Cutting The Rope

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CONCEPTS: Algebra, Data Analysis, Mathematical Reasoning
SKILLS: Exploring patterns using pictures and numbers, solving problems using multiple representations and strategies, finding explicit rules using algebraic notation
MATHEMATICS STANDARDS: Gr 6: AF 1.0, 1.2, 2.0, 2.2, MR 2.4; Gr 7: AF 1.1, 1.5, 3.0, 3.3, 3.4, SDAP 1.0, 1.2; AlgI 6.0, 7.0, 8.0
STANDARDS FOR MATHEMATICAL PRACTICE: MP1, MP2, MP3, MP6, MP8
GRADES: 6–10
MATERIALS: Colored pencils, string or thin rope (optional), Student Activity Sheets (pgs 33–34)

BACKGROUND

Many of our high school students are struggling to complete secondary school mathematics literacy requirements and high school exit exams. They need experience in representing mathematical ideas visually, numerically, symbolically, and verbally (the fourfold way), giving them multiple ways to communicate their mathematical knowledge. They need simplified verbal instructions, supplemented with written and/or visual clues. Instructions given both verbally and in written form address the needs of students who may have difficulties with auditory discrimination (since they will also be able to see the assignment in writing), who have weaknesses visually (since they may have relative auditory strengths), and who have difficulty with organizational issues (since they are reminded in two different ways).

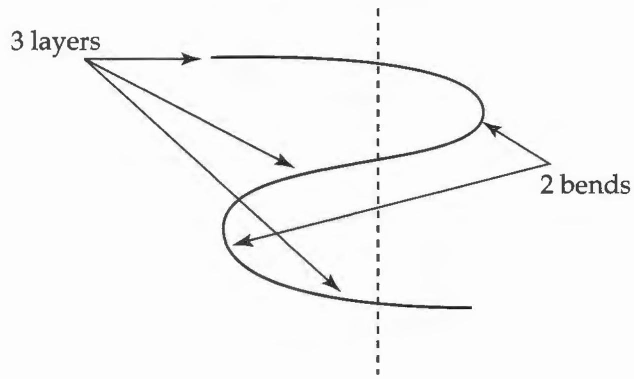
This activity is one of a series of lessons developed by mathematicians and experienced middle school teachers who understand the mathematical and pedagogical needs of struggling students. Additional activities and units designed by these authors can be found at MathLinks at **www.mathandteaching.org.**

DESCRIPTION

In this lesson the students begin with a concrete activity, discuss their work along the way—in writing and verbally—and then progress to more abstract representations, including tables, graphs, and equations. Students will perform a rope-cutting investigation and look for patterns that can be explained by algebraic equations (explicit rules).

PROCEDURE

1. Introduce the lesson. Discuss important vocabulary as relevant—layers, cuts (vertical lines), bends, and pieces. Additional terminology for discussion could include graphs, symbols, plot points, grid, *y*-intercept, slope, and rule.



2. Students will examine the number of pieces created when a rope forms snake-like patterns with various layers, and is then cut. Make sure students understand that the rope must not overlap itself. (That would be a different investigation.)
3. Ask students, "What would a snake-like pattern with one layer look like? Two layers? Three layers? Four layers?" Invite students to demonstrate with a rope or by making sketches.
4. Hand out copies of Student Activity Sheet 1. Using a colored pencil/marker, complete the first table as a group with the students so students understand how to record the

Continued on page 32 >- >-

number of cuts and number of pieces for a rope that has one layer. Have students generate a rule for the number of pieces when there is one layer. Do NOT go on to the second table.

5. Ask students, "Although every table is almost exactly the same, what is different about the next three?" [*Each table is used to record the number of cuts and pieces for a different number of layers.*]
6. Hand out copies of Student Activity Sheet 2. Have students graph the information from the first table on Student Activity Sheet 1, using the same color as the table. Be sure they label the graph clearly. Some students / classes may need help with axis labeling.
7. If needed, begin to complete Table 2 (using a second color), but refrain from doing too much of the investigation as a whole class because it will spoil the exploration.
8. Have students complete the remaining tables and answer questions as they work toward finding a general formula that will give the number of pieces obtained for any number of layers and cuts.
9. Invite students to share patterns from their tables.
10. Discuss the graphs by asking the following questions:
    - ✔ What is the same about the graphs? [*They are all lines. They all begin at the point (0, 1), which is their y-intercept.*]
    - ✔ What is different about the graphs? [*Some are steeper/flatter than others. In other words, they have different slopes. The more layers in the rope, the steeper the slope.*]
    - ✔ What does the y-intercept represent for each graph? [*This shows that there is one piece when there are no cuts.*]
    - ✔ What does the slope mean? [*The slope is the number of layers.*]
    - ✔ What is the rule for the number of pieces (p) for any number of layers (l) and any number of cuts (c)? [$p = lc + 1$]

Student Activity Sheets, pages 33–34. . .

An Easy Way to Calculate the Binomial Expansion

by Meta Davidson, Santa Paula High School
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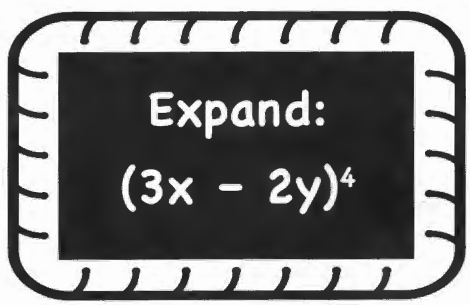

CONCEPTS: Algebra, Mathematical Reasoning
SKILLS: Displaying data in tables, using exponential functions, operating with polynomials, organizing information, using Pascal's triangle.
MATHEMATICS STANDARDS: Algebra II: 20.0
STANDARDS FOR MATHEMATICAL PRACTICE: MP5, MP6, MP7, MP8
GRADES: 9–12
MATERIALS: Pencil and Paper

BACKGROUND

My Algebra 2 students were having difficulty accurately using the Binomial Theorem to expand an expression such as $(a + b)^n$. While helping a student at lunch one day, we came up with an organized method to create the expansion. On a recent test, every student who used the new technique (which most did) calculated the expression 100% accurately.

See Binominal Expansion, page 35 ➢ ➢

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Activity 1

by Shelley Kriegler

Explore cutting the rope for different numbers of layers and cuts.

Number of layers = 1

Number of cuts (c)	Number of pieces (p)

Rule for any number of cuts:
$P =$

Number of layers = 2

Number of cuts (c)	Number of pieces (p)

Rule for any number of cuts:
$P =$

Number of layers = 3

Number of cuts (c)	Number of pieces (p)

Rule for any number of cuts:
$P =$

Number of layers = 4

Number of cuts (c)	Number of pieces (p)

Rule for any number of cuts:
$P =$

Cutting the Rope

Activity 2

by Shelley Kriegler

1. For each table in Activity 1, plot the points on the grid. Graph each set of points using a different color.

2. What is the same about each graph?

3. What is different?

4. What does the y-intercept represent for each graph in terms of the cutting rope experiment?

5. What does the slope tell us?

6. Looking at all four tables, write a rule that can be used to find the total number pieces (p) for any number of layers (l) and any number of cuts (c).