$\qquad$
Mathinks

## MATHLINKS: GRADE 7 STUDENT PACKET 13 SAMPLING

| 13.1 | Introduction to Sampling |  |
| :--- | :--- | :--- |
|  | - Review statistics used to summarize center and spread of a | 1 |
|  |  |  |
|  | numerical data set. |  |
|  | - Use random sampling to make valid inferences about populations. |  |

13.3 Fish in a Lake

- Create a mathematical model based on proportional reasoning.
- Use random sampling to make valid inferences about populations.
- Use measures of center and spread to compare two populations.
- Use dot plots, box plots, and tables to understand and compare numerical data sets.
13.4 Skill Builders, Vocabulary, and Review


## WORD BANK

| Word or Phrase | Definition or Description | Example or Picture |
| :--- | :--- | :--- |
| box plot |  |  |
| dot plot |  |  |
| five-number |  |  |
| summary |  |  |
| interquartile range |  |  |
| mean |  |  |
| mean absolute |  |  |
| meviation |  |  |
| mopulation |  |  |
| mode |  |  |
| samulation |  |  |

## INTRODUCTION TO SAMPLING

## Summary

We will use measures of center and spread to describe data sets. We will make predictions about a population based on a sample of the population. We will display data sets using dot plots, box plots, and other visual representations.

## Goals

- Review statistics used to summarize center and spread of a numerical data set.
- Use random sampling to make valid inferences about populations.
- Use dot plots, box plots, and other visual representations to understand and compare numerical data sets.

Pretend you are doing a probability experiment with a paper bag that has 5 blue and 3 red counters inside of it. Each time you draw a counter, you record its color and then replace it.

1. What is the probability you will draw a blue counter?
2. What is the probability you will draw a red counter?
3. If you draw 80 counters, about how many would you expect to be blue?
4. If you draw 800 counters, about how many would you expect to be red?
5. For problems 3 and 4 above, do you think you would get exactly the numbers you wrote as your answers? Explain.
6. Eddie drew 10 counters and all of them were blue. For his $11^{\text {th }}$ turn, what is the probability of drawing a blue counter? Explain.

## NAME SCORE STATISTICS

1. Write your first name here as you are usually called in class. $\qquad$
2. Use the values below to determine your name score. $\qquad$ Example: FRED $\rightarrow$
$F+R+E+D$
$4+1+1+2=8$

| A = 1 | D $=2$ | G = 3 | J = 10 | $\mathrm{M}=4$ | $\mathrm{P}=4$ | $\mathrm{S}=1$ | $\mathrm{V}=5$ | $\mathrm{Y}=4$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{B}=4$ | $\mathrm{E}=1$ | $\mathrm{H}=3$ | $\mathrm{K}=5$ | $\mathrm{N}=2$ | $\mathrm{Q}=10$ | $\mathrm{T}=1$ | W $=4$ | $Z=10$ |
| $\mathrm{C}=4$ | $\mathrm{F}=4$ | $\mathrm{l}=1$ | L = 2 | $\mathrm{O}=1$ | $\mathrm{R}=1$ | $\mathrm{U}=2$ | $\mathrm{X}=8$ |  |

Your teacher will lead you through an exercise. Then finish this page and the next.
3. Record all the class name scores below from least to greatest.

4. Make a dot plot of the class name scores.


## NAME SCORE STATISTICS (Continued)

Match the letters A-K to each description below, then write the values of each measure based upon the class name scores.

| A. Mean $\bar{x}$ | B. $1^{\text {st }}$ Quartile $\left(Q_{1}\right)$ | C. number of observations $(n)$ | D. minimum (min) |
| :--- | :--- | :--- | :--- |
| E. range | F. median $\left(\right.$ med $\left.=Q_{2}\right)$ | G. five-number summary | H. maximum (max) |
| I. mode | J. $3^{\text {rd }}$ Quartile $\left(Q_{3}\right)$ | K. interquartile range (IQR) |  |
|  |  |  |  |


|  | A - K | Description | Value(s) |
| :--- | :--- | :--- | :--- |
| 5. |  | The number of student scores. |  |
| 6. |  | The middle number, when the observations are ordered <br> from least to greatest. |  |
| 7. | The arithmetic average. |  |  |
| 8. |  | The lowest score. |  |
| 9. | The highest score. |  |  |
| 10. | The difference between the highest and lowest score. |  |  |
| 11. | The middle of the lower half of the data set. |  |  |
| 12. |  | The middle of the upper half of the data set. |  |
| 13. |  | The following five numbers: (min, Q1, med, Q3, max). |  |
| 14. |  | The most common observation (there may be more than <br> one). |  |
| 15. |  | The difference between $Q_{3}$ and $Q_{1}$. |  |

16. Make a box plot of the class name scores.

## MORE NAME SCORES

Suppose every student in the class calculated the name score for his or her LAST name using the same method we used to determine first name scores.

1. Predict the mean score and median score for the last names of students in your class. Explain your reasoning.

Mean $(\bar{x})_{\text {prediction }}=\underline{M e d i a n ~}\left(Q_{2}\right)_{\text {prediction }}=$ $\qquad$

Your teacher will give you another slip of paper on which to clearly write your last name and name score.
2. Ten slips will be drawn at random. Record the scores below from least to greatest. Then calculate the mean and median for this sample.

|  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

3. Ten slips will be drawn at random a second time. Record the scores below from least to greatest. Then calculate the mean and median for this sample.

|  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

4. Ten slips will be drawn at random a third time. Record the scores below from least to greatest. Then calculate the mean and median for this sample.

|  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## MORE NAME SCORES (Continued)

5. Given the data on the previous page, write an estimate for the mean and median class last name scores. Explain how you arrived at these estimates.

Mean $(\bar{x})_{\text {estimate }}=\square \quad$ Median $\left(Q_{2}\right)_{\text {estimate }}=$ $\qquad$
6. Compare your prediction in problem 1 to your estimate in problem 5. Did your prediction hold up? Explain.
7. Record all the students last name scores below from least to greatest. Then find the mean and median name scores.

|  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |

8. Find your errors as percents. Use this formula to find the decimal values first, then write as percents:

$\left\lvert\,$| estimate-actual |
| :---: |
| actual |$\quad \rightarrow \quad \bar{x}\right.$ error: $\_\quad$ Q2 error:

9. Challenge: What are some other ways to analyze the last name scores you collected? Use numerical statistics, data displays, and summarizing statements. Show your work on a separate sheet of paper.

## SCHOOL LUNCH SURVEY

Kyle is planning a survey to find out how much $7^{\text {th }}$ grade students like school lunches. He will survey 14 students and ask them to rate school lunches on a scale of 1-10.

1. How should Kyle select the students to survey in order to have a sample that represents the entire $7^{\text {th }}$ grade class? Circle an appropriate method below and explain your choice.
a. Survey the first 14 students in line for lunch.
b. Survey the last 14 students in line for lunch.
c. Survey 14 students whose names are pulled out of a bag containing the names of all $7^{\text {th }}$ graders in the school.
d. Survey 14 students staying after school for tutoring.
e. Survey his 14 best friends in the $7^{\text {th }}$ grade.
2. Kyle conducted his survey. Here are his results. Make a dot plot for this data.

| 3 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 7 | 7 | 4 | 5 | 6 | 8 | 10 |

Find the value of each statistical measure.

| Statistic | Value | Statistic |  | Value |
| :--- | :--- | :--- | :--- | :--- |
| 3. $\quad$ mean $\bar{x}$ |  | 7. $\quad$ median $\left(\right.$ med $\left.=Q_{2}\right)$ |  |  |
| 4. $\quad$ mode |  | 8. $\quad 1^{\text {st }}$ quartile $\left(Q_{1}\right)$ |  |  |
| 5. $\quad$ minimum $(\min )$ |  | 9. $\quad 3^{\text {rd }}$ quartile $\left(Q_{3}\right)$ |  |  |
| 6. $\quad$ maximum $(\max )$ |  | 10. interquartile range (IQR) |  |  |

11. Make a box plot of Kyle's data. Use tools appropriately.

12. If there were 126 seventh grade students at Kyle's school, predict the number of students who rated lunches a 7 . Show why you think this is a valid prediction.
13. When all $7^{\text {th }}$ grade students were surveyed, the mean of the ratings was 7.5. Kyle's survey predicted this mean with an error of what percent?

## COMPARING SAMPLES

## Summary

We will compare two data sets using visual displays and statistics. We will compute and interpret the mean absolute deviation for samples. We will explore the effect of outliers on a data set.

## Goals

- Use visual displays to understand and compare numerical data sets.
- Use measures of center and spread to compare two samples.
- Explore the effect of outliers on a data set.
- Draw inferences about populations based on samples.

Write each fraction or decimal as an equivalent percent.

| 1. 0.56 | 2. | 0.4 |  | 3. | 0.07 |
| :--- | :--- | :--- | :---: | :--- | :--- |
| 4. | $\frac{1}{4}$ | 5. | $\frac{1}{5}$ |  | 6. |

7. Julio was having a debate with Anthony and replied to one of his comments by saying, "I agree with you 100\%." What did Julio mean by that?
8. Kim challenged Evelyn to guess how many crackers she had in a bag. When Evelyn guessed 10, Kim replied, "You're off by $50 \%$." How many crackers could Kim have in the bag? Explain.

## MATH SCORE SAMPLES: DATA DISPLAYS

The students in Robinsdale School District took benchmark assessments. Here are samples of student scores selected randomly from the $7^{\text {th }}$ grade for two of the middle schools.

1. Rewrite the lists of scores in order from least to greatest.

2. What conclusions can you draw at this point about the results of the benchmark assessments?

## MATH SCORE SAMPLES: STATISTICS

Calculate each statistic below using the test scores from page 8.

| Statistic | Burkhart MS | Starke MS |
| :---: | :---: | :---: |
| 1. number of observations ( $n$ ) |  |  |
| 2. minimum (min) |  |  |
| 3. maximum (max) |  |  |
| 4. range |  |  |
| 5. mode |  |  |
| 6. mean $(\bar{x})$ |  |  |
| 7. median ( $\mathrm{med}=\mathrm{Q}_{2}$ ) |  |  |
| 8. $1^{\text {st }}$ quartile $\left(Q_{1}\right)$ |  |  |
| 9. $3^{\text {rd }}$ quartile $\left(Q_{3}\right)$ |  |  |
| 10. interquartile range (IQR) |  |  |
| 11. five-number summary |  |  |

12. After reviewing the statistics above, what are some new conclusions or questions you might have about the scores?

## MEAN ABSOLUTE DEVIATION

The mean absolute deviation (MAD) is a measure of spread of a data set. It is the arithmetic average of the distance of each data point to the mean. To calculate the MAD statistic:

- Find the mean of the sample.
- Find the distance from each data point to the mean.
- Find the sum of the distances.
- Divide the sum of the distances by the number of data points to find the average distance from the mean.

1. Find the mean scores for the class samples: Burkhart $\qquad$ ; Starke $\qquad$
2. Compute the MAD statistics for the two samples.

| Burkhart data | Distance from data point to mean | Starke data | Distance from data point to mean |
| :---: | :---: | :---: | :---: |
| 13 | $13-\ldots$ | 14 | $\mid 14-\ldots$ |
| 18 |  | 22 |  |
| 13 |  | 21 |  |
| 14 |  | 14 |  |
| 13 |  | 12 |  |
| 16 |  | 15 |  |
| 14 |  | 20 |  |
| 21 |  | 21 |  |
| 13 |  | 17 |  |
| 15 |  | 22 |  |
| 21 |  | 14 |  |
| 22 |  | 15 |  |
| Sum = | ; MAD = | Sum = _ ; MAD = |  |

3. Name the measure of spread associated with each measure of center.

Measure of center: mean
Measure of center: median

Measure of spread: $\qquad$
Measure of spread: $\qquad$

## MATH SCORE SAMPLES: WHAT IF?

Suppose one more score from each school is chosen at random.

- The new score from Burkhart Middle School is 3.
- The new score from Starke Middle School is 18.

1. Predict what will happen to the mean and the MAD for each data set with the addition of these scores.
2. Find the new mean scores for the samples: Burkhart $\qquad$ ; Starke $\qquad$
3. Compute the new MAD statistics for the two samples.

| $\begin{gathered} \text { Burkhart } \\ \text { data } \end{gathered}$ | Distance from data point to mean | Starke data | Distance from data point to mean |
| :---: | :---: | :---: | :---: |
| 13 |  | 14 |  |
| 18 |  | 22 |  |
| 13 |  | 21 |  |
| 14 |  | 14 |  |
| 13 |  | 12 |  |
| 16 |  | 15 |  |
| 14 |  | 20 |  |
| 21 |  | 21 |  |
| 13 |  | 17 |  |
| 15 |  | 22 |  |
| 21 |  | 14 |  |
| 22 |  | 15 |  |
| 3 |  | 18 |  |
| Sum = | ; MAD = | Sum = _ ; MAD = |  |

4. Were your predictions correct? $\qquad$ Explain.

## MATH SCORE SAMPLES: WHAT IF? (Continued)

State whether each statistic will increase, decrease, or stay the same when the additional scores are included in the samples.

| Statistic | Burkhart MS | Starke MS |
| :--- | :--- | :--- |
| 5. number of observations $(n)$ |  | increase |
| 6. minimum (min) |  |  |
| 7. maximum (max) |  |  |
| 8. range |  |  |
| 9. mean $(\bar{x})$ |  |  |
| 10. median $\left(\operatorname{med}=Q_{2}\right)$ |  |  |
| 11. 1 $^{\text {st }}$ quartile $\left(Q_{1}\right)$ |  |  |
| 12. 3 $^{\text {rd }}$ quartile $\left(Q_{3}\right)$ |  |  |
| 13. interquartile range (IQR) |  |  |
| 14. mean absolute deviation (MAD) |  |  |

An outlier is a data value that is a striking deviation from the overall pattern of the values in the data set.
15. Name any data point(s) that are potential outlier(s).
16. Which statistics seem resistant (unlikely to change) to outliers?
17. Which statistics seem more likely to change when an outlier is introduced?

## COMPARING HEIGHTS

Below are the heights (in inches) of athletes at UCLA in 2014, selected randomly from the rosters and organized in order from least to greatest.

Girls' softball team (S)

| 60 | 62 | 63 | 64 | 64 | 67 | 67 | 67 | 68 | 69 | 69 | 72 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Girls' basketball team (B)

| 66 | 68 | 69 | 69 |
| :--- | :--- | :--- | :--- |


| 61 | 61 | 62 | 62 | 63 | 63 | 64 | 64 | 64 | 65 | 65 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Girls' volleyball team (V)

| 62 | 67 | 69 | 71 | 73 | 75 |
| :--- | :--- | :--- | :--- | :--- | :--- |

1. Create dot plots to compare heights of players on each team. Label clearly.

2. Create box plots to compare the heights of players on each team. Label clearly.


## COMPARING HEIGHTS (Continued)

3. Based on the visual overlap of the dot and box plots, make three statements comparing the center and spread (variability) of the heights of players on the different teams.

Rachel used a MAD calculator online to compute the number of observations, the mean, and the MAD statistic for each data set, but she didn't organize her work or label it. Here are the twelve numbers she wrote down.

| 2.8 | 10 | 63.1 | 10 | 4.1 | 11 | 1.2 | 2.6 | 71.3 | 72.4 | 66.1 | 12 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

4. Use the visual displays and Rachel's numbers to complete the table below.

|  | Softball (S) | Basketball (B) | Gymnastics (G) | Volleyball (V) |
| :---: | :---: | :---: | :---: | :---: |
| number of <br> observations |  |  |  |  |
| Mean |  |  |  |  |
| MAD |  |  |  |  |

Write in the name of the appropriate sport to complete each statement.
5. The spread of athlete heights in $\qquad$ and $\qquad$ is about the same.
6. The MAD of heights in $\qquad$ is more than 3 times the MAD height for $\qquad$ .
7. The average heights of girls playing $\qquad$ and $\qquad$ are about the same.
8. If an athlete is very short, she is likely to participate in $\qquad$ or $\qquad$ .
9. If an athlete is very tall, she is more likely to participate in $\qquad$ than $\qquad$ .
10. The greatest variability in heights occurs among girls playing $\qquad$ .
11. The least variability in heights occurs among girls playing $\qquad$ .

## FISH IN A LAKE

## Summary

We will do a sampling experiment to estimate the fish population in a lake. We will make inferences about fish populations using data displays and measures of center and spread.

## Goals

- Create a mathematical model based on proportional reasoning.
- Use random sampling to make valid inferences about populations.
- Use measures of center and spread to compare two populations.
- Use dot plots, box plots, and tables to understand and compare numerical data sets.

Warmup
Solve each equation.

| $\frac{2}{3}=\frac{x}{27}$ | 2. | $\frac{x}{5}=\frac{-3}{10}$ |
| :--- | :--- | :--- |
| $\frac{6}{x}=\frac{4}{7}$ | 4. | $\frac{2.5}{8}=\frac{4.5}{x}$ |
| 3. |  |  |

## ESTIMATING FISH POPULATIONS

You will do an experiment that simulates (imitates) a method scientists use to estimate a fish population in a lake. Follow the directions given by your teacher to complete the fish population experiment. Use this sheet to record and analyze data.
$S_{\text {tagged }}=$ number of tagged fish in Sample $\quad \boldsymbol{P}_{\text {tagged }}=$ number of tagged fish in Population
$S_{\text {total }}=$ total number of fish in Sample $\quad P_{\text {total }}=$ total number of fish in Population

1. How many fish did you capture and tag from the lake? $\qquad$
What variable described above does this represent? $\qquad$
2. "Recapture" samples of fish from the lake. Record your data for $\boldsymbol{S}_{\text {tagged }}$ and $\boldsymbol{S}_{\text {total }}$.

|  | Sample 1 | Sample 2 | Sample 3 | Sample 4 | Sample 5 | Sample 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $S_{\text {tagged }}$ |  |  |  |  |  |  |
| $S_{\text {total }}$ |  |  |  |  |  |  |

3. Create a mathematical model to estimate the total number of fish in the lake based on your samples. Use the variables defined at the top of this page.
4. Use your model to estimate the number of fish in the lake.

| Sample 1 | Sample 2 | Sample 3 |
| :---: | :---: | :---: |
| Sample 4 | Sample 5 |  |
|  |  | Sample 6 |

## ESTIMATING FISH POPULATIONS (Continued)

5. Based on your experiment, estimate the number of fish in the population. $\qquad$ Explain how you arrived at this estimate.
6. Count all the fish. Find the actual number of fish in the population. $\qquad$
7. Did you overestimate or underestimate? $\qquad$ Was your estimation close? $\qquad$
8. Find the error in your estimate as a percent of the actual population. Recall the computation: $\square$
9. What assumptions do you think scientists make when estimating fish populations in this way?
10. What might you do to improve your estimate of the number of fish in the lake?
11. In real life, why might it be difficult to collect a representative sample using this "capturerecapture" method?
12. This experiment helped you make inferences about the number of fish in a lake. What other experiments could you try to conduct about the fish in this lake?

## FISHY STATISTICS

Suppose that when fish were tagged, they were also measured. Here are fish lengths in centimeters from two different random samples from two different lakes:

$$
\begin{aligned}
& \text { Sample A: } 75,32,38,42,47,68,51,51,58,31,51,65 \\
& \text { Sample B: } \quad 49,45,51,49,63,56,51,48,52,42,51,52
\end{aligned}
$$

1. Rewrite each list in order from least to greatest.


## A FISHY DATA DISPLAY: BOX PLOT

1. Create box plots from the data on the previous page for Sample A and Sample B using the scale provided below.

Sample A:


Sample B:

2. Why is it important to use the same scale when comparing the data?
3. About what percent of the data is included between $Q_{1}$ and $Q_{3}$ ?
4. What do the box plot and interquartile range tell us about fish lengths in the lakes?
5. Are there any other statistics not included in the box plot that you think are relevant or useful? Explain.

## A MAD EXTENSION

1. Here are the lengths of fish for Sample $A$ (in order). Recall the mean ( $\bar{x}$ ) for Sample $A$ was 50.75 cm . Draw an arrow to locate $(\bar{x})$.

| 31 | 32 | 38 | 42 | 47 | 51 | 51 | 51 | 58 | 65 | 68 | 75 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

2. Recall the MAD statistic for Sample A was 10.6. Determine which fish lengths fall within this distance from the mean.
$\bar{x}-\mathrm{MAD}=50.75-10.6=\square$
$\bar{x}+\mathrm{MAD}=\square$

Interval representing one MAD unit from the mean: $\qquad$ , (1)
(2)
3. Draw an oval around the fish lengths that fall in this interval.
4. What percent of the fish lengths fall in this interval? $\qquad$
5. Here are the lengths of fish for Sample B (in order). Recall the mean ( $\bar{x}$ ) for Sample B was 50.75 cm . Draw an arrow to locate $(\bar{x})$.

| 42 | 45 | 48 | 49 | 49 | 51 | 51 | 51 | 52 | 52 | 56 | 63 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

6. Recall the MAD statistic for Sample B was 3.5. Determine which fish lengths fall within this distance from the mean.

$$
\begin{align*}
& \bar{x}-\mathrm{MAD}=50.75-3.5=\square  \tag{1}\\
& \bar{x}+\mathrm{MAD}=\square+\frac{1}{(2)}  \tag{2}\\
& \text { Interval representing one MAD unit from the mean: }[\ldots \ldots(1)
\end{align*}
$$

7. Draw an oval around the fish lengths that fall in this interval.
8. What percent of the fish lengths fall in this interval? $\qquad$
9. About what percent of the data falls in the interquartile range (IQR)? $\qquad$ Is this always true for MAD? $\qquad$
10. How does the MAD statistic help us describe the lengths of fish in the lake?

## THE FISHING COMPETITION

You are consulting with a fishing club committee that wants to run its first annual fishing competition in Monterey County, CA.

The organizers have set aside 4 hours for the competition and want to know if that is appropriate.

They give you the following information:

- A park ranger says that the following fish live in the lake, they populate the lake in roughly equal numbers, and they are roughly equally likely to catch.

| Bluegill | Coastal rainbow <br> trout | Golden <br> shiner | Largemouth <br> sass |
| :---: | :---: | :---: | :---: | | Redear |
| :---: |
| sunfish |

- The winner is the first to catch at least one of each type of fish.
- During the competition, when a fish is caught, it is recorded and released back into the lake.

You know from past consulting experiences that the competitors typically catch one to two fish every hour.

It's your job to create a short report to present to the organizing committee. Your main goal is to justify a reasonable amount of time to set aside for the competition. Your report must include:

- An explanation of the simulation you create.
- The number of times you carry out your simulation, and reasons why you chose that number.
- The results of your simulation.
- An explanation of why you think the results of the simulation are reasonable, in view of the information given to you.
- What your initial recommendation to the organizing committee is.
- What information might be important to have in order to provide a better time estimate?


## SKILL BUILDERS, VOCABULARY AND REVIEW

## SKILL BUILDER 1

1. Find and circle the mistakes in the work shown below. Rework the solution so that it is correct.

| Equation/Steps | Circle the error and rework the solution |
| :---: | :---: |
| $-4(x+2)=12$ |  |
| $-4 x+8=12$ |  |
| $-4 x+8=12$ |  |
| $-8 \quad-8$ |  |
| $-4 x$ | $=\frac{4}{4}$ |
| $-x=$ |  |
| $x$ | $=1$ |

2. Javier says that $5-7 \frac{1}{2}$ is equivalent to $2 \frac{1}{2}$. Is Javier correct? Justify your answer using a calculation or a valid argument.

Solve.

| 3. $\frac{5}{x}=\frac{15}{21}$ | 4. $\frac{5}{6}=\frac{x}{10}$ | 5. $\frac{6}{-9}=\frac{9}{x}$ |
| :--- | :--- | :--- |

6. Grace is making orange dye (paint). She uses red and yellow in a ratio of $1: 3$. How many quarts of red dye and yellow dye will she need if she wants to make 20 quarts of paint?

## SKILL BUILDER 2

| Solve. |
| :--- |
| 1. $6.8 x-30.8=30.4$ 2. $3.5(y-4)=-56$ 3. $167.2=-5.6+7.2 x$ <br> 4. $2 \frac{1}{5}+\frac{3}{4} x=9 \frac{1}{10}$ 5. $-3 \frac{1}{2} x-4=-38 \frac{1}{8}$ 6. |

7. You flip a coin 3 times. Draw a tree diagram to find all the possible outcomes. Then find the probability of getting tails on all three coin flips.
8. The balances in four different bank accounts were \$326.98, \$24.98, -\$123.45, and $-\$ 10.85$. Find the average of the four bank balances.
9. An architect designed a building so that the elevation of the parking garage is -25 feet. The elevation of the roof is 104 feet. What is the distance from the roof to the floor of the parking garage?

## SKILL BUILDER 3

1. Sophie can solve 18 problems in 65 minutes. At that rate, how long will it take her to solve 2 problems?

Assume Raina runs at a constant speed.
2. Complete the table to the right.
3. Is Raina's distance run proportional to the time spent running? $\qquad$ If so, what is the constant of proportionality?
4. Write an equation for the distance, $d$, in miles, that Raina runs in $h$ hours.

| Time <br> (Hours) | Distance Run <br> (Miles) |
| :---: | :---: |
| 1 |  |
| 3 | 32.8 |
| 8 |  |

Compute.

| 5. $-5(3)+13$ | 6. | $2(7-9)+15$ | $-4.5(3.2-5.82)+7$ |  |
| :--- | :--- | :--- | :--- | :--- |
| 8. | $\frac{-2(9)-4}{13-11}$ | 9. | $\frac{4\left(5-2^{2}\right)}{-8+6}$ | 10. |

## SKILL BUILDER 4

1. Measure the line segment below to the nearest one-quarter inch and also the nearest one-tenth of a centimeter.

Find the percent change.
2. From 18 to 20 3. From 50 to 45.

## Solve.

| 4. $m+7>-3$ | 5. $16<-4 m$ | 6. $\frac{m}{4} \geq \frac{5}{8}$ |  |
| :--- | :--- | :--- | :--- |
| 7. $2+\frac{m}{-3} \leq-9$ | 8. | $-18<3 m-6$ | 9. |
|  |  | $1.8-x \leq 4.9$ |  |

10. Roza took a test and 13 out of 45 answers were incorrect. What percentage did she get correct?
11. Mariam borrowed $\$ 800$ from her uncle for two years. She paid simple interest on the loan, at an annual interest rate of $3.2 \%$. At the end of the two years, how much interest did she pay?

Classify the triangles by their angles and sides.
12.

## SKILL BUILDER 5

1. Kim and Sunny each babysit for 10 different families. Each family pays different amounts as seen in the charts below. Draw dot plots for Kim and Sunny. Label and scale appropriately.


Calculate the following.

| Statistic | Kim's Earnings | Sunny's Earnings |
| :--- | :--- | :--- |
| 2. number of observations $(n)$ |  |  |
| 3. range |  |  |
| 4. mode |  |  |
| 5. mean $(\bar{x})$ |  |  |

6. Find the five-number summary for earnings data for Kim and Sunny and draw box plots.

|  | $\min$ | $Q_{1}$ | $Q_{2}$ | $Q_{3}$ | $\max$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Kim |  |  |  |  |  |
| Sunny |  |  |  |  |  |



## SKILL BUILDER 6

Use the babysitting earnings data from the previous page.

| 1. Find the MAD for Kim and Sunny's data. |
| :--- |
| Kim Distance from <br> data point to mean <br> $\$ 8$  <br> $\$ 10$  <br> $\$ 8$  <br> $\$ 9$  <br> $\$ 10$  <br> $\$ 7$  <br> $\$ 11$  <br> $\$ 8$  <br> $\$ 9$ MAD = |


| Sunny | Distance from <br> data point to mean |
| :---: | :---: |
| $\$ 5$ |  |
| $\$ 8$ |  |
| $\$ 12$ |  |
| $\$ 6$ |  |
| $\$ 20$ |  |
| $\$ 10$ |  |
| $\$ 12$ |  |
| $\$ 4$ |  |
| $\$ 5$ |  |
| $\$ 12$ |  |
| Sum $=$ |  |

Explain your answers to problems 2-4 by referencing work on this page and page 26.
2. What do you think is the "most typical" earnings for a babysitting job? $\qquad$
3. What do you think expresses the spread of the earnings? $\qquad$
4. Who do you think makes the better earnings? $\qquad$

## SKILL BUILDER 7

1. McNealy and Nelson are a high school basketball team's top scorers. These dot plots show how many points each of them scored over the last 10 games of the season. Find the mean of this data.


2. Based on a visual inspection of the dot plots, which player appears to have the greater variability in points scored? Explain.
3. Use the tables below to help you calculate the MAD statistics for McNealy and Nelson.

| McNealy | Distance from <br> data point to mean |
| :---: | :---: |
| 12 |  |
| 16 |  |
| 16 |  |
| 18 |  |


| Nelson | Distance from <br> data point to mean |
| :---: | :---: |
| 14 |  |
| 14 |  |
|  |  |
|  |  |
| Sum $=$ |  |

## SKILL BUILDER 8

Use the basketball scoring data from page 28.

1. Compare McNealy's and Nelson's MAD values. Which has slightly less variability in data points from the mean? Explain why that might be.
2. If you were a college recruiter, which player would you want on your team? Use statistics to support your answer.
3. Create box plots for McNealy and Nelson using the scale provided.

McNealy:

Nelson:

4. About what percent of the data is included in the IQR? $\qquad$
5. What do the box plots tell us about the points scored?

## FOCUS ON VOCABULARY



2 measure of spread associated with mean (abbreviation)

Down
1 a portion of the population

3 difference between the minimum and maximum values

4 display created from the five-number summary: $\qquad$ plot

6 arithmetic average
$\left.\overline{(\min }, Q_{1}, Q_{2}, Q_{3}, \max \right)$

9 middle number in a data set when the data is listed in order
$8 \quad Q_{3}-Q_{1}$ (abbreviation)

10 display similar to a bar graph:
$\qquad$ plot

## SELECTED RESPONSE

Show your work on a separate sheet of paper and select the best answer(s).
The students in Mr. Walker's $1^{\text {st }}$ and $2^{\text {nd }}$ period classes collected data on how many text messages students sent over the weekend. They displayed their data in the graphs below. Use the graphs to answer problems 1-3.


1. Which of the following statements are true?
A. About a quarter ( $25 \%$ ) of $1^{\text {st }}$ period students sent fewer than 25 texts.
C. The median for $1^{\text {st }}$ period is 30 texts.

Choose all that apply.
B. About half $(50 \%)$ of $2^{\text {nd }}$ period students sent 15 or more texts.
D. The IQR for $1^{\text {st }}$ period is 5 texts.
2. Choose all the statistical measures that are greater for $2^{\text {nd }}$ period.
A. median
B. range
C. $I Q R$
D. maximum
E. minimum
F. $\quad Q_{1}$
3. Which of the following statements are true? Choose all that apply.
A. $\quad 1^{\text {st }}$ period has a greater MAD than $2^{\text {nd }}$ period.
B. The student who sent the most number of texts was in $2^{\text {nd }}$ period.
C. The student who sent the least number of texts was in $1^{\text {st }}$ period.
D. The mean for $2^{\text {nd }}$ period is greater than the mean for $1^{\text {st }}$ period.
4. A city conducted a survey about car ownership. Of the 200 households surveyed, 75 of them owned 2 or more cars. If the city has 24,000 households, how many of them can the city expect have 2 or more cars?
A. 9,000
B. 6,250
C. 625
D. 90

## KNOWLEDGE CHECK

Show your work on a separate sheet of paper, and write your answers on this page.

### 13.1 Introduction to Sampling

Susanna was doing research on the number of pets in the households in her town of Milford. She selected 20 names randomly from the phonebook and called them to ask, "How many pets do you own in your home?"

This was her data: $0,0,1,2,2,2,2,2,2,2,3,3,3,3,3,3,4,4,5,10$.

1. Calculate the mean of this data set. What does the mean represent in the context of this problem?
2. Calculate the five-number summary for this data set and use it to make a box plot on the number line below.


### 13.2 Comparing Samples

Rebecca conducted the same research in her town of Norton, also selecting 20 names randomly from the phonebook. She compiled her data in the box plot to the right.

3. Describe how the center and spread of the data compare for Milford and Norton.
4. Which data has a greater MAD? Explain your reasoning.
5. What does the MAD mean in the context of this problem?

### 13.3 Fish in a Lake

6. Out of 20 households, how many had 5 or more pets in Milford? In Norton?
7. Out of 15,000 households in Milford, how many would you expect to have 5 or more pets?
8. Out of 8,000 households in Norton, how many would you expect to have 5 or more pets?

## HOME-SCHOOL CONNECTION

Here are some problems to review with your young mathematician.
Ms. Stevenson's students are preparing for a math competition. The students took their first practice test and their scores are below.
$\begin{array}{llllllllllll}45 & 45 & 50 & 55 & 55 & 55 & 55 & 60 & 60 & 75 & 80 & 80\end{array}$

1. Find the mean, median, and mode for the data. Explain what each measure of center means in the context of the problem.
2. Make a dot plot of the data.

A few weeks later, they took a second practice test. Their scores are below.

| 35 | 40 | 55 | 55 | 60 | 65 | 75 | 80 | 80 | 80 | 85 | 90 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

3. Find the mean, median, and mode and make a dot plot of the data.
4. How do their scores on the second test compare to the first? Are they improving their scores? Explain.
5. Discuss which data set has a greater MAD. What does the MAD mean in the context of the problem?
$\qquad$

## COMMON CORE STATE STANDARDS - MATHEMATICS

|  | STANDARDS FOR MATHEMATICAL CONTENT |
| :---: | :---: |
| 6.SP.A* | Develop understanding of statistical variability. |
| 6.SP.3* | Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. |
| 6.SP.B* | Summarize and describe distributions. |
| 6.SP.4* | Display numerical data in plots on a number line, including dot plots, histograms, and box plots. |
| 6.SP.5c* | Summarize numerical data sets in relation to their context, such as by giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. |
| 6.SP.5d* | Summarize numerical data sets in relation to their context, such as by relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. |
| 7.RP.A | Analyze proportional relationships and use them to solve real-world and mathematical problems. |
| 7.RP. 3 | Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error. |
| 7.SP.A | Use random sampling to draw inferences about a population. |
| 7.SP. 1 | Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. |
| 7.SP. 2 | Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. |
| 7.SP.B | Draw informal comparative inferences about two populations. |
| 7.SP. 3 | Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable. |
| 7.SP. 4 | Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourthgrade science book. |

* Review of content essential for success in $7^{\text {th }}$ grade.


## STANDARDS FOR MATHEMATICAL PRACTICE

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP4 Model with mathematics.
MP5 Use appropriate tools strategically.
MP6 Attend to precision.


