## A FISHY WAY TO CONNECT SAMPLING TO PROPORTIONAL RELATIONSHIPS

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## RATIOS $\rightarrow$ EQUIVALENCE $\rightarrow$ EQUATIONS



FOUR IN A ROW: PROPORTIONS "CHEAT SHEET"

|  |  | BOX B |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Solve for $x$ | $\frac{x}{40}$ | $\frac{24}{x}$ | $\frac{x}{200}$ | $\frac{300}{x}$ | $\frac{x}{100}$ | $\frac{120}{x}$ |
| $\begin{aligned} & \mathbb{4} \\ & \times \\ & \text { O } \end{aligned}$ | $\frac{2}{4}$ |  |  |  |  |  |  |
|  | $\frac{3}{2}$ | 60 | 16 | 300 | 200 | 150 |  |
|  | $\frac{2}{5}$ | 16 | 60 | 80 | 750 | 40 |  |
|  | $\frac{1}{5}$ | 8 | 120 | 40 | 1500 | 20 |  |
|  | $\frac{6}{8}$ | 30 | 32 | 150 | 400 | 75 |  |
|  | $\frac{6}{10}$ | 24 | 40 | 120 | 500 | 60 |  |

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## FOUR IN A ROW: PROPORTIONS

Players: 2+
Objective: Be the first player to claim 4 spaces in a row, column, or diagonal to win the game.
Materials: Board game, 2 sets of colored counters (for the game board), 2 objects (e.g. cubes, paperclips, cut up paper) that will cover numbers in Box $A$ and Box $B$

Rules: Two players alternate solving for $x$ by choosing constant from Box $A$, an expression from Box $B$, and setting them equal to one another. (Example: $\frac{4}{2}=\frac{x}{40}$.) Players check the solution and, if successful, place their colored counter on a space with the appropriate solution.

| BOX A: CONSTANT |  |  |
| :---: | :---: | :---: |
| $\frac{2}{4}$ | $\frac{3}{2}$ | $\frac{2}{5}$ |
| $\frac{1}{5}$ | $\frac{6}{8}$ | $\frac{6}{10}$ |


| BOX B: EXPRESSION |  |  |
| :---: | :---: | :---: |
| $\frac{x}{40}$ | $\frac{24}{x}$ | $\frac{x}{200}$ |
| $\frac{300}{x}$ | $\frac{x}{100}$ | $\frac{120}{x}$ |


| GAME BOARD: PROPORTIONS (SOLVE FOR $x$ ) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 16 | 300 | 8 | 120 | 60 | 600 |
| 20 | 30 | 100 | 80 | 200 | 20 |
| 400 | 40 | 300 | 1500 | 150 | 16 |
| 50 | 240 | 60 | 24 | 40 | 750 |
| 120 | 80 | 500 | 200 | 75 | 32 |
| 600 | 150 | 48 | 40 | 160 | 60 |

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## MATCH AND COMPARE SORT: SAMPLING

1. Individually, match words with descriptions. Record results.

| Card set $\triangle$ |  |  | Card set $\bigcirc$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Card <br> number | word | Card <br> letter | Card <br> number | word | Card <br> letter |
| I |  |  | I |  |  |
| II |  |  | II |  |  |
| III |  |  | III |  |  |
| IV |  |  | IV |  |  |

2. Partners, choose a pair of numbered matched cards and record the attributes that are the same and those that are different.

3. Partners, choose another pair of numbered matched cards and discuss the attributes that are the same and those that are different.

## ESTIMATING FISH POPULATIONS

Follow your teacher's directions for (1) - (3).
(1) - (2)

| Total number of marked fish: | Marked fish sample: |
| :--- | :--- |
| Total fish population: | Total fish sample: |

(3)

|  | Sample 1 | Sample 2 | Sample 3 | Sample 4 | Sample 5 | Sample 6 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{S}_{\text {marked }}$ |  |  |  |  |  |  |
| $\boldsymbol{S}_{\text {total }}$ |  |  |  |  |  |  |
| $\boldsymbol{P}_{\text {marked }}$ |  |  |  |  |  |  |

3. Create and solve equations to estimate the number of fish in the lake for each sample.

| Sample 1 | Sample 2 | Sample 3 |
| :---: | :---: | :---: |
| Sample 4 | Sample 5 | Sample 6 |
|  |  |  |

4. Based on your experiment, estimate the number of fish in the population.

Explain how you arrived at this estimate.
5. Count all of the fish. The actual number of fish in the population is $\qquad$ .
6. Find the error in your estimate as a percent of the actual population. $\qquad$

## FOUR IN A ROW

Four in a Row games provide skills practice. The object is to get four spaces across, down, or diagonally. Players earn spaces by completing computation problems. Four in a Row games appear in Packet Resources (Essential Skills and Nonroutine Problems).

Why: Attain skills so that problem solving is not derailed by lack of arithmetic fluency. Practice in a motivating game format.

Players: 2
Materials: Board game, 2 sets of colored counters (for the game board), 2 small objects (e.g. cubes, paperclips, cut up paper) that will cover numbers in Box A and Box B

Launch Activity: Use one or more of the provided "review" Four in a Row games to introduce the rules and procedures. Play first as a class, where teacher is "Player 1" and class is "Player 2". Multiplying Single Whole Digit Numbers is described here.

- To start the game, Player 1 (teacher) chooses one number from Box A and one number from Box $B$, and put a small object on each.
- Player 1 finds the product of the selected numbers from Box A and Box B and puts their colored counter on the product on the game board. Player 2 confirms that the result is correct. (Answer key is provided.) If Player 2 can demonstrate that the result is incorrect, and produce the correct result, then Player 2 gets the square.
- Player 2 moves ONE of the markers (EITHER from Box A OR from Box B), finds the product, and puts their colored counter on that product on the game board. Player 1 confirms that the answer is correct, or corrects it to claim that square.
- Play continues until one player gets four spaces in a row - across, down, or diagonally.


## Differentiation Ideas:

- 3 players: Player 1, Player 2, and the Answer Checker. Rotate roles after each game.
- 4 players play in teams of 2 to allow pair collaboration during the game.
- Allow tools (i.e., calculators or multiplication charts) when applicable.
- When two Four in a Row games are in the same packet, copy them front to back, so players can play both or choose one to play.


## Accountability/Follow-up Ideas:

- Ask students to make an "Answer Key Grid" of all possible products prior to playing a Four in a Row game. An Answer Key Grid template is provided here.
- Encourage students to create their own Four in a Row games. First make an answer key grid, then make a game board. A Four in a Row Game Board template is provided here.


## FOUR IN A ROW: MULTIPLYING SINGLE DIGIT WHOLE NUMBERS

Players: ${ }^{2+}$
Objective: Be the first player to claim 4 spaces in a row, column, or diagonal to win the game.
Materials: Game board, 2 sets of colored counters (for the game board), 2 objects (e.g. cubes, paperclips, cut up paper) that will cover numbers in Box A and Box B

Rules: Two players alternate finding the product by choosing a factor from Box $A$ and a factor from Box $B$. Players check the product (answer key provided) and, if successful, place their colored counter on a space with the appropriate value.

| BOX A: FACTOR |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 3 | 4 | BOX B: FACTOR |  |  |  |
| 5 | 6 | 7 | 5 | 6 |  |  |
| 7 | 7 | 8 | 9 |  |  |  |


| GAME BOARD: FIND THE PRODUCT $(A \times B)$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | 24 | 32 | 18 | 48 | 25 |
| 35 | 42 | 10 | 30 | 18 | 24 |
| 20 | 24 | 21 | 56 | 35 | 36 |
| 15 | 28 | 63 | 16 | 12 | 54 |
| 42 | 14 | 45 | 30 | 49 | 28 |
| 27 | 20 | 36 | 12 | 40 | 16 |

FOUR IN A ROW: MULTIPLYING SINGLE DIGIT WHOLE NUMBERS ANSWER KEY

|  |  | BOX B |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $A+B$ | 4 | 5 | 6 | 7 | 8 | 9 |
| $\begin{aligned} & \mathbb{\nwarrow} \\ & \times \\ & 0 \\ & 0 \end{aligned}$ | 2 | 8 | 10 | 12 | 14 | 16 | 18 |
|  | 3 | 12 | 15 | 18 | 21 | 24 | 27 |
|  | 4 | 16 | 20 | 24 | 28 | 32 | 36 |
|  | 5 | 20 | 25 | 30 | 35 | 40 | 45 |
|  | 6 | 24 | 30 | 36 | 42 | 48 | 54 |
|  | 7 | 28 | 35 | 42 | 49 | 56 | 63 |

## FOUR IN A ROW: EQUIVALENT RATIOS

Players: 2+
Objective: Be the first player to claim 4 spaces in a row, column, or diagonal to win the game.
Materials: Board game, 2 sets of colored counters (for the game board), 2 objects (e.g. cubes, paperclips, cut up paper) that will cover numbers in Box A and Box B.

Rules: Two players alternate finding the missing quantity to make the two ratios equivalent by choosing a ratio from Box A and a ratio from Box B. Players check the quantity and, if successful, place their colored counter on a space with the appropriate quantity.

| BOX A: RATIO |  |  | BOX B: RATIO |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 4:6 | 8:48 | 16: 4 | $4: x$ | $x: 24$ | 32 : $x$ |
| 2 : 1 | 8:24 | $1: 2$ | 80 : $x$ | $x: 12$ | $x: 72$ |


| GAME BOARD: EQUIVALENT RATIOS |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 16 | 192 | 20 | 24 | 8 | 2 |  |
| 24 | 96 | 48 | 240 | 6 | 12 |  |
| 48 | 8 | 12 | 4 | 288 | 64 |  |
| 2 | 144 | 40 | 36 | 4 | 96 |  |
| 8 | 48 | 160 | 24 | 16 | 12 |  |
| 120 | 16 | 8 | 6 | 48 | 480 |  |

## FOUR IN A ROW: EQUIVALENT RATIOS ANSWER KEY

|  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Equivalent Ratios | 4:x | $x: 24$ | 32 : $x$ | 80 : $x$ | $x: 12$ | $x: 72$ |
| $\begin{aligned} & \text { « } \\ & \text { X } \\ & 0 \end{aligned}$ | $4: 6$ | 6 | 16 | 48 | 120 | 8 | 48 |
|  | 8:48 | 24 | 4 | 192 | 780 | 2 | 12 |
|  | $16: 4$ | 16 | 96 | 8 | 20 | 48 | 288 |
|  | $2: 1$ | 2 | 48 | 16 | 40 | 24 | 144 |
|  | 8:24 | 12 | 8 | 96 | 240 | 4 | 24 |
|  | 1:2 | 8 | 12 | 64 | 160 | 6 | 36 |

## FOUR IN A ROW: PROPORTIONS

Players: 2+
Objective: Be the first player to claim 4 spaces in a row, column, or diagonal to win the game.
Materials: Board game, 2 sets of colored counters (for the game board), 2 objects (e.g. cubes, paperclips, cut up paper) that will cover numbers in Box $A$ and Box $B$

Rules: Two players alternate solving for $x$ by choosing constant from Box $A$, an expression from Box $B$, and setting them equal to one another. (Example: $\frac{4}{2}=\frac{x}{40}$.) Players check the solution and, if successful, place their colored counter on a space with the appropriate solution.

| BOX A: CONSTANT |  |  |
| :---: | :---: | :---: |
| $\frac{2}{4}$ | $\frac{3}{2}$ | $\frac{2}{5}$ |
| $\frac{1}{5}$ | $\frac{6}{8}$ | $\frac{6}{10}$ |


| BOX B: EXPRESSION |  |  |
| :---: | :---: | :---: |
| $\frac{x}{40}$ | $\frac{24}{x}$ | $\frac{x}{200}$ |
| $\frac{300}{x}$ | $\frac{x}{100}$ | $\frac{120}{x}$ |


| GAME BOARD: PROPORTIONS (SOLVE FOR $x$ ) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 16 | 300 | 8 | 120 | 60 | 600 |
| 20 | 30 | 100 | 80 | 200 | 20 |
| 400 | 40 | 300 | 1500 | 150 | 16 |
| 50 | 240 | 60 | 24 | 40 | 750 |
| 120 | 80 | 500 | 200 | 75 | 32 |
| 600 | 150 | 48 | 40 | 160 | 60 |

FOUR IN A ROW: PROPORTIONS ANSWER KEY

|  |  | BOX B |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Solve for $x$ | $\frac{x}{40}$ | $\frac{24}{x}$ | $\frac{x}{200}$ | $\frac{300}{x}$ | $\frac{x}{100}$ | $\frac{120}{x}$ |
| $\begin{aligned} & \mathbb{X} \\ & \text { ó } \end{aligned}$ | $\frac{2}{4}$ | 20 | 48 | 100 | 600 | 50 | 240 |
|  | $\frac{3}{2}$ | 60 | 16 | 300 | 200 | 150 | 80 |
|  | $\frac{2}{5}$ | 16 | 60 | 80 | 750 | 40 | 300 |
|  | $\frac{1}{5}$ | 8 | 120 | 40 | 1500 | 20 | 600 |
|  | $\frac{6}{8}$ | 30 | 32 | 150 | 400 | 75 | 160 |
|  | $\frac{6}{10}$ | 24 | 40 | 120 | 500 | 60 | 200 |

## FOUR IN A ROW:

$\qquad$
Players: 2
Materials: Board game, 2 sets of colored counters (for the game board), 2 objects (e.g. cubes, paperclips, cut up paper) that will cover numbers in Box $A$ and Box $B$

Object of the Game:

|  | Box A |  |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |


| Box B |  |  |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |



## MATCH AND COMPARE SORTS

Match and Compare Sorts challenge students to make connections among vocabulary in multiple ways through partner work and discussions. They are located in multiple locations in MathLinks.

Why: Encourage student engagement and discussion, develop critical thinking skills about compare-contrast situations, and connect concepts to vocabulary words and phrases.

## Prepare ahead:

- Cut apart Match and Compare Sorting Cards and paper clip together (or ask students to do this). One set per partner is sufficient, and they may be reused.
- Reproduce (or project for students to copy) the Match and Compare Sort Template - one per student.

Launch the activity: Use one of the math or non-math examples to establish the steps of the activity and routines for discussion. First, students match a word or phrase to a description. Then they work with a partner to find similarities and differences among related vocabulary. And a short explanatory slide deck is included for teachers to view as desired.

- For Part 1, one partner has the triangle cards, and the other has the circle cards. Individually, each student matches the I-II-III-IV word cards to the A-B-C-D description cards. Partners discuss both sets of cards and agree that matches are correct. Both students record the matched card numbers, words, and card letters in the table. Discuss as a class if desired.
- For Part 2, with the whole class, choose one pair of matched number cards and discuss what is the same and what is different about the words or phrases on them. Show students how to record similarities and differences in a sorting circle (Venn) diagram. Ask partners to choose another pair of matched number cards. Students discuss and record what is the same and what is different about them in a Venn diagram. Students share with the class.


## Accountability/Follow-up Ideas:

- Circulate around the room and ask specific pairs to share their Venn diagrams.
- Ask a compare-contrast question as an exit slip or on a quiz.


## MATCH AND COMPARE SORT:

$\qquad$

1. Individually, match words with descriptions. Record results.

| Card set $\triangle$ |  | Card set $\bigcirc$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Card <br> number | word | Card <br> letter | Card <br> number | word | Card <br> letter |
| I |  |  | I |  |  |
| II |  |  | II |  |  |
| III |  |  | III |  |  |
| IV |  |  | IV |  |  |

2. Partners, choose a pair of numbered matched cards and record the attributes that are the same and those that are different.

3. Partners, choose another pair of numbered matched cards and discuss the attributes that are the same and those that are different.

MATCH AND COMPARE SORTING CARDS: ACTIVITIES

| GYMNASTICS | DANCING |
| :---: | :---: |
| $\begin{array}{\|l\|} \hline \text { II } \\ \hline \end{array}$ <br> TRACK AND FIELD | II <br> RUNNING |
| WRESTLING | III BOXING |
| IV <br> WATER POLO | SWIMMING |
| A <br> $\checkmark$ Participants are usually competitors or sparring partners <br> $\checkmark$ Techniques include clinch fighting, throws and takedowns, joint locks, and pins | A <br> $\checkmark$ Involves music <br> $\checkmark$ Burns about the same number of calories as Jogging <br> $\checkmark$ Increases balance and coordination |
| B <br> $\checkmark$ Events include short, middle, and long distance running <br> $\checkmark$ Some events require jumping and weight throwing | B <br> $\checkmark$ Increases heart rate without impact on the body <br> $\checkmark$ Builds endurance and muscle strength through water resistance |
| C <br> $\checkmark$ Events include uneven bars, floor, and vault for women <br> $\checkmark$ Events include horizontal and parallel bars, rings, floor, and pommel horse for men | C <br> $\checkmark$ Participants are usually competitors or sparring partners <br> $\checkmark$ Includes jabbing, punching, and slugging <br> $\checkmark$ Gloves are typically worn |
| D <br> $\checkmark$ Played in the water <br> $\checkmark$ Activity has similarities to basketball and soccer | D <br> $\checkmark$ Aerobic activity typically done outside <br> $\checkmark$ Includes jogging and sprinting |

MATCH AND COMPARE SORT: ACTIVITIES

Answer Key

1. Individually, match words with descriptions. Record results.

| Card set $\triangle$ |  | Card set $\bigcirc$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Card <br> number | word | Card <br> letter | Card <br> number | word | Card <br> letter |
| I | gymnastics | C | I | dancing | A |
| II | track and field | B | II | running | D |
| III | wrestling | A | III | Boxing | C |
| IV | water polo | D | IV | swimming | B |

2. Partners, choose a pair of numbered matched cards and record the attributes that are the same and those that are different.
Choice of vocabulary words to compare will vary. One possibility: example is shown.

3. Partners, choose another pair of numbered matched cards and discuss the attributes that are the same and those that are different.

| MEAN | MEDIAN |
| :---: | :---: |
| II <br> MEAN ABSOLUTE DEVIATION | II <br> INTERQUARTILE RANGE |
| III <br> DOT PLOT | BOX PLOT |
| IV <br> MEASURES OF CENTER | IV <br> MEASURES OF SPREAD |
| A <br> $\checkmark$ A single number that describes the middle of a data set <br> $\checkmark$ Includes mean, median, or mode | A <br> $\checkmark$ Data display of the five-number summary <br> $\checkmark$ The "whiskers" represent values in the first and fourth quartiles. |
| B <br> $\checkmark$ Average of a data set <br> $\checkmark$ Found by adding data values and dividing by the number of observations | B <br> $\checkmark$ Measures that describe the spread of a data set <br> $\checkmark$ Includes IQR and MAD |
| c <br> $\checkmark$ A data display that shows values and frequencies with dots or x's <br> $\checkmark$ Display is typically above a number line | C <br> $\checkmark$ Middle data value when data values are organized from least to greatest <br> $\checkmark$ Considered a reliable way to report the center of a data set |
| D <br> $\checkmark$ Measure of spread associated with the mean <br> $\checkmark$ Average of the distances of the values in a data set from the mean | D <br> $\checkmark$ Measure of variability associated with the median <br> $\checkmark$ Half of the data is captured in this range |

## MATCH AND COMPARE SORT: STATISTICS

See Activity Routines in the Teacher Portal for directions.

1. Individually, match words with descriptions. Record results.

| Card set $\triangle$ |  | Card set $\bigcirc$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Card <br> number | word | Card letter | Card <br> number | word | Card letter |
| I | mean | B | I | median | C |
| II | mean absolute <br> deviation | D | II | interquartile <br> range | D |
| III | dot plot | C | III | box plot | A |
| IV | measures of center | A | IV | measures of <br> spread | B |

2. Partners, choose a pair of numbered matched cards and record the attributes that are the same and those that are different. Answers will vary. One possible answer:


MATCH AND COMPARE SORTING CARDS: SAMPLING

| POPULATION | SAMPLE |
| :---: | :---: |
| BIASED SAMPLE | RANDOM SAMPLE |
| MAD | III <br> IQR |
| MEASURES OF CENTER | MEASURES OF VARIABILITY |
| A <br> $\checkmark$ A single number that describes the middle of a data set <br> $\checkmark$ Includes mean, median, mode | A <br> $\checkmark$ A specific group of objects or people to collect data from <br> $\checkmark$ A subset of the population |
| B <br> $\checkmark$ A subset of the population <br> $\checkmark$ A sample in which every person or object doesn't have an equal chance of being selected | B <br> Measures that describe the spread of a data set <br> $\checkmark$ Includes range, IQR, MAD |
| C <br> C <br> The entire group of objects or people the statistics are about <br> $\checkmark$ Typically, larger than what can reasonably be surveyed or measured | C <br> $\checkmark$ A subset of the population <br> $\checkmark$ A sample in which every person or object has an equal chance of being selected |
| D <br> $\checkmark$ Measure of spread associated with the mean <br> $\checkmark$ Average of the distances of the values in a data set from the mean | D <br> $\checkmark$ Measure of spread associated with the median <br> $\checkmark$ Half of the data is captured in this interval |

## MATCH AND COMPARE SORT: SAMPLING

See Activity Routines in the Teacher Portal for instructions.

1. Individually, match words with descriptions. Record results.

| Card set $\triangle$ |  |  | Card set $\bigcirc$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Card <br> number | word | Card <br> letter | Card <br> number | word | Card <br> letter |
| I | population | C | I | sample | A |
| II | biased sample | B | II | random sample | C |
| III | MAD | D | III | IQR | D |
| IV | measures of center | A | IV | measures of <br> variability | B |

Choice of vocabulary words to compare will vary. Some possible answers:
2. Partners, choose a pair of numbered matched cards and record the attributes that are the same and those that are different.

3. Partners, choose a pair of numbered matched cards and record the attributes that are the same and those that are different.


| I <br> BIVARIATE DATA | I <br> LINE OF BEST FIT |
| :---: | :---: |
| II <br> NUMERICAL DATA | II CATEGORICAL DATA |
| III FREQUENCY TABLE | III <br> RELATIVE FREQUENCY TABLE |
| IV <br> ASSOCIATION | IV <br> OUTLIER |
| A <br> $\checkmark$ A relationship between variables <br> $\checkmark$ Can be linear or not <br> $\checkmark$ Can be strong or weak | A <br> $\checkmark$ Data sorted into categories <br> $\checkmark$ Categories may be colors, sports, etc. If numbers may be things like ages, dates, etc. |
| B <br> $\checkmark$ Data that has two variables <br> $\checkmark$ Can be listed as ordered pairs <br> $\checkmark$ Can be graphed in the coordinate plane as a scatter plot | B <br> $\checkmark$ A straight line that best represents the data on a scatter plot <br> $\checkmark$ Shows that data has a linear association |
| C <br> $\checkmark$ A table that lists items and the number of times they occur <br> $\checkmark$ Can be two-way | C <br> $\checkmark$ A data value that is unusually large or small compared to the other values in the data set |
| D <br> $\checkmark$ Data consisting of numbers <br> $\checkmark$ Can be of measurements | D <br> $\checkmark$ A table that lists items and the percentage of times they occur <br> $\checkmark$ Can be two-way |

MATCH AND COMPARE SORT: BIVARIATE DATA
See Activity Routines in the Teacher Portal for directions.
Your teacher will give you some cards. Cut them out.

1. Individually, match words with descriptions. Record results.

| Card set $\triangle$ |  |  | Card set $\bigcirc$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Card <br> number | word | Card letter | Card <br> number | word | Card letter |
| I | bivariate data | B | I | line of best fit | B |
| II | numerical data | D | II | categorical data | A |
| III | frequency table | C | III | relative frequency <br> table | D |
| IV | association | A | IV | outlier | C |

2. Partners, choose a pair of numbered matched cards and record the attributes that are the same and those that are different. Answers will vary. One possible answer:

3. Partners, choose another pair of numbered matched cards and discuss the atributes that are the same and those that are different.

## ESTIMATING FISH POPULATIONS

Follow your teacher's directions for (1) - (3).
(1) $-(2)$

| Total number of marked fish: | Marked fish sample: |
| :--- | :--- |
| Total fish population: | Total fish sample: |

(3)

|  | Sample 1 | Sample 2 | Sample 3 | Sample 4 | Sample 5 | Sample 6 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{S}_{\text {marked }}$ |  |  |  |  |  |  |
| $\boldsymbol{S}_{\text {total }}$ |  |  |  |  |  |  |
| $\boldsymbol{P}_{\text {marked }}$ |  |  |  |  |  |  |

3. Create and solve equations to estimate the number of fish in the lake for each sample.

| Sample 1 | Sample 2 | Sample 3 |
| :---: | :---: | :---: |
| Sample 4 | Sample 5 |  |
|  |  | Sample 6 |
|  |  |  |

4. Based on your experiment, estimate the number of fish in the population. $\qquad$
Explain how you arrived at this estimate.
5. Count all of the fish. The actual number of fish in the population is $\qquad$ .
6. Find the error in your estimate as a percent of the actual population. $\qquad$

## LESSON NOTES S10.3: ESTIMATING FISH POPULATIONS

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.
Students learn about a "mark-recapture" technique used to estimate the population of fish in a lake. In this procedure, biologists use traps to capture the fish and safely mark them in some way. The fish are then returned to the lake. Over a period of time, the fish are trapped again and recorded as a ratio of marked fish to the total number trapped to estimate the total population. See Prepare Ahead for set up ideas. If desired, introduce the simulation with a YouTube video about the mark-recapture process with ants: (https://youtu.be/ekSjRsnhJrk).

- Slide 1: Discuss the mark-recapture process. Make sure students understand the goal (research question) and the materials that will be used. This slide gives examples. Any two colors of small, equal-sized objects will do.

How is a mark-recapture method like random sampling? Trapping the fish is a random collection process. We can't know the number of marked fish versus unmarked fish for any given trapping.

- Slide 2: Show students how to set up their "lake" for the experiment. Students may mark as many fish as they like.

How many fish do you think you will need to mark to get good estimates from sampling? Answers will very. Typically, the more fish that are marked, the more accurate the estimates. Sample size will also affect accuracy. Students will learn this from experience.

Distribute supplies so groups can set up their lakes.


- Slide 3: For (1), students identify and define the variables and copy them on their paper. Discuss subscript notation. Students may recall this notation was used in the trapezoid area formula $\left[A=\frac{1}{2} h\left(b_{1}+b_{2}\right)\right]$.

For (2), students make note of the known value. Which variable's value is already known? The number of marked fish in the population ( $\rho_{\text {marked }}$ ). Why? We removed some fish from the bag, counted them, and changed their color.


Which variable will we need to solve for? The total number of fish in the population. Why? It is the value we are looking for in this research question.

## LESSON NOTES S10.3: ESTIMATING FISH POPULATIONS <br> Continued

- Slide 4: Use the slide to further explain the simulation and recording process. Specifically, students:
$\checkmark$ shake the lake and grab a sample;
$\checkmark$ record the number of marked fish in the sample ( $S_{\text {marked }}$ ) and the total fish in the sample ( $S_{\text {total }}$ );
$\checkmark$ record the number of marked fish in the population
( $P_{\text {marked }}$ ), which is a known value from the setup;
$\checkmark$ return the sample to bag;
$\checkmark$ repeat-collect a total of 6 samples.
For (3), students perform the simulation and collect data. The remainder of the page guides students to solve for the unknown value in each sample, draw conclusions, compare estimates to the actual number of fish in the lake, and compute percent error.
- Slide 5: Use this slide as desired as an example of how to estimate the total population with data provided. We assume a proportional relationship.

Which variable is not in the table? The total population of fish. Why? This is what we are trying to determine.

What are some ways to make sense of the six estimates computed from the equations? Answers will vary. Students may want to discard outliers, or highs and lows. They may want to compare the mean and median of the estimates. They may want to combine all data. Allow students to try different strategies and share methods as part of this modeling process.

- Slide 6: Discuss results. How did groups compute estimates? Some possible answers: finding an average or median of one group's computations or among groups.


## Would changing the number of samples change results?

 Possibly. More samples may lead to better estimates.What are some assumptions scientists might make when using a mark-recapture technique? Each fish is equally likely to be caught. The population does not change between samples. Tags/markings will not fall off the fish.

## PERFORM THE EXPERIMENT

Here is how to perform the simulation

- Shake the lake and grab a scoop of fish.

Which variables are represented in this sample?


- Fill in the chart for YOUR Sample 1.
- Return the fish to the lake and repeat this process for Samples 2-6.
(3) Perform the simulation. Fill in the table as you go.
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## SLIDE DECK ALTERNATIVE S10.3: ESTIMATING FISH POPULATIONS

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slide 1
Your group will answer the question, "How many fish are in Lake Calculus?" using a "mark-recapture" simulation method.

The Supplies

| Actual Objects | Lake | Net | Fish in the lake <br> (unmarked) | Marked fish |
| :---: | :---: | :---: | :---: | :---: |
| Simulated <br> Materials |  |  |  |  |

Slide 2
The Set Up


Slide 3
(1) Identify and define the variables.

| Total number of marked fish: | Marked fish sample: |
| :--- | :--- |
| Total fish population: | Total fish sample: |

(2) Write in your known value.

## SLIDE DECK ALTERNATIVE S10.3: ESTIMATING FISH POPULATIONS Continued

Slide 4
(3) Perform the simulation. Fill in the table as you go.

- Shake the "lake" and grab a scoop of fish.
- Fill in the chart for Sample 1
- Return the fish to the lake and repeat for Samples 2-6.

Slide 5
An Example for a Sample

| $S_{\text {marked }}$ | 2 |
| :---: | :---: |
| $S_{\text {total }}$ | 14 |
| $P_{\text {marked }}$ | 10 |

$$
\begin{array}{llll}
S_{\text {marked }}: S_{\text {total }} & \rightarrow 2: 14 & \rightarrow & \frac{2}{14} \\
P_{\text {marked }}: P_{\text {total }} & \rightarrow 10: x & \rightarrow & \frac{10}{x}
\end{array}
$$

Slide 6
How did you compute your estimate? How close were you?
What assumptions might scientists make when estimating fish using a mark-recapture technique?

Explain in what ways our process followed this modeling cycle.


