-7 or Reproducible 26. This table includes six possible representations. Students need not generate all of these, and they may find representations not listed here.

12 × 12	**8 × 8**	**4 × 4**	***n* × *n***
4(10) + 4	4(6) + 4	4(2) + 4	$4(n – 2) + 4$
4(12) – 4	4(8) – 4	4(4) – 4	$4n – 4$
4(11)	4(7)	4(3)	$4(n – 1)$
2(12) + 2(10)	2(8) + 2(6)	2(4) + 2(2)	$2n + 2(n – 2)$
$12^2 – 10^2$	$8^2 – 6^2$	$4^2 – 2^2$	$n^2 – (n – 2)^2$
(simplified expressions)			
4(3) + 4(8)	4(3) + 4(4)	4(3) + 4(0)	$4(3) + 4(n – 4)$
= 44	= 28	= 12	$= 4n – 4$

What do you notice about the simplified expressions? They are equal.

Why does this make sense? These expressions represent different methods of counting the same number—the number of border squares for this particular grid.

The fact that different expressions are equivalent is a really big deal in mathematics. Hopefully this connection will result from a lively classroom discussion.

In the expression, we call 4n the <u>linear term</u>? What might this linear term represent on the grid? The sum of the number of square tiles on each side of the square.

Why is it necessary to subtract 4 from this number of squares? The squares in the corners are counted twice in the expression $4n$.

- If desired, show students a representation listed above that they did not find, and ask them to interpret its meaning.

Some answers are in the teacher packet. All answer keys are available for download at www.mathandteaching.org through a secure teacher login. Printed answer keys are available for purchase.

PRACTICE 2	
Partners / Individuals Page 8 Practice 2	• This page is appropriate for class work or homework. It extends the work done on Practice 1 earlier in this lesson.
EXTEND 2	
Partners /Individuals Page 9 A Different Border Problem Task, Page 12 Border Tile Extensions	• Challenge students to generalize the number of tiles needed for a different rectangular border problem with dimensions $n \times (n + 1)$. • Use the task to challenge students to generalize other border tile patterns.
CLOSURE	
Whole Class Page 0 Word Bank Page 1 Equivalent Expressions	• Review the vocabulary, goals, and the standards of the lesson.

HUNDREDS CHART PATTERNS

Summary	Goals
Students investigate patterns on the hundreds chart. Students write algebraic expressions and use them to prove conjectures based on the patterns. Students find equivalent expressions that involve variables and rational numbers. *6.NS.B, 7.EE.A, 7.EE.B*	• Make conjectures about number patterns. • Use algebraic expressions to prove conjectures. • Write and simplify algebraic expressions that include rational numbers. • View algebra as a useful mathematical tool.

PREVIEW / WARMUP

Whole Class

Page 0
Word Bank

Page 10
Hundreds Chart Patterns

Reproducible 27
Hundreds Chart (optional)

- Introduce the goals and standards of the lesson. Discuss important vocabulary as relevant.
- Students list at least three patterns that they observe on the hundreds chart. Ask them share patterns, and ask follow up questions that promote generalization.

If student says:	Possible follow up questions are:
Count by ones in each row	***Starting at any number in any row, describe how to get to the next number in that row.*** Go right 1, +1, add 1 ***Describe how to get to the previous number in that row.*** Go left 1, -1, subtract 1
Count by tens in each column	***Starting at any number in any column, describe how to get to the next number in that column.*** Go down 10, +10, add 10 ***Describe how to get to the previous number in that column.*** Go up 10, -10, subtract 10
Each diagonal moving downward from left to right increases by 11	***Starting at any number in any diagonal (moving down/right), describe how to get to the next number in that diagonal.*** Go down 10 and right 1, go +10 and +1, add 11 ***Describe how to get to the previous number in that diagonal.*** Go up 10 and left 1, go -10 and -1, subtract 11.
Each diagonal moving downward from right to left increases by 9	***Starting at any number in any diagonal (moving down/left), describe how to get to the next number in that diagonal.*** Go down 10 and left 1, go +10 and -1, add 9 ***Describe how to get to the previous number in that diagonal.*** Go up 10 and right 1, go -10 and +1, subtract 9.

INTRODUCE

Whole Class / Partners / Individuals

Pages 10 Hundreds Chart Patterns

Page 11 Hundreds Chart

Reproducible 27 Hundreds Chart (optional)

- Ask students to fill in the blanks with the appropriate variable expressions. Consider doing the first problem with them if the warmup isn't enough of a head start. Share and discuss as needed.

  Why are $n + 3$ and $3n$ not equivalent expressions? Possible answers:
  (1) $n + 3$ represents adding 3 to a number, and $3n$ represents multiplying 3 by a number.
  (2) On the hundreds chart, $n + 3$ represents the number 3 spaces to the right of n, and $3n$ represents the number obtained when "skip counting" by the value of n three times.
  (3) For any number, n, on the hundreds chart, substituting that value into both expressions yields different results.

  For Enrichment:

  Challenge students to find a value of n such that $n + 3$ and $3n$ are equal. The number (1.5) is easily found by solving the equation $n + 3 = 3n$, but students probably have not learned this algebra skill yet. If they find the number, encourage them NOT to tell their friends – as this would spoil the fun!

- Students experiment with the diagonal numbers in the given 2×2 square portion of the hundreds chart and list observations. Make sure that "diagonals have equal sums" is discussed as this conjecture will be proven on the next page. Acknowledge other patterns and try to prove them later if desired.

- Students choose at least four more such sets of numbers, check to see if their observation holds, and make a conjecture (the sums of the diagonals of all such squares are equal).

  How many examples did you try? Answers will vary.

  About how many examples did the whole class try? Collectively it should be a large number of examples.

  Did the class try ALL possible examples on the hundreds chart? Possibly, but probably not.

  If we added another hundreds chart on the bottom of yours to continue it, how would the first and last squares be numbered? 101 and 200. ***What about the first and last squares of the next hundreds chart?*** 201 and 300.

  Do you think your conjecture would work on these hundreds charts? The class may choose to try a few examples to convince them that it appears to work.

  How many hundreds charts, in theory, could be added this way? Infinitely many.

  How many examples would there be to try? Infinitely many ***Is this possible to do?*** No. Therefore, even though our conjecture seems quite reasonable, it is not proven to be true.

INTRODUCE (Continued)

Page 12
2 by 2 Square Conjectures

- Explain the difference between inductive reasoning and deductive reasoning.

> We use inductive reasoning to arrive at a conjecture (or hypothesis) based on examples. In this problem, students used numerical examples to arrive at the diagonal conjecture. Proof by inductive reasoning would be impossible because there are an infinite number of examples to check.
>
> We use deductive reasoning when we use algebra as a tool to prove a statement is true for all cases. In this problem, students will use variables to generalize patterns in the table and then prove the statement true.

In algebra, when we want to use a symbol to represent any number, what do we usually choose? Typically we use n to represent an integer. It is also traditional to use x to denote an unknown. Other notations are acceptable.

If n is a number on the hundreds chart, what is an expression for the number that is directly to the right of it? $n + 1$

What is an expression for the number that is directly below n on the hundreds chart? $n + 10$

What is an expression for the number that is directly to the right of n + 10 on the hundreds chart? $(n + 10) + 1 = n + 11$

Looking diagonally, what is the sum of n and n + 11? $n + (n + 11) = 2n + 11$.

What is the sum of n + 1 and n + 10? $(n + 1) + (n + 10) = 2n + 11$.

What does this tell us about the sums of these diagonals? Since both sums are equivalent to $2n + 11$, the diagonal sums we are exploring must always be equal, for all numbers n. This proves our conjecture.

EXPLORE

Individuals / Partners

Page 13
3 By 3 Square Conjectures

Reproducible 27
Hundreds Chart (optional)

- Students investigate an extension of the same problem with 3×3 number squares for more practice. Again, the intended conjecture is that the diagonals have equal sums. If others arise, be sure to acknowledge them as well.

> **For enrichment:**
>
> Challenge students to choose to place their n in a different location and then write all of the expressions for the other squares. They may be surprised when they find that the result is the same.

$n - 11$	$n - 10$
$n - 1$	n

SUMMARIZE

Whole Class

Page 13
3 By 3 Square Conjectures

- Discuss patterns in the 3 × 3 number squares.

  How did you represent numbers in a 3 × 3 square? One example is to the right.

n	$n + 1$	$n + 2$
$n + 10$	$n + 11$	$n + 12$
$n + 20$	$n + 21$	$n + 22$

  What are the sums of the diagonals? One is $n + (n + 11) + (n + 22) = 3n + 33$; the other is $(n + 2) + (n + 11) + (n + 20) = 3n + 33$.

  Does the same conjecture hold for the 3 × 3 number squares? Yes. ***Why?*** Because both diagonals are represented by equivalent expressions, $3n + 33$. They are equal for all values of n.

PRACTICE

Partners / Individuals

Page 14
Plus Pattern Conjectures

Reproducible 27
Hundreds Chart (optional)

- Students make more conjectures and try to prove or disprove them. This is appropriate for homework.

EXTEND

Partners / Individuals

Pages 15-16
Rewriting Expressions

Page 17
Practice with Equivalent Expressions

Task, Page 13
Hundred Chart Explorations

- Challenge students to simplify expressions with non-integer coefficients and to identify equivalent expressions. Do some problems with students as needed before asking them work individually or with partners.
- For the task, students identify other patterns on the hundreds chart, make a conjecture, and try to prove it.

CLOSURE

Whole Class

Page 0
Word Bank

Page 10
Hundreds Chart Patterns

- Review the goals, standards, and vocabulary of the lesson.

POLYGON AREA PUZZLE

Summary	Goals
Students create expressions and equations based upon the areas of polygon puzzle pieces. Students evaluate expressions, solve equations, and solve problems. *6.NS.B, 6.EE.B, 7.NS.A, 7.EE.A, 7.EE.B*	• Write algebraic expressions and equations. • Evaluate algebraic expressions. • Solve equations. • Use algebra to solve problems.

PREVIEW / WARMUP

Whole Class

Page 0
Word Bank

Page 18
Polygon Area Puzzle

- Introduce the goals and standards of the lesson. Discuss important vocabulary as relevant.
- Students explain the relationships between the areas of the triangles and find unknown values using given information.

Management Idea:

Ask students who arrive early to class to cut up the puzzle pieces and put them in envelopes.

INTRODUCE 1

Whole Class

Page 19
Writing Equations

Reproducible 28
Polygon Puzzle Pieces (for demonstration)

- Show students puzzle pieces with areas A, B, and C. Explain to students that they will be writing equations about the areas of puzzle pieces where the letter inside of each piece represents its area. Demonstrate the equation $A = 2B$.

Is it a label or is it a variable?

For the polygon puzzle, the letter inside each piece could be interpreted as a label to identify the object or a variable that represents its area. For problems in this lesson, the italic letter inside is a variable that represents the area of the piece.

To describe the area, we may say, “the shape with area *A*,” “area *A*,” or simply “*A*.”

What are some other equations we might write? Some possibilities are: $B = 2C$, $A = 4C$, $A = B + 2C$, and $A + B = 6C$.

EXPLORE 1

Groups / Individuals

Page 19
Writing Equations

Reproducible 28
Polygon Puzzle Pieces
(1 set per group – cut up)

- Distribute puzzle pieces to groups. Ask students to work individually for about 5 minutes and create as many equations as they can.
- Invite students to share equations with each other. This is a good opportunity for students to make viable arguments and critique the reasoning of others.

SUMMARIZE 1

Whole Class

Page 19
Writing Equations

- Ask students to share many equations by stating them orally or demonstrating them in front of the class. Keep a list of all true equations. Set a time limit since the number of possible equations is immense.

- Ask questions that focus on interesting relationships.

  What is the relationship between D and Q? They have the same area. ***Why?*** A square created with two *D* pieces exactly covers a square created with two *Q* pieces. If $2D = 2Q$, then $D = Q$.

  Write an equation using G and H that includes at least one fraction. $\frac{1}{6}H = G$.

  Find two F pieces and four D pieces. Write an expression for this area as the sum of two terms. $2F + 4D$. ***Factor this expression and write it as the product of two terms.*** $2(F + 2D)$.

EXTEND 1

Whole Class / Individuals / Groups

Page 19
Writing Equations

- Challenge students to sort or categorize equations that they or their classmates created, and to explain their organizational structure. A few examples are provided.

  (1) Equations with 2 variables ($2Q = 2D$), 3 variables ($B = C + 2D$), or 4 variables ($A = 2K + L + D$), etc.

  (2) Equations with only sums ($B + B = A$), only products ($A = 2B$), or combinations ($2Q + 2D + M = J$).

  (3) Equations with a variable that has a coefficient of 1 on one side ($A = 2K + L + D$).

  (4) Equations with variables on both sides of the equal sign that have coefficients other than 1 ($2Q = 2D$).

  (5) Equations that have non-integer coefficients, like $N = \frac{1}{4}P$.

INTRODUCE 2 / EXPLORE 2

Whole Class / Partners / Individuals

Page 20
Relationships Among Polygon Areas

Pages 21-22
Solving Polygon Area Equations

- Students refer to the intact Puzzle Pieces located at the end of this packet. Encourage student exploration as individuals or with partners. Discuss one or two problems as needed.

SUMMARIZE 2

Whole Class / Partners / Individuals Page 20 Relationships Among Polygon Areas Pages 21-22 Solving Polygon Equations	• Share and discuss strategies and solutions. Encourage students who solved problems in different ways to share solutions. Require students to politely challenge each other when they disagree, and ask students to defend their reasoning. Promote discussion with follow-up questions. (Page 20, problem 6) ***How do Q, D, and F relate to N?*** $Q = D = F$; $N = \frac{1}{2}Q$; $N = \frac{1}{2}D$; $N = \frac{1}{2}F$. (Page 21, problem 6) ***How can we find the values of Q, D, and F if we know $N = \frac{1}{3}$?*** By substitution, $\frac{1}{3} = \frac{1}{2}Q$**.** This is a 6th grade level equation. Students may reason that one-half of $\frac{2}{3}$ is $\frac{1}{3}$, or they may recall that they can solve for Q by multiplying both sides of the equation by 2. (Page 21, problem 4) ***Why does $2G + H = P + 2(N + D)$?*** One way is to notice that the expressions on either side of the equal sign are both equal to J. (Page 21, problem 4) ***How can we find D?*** After substituting, students may arrive at something like $6 + 18 = 12 + 2(2 + D)$, which may be simplified to $24 = 16 + 2D$. This is a 7th grade level equation and this topic is formally addressed in the next packet. Encourage sense making strategies, such as guess and check. Students may reason that "$24 = 16 + 8$, therefore $2D = 8$, and $D = 4$."

EXTEND 2

Whole Class / Individuals / Partners Page 23 Writing Equations Revisited	• Challenge students to write equations in terms of the given variables using the areas of puzzle pieces. Discuss the two examples with students, and then ask them to work the remaining problems individually and with partners.
Page 24 Paintings on the Wall	• Paintings on the Wall offers an opportunity for students to solve a real life problem that includes rational numbers, algebraic expressions, and equations. This is appropriate for class work or homework.
Task, Page 14 Polygon Area Puzzle Challenge	• Use the task to challenge students to prove triangles have equal area using the area formula.

CLOSURE

Whole Class Page 0 Word Bank Page 18 Polygon Area Puzzle	• Review the goals, standards, and vocabulary of the lesson.

Name ______________________________ Period ___________ Date _____________

QUIZ 8A

1. On the hundreds chart, Jaxon conjectured that for three consecutive whole numbers in a row, the sum of all three numbers is equal to 3 times the middle number.

   a. Demonstrate that this conjecture is true for this set of numbers.

4	5	6

Two parallel forms of quizzes assess basic knowledge in the packet. They are typically two pages long, contain structured workspace (as in the packet), and ask for short answer responses.

   b. Fill in the row for the
   of 23 in the hundred
   conjecture is true fo

	23	

   c. Fill in the row with the variable expressions that should be on either side of n in the hundreds chart. Then prove that Jaxon's conjecture is true.

	n	

2. Simplify the expression $w + 6y + 2w + y + 6w + 5y$.

3. Rewrite the expression in problem 2 as a product of 3 and an expression with two terms.

   _____ • (_____ + _____)

4. Use the distributive property to rewrite each expression so that it is a sum of terms.

a. $2(f + 7)$	b. $5(-w + x)$	c. $(-2y – 3)(-4)$

5. Use the distributive property and the GCF of the terms to rewrite each expression so that it is a product of factors.

a. $8v + 24$	b. $-4w – 10$	c. $-36 – 15u$

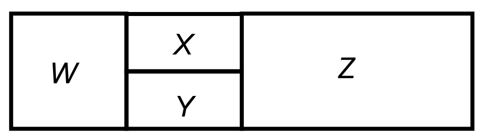
Name ____________________________ Period __________ Date ____________

QUIZ 8A (Continued)

6. Simplify.

a. $-2(x + 8) - 4x$	b. $3(m + 2) - 2(3 - m)$
c. $-3.5 + 2m - 3.64m + 1$	d. $\frac{1}{4}(g + y) - \frac{2}{5}(g - y)$

7. Let the letter inside each figure represent its area. The shape with area X exactly covers the shape with area Y. The shape with area W exactly covers the shapes with areas X and Y combined. The shape with area Z exactly covers the shapes with areas W, X, and Y combined.



a. Write an equation using any of W, X, Y, or Z that has variables on both sides.	b. Write an equation that has at least one fraction in it.
c. Given: $W = 12$ Z = ________ $\frac{1}{2}Z + Y$ = ________	d. Given: $X = \frac{1}{4}$ Z = ________ $Y + 2W$ = ________

Name ____________________________ Period __________ Date ____________

QUIZ 8B

Form A and Form B are parallel

1. On the hundreds chart, Jalen conjectured that for three consecutive whole numbers in a row, the sum of all three numbers is equal to 3 times the middle number.

   a. Demonstrate that this conjecture is true for this set of numbers.

7	8	9

   b. Fill in the row for the numbers that should be on either side of 35 in the hundreds chart. Then demonstrate that this conjecture is true for this set of numbers.

	35	

   c. Fill in the row with the variable expressions that should be on either side of n in the hundreds chart. Then prove that Jalen's conjecture is true.

	n	

2. Simplify the expression $x + 8z + 7x + z + 7z + 4x$.

3. Rewrite the expression in problem 2 as a product of 4 and an expression with two terms.

_____ • (_____ + _____)

4. Use the distributive property to rewrite each expression so that it is a sum of terms.

a. $4(w + 5)$	b. $4(-y + x)$	c. $(-5y - 2)(-3)$

5. Use the distributive property and the GCF of the terms to rewrite each expression so that it is a product of factors.

a. $5v + 30$	b. $6w - 8$	c. $-20 - 5u$

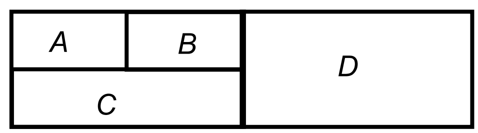
Name ____________________________ Period __________ Date ____________

QUIZ 8B (Continued)

6. Simplify.

a. $-3(x + 4) - 8x$	b. $5(m + 1) - 2(6 - m)$
c. $-3.8 + 5m - 7.62m + 5$	d. $\frac{1}{2}(g + y) - \frac{4}{7}(g - y)$

7. Let the letter inside each figure represent its area. The shape with area A exactly covers the shape with area B. The shape with area C exactly covers the shapes with areas A and B combined. The shape with area D exactly covers the shapes with areas A, B, and C combined.



a. Write an equation using any of A, B, C, or D that has variables on both sides.	b. Write an equation that has at least one fraction in it.
c. Given: $A = 8$ C = ________ $\frac{1}{2}C + B$ = ________	d. Given: $C = \frac{1}{4}$ D = ________ $D + 2B$ = ________

Name ______________________________ Period ___________ Date _____________

TEST PART 8

Show your work on a separate sheet of paper.

Since students continue to practice what they've learned in skill builders, we recommend that summative tests be given at least a few weeks after is completed.

Tests (written in parts to correspond with packets) are intended to be combined to create periodic assessments. For example, Test Parts 8, 9, and 10 may be combined for a summative test, given after Packet 11.

1. Choose ALL expressions below that are equivale

   A. $10 - w$ B. $-2w + 10$ C.

---

2. Choose ALL expressions below that are equivale

   A. $-4(3y + 2x)$ B.

   C. $4(-3y - 8x)$ D.

---

3. Choose ALL expressions below that are equivale

   A. $12g + 11$ B. $-2g + 11$ C. $-2g + (-7)$ D. $-2g - 7$

---

4. Choose ALL expressions below that are equivalent to $3(-d + 1.8x) - 4.2d$.

   A. $-1.2d + 5.4x$ B. $5.4x + 1.2d$

   C. $5.4x - 7.2d$ D. $7.2d + 5.4x$

---

5. Choose ALL expressions below that are equivalent to $\left(-2\frac{2}{5}\right)\left(15y + 1\frac{5}{6}\right) - \frac{3}{5}$.

   A. $36y - 5$ B. $-36y - 5$

   C. $36y - 3\frac{4}{5}$ D. $-36y - 3\frac{4}{5}$

---

6. Find A in the equation $\frac{A}{B} = C$ if $B = -1.5$ and $C = \frac{3}{4}$.

   A. 0.5 B. 1.125

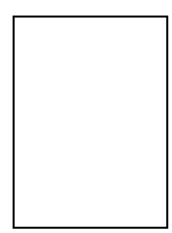
   C. -0.5 D. -1.125

---

7. The rectangle to the right has one side length equal to $3x$ units and area equal to $15x$ square units.

   a. Find the missing side length.

   b. Find the perimeter.



$3x$

Name ____________________________ Period __________ Date ____________

PROFICIENCY CHALLENGE 8

1. Riley said, "If I multiply $\frac{x}{6} + \frac{2}{3}$ by 6, I get $x + 4$. Therefore, $\frac{x}{6} + \frac{2}{3}$ and $x + 4$ are equivalent expressions." Use reasoning to explain or examples to show that Riley is incorrect.

Each packet includes a Proficiency Challenge. Questions assess skills, concepts, and applications with more rigorous questions than those found on quizzes. Many questions are based on released items from the Smarter Balanced Assessment Consortium.

2. Show that none of these expressions are equivalen

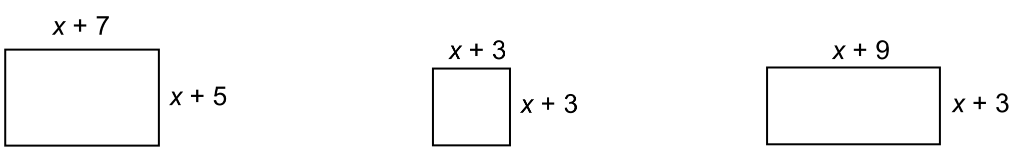
   2[4 + 2 • 3 + 4]   2[4 + 2(3 + 4)]

   Can a set of grouping symbols in any of the express changing the value of the expression? Explain.

3. Determine which of the following expressions are equivalent to 2(3*x* + 2).

6*x* + 2	2(2 + 3*x*)	3*x* + 2 + 3*x* + 2	2(*x* + *x* + *x* + 1 + 1)	*x*(2) + 2(2)	(3*x* + 2)(3*x* + 2)

4. Find the perimeter of the rectangles below. The rectangles are not drawn to scale.



   Do any of the rectangles have equivalent perimeters for any given value of *x*? Explain.

5. William finds the perimeter of a rectangle by adding the length and the width and then doubling this sum. Matthew finds the perimeter of a rectangle by doubling the length, doubling the width and then adding the doubled amounts.

   Write an expression that shows how William finds the perimeter.

   Write an expression that shows how Matthew finds the perimeter.

   Explain why their expressions are equivalent.

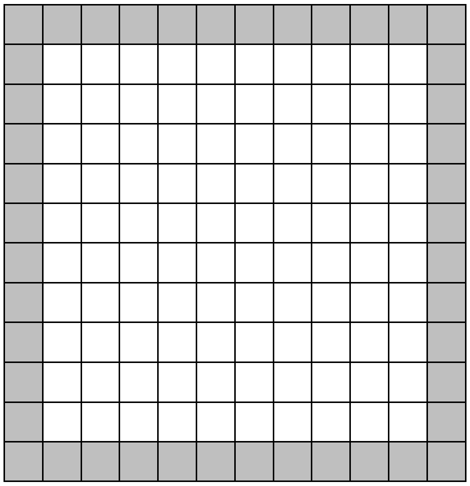
6. Valerie has a room with a wall that is $10\frac{3}{4}$ feet wide.

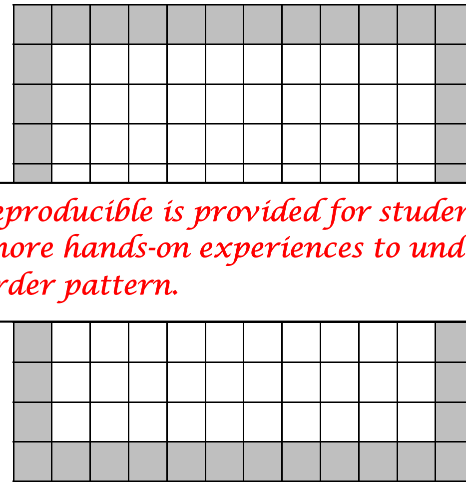
   - She wants to hang a TV in the middle of the wall that is 3.25 feet wide.
   - She wants to hang one picture on each side of her TV that are both the same size.
   - She wants 1-and-one-quarter feet between the TV and each picture.
   - She wants to leave 6 inches between the outer wall edges and each picture.

   Find the maximum width of **each** picture.

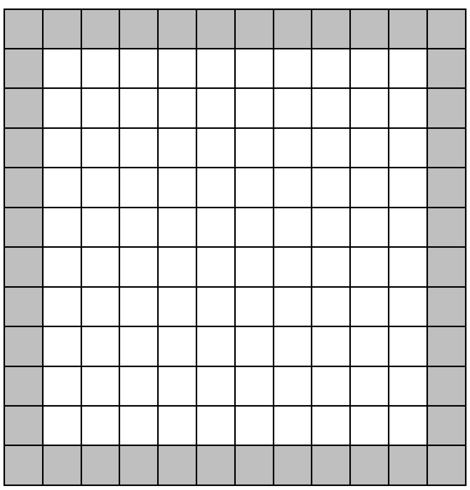
EXTRA BORDER GRIDS

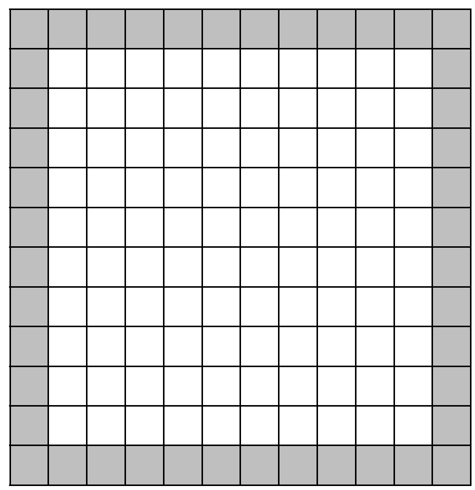
(12×12 and 8×8)

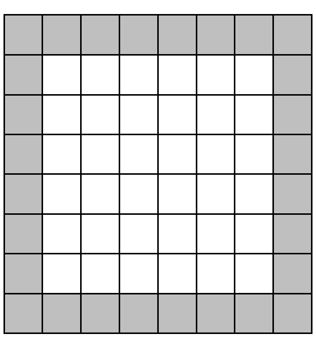


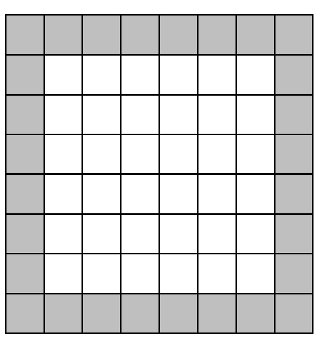


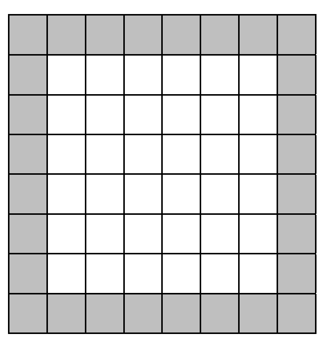
This reproducible is provided for students who need more hands-on experiences to understand the border pattern.

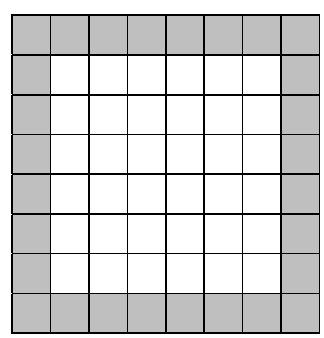












ANALYZING THE BORDER PROBLEM

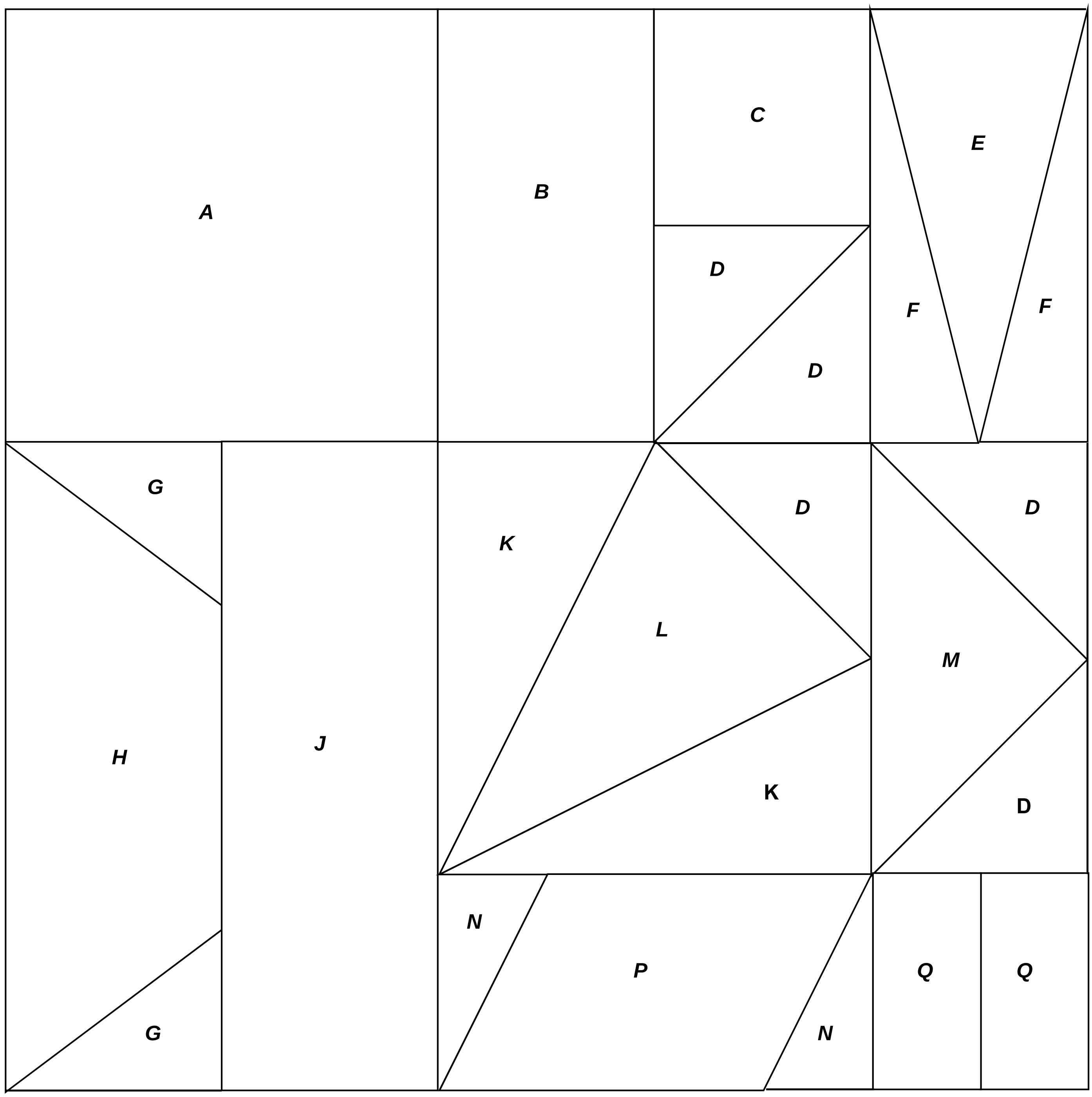
Turn your paper to “landscape” orientation to complete this table. List some of the different expressions that the class generated for each square grid. Organize the expressions so that the related ones from each grid size are in the same row. Then write each expression in simplest form. What do you notice about all of the expressions for each grid?

diagram	12 x 12 grid	8 x 8 grid	5 x 5 grid	*n* x *n* grid

HUNDREDS CHART

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

POLYGON PUZZLE PIECES



Since the Polygon Puzzle will be cut up for use with groups, we also included the puzzle on the inside cover of the Student Packet for reference as the lesson progresses.

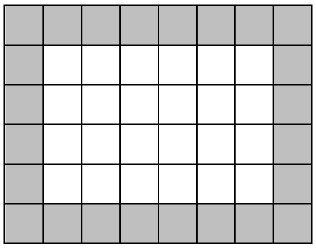
BORDER TILE EXTENSION

Lesson 8.1

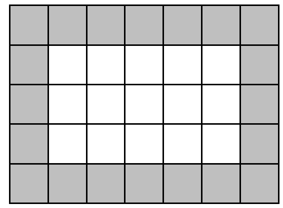
Part 1:

1. Two different grids are pictured below. Write several numerical expressions for the number of shaded border squares for each grid.

6 × 8



5 × 7



2. Write several numerical expressions for the number of shaded border squares for a 4 × 6 grid.

3. Consider the established pattern of grids above. If the length of the shorter side has a length of *n*, what is the length of the longer side? Write several variable expressions for the number of shaded border squares, and simplify them all. Circle the simplified expressions to show that they are all equ

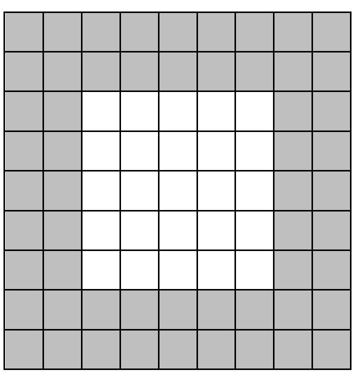
Every packet includes at least one task. The tasks are made of skill building challenges, conceptual development extensions, real world problems, performance tasks, or open-ended questions.

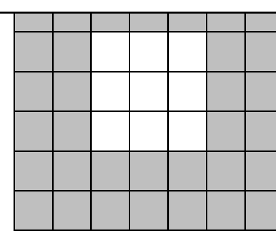
This particular task extends concepts introduced in the "The Border Problem" in lesson 8.1.

Part 2:

4. Two different grids are pictured below. W number of shaded border squares for ea

9 × 9





5. Write several numerical expressions for the number of shaded border squares for a 5 × 5 grid.

6. Consider the established pattern of grids above. If the length each side is *n*, write several variable expressions for the number of shaded border squares, and simplify them all. Circle the simplified expressions to show that they are all equivalent.

HUNDREDS CHART EXPLORATIONS

Lesson 8.2

Find more patterns on the hundreds chart. Try several numerical examples for each. Make a conjecture for each pattern you recognize, and try to prove or disprove the conjecture algebraically.

POLYGON AREA PUZZLE CHALLENGE
Lesson 8.3

- These pictures are portions of the Polygon Area Puzzle. Areas are indicated inside each triangle.
- The rectangle with area $E + 2F$ exactly covers the rectangle with area $M + 2D$.
- For both rectangles, the longer side is twice the length of the shorter side.
- Use the area of a triangle formula to prove:

  1. $E = M$

  2. $F = D$