Number Sense through Olympiad Problems

by the ComMuniCator Editorial Panel

CONCEPTS: Number Sense, Algebra, Geometry
SKILLS: Solving problems
MATHEMATICS STANDARDS: Gr 4: NS 1.9, 3.1, 4.2; Gr 5: MG 1.0, MR 1.0, 2.0; Gr 6: NS 2.3, AF 3.2, MR 1.0, 2.0; Gr 7: MG 2.0, 2.2, MR 1.1, 2.0
GRADES: 4–8
MATERIALS: *Math Olympiad Contest Problems Volume 2*; Student Activity sheets, pages 53–54

DESCRIPTION
Students develop mathematical power by having time to work on interesting and engaging problems that have a variety of solution methods. A problem, unlike an exercise, presents an unfamiliar situation for which students need to devise a procedure. Mathematical standards are covered in a deeper way than with the "two-page spread" when students have time to explore, use multiple approaches, and discuss various solution methods.

The *ComMuniCator* Editorial Panel recommends the *Math Olympiad Contest Problems Volume 2* book (Richard Kalman, editor) to upper elementary and middle school teachers. The book contains 85 sets of five problems each. Each set provides a page of hints that may be duplicated for students, pages of correctly worked out solutions, a list of all of the answers (with no solutions), and previous results of what percent of contest students had solved each of the problems correctly.

We have provided two pages of Math Olympiad problems on pages 53 and 54 for you to use with your students. Go to **www.moems.org** if you wish to follow-up on these problems.

Student Activity Sheets, pages 53–54. . .

Approximating Square Roots by Linear Interpolation

by Shelley Kriegler and Ted Gamelin, UCLA
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CONCEPT: Number Sense
SKILLS: Estimating, finding square roots, working with fraction and decimal conversions and equivalences
MATHEMATICS STANDARDS: Gr 7: NS 1.2, 1.3, 2.4, MR 2.1, 2.7
GRADES: 7–12

BACKGROUND
This activity is part of a collection of eight sample lessons from *Introduction to Algebra* available online at **www.introtoalg.org/program_sampler.htm**. At this site you can download student pages for lessons leading to this activity where students learn the connection between squaring a number and finding the square root of a number. They approximate a square root by locating it between two consecutive integers and use fractions and decimals to approximate square roots.

A reprint of the teacher page for Approximating Square Roots by Linear Interpolation provides background information and a visual representation of linear interpolation that will enhance a lesson on approximating square roots.

Teacher Page Reprint, page 55. . .

MATH BACKGROUND

Approximating Square Roots by Linear Interpolation

Math Background 1

Teacher Mathematical Insight

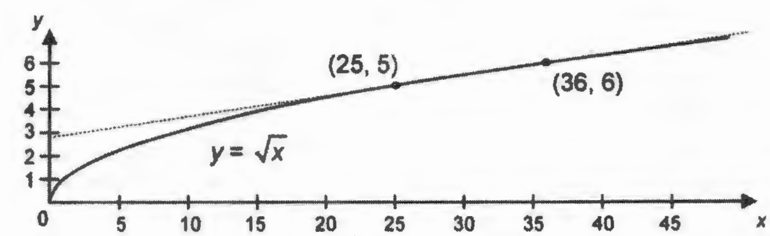
Linear interpolation is a method by which the values $y = f(x)$ of a function f on an interval $x_1 < x < x_2$ are estimated by the values of the linear function $y = mx + b$ that matches the values of f at the endpoints of the interval. The parameters m and b satisfy the two equations $f(x_1) = mx_1 + b$ and $f(x_2) = m x_2 + b$. The graph of the linear approximation $y = mx + b$ is then the straight line segment joining the points $(x_1, f(x_1))$ and $(x_2, f(x_2))$ on the graph of f.

The linear approximation can be found directly through proportional reasoning, without writing down any equations of lines. To illustrate, we approximate values of $\sqrt{x}$.

To find an approximate value for $\sqrt{27}$:

First find the closest perfect square (25) that is less than 27 and the closest perfect square (36) that is greater than 27. Then 25 < 27 < 36. The points 25 and 36 will be the endpoints of the interval of interpolation.

Take the square root of each number: $\sqrt{25} = 5$, $\sqrt{36} = 6$. We aim to approximate $\sqrt{27}$ using the y-coordinate of the straight line through (25,5) and (36,6).



Now 27 is 2 units larger than 25, and 36 is 11 units larger than 25. Thus 27 is two elevenths $\left(\frac{2}{11}\right)$ of the distance from 25 to 36. We approximate $\sqrt{27}$ by the number that is two elevenths of the distance from $\sqrt{25}$ to $\sqrt{36}$, that is, two elevenths of the distance from 5 to 6 (proportional reasoning!!). This number is $5 + \frac{2}{11} = 5\frac{2}{11}$.

Summary: On a number line, 27 is $\frac{2}{11}$ of the distance from 25 to 36. Therefore, $\sqrt{27}$ is approximately equal to $\frac{2}{11}$ of the distance from $\sqrt{25} = 5$ to $\sqrt{36} = 6$, which is $5 + \frac{2}{11}$. The approximation to $\sqrt{27}$ is $5\frac{2}{11}$.

By linear interpolation to the nearest thousandth: $5\frac{2}{11} \approx 5.182$

Accurate to the nearest thousandth: $\sqrt{27} \approx 5.196$