PROPORTIONAL REASONING 1
STUDENT PACKET

RATIO REPRESENTATIONS

Parent (or Guardian) signature ___________________________________

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. (See section 1.5.) Key mathematical vocabulary is underlined throughout the packet.

double number line	equivalent ratios
ratio	tape diagram
unit price	unit rate

NANA'S CHOCOLATE MILK

Follow your teacher's directions. How would you fix the recipe?

What I should have done:	What I did:

TABLES AND TAPE DIAGRAMS

We will use ratios, tables, and tape diagrams to solve problems.

GETTING STARTED

1. According to the directions on a can of frozen orange juice concentrate, 3 cans of water are to be mixed with 1 can of concentrate to make orange juice.

   Draw a picture to illustrate this mixture.

2. Complete the table.

	Draw a picture to illustrate this mixture.	Will this mixture be "more orangey" than, "less orangey" than, or "the same" as recommended in the directions?
Maria adds 4 cans of water to one can of concentrate.		
Gustave adds 2 cans of water to one can of concentrate.		

PAINT MIXTURES

Your teacher will give you cards that indicate different ways to mix red paint (shaded) with white paint (un-shaded) to form shades of pink. Discuss with your partners and record your thinking.

1. Arrange the mixture cards from lightest pink to darkest pink and draw the results here.

Mixture Card:	Mixture Card:	Mixture Card:	Mixture Card:

Critique the reasoning of each student.

2. Jody said, “Mixture B and mixture C will be the same because they both have the same number of parts of red.

3. Ed said, “Mixture A and mixture D will be the same because mixture A has one more cup of white than red, and mixture D has one more cup of white than red.”

4. Choose a different pair of mixture cards than the ones discussed in 2 and 3 above and explain how you know which mixture represents the darker pink.

TAPE DIAGRAMS

1. On this page we will work with tape diagrams. Find the definition and examples in section 1.5, discuss them with your teacher and the class, and record in My Word Bank.

Here are two ways to draw a tape diagram to represent the mixture card B.

Draw tape diagrams to represent each paint mixture.

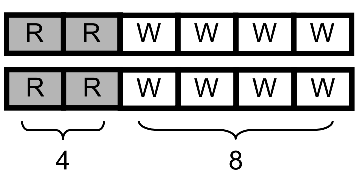
2. Mixture A:

3. Mixture C:

4. Mixture D:

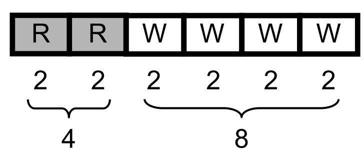
Alex and Lily were asked how many gallons of red paint and white paint were needed to make 12 gallons of paint that is the same shade as the mixture on card B.

5a. Explain Alex's method:



5b. Use Alex's method to find the amount of each ingredient needed to make 9 quarts of mixture C.

6a. Explain Lily's method:



6b. Use Lily's method to find the amount of each ingredient needed to make 72 ounces of mixture D.

PRACTICE 1

Use tape diagrams to solve these problems.

Zachary likes to make fruit soda when he has friends over to his house. He uses 4 parts juice for every 3 parts sparkling water.

1. Make a tape diagram to illustrate this mixture.

2. How much juice and how much sparkling water will he need if he wants to make 14 cups of fruit soda?

3. How much sparkling water should he use if he has 12 cups of juice?

4. How much juice should he use if he wants to make 70 cups of fruit soda?

Jane likes to make fruit soda too. Her recipe uses 2 parts juice and 1 part sparkling water.

5. Who makes a fruitier soda, Zachary or Jane? Explain how you know.

RATIOS AND TABLES

1. On this page we will work with ratios. Find the definition and examples in section 1.5, discuss them with your teacher and the class, and record in My Word Bank.

2. Use mixture cards A, B, C, and D. Write the ratios for each of the mixtures.

	Mixture A	**Mixture B**	**Mixture C**	**Mixture D**
Red to White				
White to Red				
Red to Total				

3. Complete each table below based on mixtures cards B and D.

Mixture B					
cups red	2	1	10		
cups white				1	7

Mixture D					
cups red	4	2		8	
cups white			20	10	1

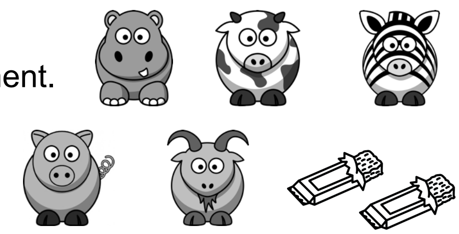
4. Use the entries in the table above to explain which is a darker pink: mixture B or mixture D?

5. Circle all the representations of paint mixtures that make sense to you.

pictures of mixtures tape diagrams ratios tables

PRACTICE 2

Blakely is making “goodie bags” to give out at her younger brother Graham’s birthday party. She wants to put 5 stickers and 2 granola bars in each one.



1. Describe ratios in different ways by completing each statement.

   For every 5 stickers, there will be _____ granola bars.

   The ratio of stickers to granola bars is _____:_____.

   The ratio of granola bars to total items in the bags is _____ to _____.

2. Complete the table below based on the given ratio of stickers to granola bars in the goodie bags. Leave the last column blank for problem 3 below.

# of stickers	5		20		50	
# of granola bars	2					
# of goodie bags		2		8		

3. Blakely uses 60 stickers to fill bags.

   Enter this quantity in your table in the last column.

   Explain how the table can be used to help you determine how many granola bars she will need.

   How many bags does she plan to fill? _____ How many granola bars will she need? _____

4. Suppose there were 20 people coming to Graham’s party. How many stickers and granola bars would be needed? Show or explain how you know.

PRACTICE 3

1. A purple paint is made with 2 parts blue and 6 parts red.

   a. Make a tape diagram to illustrate this relationship.

   b. Express three or more different ratios that could be represented based on the tape diagram above. Try to use symbols for some and words for others.

2. Antonia makes tie-dyed shirts. Her most frequently used colors are orange and green.

   a. For the orange dye, she uses red and yellow in a ratio of 3 : 2. How many scoops of red and yellow dye will she need if she wants to make 80 scoops of orange dye? Use a tape diagram.

   b. For the green dye, she uses blue and yellow in a ratio of 5 : 2. How many scoops of yellow dye will she need if she is using 40 scoops of blue dye? Use a table.

TABLES AND DOUBLE NUMBER LINES

We will use tables and double number lines to solve problems.

GETTING STARTED

Sienna earns $32 for every four hours of babysitting.

1. Complete the following table.

dollars	32	64				
hours			1	2		5
dollars : hours					48 : 6	
$\frac{\text{dollars}}{\text{hours}}$						

2. Explain how you found the number of dollars earned for 5 hours of babysitting.

3. How much does Sienna earn per hour? How do you know from the table?

4. The number in the bottom row $\left(\frac{\text{dollars}}{\text{hours}}\right)$ is called the value of the ratio or the unit rate. What is Sienna's unit rate?

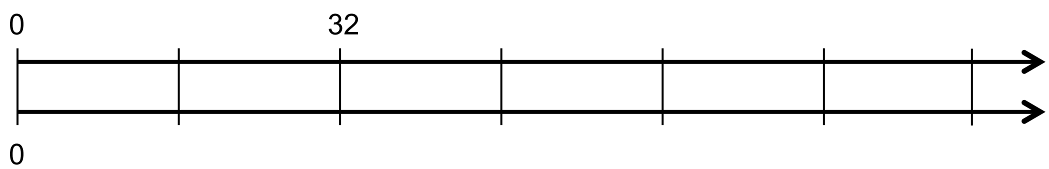
5. How is a ratio and the value of a ratio different? Use section 1.5 to help you.

DOUBLE NUMBER LINES

1. On this page we will work with double number lines. Find the definition and examples in section 1.5, discuss them with your teacher and the class, and record in My Word Bank.

Recall that Sienna earned $32 for each four hours of babysitting.

2. Represent this relationship on the double number line below. Include all the values in table on the previous page.



3. How are the quantities on a double number line organized differently than those in the table?

4. What is the unit rate? __________ Circle it on the double number line.

5. What patterns do you notice in the double number line?

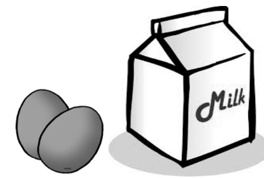
6. Sienna wants to buy a jacket. Add entries to the double number line to help you answer these questions.

   a. Suppose the jacket costs $88. How many hours will she need to work? How do you know?

   b. Suppose the jacket costs $84. How many hours will she need to work? How do you know?

BACK IN NANA'S KITCHEN

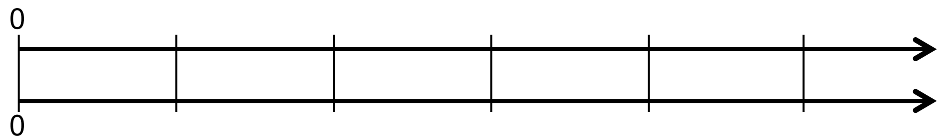
Follow your teacher's directions. How would you fix the recipe?



PRACTICE 4

Create double number lines to help you solve each problem. Assume constant rates for each problem.

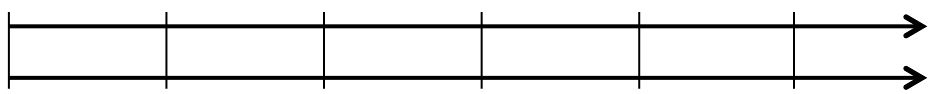
1. Charlotte pays $60 for 10 sandwiches.



   a. What is the price for 25 sandwiches at this rate?

   b. What is cost for 1 sandwich (called unit price)?

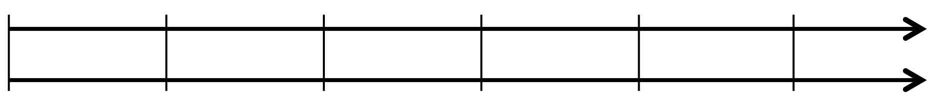
2. Sofia read 5 books in 2 weeks.



   a. At this rate, how many books will she read in 9 weeks?

   b. How many books did she read per week?

3. A workday is 8 hours. You earn $48 for one work day.



   a. What is the hourly pay rate?

   b. At this rate, how much would you earn in 5 hours?

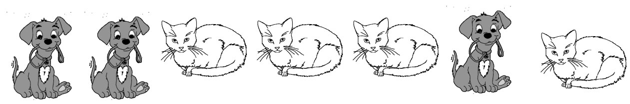
4. Describe a general process you used to create the double number lines above.

REPRESENTATIONS OF EQUIVALENT RATIOS

We will learn how to determine if ratios are equivalent. We use different representations of ratios to solve problems.

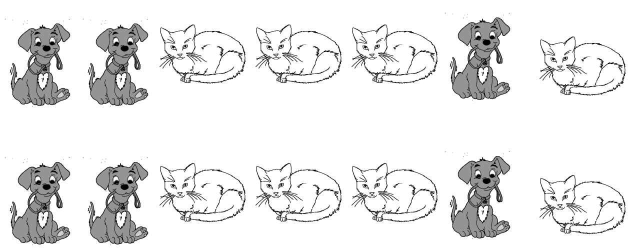
GETTING STARTED

1. Here is a picture of some cats and dogs. Write the ratios for this diagram.



Number of cats to number of dogs	4 to _____	or	4 : _____
Number of cats to total number of animals	_____ to _____	or	_____ : _____
Number of total animals to number of cats	__________	or	__________

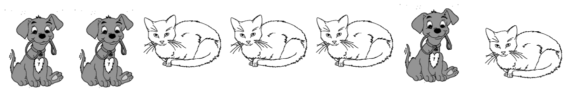
2. The original picture is repeated here. Write ratios for this diagram.



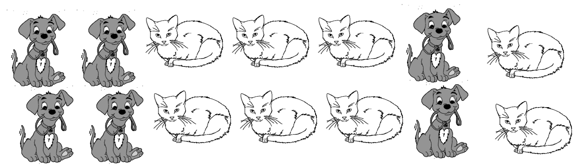
Number of dogs to number of cats	6 to _____	or	6 : _____
Number of cats to total number of animals	_____ to _____	or	_____ : _____
Number of cats to number of dogs	__________	or	__________

EQUIVALENT RATIOS

Recall the pictures and ratios of cats and dogs.



4 to 3



8 to 6

1. On this page we will work with equivalent ratios. Find the definition and examples in section 1.5, discuss them with your teacher and the class, and record in My Word Bank.

2. One way to show equivalent ratios is with an arrow diagram.

   Write the multiplier that can be used to justify that the ratios are equivalent.

   4 to 3

   ×____ ×____

   8 to 6

Recall that Sienna earned $32 for every four hours of babysitting. Here are some ratios you wrote previously to represent that relationship:

dollars : hours	32 : 4	64 : 8	8 : 1	16 : 2	40 : 5

3. Find the multiplier for each pair of ratios that can be used to justify that they are equivalent.

8 : 1 ×____ ×____ 32 : 4	64 : 8 ____ ____ 32 : 4	16 : 2 ____ ____ 40 : 5

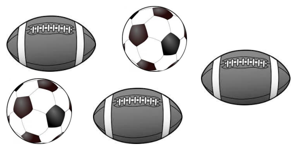
Show your work to determine if each pair of ratios is equivalent.

4. 3 to 5 12 to 30	5. 8 : 5 24 : 15	6. 8 : 6 and 12 : 9

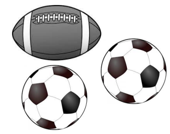
PRACTICE 5

1. Find the ratios of soccer balls to footballs in each collection. Then draw arrows to match collections that represent equivalent ratios. Not every collection has a match.

A.



B.



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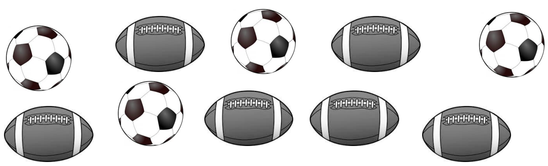

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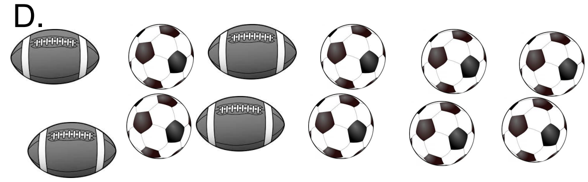
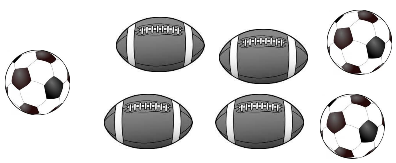
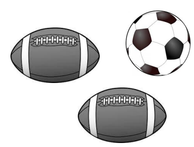


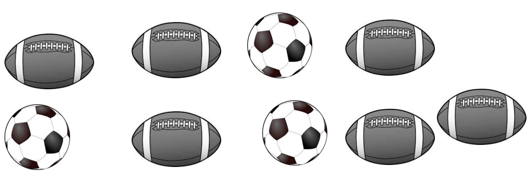
E.



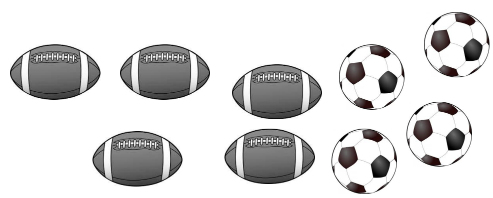
F.



G.



H.

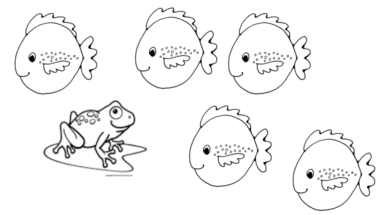


2. Choose one pair of equivalent ratios above. Show or explain how you know they are equivalent.

3. Charlie says that collection A and collection H represent equivalent ratios because each one has one more football than soccer ball. Explain why Charlie is wrong.

EQUIVALENT RATIOS IN TABLES

The ratio of the number of fish to the number of frogs in the science lab is 5 to 1, or 5 fish for every 1 frog.



1. Complete the table below for possible numbers of fish and frogs that could be in the lab. Write in the multipliers for the arrows.

# of fish	5		30		
# of frogs	1	3		4	12

2. Explain two ways that you could use constant multipliers to determine the number of fish if there are 3 frogs.

3. Use the table above. Choose two different pairs of entries and form ratios. Find the multiplier for each pair of ratios that can be used to justify that they are equivalent.

Example:

1 : 5

$\times 3$ $\times 3$

3 : 15

Suppose you know there are less than 20 fish in the lab, and the science teacher adds a new frog to the animals already in the lab.

4. Make a table that shows possible numbers of fish and frogs in the lab after adding the new frog.	5. Are the possible ratios of fish to frogs in the lab still equivalent after adding the new frog? Justify your answer with an example.

PRACTICE 6

In a soccer tournament, the ratio of the number of 12-year-olds to the number of 11-year-olds is 1 to 2.

1. What is the ratio of 11-year olds to 12-year olds?

2. Complete the table.

3. Choose two different ratios for the number of 11-year-olds to the number of total players. Then use an arrow diagram to show that these two ratios are equivalent.

# of 11-year-olds	# of 12-year-olds	Total Players
20		
	25	
		150
	20	
350		

4. Make a tape diagram to solve this problem: If there are 78 players in the tournament, how many 11-year olds will there be?

5. Suppose the teams play six games every four weeks. How many games will they play in ten weeks? Make a double number line.

5. Circle all the representations for the soccer tournament that make sense to you. Put a star by the representation(s) you prefer.

table arrows / arrow diagram tape diagram double number line

REVIEW

BIG SQUARE PUZZLE: EQUIVALENT RATIOS

Your teacher will give you a “big square puzzle” to complete. Assemble it so that equivalent ratios match on the edges. Tape it here.

POSTER PROBLEM: RATIO REPRESENTATIONS

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _______.
- Each group will have a different colored marker. Our group marker is _________.

Part 2: Do the problems on the posters by following your teacher's directions.

Poster 1 (or 5)	Poster 2 (or 6)	Poster 3 (or 7)	Poster 4 (or 8)
A recipe calls for 2 cups sugar and 4 cups flour.	You can buy 3 cookies for $1.00.	Greg ran 10 kilometers in 50 minutes.	A project needs 5 colored pencils and 10 index cards.

A. Copy the statement and use the information to write two different ratios.

B. Make a table that displays different equivalent ratios for the statement.

C. Make a double number line or a tape diagram to display different equivalent ratios. Include some of the data in your table.

D. Use an arrow diagram to show that any two of the ratios are equivalent.

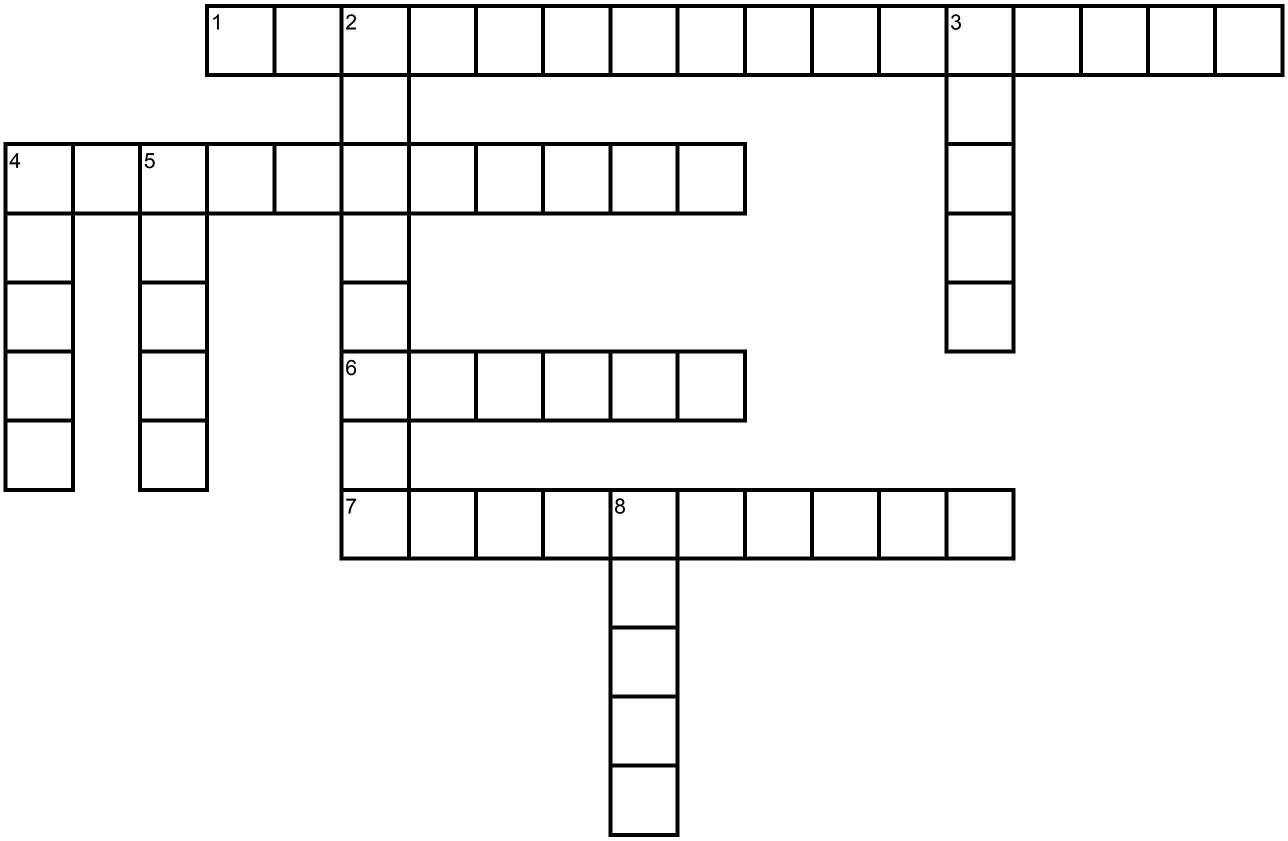
Part 3: Return to your seats. Work with your group.

Use your "start problem."

1. Write a story problem based on the fact statement, and find its solution.

2. Exchange problems with another group. Answer the other group's problem. Check each others' solutions.

VOCABULARY REVIEW



Across

1 A picture where corresponding values are placed on two parallel lines (3 words).

4 A picture where rectangles of equal length represent the same quantities (2 words).

6 Symbols used in a diagram to show equivalent ratios.

7 The ratios 3 : 7 and 21 : 49 are _____ ratios.

Down

2 5 miles per hour is an example of a _____ (2 words).

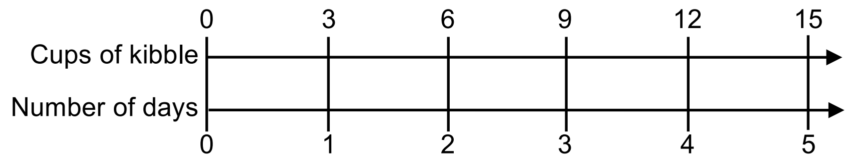
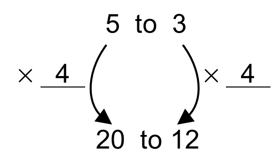
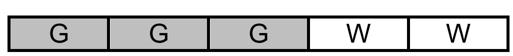
3 3 stars for every 5 circles is an example of a _____.

4 A chart with rows and columns.

5 The cost for one item is its unit _____.

8 Another name for unit rate is the _____ of the ratio.

DEFINITIONS, EXPLANATIONS, AND EXAMPLES

Word or Phrase	Definition
double number line	A double number line is a diagram made up of two parallel number lines that visually depict the relative sizes of two quantities. Double number lines are often used when the two quantities have different units, such as miles and hours. The proportional relationship “Wrigley eats 3 cups of kibble every 1 day” can be represented in the following double number line diagram. 
equivalent ratios	Two ratios are equivalent if each number in one ratio is obtained by multiplying the corresponding numbers in the other ratio by the same positive number. 5 : 3 and 20 : 12 are equivalent ratios because both numbers in the ratio 5 : 3 are multiplied by 4 to get to the ratio 20 : 12. An arrow diagram can be used to show equivalent ratios. 
ratio	A ratio is a pair of nonnegative numbers, not both zero, in a specific order. The ratio of a to b is denoted by $a : b$ (read “*a* to *b*,” or “*a* for every *b*”). The ratio of 3 to 2 is denoted by 3 : 2. The ratio of dogs to cats is 3 to 2. Use 3 cups of water for every 2 cups of juice. The fraction $\frac{3}{2}$ does not represent this ratio, but it does represent the value of the ratio (or the unit rate).
tape diagram	A tape diagram is a graphical representation that uses length to represent relationships between quantities. We draw rectangles with a common width to represent quantities, and rectangles with the same length to represent equal quantities. Tape diagrams are typically used to represent quantities expressed in the same unit. This tape diagram represents a drink mixture with 3 parts grape juice for every 2 parts water. 
unit rate	The unit rate associated with a ratio a : b of two quantities a and b, $b \neq 0$, is the number $\frac{a}{b}$, to which units may be attached. This is sometimes referred to as the value of the ratio. The ratio of 40 miles each 5 hours has a unit rate of $\frac{40}{5}$ = 8 miles per hour.
unit price	A unit price is a price for one unit of measure. If 4 apples cost \$1.00, then the unit price is $\frac{\$1.00}{4}$ = \$0.25 for one apple, or 0.25 dollars per apple or 25 cents per apple.

Ratios: Language and Notation

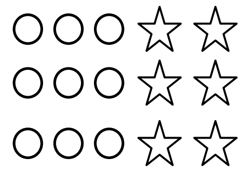
The ratio of *a* to *b* is denoted by $a : b$ (read " *a* to *b*," or "*a* for every *b*").

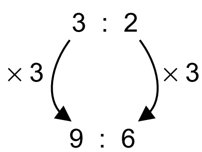
Note that the ratio of *a* to *b* is not the same as the ratio of *b* to *a* unless $a = b$.

We can identify several ratios for the objects in the picture to the right.

- There are 3 circles for every 2 stars.
- The ratio of stars to circles is 2 to 3.
- The ratio of circles to total shapes is 3 : 5
- The ratio of circles to stars is 3 : 2.

If we make 3 copies of the figure above, we get the picture to the right. Now the ratio of circles to stars is 9 : 6. The ratio 9 : 6 is obtained by multiplying each number in the ratio 3 : 2 by 3 (called the multiplier).





The arrow diagram to the left shows that 3 : 2 and 9 : 6 are equivalent ratios.

A fraction formed by a ratio is called the value of the ratio (or unit rate). Equivalent ratios have the same value. In our example, $\frac{3}{2} = \frac{9}{6} = 1.5$.

Tables of Number Pairs

Tables are useful for recording number pairs that have equivalent ratios. In the case of a ratio of three circles for every two stars, there are two ways that number pairs with equivalent ratios might be recorded in a table. Table 1 is aligned horizontally. Table 2 is aligned vertically. Entries may be in any order.

Table 1

Circles	3	9	6
Stars	2	6	4

Table 2

Circles	**Stars**
3	2
9	6
6	4

Tape Diagrams

A <u>tape diagram</u> is a visual model consisting of strips divided into rectangular segments whose areas represent relative sizes of quantities. Tape diagrams are typically used when quantities have the same units.

Here are two versions of tape diagrams that show that the ratio of grape juice to water in some mixture is 2 : 4.

Diagram 1

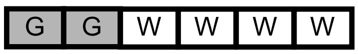
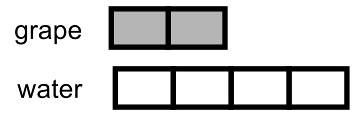
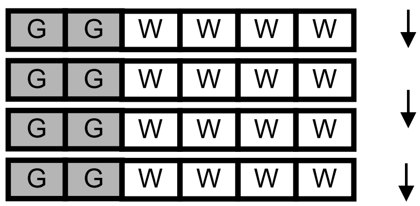


Diagram 2



Suppose we want to know how much grape juice is needed to make a mixture that is 24 gallons. Here are two methods using Diagram 1.

Method 1:



2 gal grape : 4 gal water → 6 gal total (not enough)
4 gal grape : 8 gal water → 12 gal total (not enough)
6 gal grape : 12 gal water → 18 gal total (not enough)
8 gal grape : 16 gal water → 24 gal total (perfect)

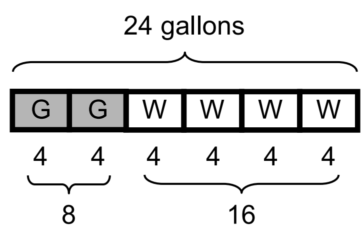
24 gallons of mixture will require 8 gallons of grape juice.

Notice here that each rectangle (piece of tape) represents 1 unit (1 gallon of liquid.)

Method 2:

Six rectangles in the tape diagram represent 24 gallons of mixture.

Since 24 ÷ 6 = 4, one rectangle in the tape diagram represents 4 gallons of liquid.



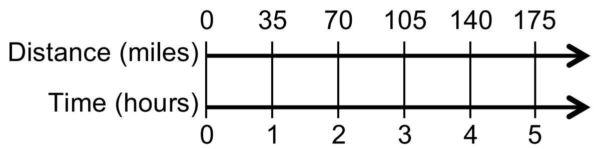
24 gallons of mixture will require 8 gallons of grape juice.

Notice here that each rectangle (piece of tape) represents more than 1 unit (4 gallons, in this case). Pieces of tape in the diagrams do not always need to represent 1 unit.

Double Number Lines

A double number line diagram is a graphical representation of two quantities in which corresponding values are placed on two parallel number lines for easy comparison. Double number lines are often used to compare two quantities that have different units.

The double number line below shows corresponding ratios if a car travels 70 miles every 2 hours.



We can see from the double number line diagram above that at the given rate, the car goes 35 miles in 1 hour (which is the unit rate of 35 miles per hour), 105 miles in 3 hours, etc. Notice the same tick marks on the number line are used to represent different quantities, and values are scaled in numerical order.

Unit Rate and Unit Price

The unit rate associated with a ratio is the value of the ratio, to which we usually attach units for clarity. In other words, the unit rate associated with the ratio $a : b$ is the number $\frac{a}{b}$, to which we may attach units. For this to make sense, we must assume that $b \neq 0$.

Suppose a car travels 70 miles every 2 hours.

- This may be represented by the ratio 70 : 2.
- The number $\frac{70}{2} = \frac{35}{1} = 35$ is the value of the ratio.
- The unit rate is then the value 35, to which we attach the units “miles per hour.” Thus the unit rate may be written:

$35 \ \frac{\text{miles}}{\text{hour}}$ or 35 miles per hour or 35 miles/hour

A unit price is the price for one unit.

Suppose it costs $1.50 for 5 apples.

- This may be represented as the ratio 1.50 : 5.
- The number $\frac{1.50}{5} = 0.30$ is the value of the ratio.
- The unit price is then the value 0.30, to which we attach the units “dollars per apple.” The unit price can be written in any of the forms below.

$0.30 \ \frac{\text{dollars}}{\text{apple}}$ 0.30 dollars per apple $.30 per apple

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