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Mattinks

## MATHLINKS: GRADE 7 STUDENT PACKET 11 PROPORTIONAL REASONING

11.1 Proportional Reasoning ..... 1

- Use sense-making strategies to solve proportional reasoning problems.
- Create tables, double number lines, and tape diagrams to represent proportional relationships.
- Understand the cross-multiplication property for solving
proportions.
- Solve problems using proportions.

| 11.2 | Rate Problems | 10 |
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|  | $\bullet$ Solve problems involving work rates. |  |
|  | $\bullet$ Solve problems involving distance, rate of speed, and time. |  |

11.3 Best Buy Problems18

- Use various methods to determine the best buy, including tables and graphs.
- Write equations that represent relationships between the quantity and cost of a purchase.
- Determine if quantities are directly proportional.
11.4 Skill Builders, Vocabulary, and Review ..... 26


## WORD BANK

| Word or Phrase | Definition or Description | Example or Picture |
| :--- | :--- | :--- |
| complex fraction |  |  |
|  |  |  |
| cross-multiplication |  |  |
| property |  |  |

## PROPORTIONAL REASONING

## Summary

We will use different strategies to solve proportional reasoning problems.

## Goals

- Use sense-making strategies to solve proportional reasoning problems.
- Create tables, double number lines, and tape diagrams to represent proportional relationships.
- Understand the cross-multiplication method for solving proportions.
- Solve problems using proportions.

Warmup
Compute.

| 1. $\$ 2.56+\$ 1.87+\$ 3.29$ | 2. | $4 \bullet(\$ 1.57)$ |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
| 3. $\$ 8.23-\$ 4.68$ | 4. | $\frac{\$ 8.61}{7}$ |

5. Using your knowledge of equivalent fractions, solve for $x: \frac{2}{9}=\frac{6}{x}$

## PENCIL PROBLEMS

Solve each pencil problem in two ways. Show your work, and state your answers clearly. Your teacher will give you the missing pieces of information one by one.

| 1. If 1 pencil costs $\$ \ldots$, what is the cost of 4 pencils? |
| :--- |
| Answer: |
| 2. If 5 pencils cost $\$ \ldots$, what is the cost of 1 pencil? |
| Answer: |

3. If 3 pencils cost $\$$ $\qquad$ , what is the cost of 6 pencils?

Answer:

## PENCIL PROBLEMS (Continued)

Solve each pencil problem in two ways. Show your work, and state your answers clearly. Your teacher will give you the missing pieces of information one by one.

5. How much will ___ pencils cost if 6 pencils cost $\$ 1.68$ ?

Answer:
6. How much will $\qquad$ pencils cost if 7 pencils cost $\$ 1.82$ ?

Answer:

## PUNCH PROBLEMS

Solve each pencil problem in two ways. Show your work, and state your answers clearly. Your teacher will give you the missing pieces of information one by one.

1. A recipe for punch calls for 1 part grape juice and 3 parts water. How much water is needed for $\qquad$ cups of juice?

Answer:
2. A recipe for punch calls for 2 parts grape juice and 5 parts water. Anna has $\qquad$ cups of grape juice. How much water should she add to make this mixture?

Answer:
3. Antonio has 10 cups of punch, which is 3 parts grape juice and 2 parts water. He decides that it is too "grapey." He wants to make $\qquad$ cups of punch. How much grape juice and water should he add to the original punch so that there is one part grape juice for every one part water?

Answer:

## PAPER PROBLEMS AND MORE

Solve each pencil problem in two ways. Show your work, and state your answers clearly. Your teacher will give you the missing pieces of information one by one.

| 1. How much will 50 copies cost if it costs $\$ 4.50$ to make 100 copies? | 2. If the cost of making 20 copies is $\$ 1.30$, how much will 1 copy cost? |
| :---: | :---: |
| 3. What will it cost to make 1,000 copies if the cost of 1 copy is $\$ 0.041$ ? | 4. If the cost of 100 copies is $\$ 3.10$, how many copies can be made for $\$ 155$ ? |
| 5. A juice recipe calls for 5 parts pear juice for every 2 parts apple juice. How much apple juice is needed to make a mixture with 10 cups of pear juice? | 6. In a fruit basket, the ratio of apples to oranges is 2 to 3 . If there are 15 total pieces of fruit, how many are apples? |

## EXPLORING PROPORTIONS

A proportion is an equation stating that the values of two ratios are equal.

1. Dana looked at the proportion $\frac{2}{3}=\frac{4}{6}$ and said, "I can invert both fractions and still have a true equation." She then wrote $\frac{3}{2}=\frac{6}{4}$
a. Prove that Dana is correct for this
example.
b. Write another pair of proportions that are true the way Dana sees it.
2. Lana looked at the proportion $\frac{3}{4}=\frac{9}{12}$, and said, "I can compare numerators and denominators and still have a true equation.

She then wrote $\frac{3}{9}=\frac{4}{12}$.
a. Prove that Lana is correct for this example.
b. Write another pair of proportions that are true the way Lana sees it.
3. Sara looked at the proportion $\frac{2}{5}=\frac{10}{25}$ and said, "I can multiply diagonals and still have a true equation." She then wrote $2 \cdot 25=10 \bullet 5$.
a. Prove that Sara is correct for this example.
b. Write another proportion and equation that are true the way Sara sees it.

## GIVE THE REASONS

1. Write what was done for each step (a)-(f)


Look carefully at (a) $\frac{x}{3}=\frac{3}{8}$ and (d) $8 x=9$. Going from (a) to (d) in one step is a process we will call the cross multiplication property.
2. Write the equation $-\frac{7}{3}=\frac{5}{x}$ using the cross-multiplication property: $\qquad$ $=$ $\qquad$ .

Use the cross-multiplication property to solve each equation. Round appropriately as needed.

| 3. $-\frac{4}{9}=\frac{5}{x}$ | $4 . \quad \frac{x}{6}=\frac{-8}{5}$ | $5 . \quad \frac{3}{7}=-\frac{x}{5}$ |
| :--- | :--- | :--- |

## PRACTICE

Solve each equation using a mental strategy (MM), paper and pencil (P), or a calculator (C). Show your work, and identify your strategy.

| 1. $\frac{2}{5}=\frac{x}{21}$ | 2. $\frac{-3}{5}=\frac{x}{55}$ | 3. $\frac{137}{5}=\frac{x}{55}$ |  |
| :--- | :--- | :--- | :--- |
| 4. $\frac{2}{x}=\frac{3}{13}$ | 5. | $-\frac{1}{2}=-\frac{5}{x}$ | 6. $\frac{2.5}{5}=\frac{x}{12}$ |

7. Some students were exploring the equation $\frac{3}{5}=\frac{6}{10}$, and rewrote it in a few different ways. Circle the two true equations. For the equation that is not true, explain to that student why it is not true and how to revise his work.
Abner: $\frac{3}{6}=\frac{5}{10}$
Mick: $\frac{6}{3}=\frac{5}{10}$
Buck: $\frac{5}{3}=\frac{10}{6}$
8. Rewrite the equation $\frac{2}{7}=\frac{6}{21}$ in three other ways to create true equations.


## POSTER PROBLEMS 4

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D. I am group member $\qquad$ .
- Each group will start at a numbered poster. Our group start poster is $\qquad$ .
- Each group will have a different color marker. Our group marker is $\qquad$ .

Part 2: Answer the problems on posters by following the directions of your teacher.
Part 3: Return to your seats.

Refer to the poster problems. Discuss and answer each question using representations on the posters and common sense.

1. School policy dictates that there must be 3 adults for every 25 students that go on a field trip. The $7^{\text {th }}$ grade class has 260 students going on the field trip. How many adults must go?
2. Zippy can read 3 books every 5 months. At that rate, how many books can he read in a year?
3. Everett exercises on average 5 days per week. At that rate, how many days of exercise will he have in May?
4. Domingo eats a lot of apples and oranges. He decides to keep track of this, and finds that for every 8 apples he eats, he also eats about 5 oranges. Over the course of several weeks he sees that he ate 42 pieces of these fruits. About how many were apples?

## RATE PROBLEMS

## Summary

We will solve a variety of rate problems.

## Goals

- Solve problems involving work rates.
- Solve problems involving distance, rate of speed, and time.

1. Joanne drew the double number line diagram to the right to reflect the amounts she might pay for copies at Copy Carousel. Write a proportion for the problems below and solve for the quantities below using the cross-multiplication property.

b. the number of copies for $\$ 1.75$

Solve each equation using a mental strategy (MM), paper and pencil (P), or a calculator (C). Show your work as needed and identify your strategy.

| 2. $-\frac{4}{5}=\frac{n}{35}$ | $3 . \quad \frac{n}{9}=\frac{16}{22.5}$ | 4. $\frac{7}{n}=\frac{1 \frac{3}{4}}{2 \frac{1}{4}}$ |
| :--- | :--- | :--- |

## PAINTING PROBLEMS

1. How many feet are in 1 yard?
2. How many square feet are in 1 square yard? $\qquad$


Solve using any method and show your work. Assume a constant rate of painting for all problems. Do not include other factors such as rest breaks.
3. It took Ping 20 minutes to paint a fence that is 3 feet tall and 10 feet long.
a. What was Ping's unit rate in square feet per minute ( $\mathrm{ft}^{2} / \mathrm{min}$ )?
b. At that rate, how long will it take him to paint a fence that is twice as tall and twice as long?
4. Phong paints 1 square yard in 2 minutes. At that rate, how long will it take her to paint a fence that is 3 feet tall and 30 feet long?
5. Tang paints 1 square foot in 1 minute 30 seconds. At that rate, how many minutes will it take her to paint a fence that is 1 yard by 10 yards?

## PAINTING PROBLEMS (Continued)

The work required to paint a fence can be measured in "painter-hours."
number of painter-hours needed to paint the fence
number of painter-hours needed to paint the fence
number of hours it takes one painter to paint the fence
number of painters it takes to paint the fence in one hour
a fence, then it takes 18 painter-hours to paint the fence.

- Suppose painting a fence requires 40 painter-hours of labor.
- Suppose a painter charges $\$ 24$ per hour (i.e. the cost is $\$ 24$ per painter-hour).

6. How many hours will it take 2 painters working together to paint the fence?
7. How many hours will it take 5 painters working together to paint the fence?
8. If a fence must be painted in 4 hours, how many painters should be hired to work together to paint it?
9. How much will it cost to paint the fence?
10. If a contractor hires 4 painters to work together paint the fence, how much will it cost the contractor per hour to have the fence painted?

## WATER SPORTS

Ed likes to surf. It takes him 10 minutes to paddle from the beach $\frac{1}{8}$ of a mile out into the ocean to catch a wave. He rides the wave back to the beach in half a minute.

1. Write rates that represent Ed paddling out into the ocean.

2. Write rates that represent Ed riding the wave back to the beach.


Jody likes to water ski. Jody skis for $\frac{1}{2}$ of a mile at a speed of 30 miles per hour.
3. What is her rate of speed? $\qquad$
4. How many hours does it take Jody to travel $\frac{1}{2}$ mile at this rate? Use the formula:

$$
\text { Distance }=\text { Rate } \times \text { Time }(D=R T)
$$

5. How many minutes does it take Jody to travel $\frac{1}{2}$ mile at this rate?
6. If Jody water skis for 5 minutes, how far does she travel?

## WATER SPORTS (Continued)

| Distance | $=$ Rate $\times$ Time |
| ---: | :--- |
| $D$ | $=R \bullet T$ |

Many sports are timed for specific distances. Find the missing information in the table below.

| Sport |  |  | Time |  | Distance | Show your work |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Minutes | Hours |  |  |
|  | canoeing |  | 4 mph |  | $\frac{1}{2}$ | ___miles | $\begin{aligned} & D=R \cdot T \\ & D=(4) \cdot\left(\frac{1}{2}\right) \\ & D=\quad \text { miles } \end{aligned}$ |
|  | swimming | 1 mph |  |  | $\frac{1}{4} \text { mile }$ |  |
|  | rowing |  | 90 min |  | 7.5 miles |  |
| 10. | jet skiing | 50 mph |  |  | 20 miles |  |
| 11. | water skiing |  |  | $1 \frac{1}{3}$ | 48 miles |  |

12. Consider the distances, rates, and times above for each activity, and list any that you think may be unreasonable.

## PRACTICE

## Solve each problem.

COMMUNITY GARDEN: Student volunteers from a local high school are turning a vacant lot into a community garden. A community beautification planner estimates the time it will take 1 person to complete each of the following tasks:

- 8 hours to prepare the soil
- 40 hours to plant the flowers
- 18 hours to build a fence
- 14 hours to paint the fence

1. How many hours will it take for 2 people to prepare the soil together?
2. How many hours will it take for 4 people to plant the flowers together?
3. If 5 people are going to work together to plant the flowers, and they work 4 hours per day, how many days will be needed to complete the job?
4. Four people are going to work together to build and paint the fence. If they want to complete the job in two days, and to work the same number of hours on the first day as the second day, how many hours does each person need to work each day?

PAINTING YOUR BEDROOM: You want to paint your bedroom with your favorite shade of purple. Making this shade requires $\frac{1}{2}$ cup blue paint for every $\frac{1}{3}$ cup red paint.
5. If you want to mix blue and red paint in the same ratio to make 20 cups of your favorite purple paint, how many cups of blue paint and how many cups of red paint will you need?

## PRACTICE (Continued)

Solve each problem.
COUNTING CARS: While waiting for the bus, you notice that 1 truck drives by for every 10 cars.
6. At this rate, about how many trucks would you see if 56 cars drove by?
7. If you saw 13 trucks drive by, about how many total vehicles drove by during that time?

PRINTERS: A school has four printers that print pages at different rates. Determine the number of pages per minute for each:
8. The printer in the main office prints $2 \frac{1}{2}$ pages per second.
9. The printer in the attendance office prints 50 pages per $\frac{1}{2}$ minute.
11. The printer in the faculty lounge prints 1 page every 2 seconds.
12. Which printer prints the fastest?

## QUIRKY WORLD RECORDS

Solve each problem and show your work.

1. The mile record for one person carrying another on their shoulders was set at a rate of four miles per hour. If the race course was one mile long, how many hours did it take them to complete the race?
2. While piggyback racing, a man completed a $1 \frac{1}{2}$ mile race in $\frac{1}{3}$ of an hour. What was his rate in miles per hour?
3. The record for the fastest sack race was set at a rate of 3 miles per hour. If the sack racer finished the race in 20 minutes, what was the length of the race course?

## BEST BUY PROBLEMS

## Summary

We will use tables, graphs, and symbolic representations to determine whether a relationship is proportional.

## Goals

- Use various methods to determine the best buy, including tables and graphs.
- Write equations that represent relationships between the quantity and cost of a purchase.
- Determine if quantities are directly proportional.


## Warmup

Circle the better buy for each situation below and explain your reasoning. No calculations are necessary.

1. 0.75 pounds of oranges for $\$ 1.00$
or
1.25 pounds of oranges for $\$ 1.00$
2. 3 pounds of bananas for $\$ 3.65$
or
3 pounds of bananas for $\$ 4.15$

Suppose you are running out of your favorite energy snacks, so you compare prices at two stores before making a purchase.

3. Without doing any calculations, explain which store offers the better buy for Healthy Crunch.

## QUIGLEY'S

Healthy Crunch: 2 for $\$ 2.75$
Super Bar: 3 for $\$ 3.25$
4. Without doing any calculations, explain which store offers the better buy for Super Bar.

## BAGELS

SHMEAR ' $N$ THINGS
4 bagels for $\$ 3.00$

$$
\frac{\text { HOLE-Y BREAD }}{5 \text { bagels for } \$ 4.00}
$$

1. Complete the tables below. Assume each shop will sell you any number of bagels at the rates shown above.

| SHMEAR ' THINGS |  |
| :---: | :---: |
| \# of bagels $(x)$ | cost in dollars $(y)$ |
| 4 |  |
| 8 |  |
| 12 |  |
| 16 |  |
| 20 |  |


| HOLE-Y BREAD |  |
| :---: | :---: |
| \# of bagels $(x)$ | cost in dollars $(y)$ |
| 5 |  |
| 10 |  |
| 15 |  |
| 20 |  |
| 25 |  |

Two variables (quantities that vary) are proportional if one is a constant multiple of the other. One method of testing for a proportional relationship is to write and compare ratios. If the values of the ratios are all equal, then the quantities are proportional.
2. Write the values of some ratios for each shop in the table below using the data tables above.

3. Which shop has the better buy?

Circle entries in your table to justify your answer and explain in words.

## TESTING FOR PROPORTIONAL RELATIONSHIPS

Each table below shows a relationship between quantities that vary. Write, simplify, and compare the values of the ratios (unit rates). Then state whether the pairs in each table represent a proportional relationship between the quantities.

1. The number of meals Venya served to the homeless and their cost.

|  | Monday | Tuesday | Wednesday | Thursday | Friday |
| :--- | :---: | :---: | :---: | :---: | :---: |
| \# of meals | 45 | 60 | 20 | 100 | 55 |
| cost | $\$ 135$ | $\$ 180$ | $\$ 60$ | $\$ 300$ | $\$ 165$ |
| unit rate |  |  |  |  |  |
| Proportional? |  |  |  |  |  |

2. The number of bags of feathers Jaime used to make pillows.

| \# of bags | 9 | 24 | 3 | 18 | 4.5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| \# of pillows | 6 | 16 | 2 | 12 | 3 |
| unit rate |  |  |  |  |  |
| Proportional? |  |  |  |  |  |

3. The number of tables LaTonya rented for a party and their cost.

| \# of tables | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| cost | $\$ 20$ | $\$ 25$ | $\$ 30$ | $\$ 35$ | $\$ 40$ |
| unit rate |  |  |  |  |  |
| Proportional? |  |  |  |  |  |

4. The size of each room in Satoshi's house and the number of plants in each room.

|  | bedroom | living room | den | kitchen | patio |
| :--- | :---: | :---: | :---: | :---: | :---: |
| square feet | 100 | 200 | 150 | 125 | 250 |
| \# of plants | 2 | 5 | 6 | 5 | 10 |
| unit rate |  |  |  |  |  |
| Proportional? |  |  |  |  |  |

## TESTING FOR PROPORTIONAL RELATIONSHIPS (Continued)

Graph the data from each of the tables page 20 . Be sure to label and scale the axes appropriately.
5. The number of meals Venya served to the homeless and their cost.

|  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

7. The number of tables LaTonya rented for a party and their cost.

8. The number of bags of feathers Jaime used to make pillows

$(0,0)$
9. The size of each room in Satoshi's house and the number of plants in each room.
$(0,0)$

|  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
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9. What do you notice about graphs that represent proportional relationships, compared to those that do not?

## TORTILLAS

## FLAT 'N ROUND

3 tortillas for $\$ 0.60$

```
WRAP IT UP
4 tortillas for $0.76
```

1. Complete the tables below. Assume each shop will sell any number of tortillas at the rates shown.

| FLA | 'N | WRAP IT UP |  |
| :---: | :---: | :---: | :---: |
| \# of tortillas <br> (X) | cost <br> (y) | \# of tortillas (x) | cost <br> (y) |
| 3 |  | 4 |  |
| 6 |  | 8 |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

2. Find the unit price at each tortilla store.
3. Write equations to relate the number of tortillas to the cost.

FLAT 'N ROUND $\quad y=$ $\qquad$
WRAP IT UP
$y=$ $\qquad$
$\qquad$
4. Label and scale the grid. Graph the data using two different colors.

5. Identify the $y$-coordinate when $x=1$.

FLAT 'N ROUND ( $1, \ldots$ )
WRAP IT UP (1, __ )
6. Explain the meaning of these coordinate pairs in this context.

The equations above are in the form $y=m x$ and represent proportional relationships. For each respective equation, $y$ is proportional to $x$. The number $m$ is called the constant of proportionality.
7. How are the coordinates for the ordered pairs in problem 5 related to the equation in the form $y=m x$ ?

## PITA BREAD

$$
6 \frac{\text { PAPA'S PITA }}{6 \text { pitas for } \$}
$$

> EAT-A PITA
> 5 pitas for \$

1. The graph for EAT-A PITA is provided below. How do you know by looking at the graph for EAT-A PITA that it depicts a proportional relationship?
2. Complete the tables below and graph for PAPA'S PITA. A partial table for PAPA'S PITA is provided. Use tables and graphs to complete the pricing information above. Assume each shop will sell any number of pitas at the rates shown.

| PAPA'S PITA |  | EAT-A PITA |  |
| :---: | :---: | :---: | :---: | :---: |
| \# of <br> tortillas <br> $(x)$ | cost <br> $(y)$ | \# of <br> tortillas <br> $(x)$ | cost <br> $(y)$ |
| 3 |  |  |  |
| 6 |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

3. Find the unit prices at both pita stores.
4. Write equations to relate the number of pitas to the cost.

PAPA'S PITA $\quad y=$ $\qquad$
EAT-A PITA
$y=$ $\qquad$
5. Verify that the relationship between the number of pitas and the price paid at EAT-A PITA is proportional by creating ratios and checking for equivalent fractions.
6. Does the graph for PAPA'S PITA go through the origin? What does that mean in this context?
7. Which graph illustrates a slower rise in price? How do you know?

## PIZZA

DOOR-TO-DOOR PIZZA \$ $\qquad$ per pizza
$\$ 5$ for delivery

1. Complete the tables and graphs. The graph for PIZZA PALACE is provided. Use tables and graphs to complete the pricing information above.

2. Using data from the table, explain/show which relationship is proportional.
3. Do both graphs represent proportional relationships? Why or why not?
4. Using the data from the tables, determine which shop offers the better buy. Explain your reasoning.
5. Identify the coordinates of the point where the two graphs intersect.
6. What does this point mean within the context of the problem?

## PROTEIN DRINKS

Suppose you are shopping for your favorite protein drink. You find there are two different size drinks available.


1. Which drink is the better buy? Justify your answer using multiple representations (tables of numbers, equations, graphs, and words).

2. Find the points $(1, y)$ on each graph above. Explain what each of these points means in the context of the problems.

| Regular: $(1, \ldots)$ | Super: $(1, \ldots, \ldots)$ |
| :--- | :--- |

3. After doing the mathematics in this problem, is there anything else you would take into consideration in choosing one of the protein drink sizes? Explain.

## SKILL BUILDERS, VOCABULARY, AND REVIEW

## SKILL BUILDER 1

Fill in the boxes below so that each row and column has a sum of 2.
1

| -2 | 5 |  |
| :--- | :--- | :--- |
| -3 | 4 | 1 |
|  | -7 |  |

2. 

|  | 8 | 2 |
| :---: | :---: | :---: |
| 6 | -9 |  |
|  |  | -5 |

3. Grace said that the value of $2 \%$ as a decimal is 0.2 . Victoria disagreed and said the value of $2 \%$ as a decimal is 0.02 . Who is correct? Explain.
4. Circle all of the following that are equal to
$\frac{4}{-2}$
$\frac{-8}{4}$
$\frac{-4}{-2}$
$\frac{-4}{2}$
$\frac{16}{-8}$

Compute.

| 5. | $(-3.4)(2.1)+15.8$ | 6. | $5+(-8)-\frac{1}{16}$ | 7. |
| :--- | :--- | :--- | :--- | :--- |

8. James wants his birthday party to be at Jump High Trampolines. It costs $\$ 120$ to host a party plus an extra $\$ 23$ per guest. James has no more than $\$ 373$ to spend on his party. Write an inequality that represents this scenario. How many guests will be able to attend the birthday party?

## SKILL BUILDER 2


7. Marcie was paid $\$ 41$ to babysit two children. She was paid a $\$ 5$ flat fee plus $\$ 8$ every hour she babysat. Write and solve an equation that expresses how many hours Marcie babysat.

Fill in the below table with equivalent percents, decimals, and fractions in each row.

|  | Percent Form | Decimal Form | Fraction Form |
| :--- | :---: | :---: | :---: |
| 8. | $25 \%$ |  |  |
| 9. |  |  |  |
| 10. |  |  | $\frac{6}{5}$ |
| 11. | $1.8 \%$ |  |  |
| 12. |  |  |  |

## SKILL BUILDER 3

1. The Midtown Basketball League has 11 players and 2 coaches for every team. Make a table of values that could represent the number of players and coaches participating in the League.
2. Explain why the numbers in the rows in this table cannot be used to represent equivalent ratios of $6^{\text {th }}$ graders to $7^{\text {th }}$ graders.

|  | $\mathbf{6}^{\text {th }}$ graders <br> in class | $\mathbf{7}^{\text {th }}$ graders <br> in class |
| :---: | :---: | :---: |
| Period 1 | 8 | 16 |
| Period 2 | 9 | 20 |
| Period 5 | 7 | 22 |
| Period 6 | 10 | 18 |

Complete the table. For cases in which the $x$ is on the right side of the number sentence, also write an equivalent number sentence with $x$ on the left.

|  | Words | Symbols | Graph the solution | Test a number |
| :---: | :---: | :---: | :---: | :---: |
| 3. | -2 is greater <br> than $x$ |  | $\longleftrightarrow$ |  |
| 4. | $x$ is an integer <br> less than or <br> equal to 5 |  | $\longleftrightarrow$ |  |
| 5. | 5 is less than <br> or equal to $x$ |  |  |  |

6. Steve owes $\$ 35$ for his meal at an Italian restaurant. He wants to leave a $20 \%$ tip for the waitress. How much money should he leave in all?

## SKILL BUILDER 4

## Solve and graph the inequalities.


4. Jane thinks the sum of -8 and $\frac{1}{3}$ is $-8 \frac{1}{3}$. Is Jane correct? Explain.

Write $<,=$, or $>$ to make the statement true.

| 5. | $\frac{-3(8-10)}{2}-19-(-12)(2)$ |
| :--- | :--- |
|  | $6 . \quad 3 \frac{1}{2}+\left(-2 \frac{3}{4}\right) \ldots(-1.23)(-2.8)$ |
| $7 . \frac{3}{8}-\frac{6}{5}-\left(-\frac{4}{9}\right)$ | $8 . \quad 4\left(22-5^{2}\right) \longrightarrow 12-2(3-9)$ |

## SKILL BUILDER 5

Solve each problem. Show your work. State your answers clearly.

1. How much will 20 pencils cost if it costs $\$ 3.50$ for 100 pencils?
2. If the cost of 20 stamps is $\$ 9.80$, how much will 1 stamp cost?
3. Rewrite the equation $\frac{4}{13}=\frac{8}{26}$ in three other ways to create true equations.

4. Which of the following could be equivalent to $-\frac{3}{4}=\frac{x}{6}$ using the cross-multiplication property?
$-18=4 x$
$-18=-4 x$
$18=4 x$
$-4 x=18$
$4 x=-18$
5. If Rachel can read 10 pages in 12 minutes, how many minutes would it take her to read 8 pages? Explain.

## SKILL BUILDER 6

Solve each equation. Round appropriately as needed.


## SKILL BUILDER 7

BASEBALL FIELD: A city's mayor has decided to build a baseball field in the middle of the city. A city planner estimates the time it will take 1 person to complete each of the following tasks:

- 3 hours to install the bases
- 20 hours to plant grass in the outfield
- 24 hours to put in the infield material
- 14 hours to put up the fence

1. How many hours will it take for 2 people to plant the grass together?
2. How many hours will it take for 6 people to put up the fence together?
3. If 4 people are going to work together to plant the grass, and they work 5 hours per day, how many days are needed to complete the job?
4. Five people work together to put in the infield material. They do the same amount of work each day, and they complete the job in two days. How many hours does each person work each day?

PAINTING YOUR BEDROOM: You want to paint your bedroom with your favorite shade of green. Making this shade requires $\frac{1}{3}$ cup blue paint for every $\frac{1}{4}$ cup yellow paint.
5. If you want to mix blue and yellow paint in the same ratio to make 35 cups of your favorite green paint, how many cups of blue paint and how many cups of yellow paint will you need?

## SKILL BUILDER 8

Suppose you are shopping for your favorite frozen yogurt. You find there are two different size packages available.


1. Which package is the better buy? Justify your answer using the fourfold way (tables of numbers, equations, graphs, and words).

2. Find the points $(1, y)$ on each graph. Explain what each of these points mean in the context of the problem.
Regular: (1, ___ )
Super: (1, $\qquad$ )

## FOCUS ON VOCABULARY



## Across

1 The $\qquad$ multiplication property states that if $\frac{a}{b}=\frac{c}{d}$, then $a d=b c(b \neq 0, d \neq 0)$.

## Down

1 A fraction with fraction(s) in numerator and/or denominator.

2 A $\qquad$ number line diagram is a tool that can be used for solving ratio problems.

6 equation for a proportional relationship
3 If pumpkins cost $\$ 5$ for 2 , then 2.5 is the $\qquad$ _.

7 value of a ratio
5 A $\qquad$ diagram visually shows relative sizes of quantities.

8 An ordered pair of numbers (written $3: 2)$.

## SELECTED RESPONSE

Show your work on a separate sheet of paper and select the best answer(s).

1. If 12 pencils cost $\$ 3$, which of the following statements are true? Choose all that apply.
A. 1 pencil costs $\$ 4$
B. 1 pencil costs $\$ 0.25$
C. 24 pencils cost $\$ 6$
D. 6 pencils cost $\$ 1$
2. Choose all the equations below that are true.
A. $\frac{3}{4}=\frac{6}{8}$
B. $\frac{4}{3}=\frac{8}{6}$
C. $\frac{8}{3}=\frac{4}{6}$
D. $\frac{8}{3}=\frac{6}{4}$
3. If $\frac{x}{5}=\frac{10}{20}$ is true, choose the statement below that is NOT true.
A. $20 x=50$
B. $10 x=100$
C. $x=2.5$
D. $\frac{20}{10}=\frac{5}{x}$
4. Marie went on a 10 mile hike. She averaged 2.5 miles per hour. How long did it take her to complete her hike?
A. 4 minutes
B. 5 hours
C. 4 hours
D. 25 hours
5. Twelve students are painting 600 square feet of fencing together. If it takes 2 hours for each student to paint 10 square feet of fencing, how long will it take them to complete the whole fence?
A. 10 hours
B. 5 hours
C. 20 hours
D. 30 hours
6. Which of the following is the best buy for rope?
A. 10 feet for $\$ 4$
B. $\$ 2$ for 5 feet
C. $\$ 1.50$ for 1 foot
D. $\$ 100$ for 30 feet
7. Marty is mixing blue and yellow paint to make a green color. He knows he needs to mix 1 part blue for every 5 parts yellow. If he only has $\frac{1}{2}$ of a container of blue paint, how much yellow paint does he need?
A. 2.5 containers
B. 10 containers
C. 2 containers
D. $\frac{1}{2}$ of a container

## KNOWLEDGE CHECK

Show your work on a separate sheet of paper and write your answers on this page.

### 11.1 Proportional Reasoning

1. If 10 pieces of gum costs $\$ 1.20$, how much do 45 pieces cost?

How much does each piece cost?
2. Rewrite $\frac{3}{5}=\frac{9}{15}$ in three different ways.

Solve each equation using any method.
3. $\frac{2}{5}=\frac{8}{x}$
4. $\frac{-7}{6}=\frac{x}{36}$
5. $\frac{x}{8}=\frac{45}{45}$
11.2 Rate Problems
6. Felix is reading about 3 pages every 2 minutes.

How long will it take him to read 21 pages at this rate?
How many pages will he have read in 1 hour at this rate?
7. Geena can read twice as fast as Felix.

How long will it take her to read 21 pages at this rate?
How many pages will she read in an hour at this rate?

### 11.3 Best Buy Problems

8. Carmella compared the prices at 3 different coffee shops. Determine if their pricing is proportional to size or not, and explain your reasoning.

| Coffee Shop A |  |
| :---: | :---: |
| Size <br> (in ounces) | Price <br> (in dollars) |
| 8 | $\$ 1.50$ |
| 12 | $\$ 3.00$ |
| 24 | $\$ 4.50$ |


| Coffee Shop B |  |
| :---: | :---: |
| Size <br> (in ounces) | Price <br> (in dollars) |
| 8 | $\$ 1.00$ |
| 12 | $\$ 1.50$ |
| 24 | $\$ 3.00$ |


| Coffee Shop C |  |
| :---: | :---: |
| Size <br> (in ounces) | Price <br> (in dollars) |
| 8 | $\$ 1.00$ |
| 12 | $\$ 2.00$ |
| 24 | $\$ 4.00$ |

## HOME-SCHOOL CONNECTION

Here are some problems to review with your young mathematician.

1. If it costs $\$ 4.50$ to make one dozen cupcakes, how much will it cost to make 5 dozen? How much will it cost to make a half-dozen?
2. It takes you $\frac{3}{4}$ of an hour to mow $\frac{1}{3}$ of your lawn. How long will it take you to mow your whole lawn at this rate?

How much of your lawn could you mow in an hour at this rate?
3. Which of the following stores offers the best buy?


MUFFIN MAYHEM
4 Muffins for $\$ 2.50$

MUFFIN UP!
5 Muffins for $\$ 3.00$
$\qquad$

## COMMON CORE STATE STANDARDS - MATHEMATICS

|  | STANDARDS FOR MATHEMATICAL CONTENT |
| :---: | :---: |
| 6.RP.A | Understand ratio concepts and use ratio reasoning to solve problems. |
| 6.RP.3a* | Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations: Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. |
| 6.RP.3b* | Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations: Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? |
| 7.RP.A | Analyze proportional relationships and use them to solve real-world and mathematical problems. |
| 7.RP. 1 | Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. For example, if a person walks $1 / 2$ mile in each $1 / 4$ hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour. |
| 7.RP.2a | Recognize and represent proportional relationships between quantities. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. |
| 7.RP.2b | Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. |
| 7.RP.2c | Represent proportional relationships by equations. For example, if total cost $t$ is proportional to the number $n$ of items purchased at a constant price $p$, the relationship between the total cost and the number of items can be expressed as $t=p n$. |
| 7.RP.2d | Explain what a point $(x, y)$ on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0,0)$ and $(1, r)$ where $r$ is the unit rate. |
| 7.NS.A | Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. |
| 7.NS. 3 | Solve real-world and mathematical problems involving the four operations with rational numbers. |
| 7.EE.B | Solve real-life and mathematical problems using numerical and algebraic expressions and equations. |
| 7.EE. 3 | Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $\$ 25$ an hour gets a $10 \%$ raise, she will make an additional 1/10 of her salary an hour, or $\$ 2.50$, for a new salary of $\$ 27.50$. If you want to place a towel bar $93 / 4$ inches long in the center of a door that is $271 / 2$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation. |

*Review of content essential for success in $7^{\text {th }}$ grade.

## STANDARDS FOR MATHEMATICAL PRACTICE

MP1 Make sense of problems and persevere in solving them.
MP3 Construct viable arguments and critique the reasoning of others.
MP4 Model with mathematics.
MP5 Use appropriate tools strategically.

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