STACKING CONES (OR CUPS)

A Middle School Lesson About Modeling Using Linear Functions

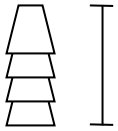
LESSON PLAN

Whole Class

Student Page
Stacking Cups: Table, Graph, and Rules

Materials
- Cups

- Explain the measurement process for stacking cones (or cups). It is intended that students measure the vertical distance, not the slant height of the cups. Discuss appropriate units of measure (e.g. centimeters, millimeters, inches), scaling of the axes, and title for the graph.



height of 4 stacked cups

ELL Strategy
Compare the common use of the word lip (the edge of the opening of a mouth) to its part on cup (the edge of the opening of a cup).

Partners/Individuals

Student Page
Stacking Cups: Table, Graph, and Rules

Student Page
Stacking Cups: Analysis

Materials
- Cups
- Rulers
- Dry Spaghetti

- Students make at least four measurements, record their measurements in the table, and graph their results.

Measuring to the nearest centimeter is easy, but not very accurate. Measuring in millimeters avoids working with decimals but requires special scaling of axes. Measuring in inches requires the use of fractions.

Are the points falling on a straight line? Possibly, but probably not due to measurement error and rounding***. Is there a reason points may not line up?*** Students may make errors reading the ruler. In any event, measurements with a ruler are approximations.

- Ask students to draw a line that represents the data as well as possible. A piece of spaghetti can be moved to help "eyeball" the estimated line of best fit.

- Students answer the questions about stacking cups based on their graphs. Question students to guide further exploration.

(continued on back)

STACKING CUPS: TABLE, GRAPH, AND RULES

You will measure vertical heights of cups as you stack them together and then analyze the data.

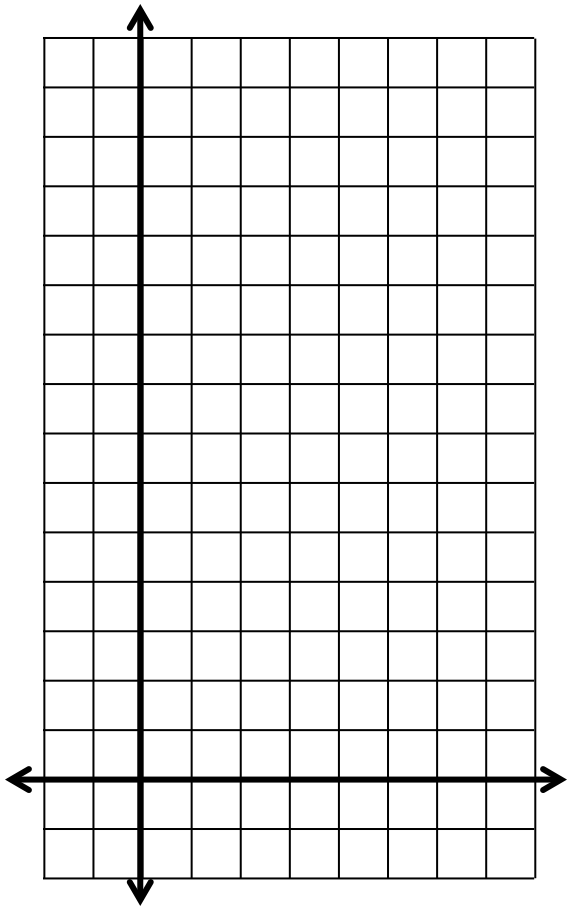
1. What is an appropriate unit of measure? _____,

   rounded to the nearest _____.

2. Measure and record the vertical height of the first cup. Place a second cup inside the first, and measure and record the new height. Continue this process a few more times with different numbers of cups.

Number of cups (x)	1	2	3	4			
Height (y)							

3. Graph the ordered pairs and sketch a trend line that fits the data. Scale the axes as needed.



4. We might say that the height of the stack depends on the number of cups in the stack, so the independent variable is

   _____,

   and the dependent variable is

   _____.

5. What might be a good recursive rule for the height (in words)?

6. What might be a good explicit rule for the height (using symbols)?

7. For the line you sketched to fit the data, what do you think is the significance of the y-values corresponding to x-values that are not integers (as in $x = 1.5$)?

STACKING CUPS: ANALYSIS

1. How did you determine a line to fit your data?

2. How did you determine the recursive rule?

3. What part of the cup represents the difference between the heights in successive cup measurements?

4. How does this difference relate to the numbers in the table?

5. Where is this difference visible on the graph?

6. Estimate the slope of your line that fits the data.

7. Estimate the height (in centimeters) of:

   a. A stack of 10 cups

   b. A stack of 100 cups

8. If you extended your line to $x = 0$, approximately where would it intersect the y-axis? In other words, name the y-intercept.

9. Although zero cups would have zero height, the line does **not** pass through the point (0, 0). Why?

Adapted especially for NCTM 2016 - from *MathLinks*: Grade 8 (Lesson 4.3)

Whole Class Student Page Stacking Cups: Table, Graph, and Rules	• Discuss the results obtained when making tables and graphs. ***What height measurements did you get?*** Answers will vary depending on the cup used, the unit of measurement (cm or mm), and the accuracy of measurements. ***When graphing the coordinates, did all the points fall in a perfectly linear path?*** Possibly, but probably not. Again emphasize that this is due to the nature of measuring with rulers and measurement errors, which are expected. ***Did you connect the points?*** Technically, these data points are discrete and should not be connected. However, an estimated line to fit the data represents a trend line for the data. • Discuss responses to the stacking cup questions. ***Where does the graph intersect the y-axis?*** Answers will vary depending on the cup used and the accuracy of measurements. ***What does this point represent?*** Some might argue (correctly) that it represents the height of the original cup without lip. This corresponds to viewing the stack as a number of lips topped by one cup without the lip. This is what one sees when one views the stack from the side. Others might say it has no meaning. In any event, it cannot be interpreted as the height of 0 cups, since the height of zero cups is 0 cm. ***What is the equation for your estimated line to fit the data? How did you get it?*** Answers may vary. Encourage students to share their thinking.

Lesson Extensions

(1) Begin by having students estimate on the front end of the lesson how tall a stack of 10 cups might be after only measuring one or two. After taking more measurements, and after doing the analysis, have students use their equations to determine the height of 10 cups in addition to actually measuring a stack of 10.

(2) Begin the lesson with two different types of cups, a shorter one with a bigger lip and a taller one with a smaller lip. Ask students to predict whether or not some number of shorter cups will ever be equal to or greater than in height as the same number of taller cups. The lip determines the growth rate, and students may be surprised at the potential quick growth rate of these shorter cups. This two-cup exploration is a nice, informal introduction to systems of equations. Go to http://www.101qs.com/1897-stacking-cups--act-1 for a "Three Act" lesson" by Andrew Stadel that includes video.

(3) As a follow up to stacking cups, students can stack other objects that "nest" and use multiple representations to investigate the relationships. They might use plates, bowls, shopping carts or carry baskets at a grocery, or come up with something on their own.