REIMAGINE ALGEBRA PROBLEM-SOLVING WITH DIFFERENT LEARNERS

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Please scan QR code for lesson plans and slides

STRATEGIES TO SUPPORT DIVERSE POPULATIONS

An Excerpt from *MathLinks* (Goldstein, Kriegler, et al, 2022)

(1) Know your Learner	✓ Understand student attributes that support or interfere with learning ✓ Determine preferred learning and interaction styles ✓ Assess student knowledge of prerequisite mathematics content ✓ Check for understanding continuously ✓ Provide differentiation opportunities for intervention or enrichment to reach more learners ✓ Encourage students to write about their attitudes and feelings towards math ✓ Use contexts that link to students' cultures*
(2) Increase Academic Language through Mathematics	✓ Provide opportunities for students to read, write, or speak about their mathematical learning ✓ Explain the academic vocabulary needed to access mathematical ideas, providing both examples and non-examples ✓ Use strategically organized groups that attend to language needs* ✓ Use rich mathematical contexts and sophisticated language to help ELs progress in their linguistic development* ✓ Use cognates and root words (when appropriate) to link new math terms to students' background knowledge*
(3) Increase Comprehensible Input	✓ Link concepts to past learning ✓ Make concepts meaningful through hands-on activities, visuals, demonstrations, and color-coding ✓ Use a think-aloud strategy to model appropriate thinking processes and academic language use ✓ Use graphic organizers to help students record information and data, see patterns, and generalize them ✓ Use multiple representations (pictures, numbers, symbols, words, contexts) of math ideas to create meaning and make connections ✓ Strategically sequence and scaffold to make mathematics accessible ✓ Simplify written instructions, rephrase explanations, and use verbal and visual clues*
(4) Promote Student Interaction	✓ Use flexible group configurations that support content objectives ✓ Use strategies and activities that promote teacher/student and student/student interactions (e.g., think-pair-share, Poster Problems) ✓ Encourage elaborate responses through questioning ✓ Allow processing time and appropriate wait time, recognizing the importance of the different requirements for speaking, reading, and writing in a new language* ✓ Allow alternative methods to express mathematical ideas (e.g., visuals, students' first language)*

CROSSING THE LAKE (VERSION 1)

____ children and ____ adults need to get across a lake in a canoe. Everyone can row. The canoe holds one child alone or two children together or one adult alone. How many one-way trips are needed to get everyone across the lake?

CROSSING THE LAKE (VERSION 2)

(Grade 7, Packet 6, Opening Problem)

Follow your teacher's directions.

(1)

(2)

BUYING A SKATEBOARD

Naomi and Karolina are saving for a skateboard. Naomi has _____ in the bank and will save _____ each month. Karolina has _____ in the bank and will save _____ each month. During what month will they have the same amount of money?

CROSSING THE LAKE

[SMP1, 4, 5, 7, 8]

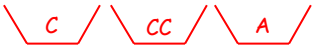
Follow your teacher's directions.

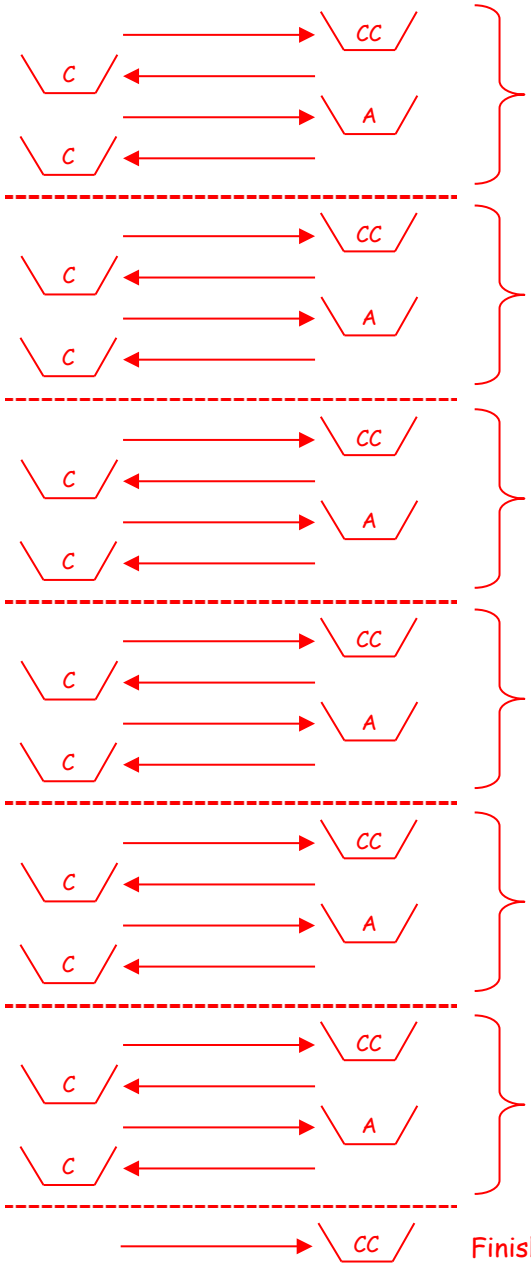
(1) ***Summarize the facts of the problem.***

- There is a canoe that holds 1 child alone, 2 children together, or 1 adult alone.
- Everyone can paddle. The only way to get across the lake is to paddle.
- There are 6 adults and 2 children who need to cross the lake.

(2) ***How many one-way trips are needed to get everyone across the lake?***

One way to illustrate the solution is:





1st cycle
Gets 1 adult over and the 2 children are back to the "starting point"

2nd cycle

3rd cycle

4th cycle

5th cycle

6th cycle

Finish with the last trip.

It takes 4 trips to get 1 adult across:

→ 2 children go over.

← 1 child comes back.

→ 1 adult goes over.

← 1 child comes back.

This cycle repeats 6 times because there are 6 adults.

Finally, the two children come over one last time.

6(4) + 1 = 25 trips.

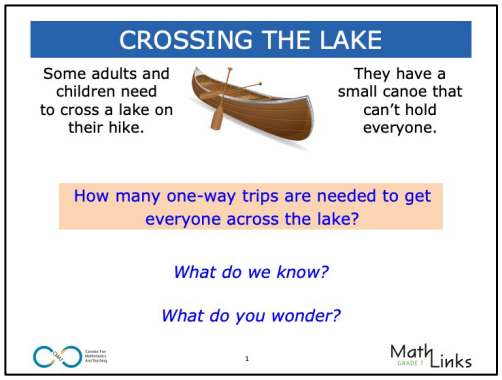
LESSON NOTES S6.0: CROSSING THE LAKE

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

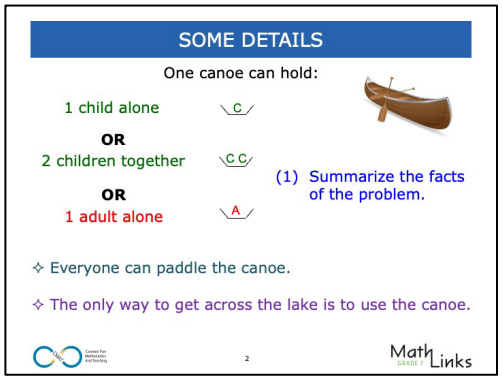
Information for **Crossing the Lake** is revealed in pieces so that students can make sense of it and become motivated to engage in the problem-solving experience. Make tools such as extra blank paper, colored pencils, and manipulatives in two colors available to be used if needed.

This investigation requires perseverance. Teachers should try not to contribute to the phenomenon of "learned helplessness" by giving too much help. Though well intended, this actually contributes to students' lack of persistence. However, if working with English learners, consider additional linguistic support if needed. The problem is revisited and extended at the end of Lesson 6.1.

- Slide 1: Reveal the context for the problem and ask students to read the information silently. Intentionally, there is not enough information. Ask students to share what we know, and chart all questions that they have without responding. This validates all student input without judgment. Even irrelevant questions, like, "What color is the canoe?" show willingness to engage. But be sure to include questions that request needed information (e.g., how many adults and how many children need to cross the lake?). When answering questions, leave out details about who the canoe can hold, who can paddle, and the number of adults and children. This information is on slides 2 and 3.



- Slide 2: Reveal more details about the problem, check for understanding, and discuss how efficient use of sketches might help at some point in the problem-solving process. For (1), students summarize these facts.



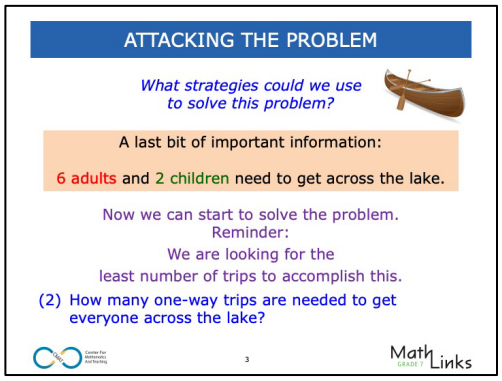
LESSON NOTES S6.0: CROSSING THE LAKE

Continued

- Slide 3: Before revealing the number of people who will cross the lake, brainstorm solution strategies.

  What strategies could we use to solve this problem? What are ways to model people crossing a lake? Act out the situation; move manipulatives back and forth across the desk; draw pictures.

  Reveal that 6 adults and 2 children must cross the lake. For (2), allow productive struggle. If using manipulatives, encourage students to record a diagram.
  What are some ways to keep track of trips? Encourage students to share recording strategies.

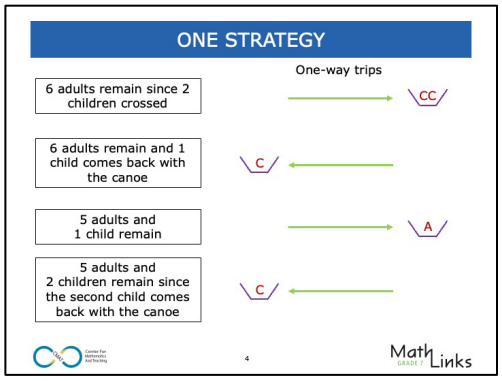


- Slide 4: Share student work and use this slide if needed. Emphasize patterns, structure, and repeated reasoning.

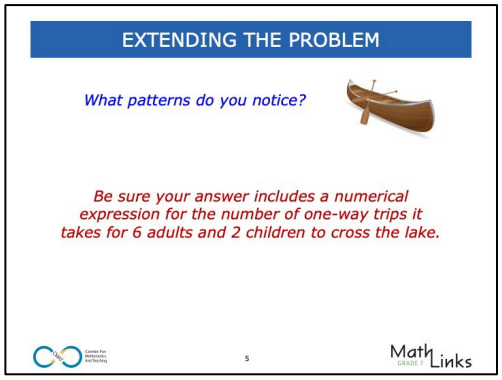
  In your diagram(s), what pattern seems to be repeating? Two children cross, one child returns, one adult crosses, the other child returns. This might be considered one "cycle" to transport one adult across the lake.

  How many of these cycles are required to get all six adults across the lake? 6 ***How many one-way trips is this?*** 4 • 6 = 24

  Why are we not done yet? We need one more trip to get the two children back across (4 • 6 + 1 = 25).



- Slide 5: If students seem ready to generalize the problem, move forward as desired. The problem will be revisited at the end of Lesson 6.1.



SLIDE DECK ALTERNATIVE S6.0: CROSSING THE LAKE

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slides 1 – 3

Some adults and children need to cross a lake on their hike.
They have a small canoe that can't hold everyone.
How many one-way trips are needed
to get everyone across the lake?

What do we know? *What do you wonder?*

Some facts:

- One canoe can hold:

C	or	C C	or	A
1 child alone		2 children together		1 adult alone

- Everyone can paddle the canoe.
- The only way to get across the lake is to use the canoe.

(1) Summarize the facts of the problem.

What strategies could we use while solving this problem?

One more fact:

- There are 6 adults and 2 children that need to get across the lake.

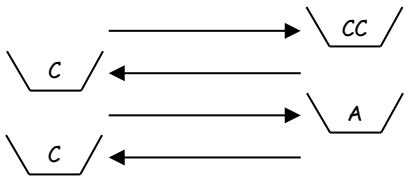
(2) Now solve the problem stated above.

Slides 4 – 5

What patterns do you notice?

Be sure your answer includes a numerical expression for the number of one-way trips it takes for 6 adults and 2 children to cross the lake.

One Strategy



CROSSING THE LAKE REVISITED

The *MathLinks* Rubric: See Activity Routines in Program Information for directions. [SMP1, 2, 4, 6, 7, 8]

1. Review your work and notes from the opening problem. Do you see any patterns? Does anything seem to be happening regularly, over and over again? Circle a repeating pattern if you see one. Write your observations below.
   Answers will vary. It takes four one-way trips to get one adult across: Two children go over, one child comes back, one adult goes over, and the other child comes back. It also takes one extra one-way trip at the end to get the children back across the lake with the canoe.

2. Write a numerical expression that represents the number of one-way trips it takes for 6 adults and 2 children to cross the lake.
   $\_\_4\_\_ \bullet \_\_6\_\_ + \_\_1\_\_$

For problems 3 – 8, write each as an expression in the form of problem 2 above. Use your diagram as needed to determine the number of one-way trips necessary to get each combination of people across the lake.

3. 4 adults and 2 children $4(4) + 1 = 17$	4. 2 adults and 2 children $4(2) + 1 = 9$	5. 0 adults and 2 children $4(0) + 1 = 1$
6. 20 adults and 2 children $4(20) + 1 = 81$	7. 100 adults and 2 children $4(100) + 1 = 401$	8. n adults and 2 children $4n + 1$

9. Explain the meaning of the expression in problem 8 above.
   If I multiply 4 times the number of adults and add one (for ANY number of adults), that results in the minimum number of trips needed to get across the lake.

10. Assume the number of children remains 2.

    a. If the number of adults is doubled, are the number of trips doubled?
       No. Compare problems 3 and 4 above.

    b. If the number of adults is multiplied by 5, are the number of trips multiplied by 5?
       No. Compare problems 6 and 7 above.

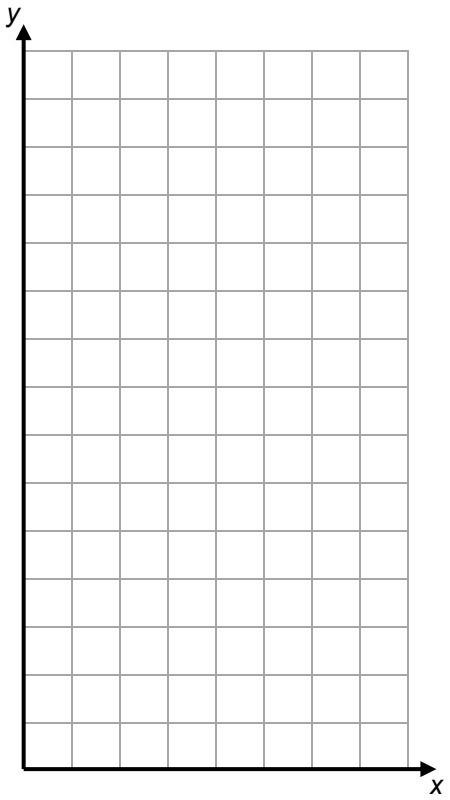
    c. Record the meaning of proportional relationship in **My Word Bank**.
       Does the crossing the lake scenario represent a proportional relationship? No

11. Suppose it takes some adults and 2 children a minimum of 201 one-way trips to get everyone across the lake. How many adults are in the group?
    If $4n + 1 = 201$, then $4n = 200$, and $n = 50$.
    There are 50 adults in the group.
    Solving equations in this form is a focus of an upcoming lesson in the next packet.

SAVING FOR A SKATEBOARD

Naomi and Karolina are saving for a skateboard. Naomi has $100 in the bank and will save $30 each month. Karolina has $40 in the bank and will save $45 each month.

1. Complete the table below, graph the data, and write the input-output equations.



Naomi		Karolina	
Month # (*x*)	**Total saved in $ (*y*)**	**Month # (*x*)**	**Total saved in $ (*y*)**
0		0	
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	
7		7	
y =		*y* =	

2. Who is saving at a faster rate? Justify your answer by referring to some problem 1 representations.

3. During which month(s)…
   a. does Naomi have more money?
   b. does Karolina have more money?
   c. do they have the same amount of money?

      What do you notice about the table entries at this month?

      What do you notice about the graphs at this month?

4. Use substitution to write one equation in *x* equating Naomi's and Karolina's savings. Use this equation to verify the month at which they have the same amount of money. State your answer in a short sentence.

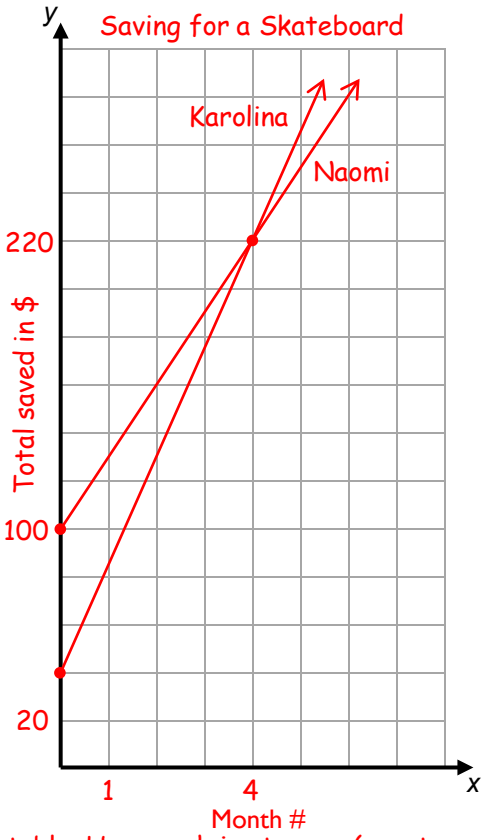
PRACTICE 3 (SAVING FOR A SKATEBOARD)

The *MathLinks* Rubric: See Activity Routines in Program Information for directions. [SMP2, 4]

Naomi and Karolina are saving for a skateboard. Naomi has $100 in the bank and will save $30 each month. Karolina has $40 in the bank and will save $45 each month.

5. Complete the table below, graph the data, and write the input-output equations.

Naomi		Karolina	
Month # (x)	**Total saved in $ (y)**	**Month # (x)**	**Total saved in $ (y)**
0	100	0	40
1	130	1	85
2	160	2	130
3	190	3	175
4	220	4	220
5	250	5	265
6	280	6	310
7	310	7	355
$y =$ $30x + 100$		$y =$ $45x + 40$	



6. Who is saving at a faster rate? Justify your answer by referring to some problem 1 representations.

   Karolina. Her savings are increasing by more at each month in the table. Her graph is steeper (greater positive slope).

7. During which month(s)…

   a. does Naomi have more money? Months 0 – 3

   b. does Karolina have more money? Months 5 and on

   c. do they have the same amount of money? Month 4

      What do you notice about the table entries at this month? They both have $220.

      What do you notice about the graphs at this month? The lines intersect at (4, 220).

8. Use substitution to write one equation in x equating Naomi's and Karolina's savings. Use this equation to verify the month at which they have the same amount of money. State your answer in a short sentence.

   $30x + 100 = 45x + 40$

   $30(4) + 100 = 45(4) + 40$

   $120 + 100 = 180 + 40$

   $220 = 220$ At month 4, both Naomi and Karolina have $220.