



# A FISHY WAY TO CONNECT SAMPLING TO PROPORTIONAL RELATIONSHIPS

Presented by:

Shelley Kriegler

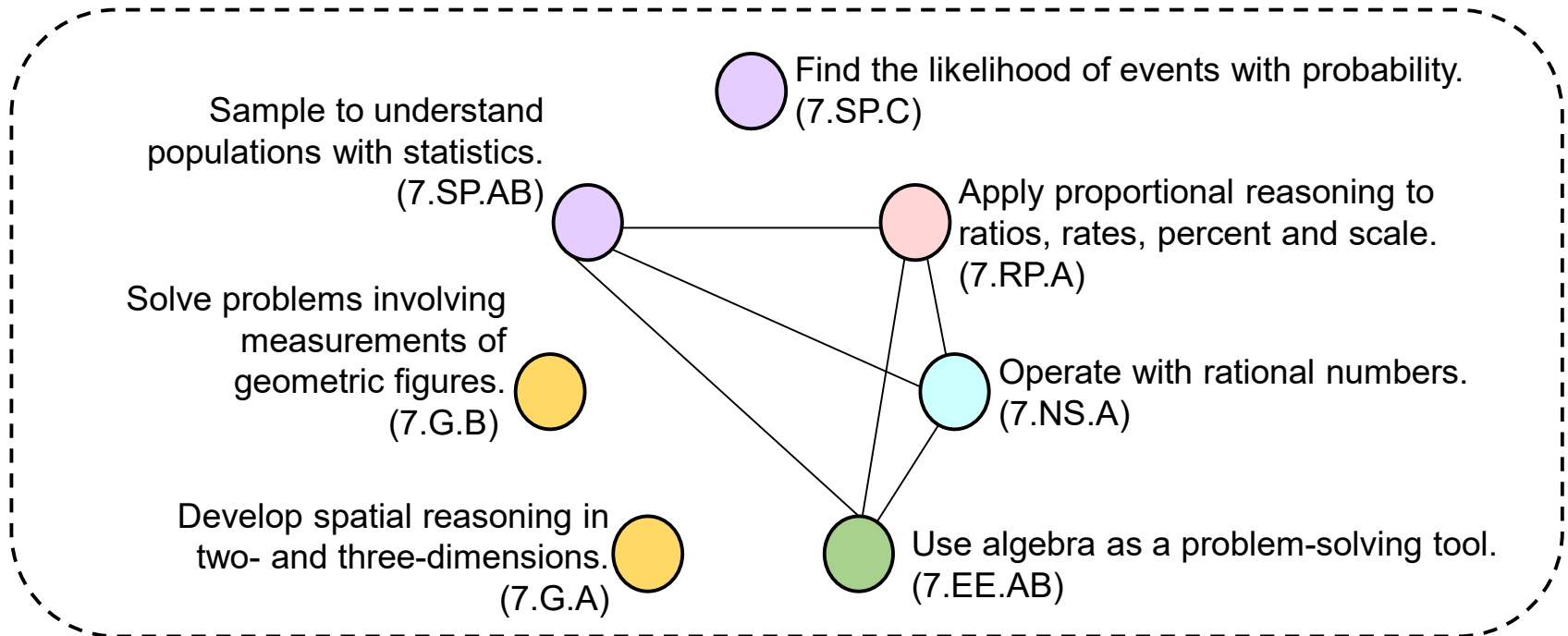
(with Cynthia Raff and Mark Goldstein)

CMC-South, 2023

# TODAY'S PLAN

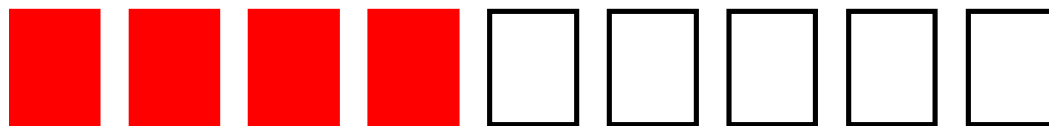
- Concept: "Proportion" Equations
- Practice: "Proportion" Equations
- Review: Statistics Vocabulary
- Simulation: A Fishy Problem

# 7<sup>TH</sup> GRADE BIG IDEAS AND CONNECTIONS

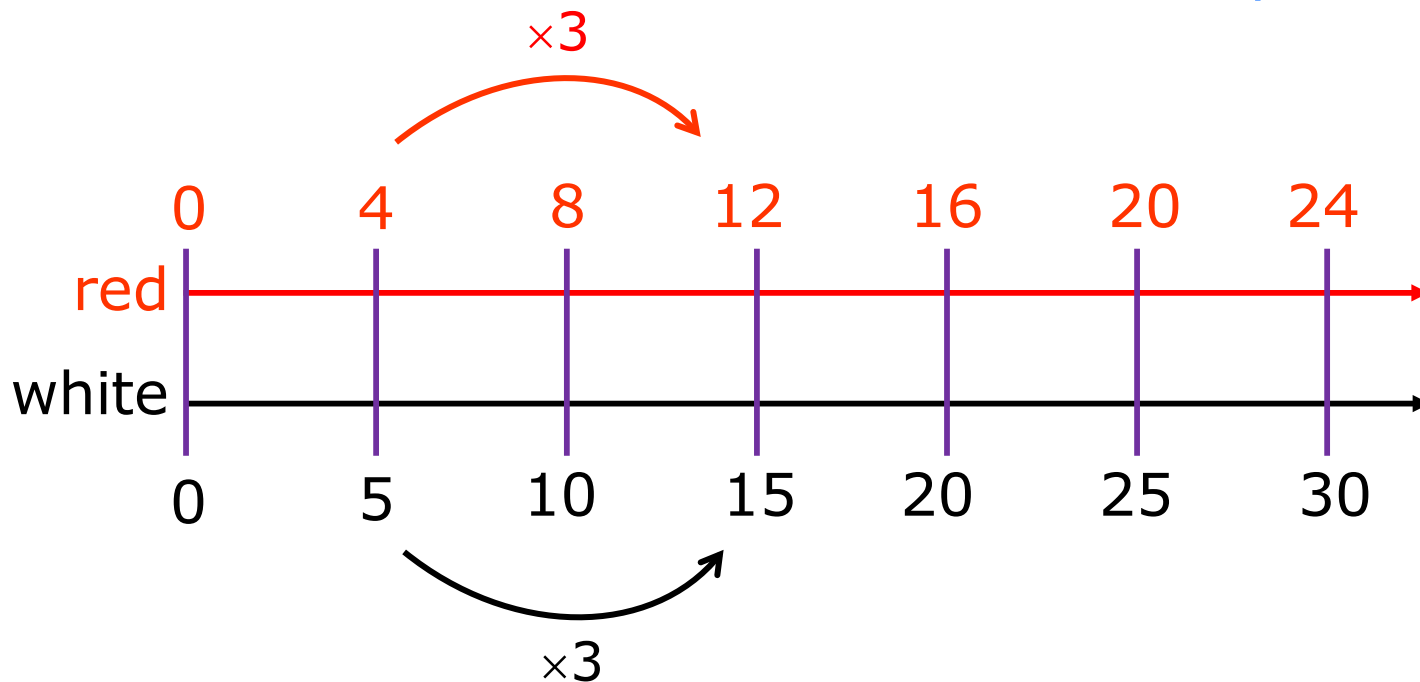


# CONCEPT: EQUATIONS $\left(\frac{a}{b} = \frac{c}{d}\right)$

# DOUBLE NUMBER LINES

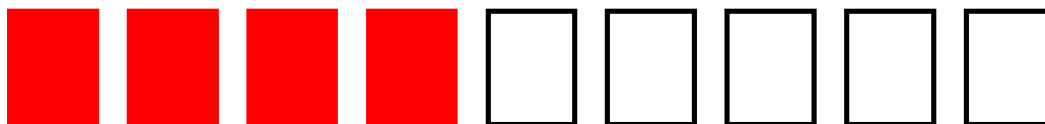


A paint mixture is 4 parts red and 5 parts white.  
Make a double number line for this relationship.

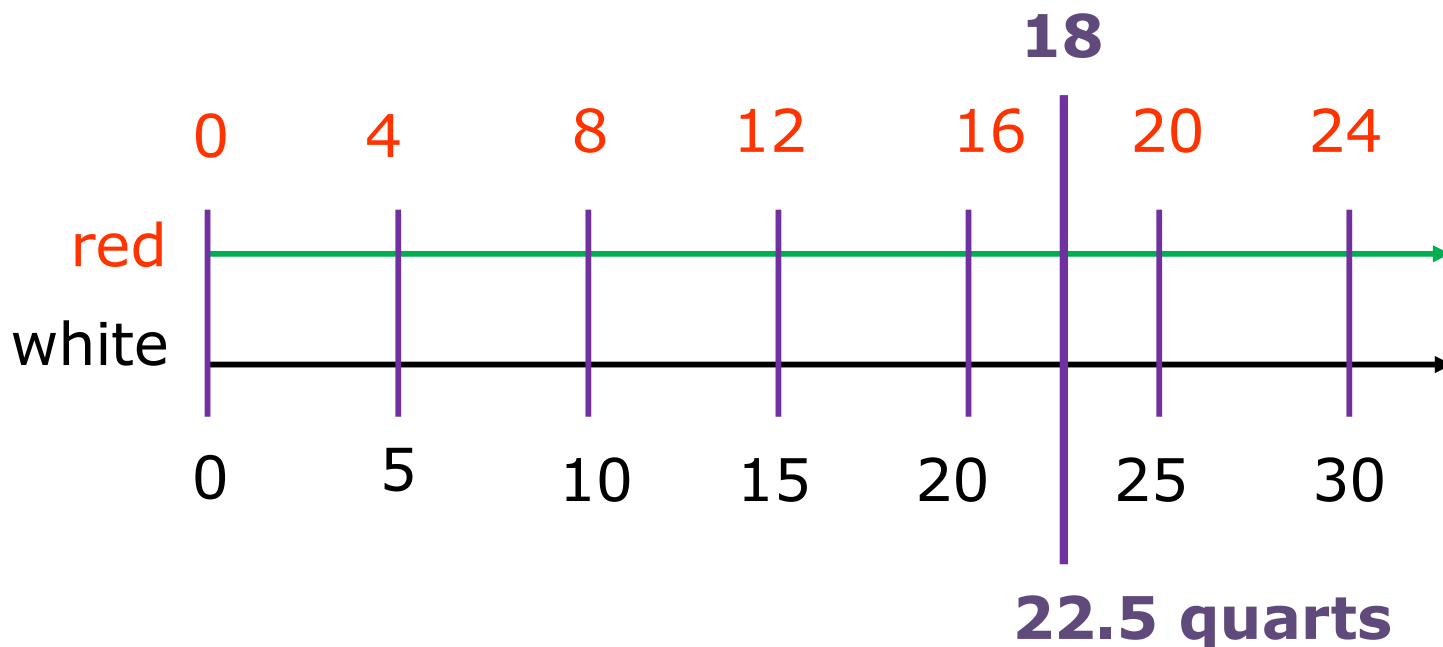


How does this connect to equivalent ratios?

# CONNECT CONCEPT TO PROBLEM

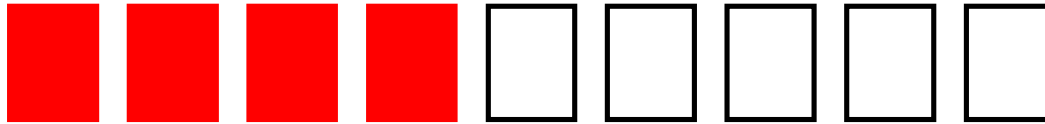


*A paint mixture is 4 parts red and 5 parts white.  
How much white paint is needed to create this  
mixture with 18 quarts of red?*

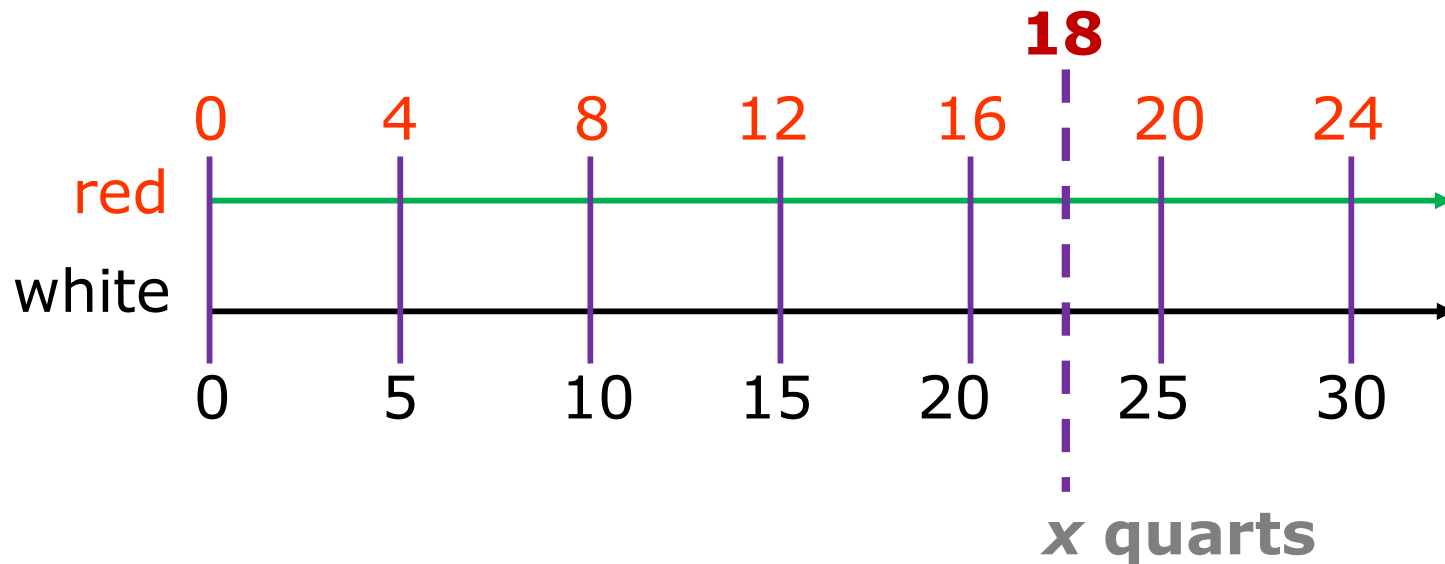


# CONNECT CONCEPT TO EQUATION

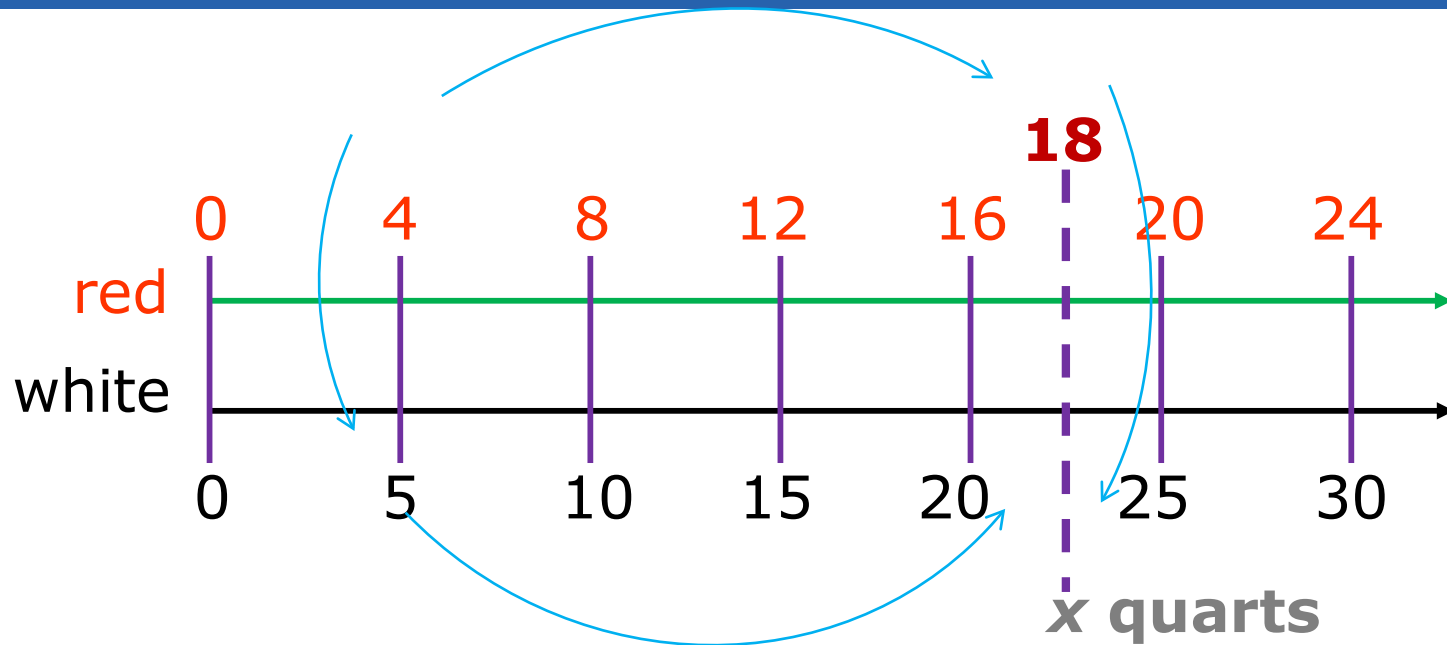
D:



*A paint mixture is 4 parts red and 5 parts white.  
How much white paint is needed to create this  
mixture with 18 quarts of red?*



# EQUATIONS(cont.)



$$\frac{4}{18} = \frac{5}{x}$$

$$\frac{4}{5} = \frac{18}{x}$$



# PRACTICE: EQUATIONS $\left(\frac{a}{b} = \frac{c}{d}\right)$

# FOUR IN A ROW

Object of the Game  
Get 4 spaces across, down,  
or diagonally.

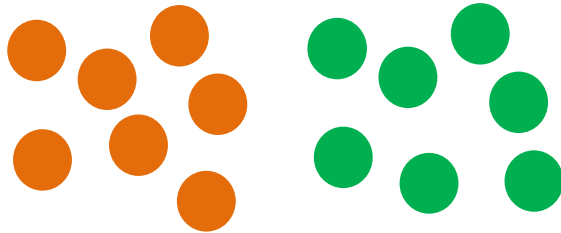
“Y”?

- Attain skill for problem solving
- Practice in a game format



# FOUR IN A ROW: MATERIALS

- Game board
- 2 sets of colored counters



- 2 other objects like:



**ADDITION**

**Players:** 2

**Materials:** Board game, 2 sets of colored counters (for the game board), 2 objects (e.g. cubes, paperclips, cut up paper) that will cover numbers in Box A and Box B

**Object of the Game:** Players alternate choosing addends from Box A and Box B to create sums. They cover sums on the game board. The winner is the first player to get four in a row.

Box A: Addend			Box B: Addend		
2	3	5	4	6	8
6	9	12	14	20	30

Game Board: Sums					
20	11	39	16	28	19
26	7	17	22	33	25
18	11	12	17	32	12
36	23	23	32	13	35
14	9	29	10	42	13
15	6	16	20	20	10

# FOUR IN A ROW: LET'S PLAY

## ADDITION

Players: 2

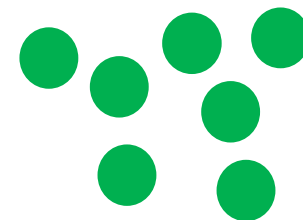
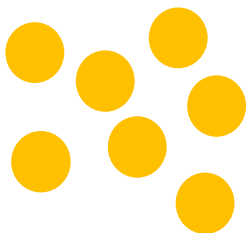
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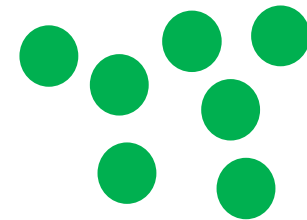
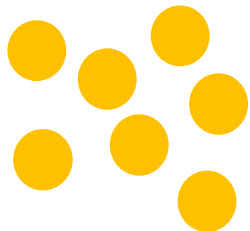
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Cynthia's  
turn



# FOUR IN A ROW: LET'S PLAY

Shelley's turn



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14	9	29	●	42	13
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# FOUR IN A ROW: LET'S PLAY

## ADDITION

**Players:** 2

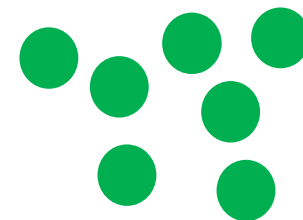
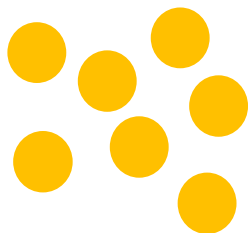
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Cynthia's  
turn



# FOUR IN A ROW: LET'S PLAY

Shelley's turn

## ADDITION

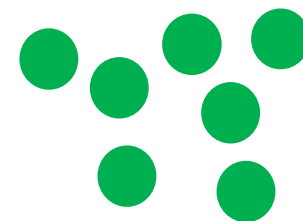
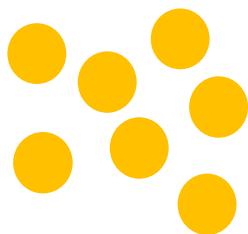
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# FOUR IN A ROW: CHEAT SHEET

		BOX B					
Solve for $x$		$\frac{x}{40}$	$\frac{24}{x}$	$\frac{x}{200}$	$\frac{300}{x}$	$\frac{x}{100}$	$\frac{120}{x}$
BOX A	$\frac{2}{4}$					50	
	$\frac{3}{2}$	60	16	300	200	150	
	$\frac{2}{5}$	16	60	80	750	40	360
	$\frac{1}{5}$	8	120	40	1500	20	
	$\frac{6}{8}$	30	32	150	400	75	
	$\frac{6}{10}$	24	40	120	500	60	



# YOUR TURN

## FOUR IN A ROW: PROPORTIONS

**Players:** 2+

**Objective:** Be the first player to claim 4 spaces in a row, column, or diagonal to win the game.

**Materials:** Board game, 2 sets of colored counters (for the game board), 2 objects (e.g. cubes, paperclips, cut up paper) that will cover numbers in Box A and Box B

**Rules:** Two players alternate solving for  $x$  by choosing constant from Box A, an expression from Box B, and setting them equal to one another. (Example:  $\frac{4}{2} = \frac{x}{40}$ .) Players check the solution and, if successful, place their colored counter on a space with the appropriate solution.

BOX A: CONSTANT			BOX B: EXPRESSION		
$\frac{2}{4}$	$\frac{3}{2}$	$\frac{2}{5}$	$\frac{x}{40}$	$\frac{24}{x}$	$\frac{x}{200}$
$\frac{1}{5}$	$\frac{6}{8}$	$\frac{6}{10}$	$\frac{300}{x}$	$\frac{x}{100}$	$\frac{120}{x}$

GAME BOARD: PROPORTIONS (SOLVE FOR $x$ )					
16	300	8	120	60	600
20	30	100	80	200	20
400	40	300	1500	150	16
50	240	60	24	40	750
120	80	500	200	75	32
600	150	48	40	160	60

# REVIEW: VOCABULARY

# A VOCABULARY MATCHING ACTIVITY

Work with a partner and two sets of cards.

One partner has the  $\triangle$  set,  
the other has the  $\bigcirc$  set.

First match **your own** I-II-III-IV **word** cards  
to **your** A-B-C-D **description** cards.

Discuss both sets of cards with your partner.  
Be sure you agree the matches are correct.

Record the matching card numbers, words,  
and card letters for each set in the table.



Mateo



Talia

# YOUR TURN

Individually, match words with descriptions and record results.

Card set $\triangle$			Card set $\bigcirc$		
Card number	word	Card letter	Card number	word	Card letter
I			I		
II			II		
III			III		
IV			IV		

# LET'S COMPARE

Here are the word cards for "II"

*What is the SAME about them?*

*What is DIFFERENT about them?*

Biased  
sample

Random  
sample

may have skewed  
or invalid results

Every person or  
object does not have  
an equal chance of  
being represented

Both are a subset of  
the population  
under consideration

Every person or  
object has an equal  
chance of being  
represented

The results will be more  
valid and more  
accurately represent  
the population

Choose another pair of number-matched cards.  
Discuss and record their similarities and differences.

# SIMULATION: A FISHY PROBLEM

# ESTIMATING FISH POPULATIONS

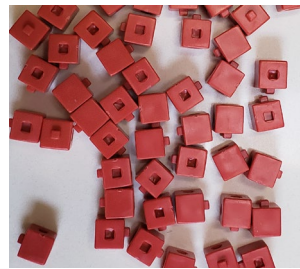
Your group will answer the question, "How many fish are in Lake Calculus?" using a "mark-recapture" simulation method.



a lake



a net



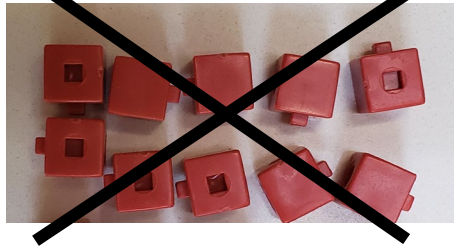
fish in the lake  
(unmarked)



marked fish

# SET UP THE EXPERIMENT

Lake Calculus  
with fish



Randomly remove  
some fish  
from the lake.



Mark the fish...  
(swap out)

"Population marked"

$P_{\text{marked}}$

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and return  
to the lake.



# THE VARIABLES

(1) Identify and define the variables.

$P_{marked}$  = number of **marked** fish in the **P**opulation

$P_{total}$  = **total** fish in the **P**opulation

$S_{marked}$  = number of **marked** fish in given **S**ample

$S_{total}$  = **total** fish in a given **S**ample

(2) Write in your known value.

*Which variable's value do we want to solve for? Why?*

# PERFORM THE EXPERIMENT

Here is how to perform the simulation.

- Shake the lake and grab a scoop of fish.

*Which variables are represented in this sample?*



For this example:

$$S_{total} = 14$$

$$S_{marked} = 2$$

- Fill in the chart for YOUR Sample 1.
- Return the fish to the lake and repeat this process for Samples 2 – 6.

(3) Perform the simulation. Fill in the table as you go.

# ANALYZE THE DATA (for the given example)



$S_{\text{marked}}$	2
$S_{\text{total}}$	14
$P_{\text{marked}}$	10

*Which variable is not in the table? Why not?*

*What is the...  
ratio of the samples?*

$$S_{\text{marked}} : S_{\text{total}} \rightarrow 2 : 14$$

*value of this ratio?*  $\frac{2}{14}$

*ratio of the populations?*

$$P_{\text{marked}} : P_{\text{total}} \rightarrow 10 : x$$

*value of this ratio?*  $\frac{10}{x}$

*If we assume a proportional relationship between the random sample and the actual lake population, what equation that can be used to estimate  $P_{\text{total}}$  ?*

$$\frac{2}{14} = \frac{10}{x}$$

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# DRAW CONCLUSIONS

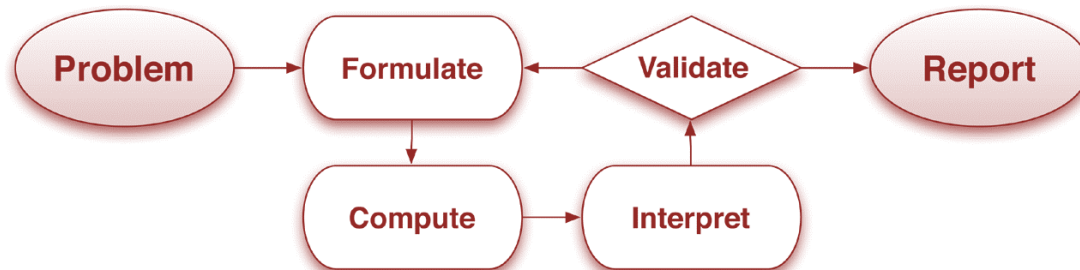
*How did you compute your estimate?*

*How close were you?*



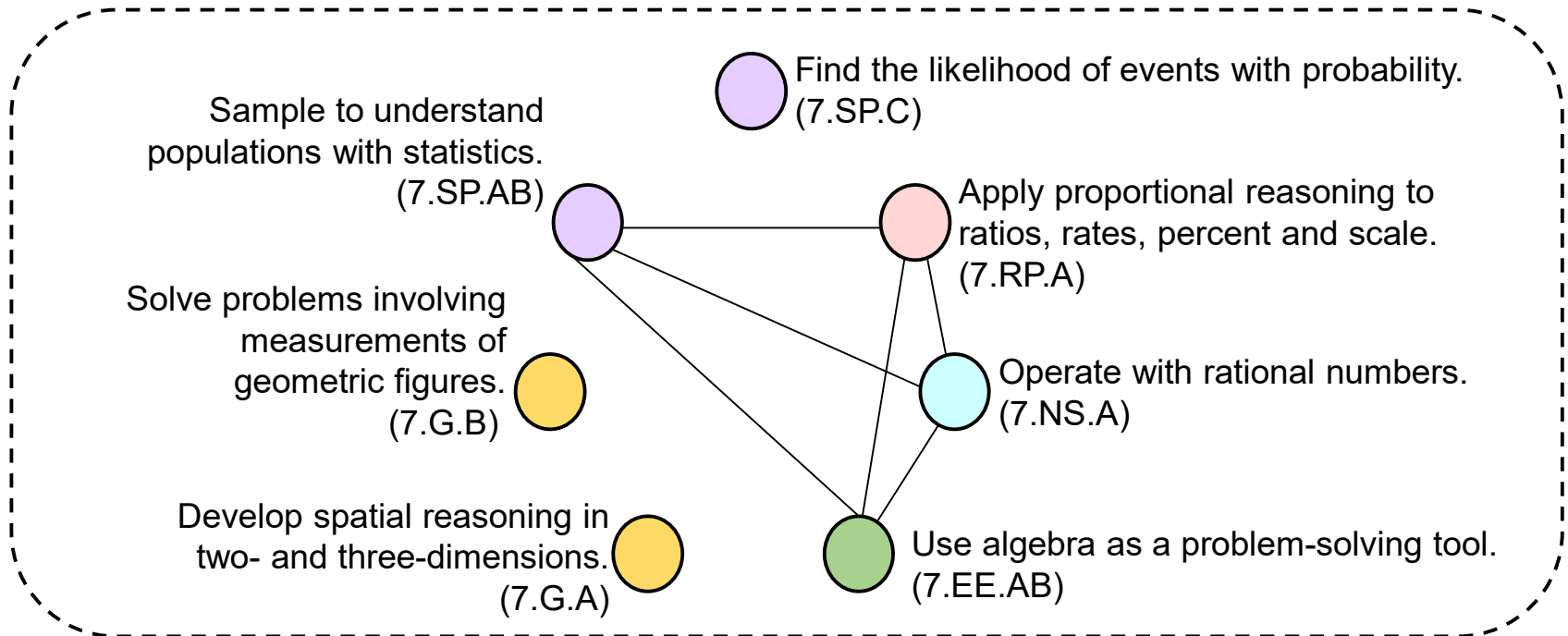
*What assumptions might scientists make when estimating fish using a mark-recapture technique?*

*Explain in what ways our process followed this modeling cycle.*



# TO WRAP UP

# 7<sup>TH</sup> GRADE BIG IDEAS AND CONNECTIONS



# REQUEST MATERIALS



- Slide Deck Presentation
- Four in a Row (3 games)
- Match and Compare Sort (4 activities)
- Estimating Fish Populations (complete lesson plan)

# THANK YOU!

Shelley Kriegler

Mark Goldstein

Cynthia Raff

Center for Mathematics and Teaching  
[www.mathandteaching.org](http://www.mathandteaching.org)

