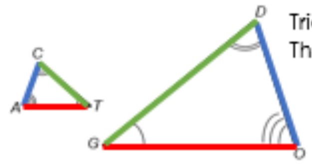
Packet 15: Geometry Discoveries

Dear Parents/Guardians,

Packet 15 builds on similarity concepts from Packet 14, extends it to new geometry ideas, and connects back to slopes of lines. Lessons 2 and 3 explore volumes of cylinders, cones and spheres.

Similar Triangles

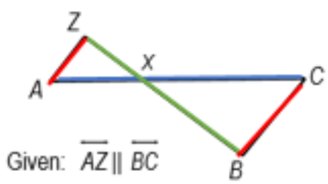
In Packet 14, students discovered that triangles are similar if the corresponding angles are congruent (≅) and the lengths of corresponding sides are proportional.



Triangle ACT is similar to triangle ODG. $\Delta ACT \sim \Delta ODG$
Therefore, the corresponding angles are congruent (≅).

$\angle ACT \cong \angle ODG$
$\angle CAT \cong \angle DOG$
$\angle ATC \cong \angle OGD$

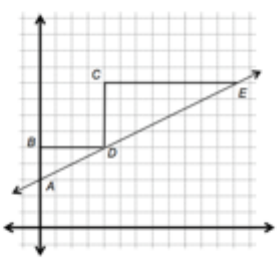
Another way to determine if two triangles are similar is the Angle-Angle Similarity Criterion. If two angles in one triangle are congruent to two angles in another triangle, then the triangles are similar. (If two of the corresponding pairs of angles are congruent, then the third pair must also be congruent.)
Students use this criterion to determine if two triangles are similar.



Statement	Reason
$\angle ZXA \cong \angle$ BXC	Vertical angles are congruent.
$\angle AZX \cong \angle$ CBX	Alternative interior angles are congruent.
$\Delta AZX \sim \Delta$ CBX	Angle-Angle Criterion

Similarity and Slope

Applying the Angle-Angle Criterion, students determine that the slope of a line is always the same as the ratio of lengths of similar right triangle legs.
Students will first prove that the two triangles are similar.



Statement	Reason
$\angle ABD \cong \angle DCE$	Both are right angles.
$\overline{BD} \parallel \overline{CE}$	Both are horizontal line segments with a slope of 0.
$\angle BDA \cong \angle CED$	Line $\overleftrightarrow{AE}$ is the transversal. Corresponding angles are congruent.
$\Delta ABD \sim \Delta DCE$	AA Criterion

The lengths of corresponding sides on similar triangles are proportional.

$\frac{|AB|}{|BD|} = \frac{2}{4} = \frac{1}{2}$ $\frac{|CD|}{|CE|} = \frac{4}{8} = \frac{1}{2}$ The slope of line $\overleftrightarrow{AE} = \frac{1}{2}$.

Volume

The volume of a three-dimensional figure is a measure of the size of the figure, expressed in cubed units (or $units^3$).
Students will solve problems involving volume of 3-D figures.

A basketball used by the NCAA can be no larger than 30 inches in circumference. Calculate the maximum volume.

First, determine the radius (r), using the formula for circumference (C).	Second, find the volume of the basketball (sphere).
$C = 2\pi r$ The basketball can be no larger than 30 inches in circumference. $2\pi r \leq 30$ Divide by 2π. $\frac{2\pi r}{2\pi} \leq \frac{30}{2\pi}$ $r \leq 4.78$ (approximately)	$V = \frac{4}{3}\pi r^3$ $V = \frac{4}{3}(3.14)(4.78)^3$ $V = 457.48\ in^3$ The basketball can have a volume no greater than $457.48\ in^3$

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MathLinks8

By the end of the packet, your student should know…

How to establish and apply the angle-angle criterion for similarity of triangles problems
Lesson 15.1

How to link similar triangle concepts to slope of lines
Lesson 15.1

How to develop the formula for volume of cylinders and use it to solve problems
Lesson 15.2

How to develop the formulas for the volumes of cones and spheres and use them to solve problems Lesson 15.3

Additional Resources

Resource Guide (RG)
Part 2, pages 44-46, 51-52