Packet 16: The Real Number System

Dear Parents/Guardians,

Packet 16 extends students' exploration of the real number system that began in Packet 11. In Lesson 1, students use their knowledge of exponents and roots to solve equations, realizing that some equations do not have solutions that are integer values. In Lessons 2 and 3, students investigate these solutions, identifying real numbers as either rational or irrational.

Exponents and Roots

Students explore relationships between exponents and roots.

Exponent	Root (which is inverse of the exponent)
$x \text{ squared} \longrightarrow x^2$ $8^2 = 64$	$\text{square root of } x \longrightarrow \sqrt{x}$ $\sqrt{64} = \pm 8$ $8 \times 8 = 64 \text{ and } (-8)(-8) = 64$
$x \text{ cubed} \longrightarrow x^3$ $5^3 = 125$	$\text{cube root of } x \longrightarrow \sqrt[3]{x}$ $\sqrt[3]{125} = 5$ $5 \times 5 \times 5 = 125, \text{but } (-5)(-5)(-5) \neq 125$

Students apply their knowledge of exponents and roots to solve equations.

$x^2 = 16$ $\sqrt{x^2} = \sqrt{16}$ $x = \pm 4$	$x^2 = 15$ $\sqrt{x^2} = \sqrt{15}$ $x = \pm\sqrt{15}$ Since the solution is not an integer, we leave it in root form.	$x^3 = \frac{8}{27}$ $\sqrt[3]{x^3} = \sqrt[3]{\frac{8}{27}}$ $x = \pm\frac{2}{3}$

Rational Numbers

Rational Numbers are quotients of integers, so long as the denominator is not zero, because dividing by zero is undefined. Rational numbers can be expressed as either repeating or terminating decimals.

A repeating decimal is a decimal that ends with repetitions of the same pattern of digits. Students will convert fractions to repeating decimals and use a "repeat bar" to indicate the digits that repeat.

Example: Express $\frac{1}{6}$ as a decimal. (Divide 1 by 6.)

$\frac{1}{6} = 0.166... = 0.1\overline{6}$ (Note the "repeat bar" is only above the repeating digit.)

A terminating decimal is a decimal whose digits are 0 from some point on. Typically, the zeros that repeat are omitted.

Example: Express $\frac{2}{25}$ as a decimal. (Divide 2 by 25.)

$\frac{2}{25} = 0.080000\ldots = 0.08$ (Note the 0 in the tenths place remains, as it does not repeat.)

Students convert fractions to decimals, identifying which numbers are rational numbers and which are not.

Irrational Numbers

The real numbers that are not rational numbers are called irrational numbers. Irrational numbers are nonrepeating decimals.

- 0.012345… is an irrational number. Even though there is a pattern, the pattern does not repeat.
- $\sqrt{5}$ is an irrational number. In fact, all roots that are not perfect squares are irrational numbers.
- A famous irrational number is π (pi). There is no exact ratio of a circle's circumference to its diameter. However, we typically use 3.14 as an approximation for π.

Students identify irrational numbers and practice locating them on a number line.

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MathLinks 8

By the end of the packet, your student should know…

How to solve equations involving exponents and roots
Lesson 16.1

How to solve problems with very large and very small numbers, using scientific notation (as well as other methods) Lesson 16.1

How to identify real numbers as rational or irrational
Lessons 16.2 and 16.3

How to locate real numbers on a number line
Lessons 16.2 and 16.3

Additional Resources

Resource Guide (RG)
Part 2, pages 29-38