

PROPORTIONAL REASONING 3 STUDENT PACKET

PROPORTIONAL REASONING APPLICATIONS

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Parent (or Guardian) signature _____

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. (See section 3.5.) Key mathematical vocabulary is underlined throughout the packet.

cross-multiplication property	proportion
proportional relationship	scale
scale drawing	scale factor

TWINKIE, THE DOG

Twinkie, the Jack Russell Terrier, pops balloons. Will she break the world record? Follow your teacher's directions to learn more about this amazing dog.



A black and white photograph of a small dog, possibly a Chihuahua, sitting on a large, dark, reflective surface. The dog is looking towards the camera. The background is dark and indistinct. The image is positioned in the top right corner of a large grid of small squares.

PROPORTIONAL REASONING

We will use proportional reasoning strategies to solve problems.

GETTING STARTED

Compute.

1. $\$2.56 + \3.29	2. $\$8.23 - \4.68	3. $4 \bullet (\$1.57)$	4. $\frac{\$8.61}{7}$
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Use your knowledge of equivalent fractions to solve for x .

5. $\frac{2}{9} = \frac{6}{x}$	6. $\frac{30}{36} = \frac{x}{6}$	7. $\frac{x}{5} = \frac{12}{20}$
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Find the cost. Try using sense-making strategies. See section 3.5 for ideas as needed.

8. If one pencil costs 35¢, what is the cost of 4 pencils?	9. If 6 pencils cost \$2.40, what is the cost of 1 pencil?
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ART SUPPLIES

Mrs. Carter is buying art supplies. Help her determine the cost and quantities for some items she needs. Assume costs and quantities are in a proportional relationship. Find this phrase in section 3.5 and record its definition in My Word bank. Then follow your teacher's directions.

[illegible]

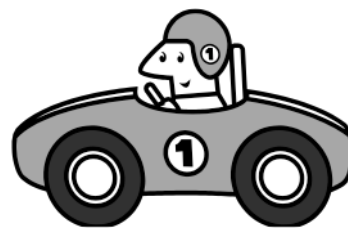
PRACTICE 1

Solve these problems using the given strategy. Check your work using another strategy of your choice.

1. Use a table.

Jake's car used 15 gallons of gas to travel 330 miles.

- a. If the car continues to consume gas at this same rate, how far can the car go on 20 gallons?
- b. How many miles per gallon does the car get?

**2. Use a double number line.**

Angie paid \$14 for 4 gallons of gas.

- a. At this rate, how many gallons of gas can she buy for \$35?
- b. How much will 14 gallons of gas cost?
- c. What is the price per gallon?



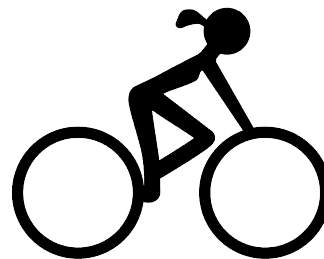
PRACTICE 2

Solve these problems using the strategy given. Check your work using another strategy of your choice.

1. Use a table or a double number line.

Samara biked 6 miles in 30 minutes.

- At that rate, how far could she go in 2 hours?
- At that rate, how far could she go in 1 hour?
- At that rate, how long would it take her to go 15 miles?



2. Use some form of arithmetic, such as a unit rate or a chunking strategy.

Greg is training for a marathon. He ran 21 miles in $3\frac{1}{2}$ hours.

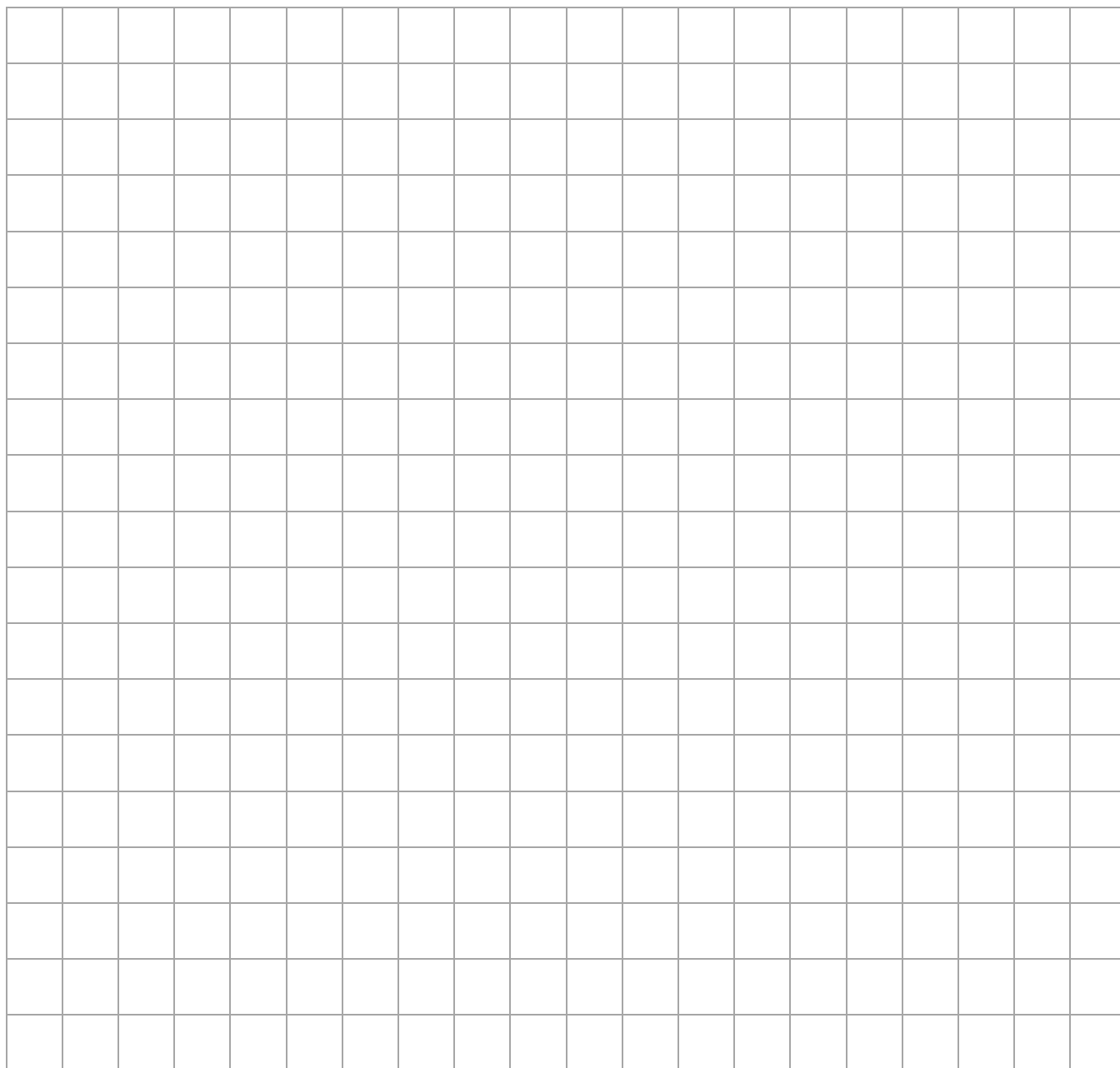
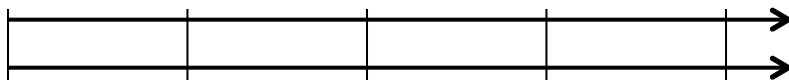
- At that pace, how far did he run in one hour?
- At that pace, *about* how long will it take him to run the marathon (26.2 miles)?



PROPERTIES OF PROPORTIONS

Follow your teacher's directions to learn about proportions. Find this word in section 3.5 and record its definition in My Word Bank.

Four baseballs cost \$_____.



PRACTICE 3

Use equivalent fractions or the cross-multiplication property to solve each equation.

1. $\frac{2}{5} = \frac{x}{20}$	2. $\frac{3}{55} = \frac{x}{55}$	3. $\frac{137}{5} = \frac{x}{55}$
4. $\frac{2}{x} = \frac{3}{13}$	5. $\frac{1}{2} = \frac{5}{x}$	6. $\frac{2.5}{5} = \frac{x}{12}$

7. Some students explored the equation $\frac{3}{5} = \frac{6}{10}$, and rewrote it in a few different ways.

Circle the two true equations. For the equation that is not true, explain to that student why it is not true and how to revise his work.

Abner: $\frac{3}{6} = \frac{5}{10}$

Mick: $\frac{6}{3} = \frac{5}{10}$

Buck: $\frac{5}{3} = \frac{10}{6}$

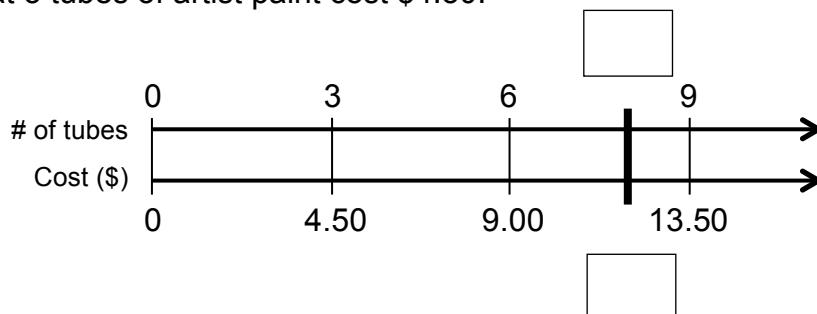
8. Rewrite the equation $\frac{2}{7} = \frac{6}{21}$ in three other ways to create true equations.

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ART SUPPLIES - REVISITED

Use a double number line to help you set up proportions and solve problems.

Recall that 3 tubes of artist paint cost \$4.50.

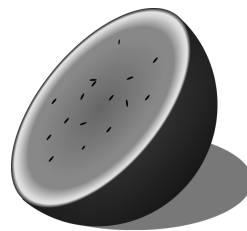


1. How many tubes can you buy for \$12?
 - a. Fill in the boxes above to indicate 12 dollars and x tubes.
 - b. Write a proportion and solve it. Then answer the question.
2. What is the cost of 50 tubes of paint?
3. How many tubes of paint can you buy for \$42?
4. What is the unit price for a tube of paint?

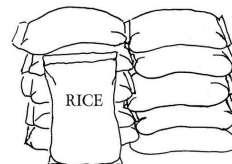
PRACTICE 4

Solve using strategies of your choice (tables, unit prices, double number lines, equivalent fractions, proportions). See section 3.5 for different strategies.

1. At the Green Grocer, 2 melons cost \$3.50, and you can purchase as many as you want at this same rate.
 - a. What is the cost per melon?
 - b. How much will 42 melons cost?
 - c. How many melons can be purchased for \$84?



2. At the Grain Grocer, 5 pounds of rice cost \$4.00, and you can purchase any amount at this rate.
 - a. What is the cost per pound?
 - b. How many pounds can be purchased for \$1.00?
 - c. Carlos needs two pounds of rice for a casserole. How much will that cost?



BEST BUY PROBLEMS

We will use tables, graphs, and equations to learn more about the behavior of proportional relationships.

GETTING STARTED

Circle the better buy for each situation below and explain your reasoning. No calculations are necessary.

1. 0.75 pounds of oranges for \$1.00 or 1.25 pounds of oranges for \$1.00	2. 3 pounds of bananas for \$3.65 or 3 pounds of bananas for \$4.15
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Suppose you are running out of your favorite energy snacks, so you compare prices at two stores before making a purchase.

BARTER JACK'S

Healthy Crunch: 2 for \$2.50
Super Bar: 3 for \$3.25

QUIGLEY'S

Healthy Crunch: 2 for \$2.75
Super Bar: 4 for \$3.25

3. Without doing any calculations, explain which store offers the better buy for Healthy Crunch.	4. Without doing any calculations, explain which store offers the better buy for Super Bar.
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SOCKS

Follow your teacher's directions to explore ways to represent which store has the better buy.

SOX 'R US

CRAZY SOCKS

This image shows a full page of blank graph paper. The grid consists of thin, light gray horizontal and vertical lines that intersect to form a uniform pattern of small squares across the entire surface. There are no margins, text, or other markings present.

TORTILLAS

FLAT 'N ROUND
3 tortillas for \$0.60

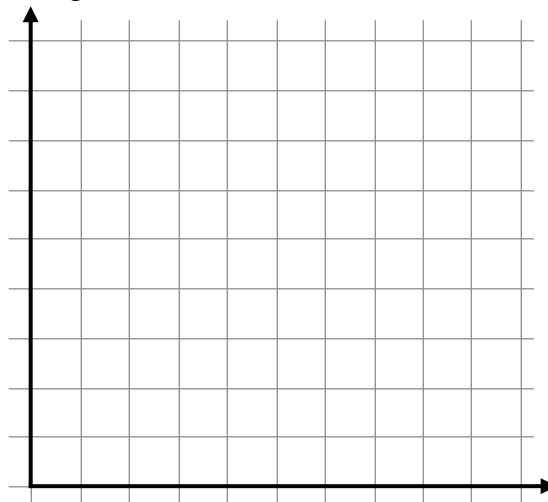
WRAP IT UP
4 tortillas for \$1.00

1. Complete the tables below. Assume each shop will sell any number of tortillas at the rates shown.

FLAT 'N ROUND	
# of tortillas (x)	cost (\$) (y)
3	
6	
9	
2	
1	

WRAP IT UP	
# of tortillas (x)	cost (\$) (y)
4	
8	
12	
2	
1	

2. Label and scale the grid. Graph the data using two different colors.



3. Identify the y-coordinate when $x = 1$.
FLAT 'N ROUND (1, _____)
WRAP IT UP (1, _____)

4. Write equations to relate the number of tortillas to the cost.

FLAT 'N ROUND: $y = \underline{\hspace{1cm}}x$

WRAP IT UP: $y = \underline{\hspace{1cm}}x$

5. How are the coordinates for the ordered pairs in problem 3 related to the equations in problem 4?
6. How do you know that the point (0, 0) satisfies the equations?
7. Why do these graphs and equations suggest proportional relationships?

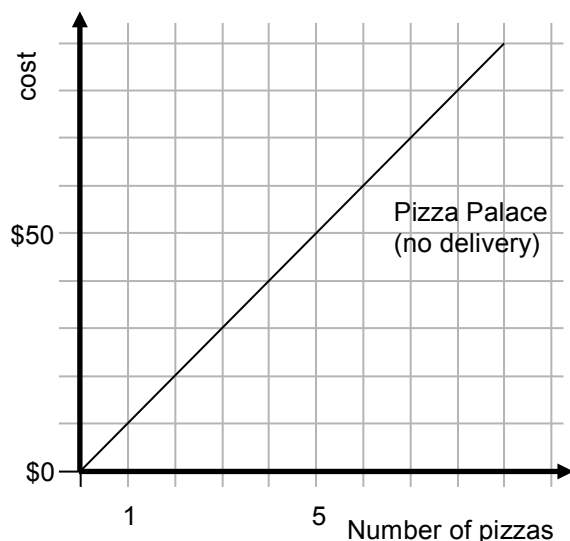
PRACTICE 5

A graph for Pizza Palace prices is given below. They also offer delivery for any number of pizzas for a fee of \$5.00.

1. Complete the tables.

Pizza Palace (no delivery)		Pizza Palace (with delivery)	
# of pizzas (x)	cost \$ (y)	# of pizzas (x)	cost \$ (y)
5		5	55
4		4	
3		3	
2		2	
1		1	

2. Graph Pizza Palace with delivery.



3. Write equations that relate cost (y) to number of pizzas (x).

Pizza Palace: $y = \underline{\hspace{1cm}}x$

Pizza Palace (with delivery): $y = \underline{\hspace{1cm}}x + \underline{\hspace{1cm}}$

4. Compare unit prices for each store. Use a calculator if needed.

no delivery	$\frac{\text{cost in dollars}}{\text{\# of pizzas}}$	$\frac{50}{5}$	$\frac{\boxed{\hspace{1cm}}}{4}$			
	Unit price (in dollars/pizza)	10				

with delivery	$\frac{\text{cost in dollars}}{\text{\# of pizzas}}$	$\frac{\boxed{\hspace{1cm}}}{5}$	$\frac{\boxed{\hspace{1cm}}}{4}$			
	Unit price (in dollars/pizza)					

5. Which of these situations represents a proportional relationship?
Why?

6. Which of these situations does not represent a proportional relationship?
Why?

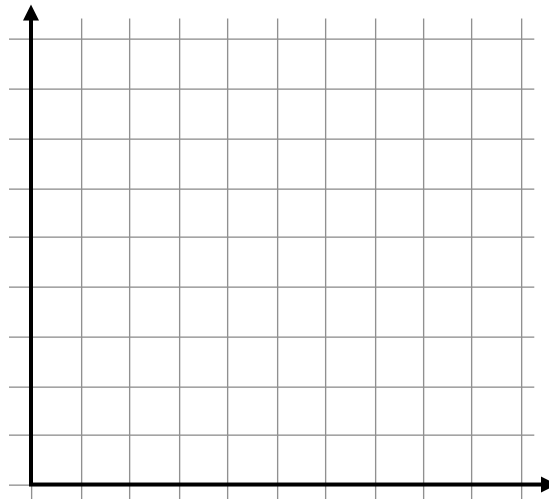
PRACTICE 6

1. Here are some ticket price options at a local amusement park. Find the unit price for the different plans.



number of tickets (x)	cost \$ (y)	$\frac{\text{cost}}{\text{ticket}}$
1	3	
5	15	
10	20	
15	25	
20	28	

2. Graph the relationship between number of tickets and cost. Be sure to label and scale axes appropriately.



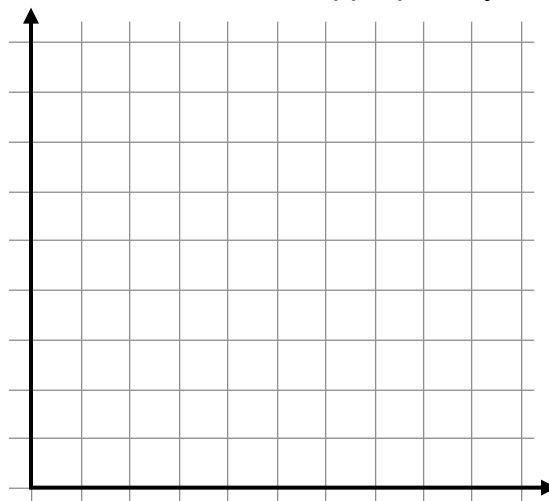
3. Does the ticket pricing represent a proportional relationship? How do you know?
4. Which ticket option would you choose? Why?

5. Here are some costs and quantities for purchasing baseballs. Find the unit price for different quantities.



# of baseballs (x)	cost \$ (y)	unit price
4	10	
16	40	
2	5	
5	12.50	

6. Graph the relationship between number of baseballs and the cost. Be sure to label and scale axes appropriately.



7. Does this represent a proportional relationship? How do you know?

SCALE DRAWINGS

We will make and interpret scale drawings. We will learn the meaning of scale factor and scale.

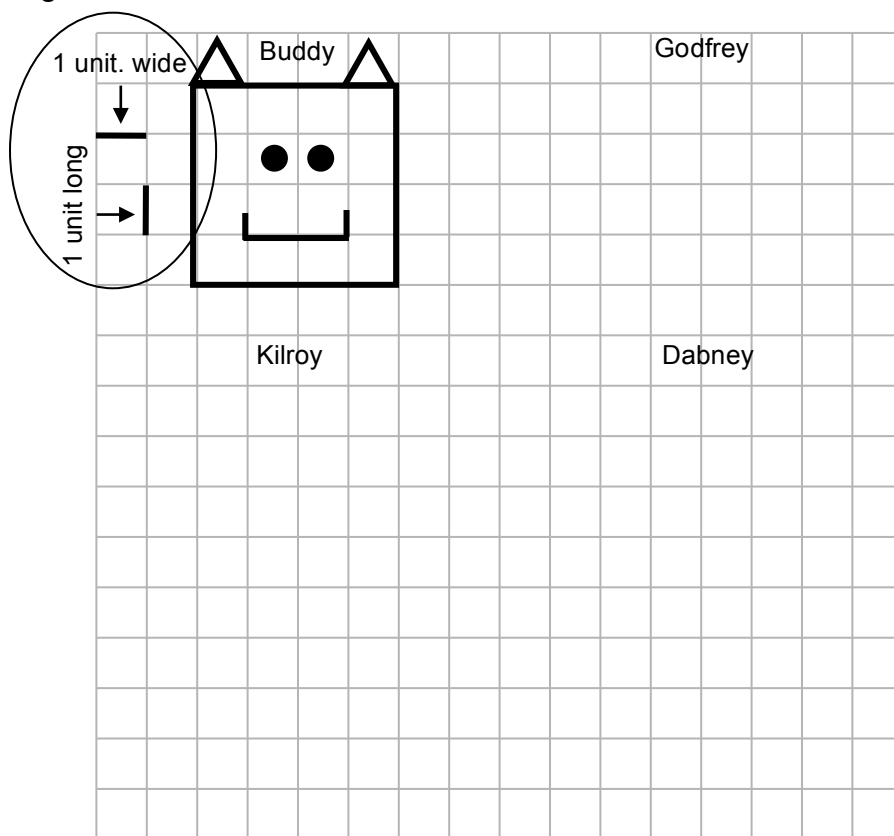
GETTING STARTED

Change all parts of Buddy's face given the following directions to create three more faces. Pay close attention to "width" and "length."

1. Godfrey's face is twice as wide and just as long as Buddy's face. Draw Godfrey's face.

2. Kilroy's face is twice as long and just as wide as Buddy's face. Draw Kilroy's face.

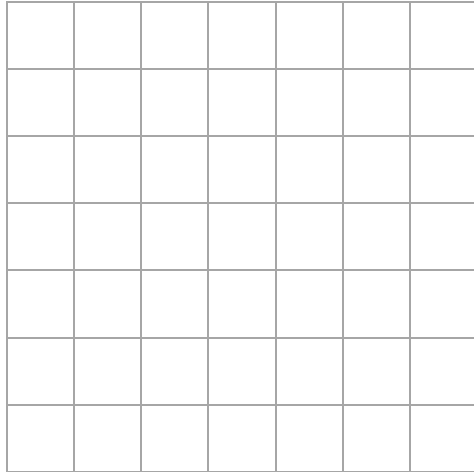
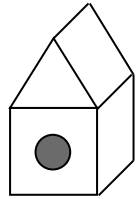
3. Dabney's face is twice as long and twice as wide as Buddy's face. Draw Dabney's face.



4. Look up scale factor in section 3.5, discuss it in class, and record it in My Word Bank.
5. Whose face represents Buddy's face scaled by a scale factor of 2?
6. Whose face represents Dabney's face scaled by a scale factor of $\frac{1}{2}$?
7. Which two faces look the most alike? _____ and _____
Be prepared to defend your opinion to your classmates.

A BIRD HOUSE

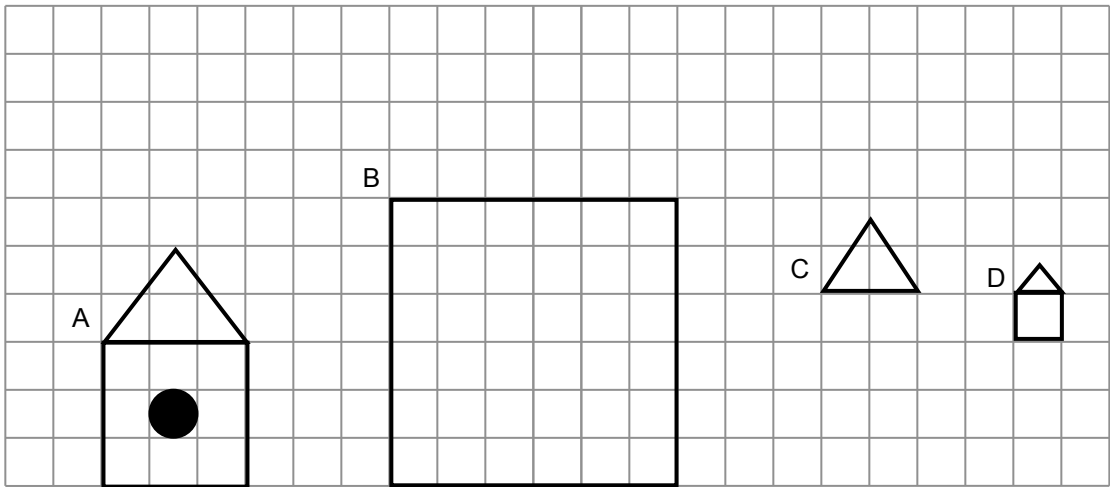
This is a birdhouse. Follow your teacher's directions to explore scale. Be sure to record this in My Word bank.



ENLARGEMENTS AND REDUCTIONS

Natasha is making scale drawings of a birdhouse she wants to build. She completed scale drawing A on graph paper for the front face of the birdhouse. Then she started drawings B, C, and D.

1. Complete drawings B, C, and D below.



2. Complete the table.

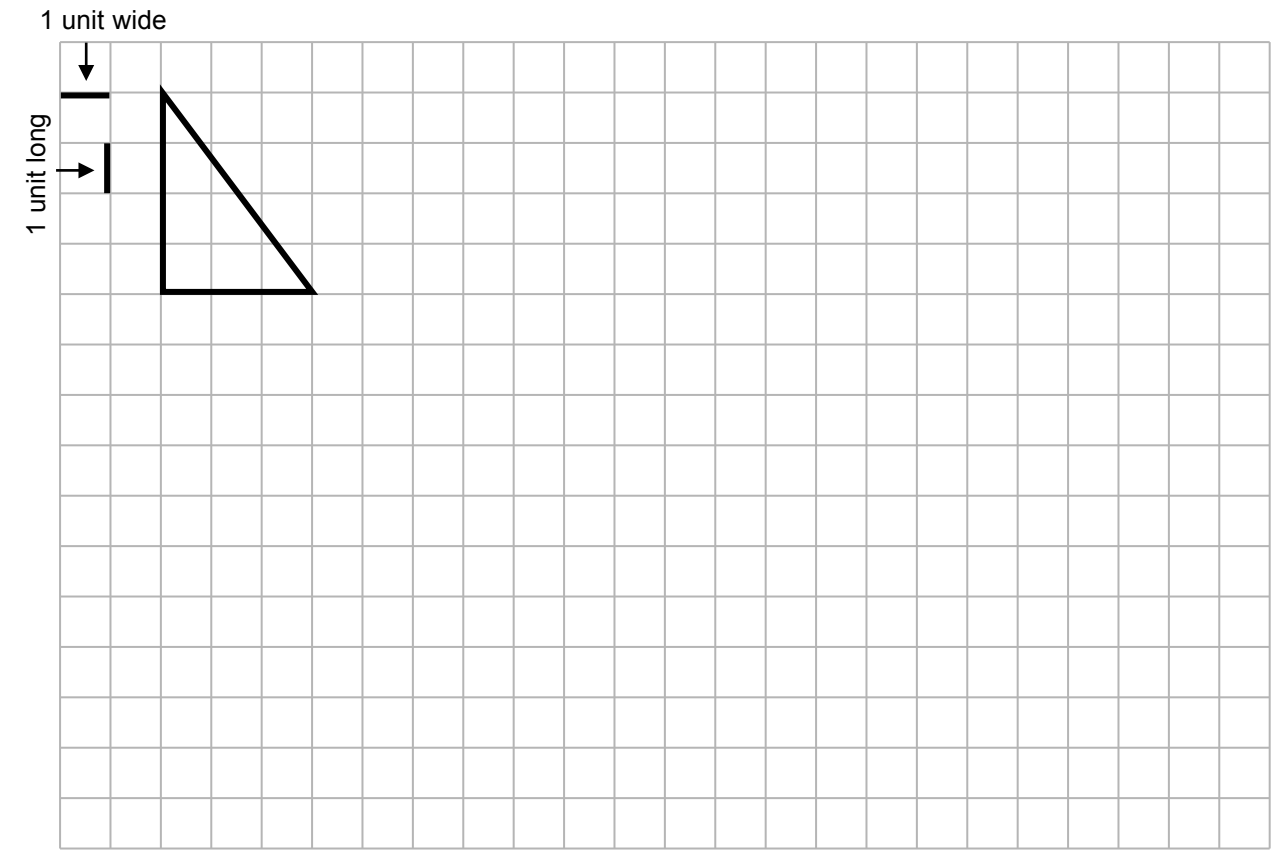
Drawing	Reduction or enlargement compared to drawing A?	Scale factor (multiplier) compared to drawing A	Scale (ratio) compared to drawing A
A			
B			
C			
D			

PRACTICE 7

Another way to describe scale factor is as a percent. For example, a scale factor of 2 could also be described as a scale factor of 200%.

Based on triangle A below, complete the table and draw each triangle on the grid paper.

Triangle	Compared to Triangle A				Height (length) “long”	Base (width) “wide”
	Scale Factor (as a percent)	Scale factor (as a number)	Scale (ratio)	Enlargement or Reduction		
A	100%			neither		
B	300%		3 : 1			
C		$0.5 = \frac{1}{2}$				
D	25%					
E		2				
F	150%					



REVIEW

POSTER PROBLEMS: IT'S ABOUT TIME!

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _____.
- Each group will have a different colored marker. Our group marker is _____.

Part 2: Do the problems on the posters by following your teacher's directions. Use a calculator as needed.

Poster 1 (or 5)	Poster 2 (or 6)	Poster 3 (or 7)	Poster 4 (or 8)
A watch gains 2 minutes in 6 hours.	Mary read 22 pages in 30 minutes.	Betsy cooks 17 hours in a 2-week period.	Hurricane Katrina dropped 14 inches of rain over a 48-hour period.

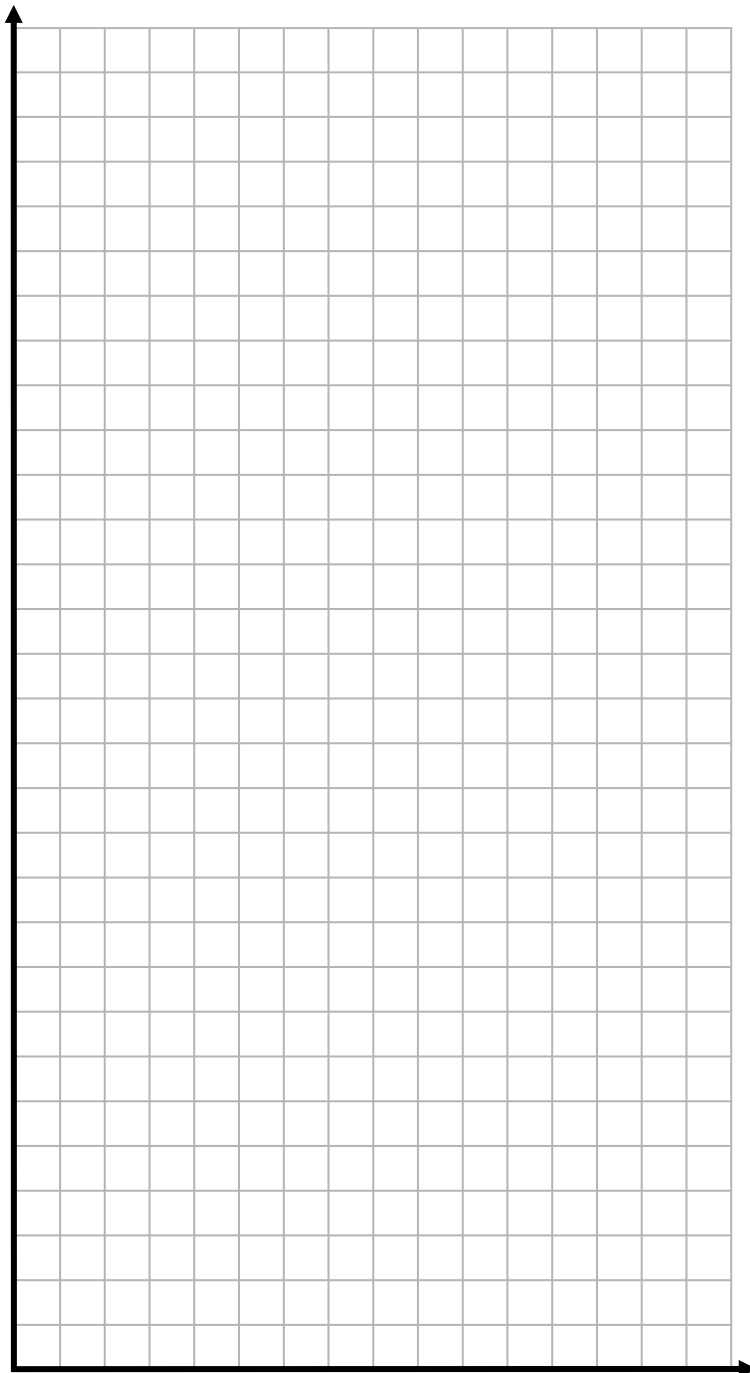
- A. Copy the fact statement. Write two unit rates to describe the situation. The first uses the given rate of time, the second is equivalent, but uses a different unit of time.
- B. Assume a proportional relationship. Make a double number line that compares the quantities for different reasonable amounts of time.
- C. Write a question that can be answered using the fact statement on your double number line.
- D. Answer the question asked in part C.

Part 3: Return to your seats. Work in partners or groups.

- Look back at each fact statement. Is it reasonable for this relationship to hold up over an extended period of time? Explain.
- If a watch gains 2 minutes every 6 hours, what time is it?

MATCHING ACTIVITY: NUTS

1. Your teacher will give you some cards that represent proportional relationships (one card has an error). Work with a partner to match cards with equivalent representations and find the error.
2. What was the error? How do you know? Fix it on the card.
3. Graph the cost vs. quantity for each mixture on the graph using different colors.
4. Do you think the points should be connected? Explain.



SPORTS PLAYING SURFACES

You will make scale drawings of sports playing surfaces.

1. Draw a double number line that shows a scale of $\frac{1}{2}$ inch : 10 feet.

Determine the dimensions of each sports surface if the scale is $\frac{1}{2}$ inch : 10 feet. You may want to use the double number line from problem 1 to help you.

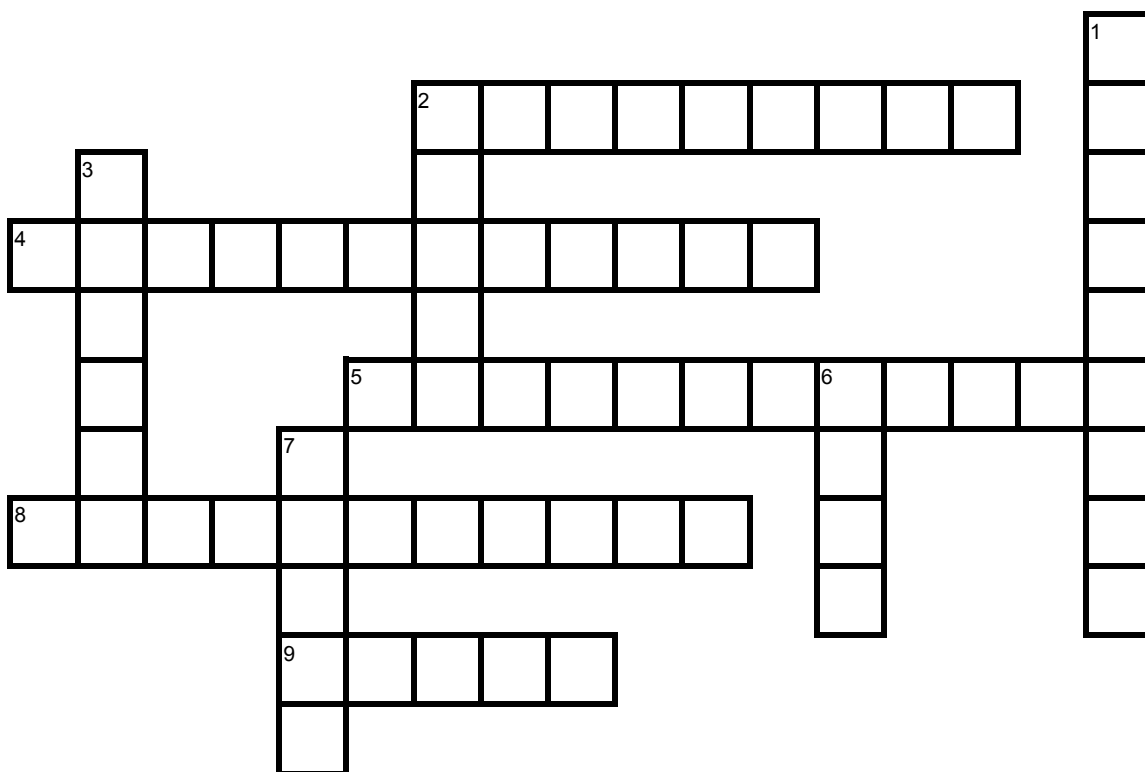
	Sport Surface	Actual Length	Actual Width	Drawing length	Drawing width
2.	Soccer Field	400 ft	300 ft		
3.	Volleyball Court	60 ft	30 ft		
4.	Football Field	360 ft	160 ft		
5.	Roller Rink	70 ft	150 ft		
6.	Bowling Lane	60 ft	4 ft		
7.	(Your choice – research on internet)				

8. **Project:** Using tools of your choice, choose two of the sports surfaces above and create scale drawings. You may want to research other features on the internet to include on your scale drawings. Cut them out and label completely.

I made scale drawings for a _____ and a _____.

9. Use your drawings and explain approximately how many copies of your smaller sports surface will fit inside your larger sports surface.

VOCABULARY REVIEW

**Across**

- 2 The result of a scale factor between 0 and 1.
- 4 The graph of a ____ relationship is a straight line through the origin.
- 5 _____ lines are parallel lines used to show a proportional relationship (two words).
- 8 The result of a scale factor greater than 1.
- 9 _____ factor (a multiplier).

Down

- 1 Cost for 1 item. (two words).
- 2 A comparison of two numbers.
- 3 The point (0,0) on a graph.
- 6 Rate for 1 item; _____ rate.
- 7 A strategy for solving proportions; _____ multiplication property.

DEFINITIONS, EXPLANATIONS, AND EXAMPLES

Word or Phrase	Definition
cross-multiplication property	<p>The <u>cross-multiplication property</u> for proportions states that if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.</p> <p>From $\frac{2}{3} = \frac{8}{12}$ we have $2 \bullet 12 = 3 \bullet 8$.</p>
proportion	<p>A <u>proportion</u> is an equation stating that the values of two ratios are equal.</p> <p>The equation $\frac{3}{25} = \frac{12}{100}$ is a proportion. It asserts that the values of the ratios $3 : 25$ and $12 : 100$ are equal. The value of both ratios is 0.12.</p>
proportional	<p>Two variables are <u>proportional</u> if the values of one are the same constant multiple of the corresponding values of the other. The variables are said to be in a <u>proportional relationship</u>, and the constant is referred to as the <u>constant of proportionality</u>.</p> <p>If Wrigley eats 3 cups of kibble each day, then the number of cups of kibble is proportional to the number of days. If x is the number of days, and y is the number of cups of kibble, then $y = 3x$. The constant of proportionality is 3.</p>
proportional relationship	See <u>proportional</u> .
scale	<p>In a scale drawing of a figure, the <u>scale</u> is the ratio of lengths in the drawing to lengths in the figure.</p> <p>A blueprint of a house floorplan has a scale of 1 inch to 5 feet, or $1 \text{ in} : 5 \text{ ft}$. Each inch on the blueprint represents 5 feet in the actual house.</p> <p>A drawing of a lady bug has a scale of 5 cm to 2 millimeters, or $5 \text{ cm} : 2 \text{ mm}$. Each 5 cm on the drawing represents 2 mm on the actual bug.</p>
scale drawing	<p>A <u>scale drawing</u> of a geometric figure is a drawing in which all lengths have been multiplied by the same scale factor, while angles remain the same.</p> <p>A blueprint of a house floorplan is a scale drawing.</p>

Word or Phrase	Definition
scale factor	<p>A <u>scale factor</u> is a positive number which multiplies some quantity.</p> <p>To make a scale drawing of a figure, we multiply all lengths by the same scale factor, keeping all angles equal to those in the original figure. If the scale factor is greater than 1, the scale drawing is an <u>enlargement</u> of the actual figure. If the scale factor is between 0 and 1, the scale drawing is a <u>reduction</u> of the actual figure.</p> <p>A drawing of a ladybug has a scale of 5 cm : 2 mm. This is equivalent to 50 mm : 2 mm. The scale factor is $\frac{50}{2} = 25$. The drawing is an enlargement.</p> <p>A blueprint of a house floorplan has a scale of 1 in : 5 ft. This is equivalent to 1 in : 60 in. The scale factor is $\frac{1}{60}$. The blueprint is a reduction.</p>
unit price	A <u>unit price</u> is a price for one unit of measure.
unit rate	<p>The <u>unit rate</u> associated with a ratio $a : b$ of two quantities a and b, $b \neq 0$, is the number $\frac{a}{b}$, to which units may be attached.</p> <p>The ratio of 40 miles each 5 hours has unit rate of 8 miles per hour.</p>
value of a ratio	<p>The <u>value of the ratio</u> $a : b$ is the number $\frac{a}{b}$, $b \neq 0$.</p> <p>The value of the ratio 6 : 2 is $\frac{6}{2} = 3$.</p> <p>The value of the ratio of 3 to 2 is $\frac{3}{2} = 1.5$.</p>

Sense-Making Strategies to Solve Proportional Reasoning Problems

How much will 5 pencils cost if 8 pencils cost \$4.40?

Strategy 1: Use a “halving” strategy

If 8 pencils cost \$4.40, then
 4 pencils cost \$2.20,
 2 pencils cost \$1.10, and
 1 pencil costs \$0.55.

Therefore, 5 pencils cost

$$\$0.55 + \$2.20 = \$2.75.$$

Strategy 2: Find unit prices

First, find the cost of one pencil.

$$\frac{\$4.40}{8} = \$0.55$$

Then, multiply by 5 to find the cost of 5 pencils,

$$(\$0.55)(5) = \$2.75.$$

Sammie can crawl 12 feet in 3 seconds. At this rate, how far can she crawl in $1\frac{1}{2}$ minutes?

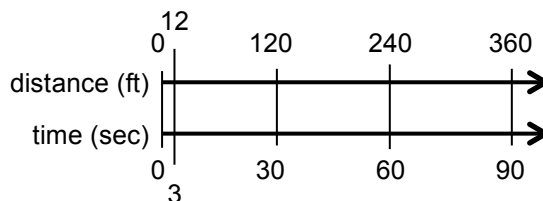
Strategy 1: Make a table

Distance	Time
12 ft	3 seconds
4 ft	1 second
240 ft	60 sec = 1 min
120 ft	30 sec = $\frac{1}{2}$ min
360 ft	90 sec = $1\frac{1}{2}$ min

Sammie can crawl 360 feet in $1\frac{1}{2}$ minutes.

Strategy 2: Make a Double Number Line

$$1\frac{1}{2} \text{ minutes} = 90 \text{ seconds.}$$



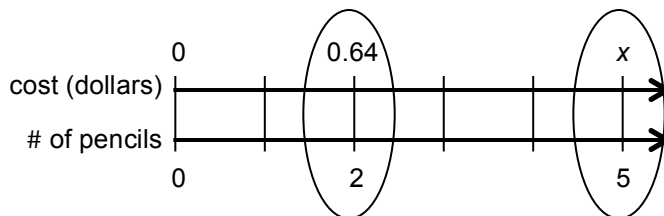
Sammie can crawl 360 feet in $1\frac{1}{2}$ minutes.

Setting Up Proportions

Here are some ways to set up a proportion to solve a problem.

If 2 pencils cost \$0.64, how much will 5 pencils cost?

Strategy 1: Compare rates. This is sometimes referred to as a “between” proportion because the ratios contain different units (i.e., between two units). Between proportions can be read directly from a double number line.



Organize known information on a double number line if desired. From the diagram we see:

$$\frac{0.64 \text{ dollars}}{2 \text{ pencils}} = \frac{0.64 \text{ dollar}}{2 \text{ pencil}}$$

$$\frac{x \text{ dollars}}{5 \text{ pencils}} = \frac{x \text{ dollar}}{5 \text{ pencil}}$$

Then, equate the two expressions and solve for x .

$$\frac{x}{5} = \frac{0.64}{2}$$

$$x = 1.60 \text{ dollars for 5 pencils.}$$

Note: The equation $\frac{5}{x} = \frac{2}{0.64}$ is another valid “between” proportion for this problem.

Strategy 2: Compare like units. This is sometimes referred to as a “within” proportion because the ratios contain the same units (i.e., within the same unit).

Use a double number line to organize information if you wish.

Create one ratio based on corresponding costs and another ratio based on the corresponding numbers of pencils.

$$\frac{\text{cost}_{\text{case 1}}}{\text{cost}_{\text{case 2}}} = \frac{0.64}{x}$$

$$\frac{\text{pencils}_{\text{case 1}}}{\text{pencils}_{\text{case 2}}} = \frac{2}{5}$$

Then, equate the two ratios, and solve for x .

$$\frac{0.64}{x} = \frac{2}{5}$$

$$x = 1.60 \text{ dollars for 5 pencils.}$$

Note: The equation $\frac{x}{0.64} = \frac{5}{2}$ is another valid “within” proportion for this problem.

Some Properties Relevant to Solving Proportions

Here are some important properties of arithmetic and equality related to proportions.


- The multiplication property of equality states that equals multiplied by equals are equal. Thus, if $a = b$ and $c = d$, then $ac = bd$.

Example: If $\frac{6}{2} = 3$ and $5 = 9 - 4$, then $\frac{6}{2}(5) = 3(9 - 4)$.

- The fraction-inverse property for proportions states that if two nonzero fractions are equal, then their inverses are equal. That is, if $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$ ($a \neq 0, b \neq 0, c \neq 0, d \neq 0$).

Example: If $\frac{5}{7} = \frac{12}{x}$, then $\frac{7}{5} = \frac{x}{12}$

- The cross-multiplication property for proportions states that if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$ ($b \neq 0, d \neq 0$).

This can be remembered with the diagram: .

Example: If $\frac{5}{7} = \frac{12}{x}$, then $5 \cdot x = 7 \cdot 12$.

To see that this property is reasonable, try simple numbers:

If $\frac{3}{4} = \frac{6}{8}$, then $3 \cdot 8 = 4 \cdot 6$.

Applying Properties to Solve Proportions

Strategy 1: Multiplication Property of Equality

Solve for x :

$$\begin{aligned} \frac{x}{12} &= \frac{3}{8} && \text{Multiplication} \\ &&& \text{Property of Equality} \\ (8 \cdot \cancel{12}) \cdot \frac{x}{\cancel{12}} &= \frac{3}{\cancel{8}} \cdot (\cancel{8} \cdot 12) \\ 8x &= 36 \\ x &= \frac{36}{8} \\ x &= 4\frac{1}{2} \end{aligned}$$

Strategy 2: Cross-Multiplication Property

Solve for x :

$$\begin{aligned} \frac{x}{12} &= \frac{3}{8} && \text{Cross-multiplication} \\ 8 \cdot x &= 3 \cdot 12 && \text{property} \\ 8x &= 36 \\ x &= \frac{36}{8} \\ x &= 4\frac{1}{2} \end{aligned}$$

Testing for a Proportional Relationship

Here are three ways to test if two variables are in a proportional relationship:

- The values of the ratios (unit rates or unit prices) created by data pairs are the same.
- An equation in the form $y = kx$ fits all corresponding data pairs.
- Graphed data pairs fall on a line through the origin (0, 0).

Alexa buys tickets when she goes to the amusement park. This chart shows the costs for different quantities of tickets.

# of tickets	10	20	25	50	100
total cost	\$40	\$60	\$75	\$125	\$200
cost per ticket	\$4	\$3	\$3	\$2.50	\$2

Since the costs per ticket (unit prices) are not the same, ticket purchasing at this amusement park does not represent a proportional relationship.

Antonio kept track of the number of miles he traveled each time he filled his tank with gas. Here is some data.

number of miles	100	200	175	300
number of gallons	4	8	7	12
miles per gallon	25	25	25	25

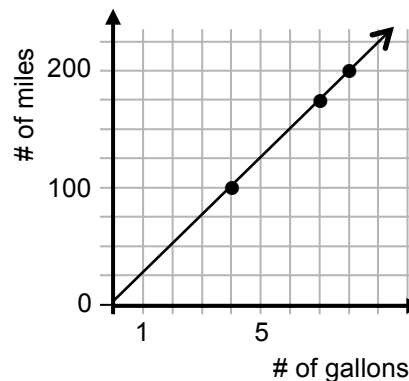
Since the miles per gallon (unit rates) created by the data pairs is the same, this situation represents quantities in a proportional relationship.

Furthermore,

Let x = the number of gallons
Let y = the number of miles

The data fits the equation $y = 25x$ (an equation in the form $y = kx$), which is an equation that represents a proportional relationship.

Finally, if the points for (gallons, miles) are graphed, they will fall on a line through the origin (0,0).



Multiple Representations and Proportional Relationships

Suppose 4 balloons cost \$6.00 and each balloon is the same price. Here are some strategies for representing this proportional relationship.

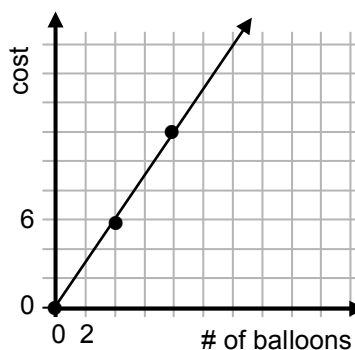
Strategy 1: Tables

Create a table to calculate unit rates. If the unit rates are the same, the variables are in a proportional relationship.

Number of Balloons	Cost	Unit Price
4	\$6.00	\$1.50
2	\$3.00	\$1.50
1	\$1.50	\$1.50
8	\$12.00	\$1.50

Strategy 2: Graphs

A straight line through the origin indicates quantities in a proportional relationship.



Strategy 3: Equations

An equation of the form $y = kx$ indicates quantities in a proportional relationship. In this case,

y = cost in dollars
 x = number of balloons
 k = cost per balloon (unit price)

To determine the unit price, create a ratio whose value is: $\frac{6 \text{ dollars}}{4 \text{ balloons}} = 1.50 \frac{\text{dollars}}{\text{balloons}}$

Therefore, $k = 1.50$ dollars per balloon, and

$$y = 1.50x.$$

This equation expresses the output as a constant multiple of the input, showing that the relationship is proportional.

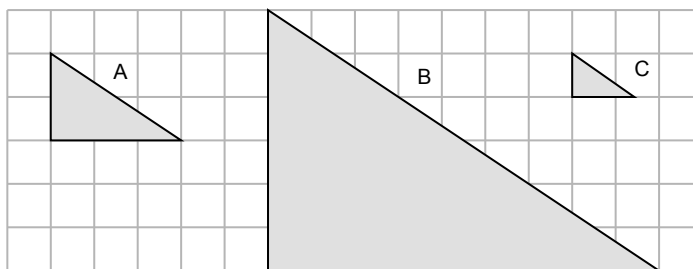
Scale Drawings

A scale drawing of a geometric figure is a drawing in which all distances have been multiplied by the same scale factor, while angles remain the same. If the scale factor is greater than 1, the figure is expanded (enlarged), and if the scale factor is between 0 and 1, the figure is reduced in size.

The ratio of lengths in the drawing to lengths in the actual figure is the scale of the drawing.

A scale of 1:1 implies that the drawing is the same as the actual object. A scale 1 : 2 implies that the drawing is smaller (half the size) than the actual object (in other words, the dimensions are multiplied by a scale factor of 0.5).

- To make Triangle B below, multiply each dimension of Triangle A by a scale factor of 3. Triangle B is a 300% enlargement of Triangle A. The scale is 3 : 1. The value of this ratio is 3.
- To make Triangle C below, multiply each dimension of Triangle A by a scale factor of $\frac{1}{2}$. Triangle C is a 50% reduction of triangle A. The scale is 1 : 2. The value of this ratio is $\frac{1}{2}$.



Scale Drawing of a Flag

The flag of Mexico is composed of three stripes (green, white, and red) that divide the flag into thirds. The national coat of arms is in the center of the white stripe. Pictured below is a (gray) scale drawing of the flag.

Suppose the original flag is 36 inches by 24 inches, and the scale drawing is 1.5 inches by 1 inch.

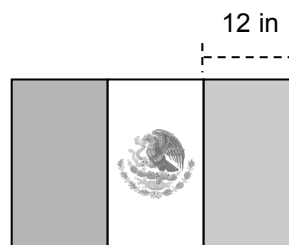
This scale may be represented as a ratio:

scale drawing : actual flag

1.5 inch : 36 inches

1 inch : 24 inches

1 : 24



The scale drawing is a reduction of the flag, with scale factor (value of the ratio) $\frac{1}{24}$.

The ratio "1 inch on the scale drawing represents 24 inches on the actual flag" is sometimes written with the technically incorrect, but convenient notation "1 inch = 24 inches." We will not do this because everyone knows that 1 inch does not really equal 24 inches!