REIMAGINE ALGEBRA PROBLEM-SOLVING WITH DIFFERENT LEARNERS

Presented by:

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THE TODOS EQUITY STRAND

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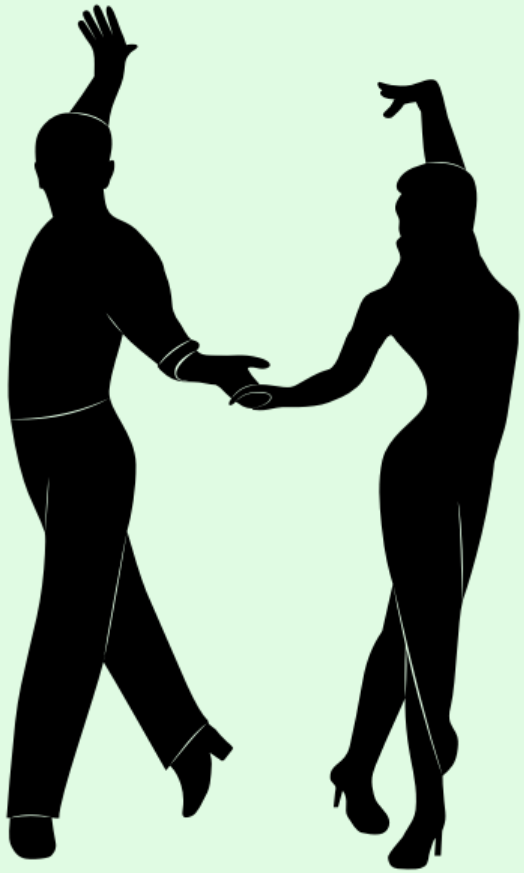

All TODOS members are invited to attend the **Salsa Party on Thursday, October 26th from 6:30pm - 8:30pm** at the Marriott Marquis Salons 9 and 10.

New Members are welcome. Wear your dancing shoes!

You won't want to miss it.

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WHAT IS PROBLEM SOLVING?

A four-step method to solve all kinds of problems:

- Understand the problem
- Make a plan
- Execute the plan
- Look back and reflect.

George Polya (1945)
How to Solve it!

WHAT IS PROBLEM SOLVING?

A problem is "any task or activity for which the students have no prescribed or memorized rules or methods, nor is there a perception by students that there is a specific 'correct' solution method."

Hiebert, et. al., (1997)

WHAT IS PROBLEM SOLVING?

The term "problem solving" refers to mathematical tasks that have the potential to provide intellectual challenges for enhancing students' mathematical understanding and development.

There is little or no evidence that students' problem-solving abilities are improved by isolating problem solving from learning mathematics concepts and procedures.

NCTM Research Brief (2021)

WHAT IS A WORTHWHILE PROBLEM?

- has important, useful mathematics embedded in it.
- requires higher-level thinking and problem solving.
- contributes to the conceptual development.
- creates an opportunity for formative or summative assessment
- can be approached in multiple ways using different solution strategies.

Lappan and Phillips (1998)
Connected Math

WHAT IS A WORTHWHILE PROBLEM?

- gives students opportunities to defend their solutions or strategies.
- encourages student engagement and discourse.
- connects to other important mathematical ideas.
- promotes the skillful use of mathematics.
- provides an opportunity to practice important skills.

Lappan and Phillips (1998)
Connected Math

CROSSING THE LAKE (Version 1)

Is this a problem? Why or why not?

2 children and 6 adults need to get across a lake in a canoe. Everyone can row. The canoe holds one child alone or two children together or one adult alone. How many one-way trips are needed to get everyone across the lake?

What challenges might students face when presented with this problem?

Let's make this accessible to more students.

CROSSING THE LAKE (Version 2)

Some adults and children need to cross a lake on their hike.

They have a small canoe that can't hold everyone.

How many one-way trips are needed to get everyone across the lake?

What do we know?

What do you wonder?

SOME DETAILS

One canoe can hold:

1 child alone

OR

2 children together

OR

1 adult alone

(1) Summarize the facts of the problem.

✧ Everyone can paddle the canoe.

✧ The only way to get across the lake is to use the canoe.

ATTACKING THE PROBLEM

What strategies could we use to solve this problem?

A last bit of important information:

6 adults and 2 children need to get across the lake.

Now we can start to solve the problem.
Reminder:

We are looking for the

least number of trips to accomplish this.

(2) How many one-way trips are needed to get everyone across the lake?

ONE STRATEGY

	One-way trips
6 adults remain since 2 children crossed	→ CC
6 adults remain and 1 child comes back with the canoe	C ←
5 adults and 1 child remain	→ A
5 adults and 2 children remain since the second child comes back with the canoe	C ←

EXTENDING THE PROBLEM

What patterns do you notice?

Be sure your answer includes a numerical expression for the number of one-way trips it takes for 6 adults and 2 children to cross the lake.

How many trips would it take for:
10 adults and 2 children?
A adults and 2 children?

STRATEGIES FOR DIVERSE LEARNERS

1) Know your learner.

2) Increase academic language through mathematics.

3) Increase comprehensible input.

4) Promote student interaction.

Goldstein and Kriegler (2022)

CROSSING THE LAKE (Wrap-up)

(Based on your strategy category)
How was Version 2 more accessible than Version 1?

Group debrief:

BUYING A SKATEBOARD

Naomi and Karolina are saving for a skateboard. Naomi has $100 in the bank and will save $30 each month. Karolina has $40 in the bank and will save $45 each month. During what month will they have the same amount of money?

Is this a problem?

How might you make this more accessible to a range of learners?

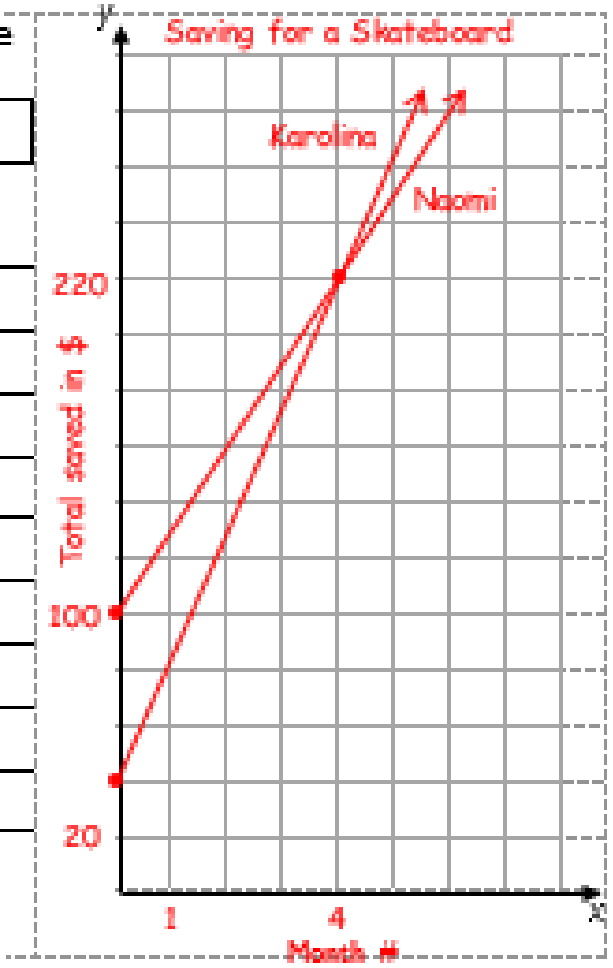
How might you use this context to deepen understanding of algebraic representations?

BUYING A SKATEBOARD (Questions 1-2)

Naomi and Karolina are saving for a skateboard. Naomi has $100 in the bank and will save $30 each month. Karolina has $40 in the bank and will save $45 each month.

1. Complete the table below, graph the data, and write the input-output equations.

Naomi		Karolina	
Month # (x)	Total saved in $ (y)	Month # (x)	Total saved in $ (y)
0	100	0	40
1	130	1	85
2	160	2	130
3	190	3	175
4	220	4	220
5	250	5	265
6	280	6	310
7	310	7	355
$y =$	$30x + 100$	$y =$	$45x + 40$



BUYING A SKATEBOARD (Questions 2-3)

2. Who is saving at a faster rate? Justify your answer using some representations from Problem 1.
   Her savings are increasing by more each month in the table. Her graph is steeper (greater positive slope).

3. During which month(s)…

   a. does Naomi have more money? Months 0 – 3

   b. does Karolina have more money? Months 5 and on

   c. do they have the same amount of money? Month 4

      What do you notice about the table entries at this month?
      They both have $220.

      What do you notice about the graphs at this month?
      The lines intersect at (4, 220)

BUYING A SKATEBOARD (Question 4)

4. Use substitution to write one equation in x equating Naomi's and Karolina's savings. Use this equation to verify the month at which they have the same amount of money. State your answer in a short sentence.

$$30x + 100 = 45x + 40$$
$$30(4) + 100 = 45(4) + 40$$
$$120 + 100 = 180 + 40$$
$$220 = 220$$

At Month 4, they will each have $220 in the bank.

REQUEST MATERIALS

- Slide Deck Presentation
- Crossing the Lake – Complete lesson plans
- Buying a Skateboard – Student Page, Answer Key

Visit us in the Exhibit Hall – Booth #636

TODOS Strand at NCTM

TODOS is part of the Equity Affiliates Strand Sessions at NCTM.

Thank you for choosing to attend a TODOS Strand session.

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