

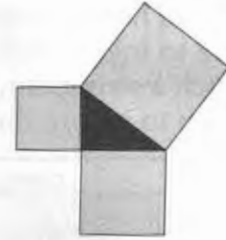
lesson as the students are putting up their cards on the number line. You can check their work by converting all the cards to decimals. The Activity Sheet, page 34, can provide an additional opportunity to assess student knowledge.

Reference
Fulton, B. Number Line: Middle School Level-Fractions, Decimals, Percent, Algebra. <https://www.teacherspayteachers.com/Product/Number-Line-Middle-School-Level-Fractions-Decimals-Percent-Algebra-1488699>.

Activity Sheet, page 34 . . .

Discovering the Pythagorean Theorem on Dot Paper

by Mark Goldstein, Shelley Kriegler, Ted Gamelin, and Chase Orton,
Center for Mathematics Teaching; shelley@mathandteaching.org



CONCEPTS: Algebra, Geometry

SKILLS: Finding areas of squares and right triangles, using square roots, exploring the meaning of the Pythagorean theorem, making conjectures

MATH CONTENT STANDARDS: 6.G.1; 7.G.6; 8.EE.2, 8.G.6

MATH PRACTICE STANDARDS: 1, 2, 6, 8

GRADES: 6–9

MATERIALS: Activity Sheets (pages 35–36), document camera

BACKGROUND

The Pythagorean theorem was discovered thousands of years ago and was understood to varying degrees in many ancient civilizations. Ancient Babylonians recorded many Pythagorean triples on clay tablets between 1900 and 1600 BCE. Although the theorem as we know it today is commonly attributed to the Greek philosopher and mathematician Pythagoras, the earliest known formal proofs of the Pythagorean theorem and its converse are recorded in Book 1 of Euclid's *Elements*, written about 300 BCE.

In this two-part activity, students explore right triangles and the areas determined by the squares on the sides to discover the Pythagorean theorem. In Part 1, they use dot paper to draw squares on each side of a right triangle and then find the areas of those squares. After repeating this for additional right triangles, students make a conjecture about the relationship between the area of the square on the hypotenuse and the areas of the squares

on the legs of a right triangle.

In Part 2, students test their conjecture on an additional right triangle and then investigate whether the same relationships exist for non-right triangles.

PREPARATION

Duplicate copies of Activity Sheets 1 and 2 (pages 35–36). Place a copy of Activity Sheet 1 under a document camera for use in Part 1

PROCEDURE

Part 1: Right triangles

- ✓ Ask students to recall the basic properties of a right triangle (three sides, two acute angles, one right angle) and the vocabulary (legs, hypotenuse) associated with the naming of the sides of a right triangle. Ask: *Which side of a right triangle is the longest? Why?*
- ✓ Using a document camera, project Activity Sheet 1 (page 35) to the whole class. Focus attention on the small triangle at the top. Ask students to help you determine how to draw squares based on the legs and how to find the areas of those squares. Use different colors for emphasis.
- ✓ Then, using the given vertices as guides, draw a square on the hypotenuse. Ask students to brainstorm ideas: *How could we find the area of the square on the hypotenuse?* Have students share some of their ideas aloud and use the document camera to illustrate their approaches. Hint: one way to

find the area of the “tilted” square on the hypotenuse is to draw a square around it and then subtract off the areas of the four right triangles that “pinwheel” around it.

- ✓ Once students agree that the area of the square on the hypotenuse is 25, ask them how they can find the length of the hypotenuse.
- ✓ Let students know that they will be developing a conjecture about the relationship between the area of the squares on the hypotenuse and on the legs.
- ✓ Distribute Activity Sheet 1 (page 35) to students. Have students work in pairs or groups to complete Triangle A on their own sheets and then fill in the information in the first column on the table at the bottom of the page.
- ✓ Circulate as students work through the tasks, noting findings to have students share later in a whole-class discussion.
- ✓ Bring the whole class back together and ask students to share their findings.
- ✓ Pose the following focal question during discussion: *What appears to be a relationship between the area of the square on the hypotenuse and the areas of the squares on the two legs? (The sum of the areas of the squares on the legs is equal to the area of the square on the hypotenuse.)*
- ✓ Share with students that this conjecture is

among the most well-known mathematical theorems, called the Pythagorean theorem.

PART 2: Non-right triangles

- ✓ Distribute Activity Sheet 2 (page 36) so that students can further their exploration. This second sheet is an important follow-up to the activities on Activity Sheet 1 since it includes an additional right triangle as well as non-right triangles as examples where the Pythagorean Theorem does not hold.
- ✓ Explain to students that they are to complete the same steps for the first triangle and fill in problems 2 and 3 at the bottom of the page.
- ✓ Then explain that they are to draw squares on each side of the next two triangles, which are non-right triangles. They then need to complete the information for problems 4 and 5 at the bottom of the page.
- ✓ Bring the whole class back together and ask students to share their findings.
- ✓ Conclude the discussion by emphasizing that the Pythagorean theorem only works for right triangles.

Reference

Kriegler, S., M. Goldstein, T. Gamelin, C. Orton. 2014. *MathLinks: Grade 8 (Packet 2)*. Van Nuys, CA: Center for Mathematics and Teaching

Activity Sheets, pages 35–36 . . .

Desmos (cont. from page 24)

signed to work on a tablet or computer, but not on a phone.


During my time teaching at-risk 8th grade students, we had additional time after a day of testing and I had the students explore the Desmos page. As the kids used the website, they began exploring the graphs of trigonometric functions and posed questions about asymptotes. Some students realized how to graph a circle (which soon became the talk of the class) and others noticed the difference between an equation’s graph and an inequality’s graph. Students who are traditionally disengaged with the material were asking wonderful questions and were sad to pack up at the

end of the day.

If you have not had the opportunity to use Desmos in your classroom, I strongly encourage you to sign up for a computer lab day and see what you and your students have been missing.

Reference

King, Ian. 2017. “Startup Targets the TI Calculators Your Kids Lug to Class.” *Bloomberg.com*, *Bloomberg* (May 12) <http://www.bloomberg.com/news/articles/2017-05-12/startup-targets-the-ti-calculators-your-kid-lugs-to-class>.

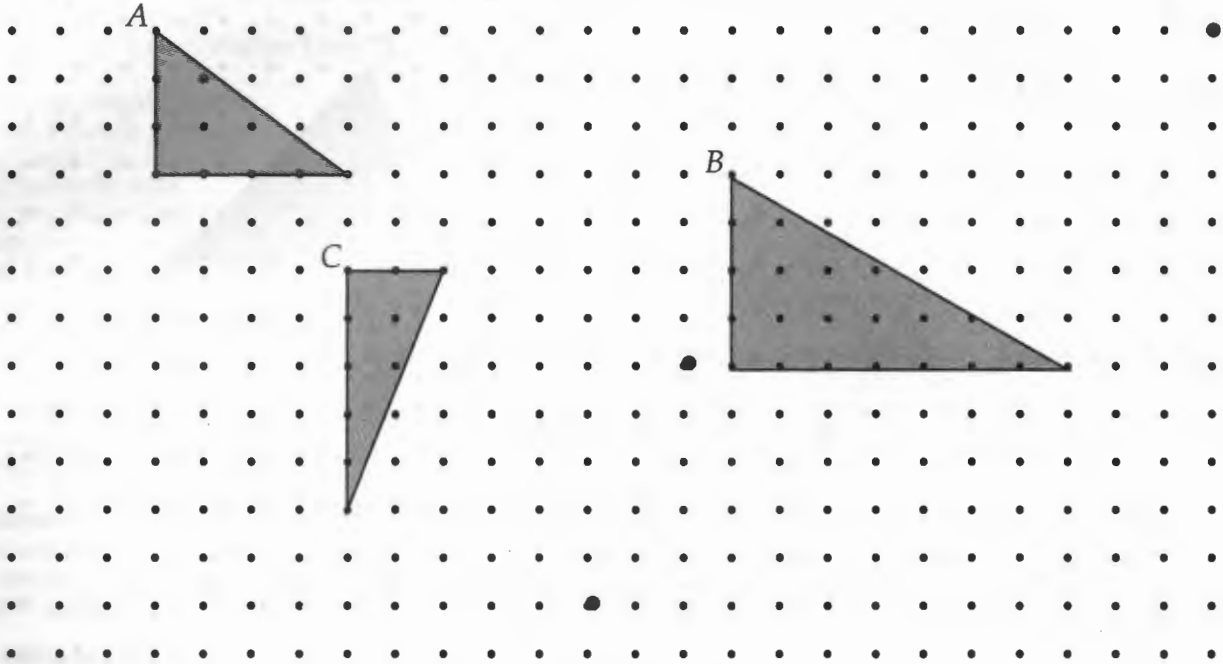
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The Pythagorean Theorem Activity 1

1. Draw the squares on the legs and the hypotenuse of each right triangle A, B, C below.

is 1 linear unit

is 1 square unit



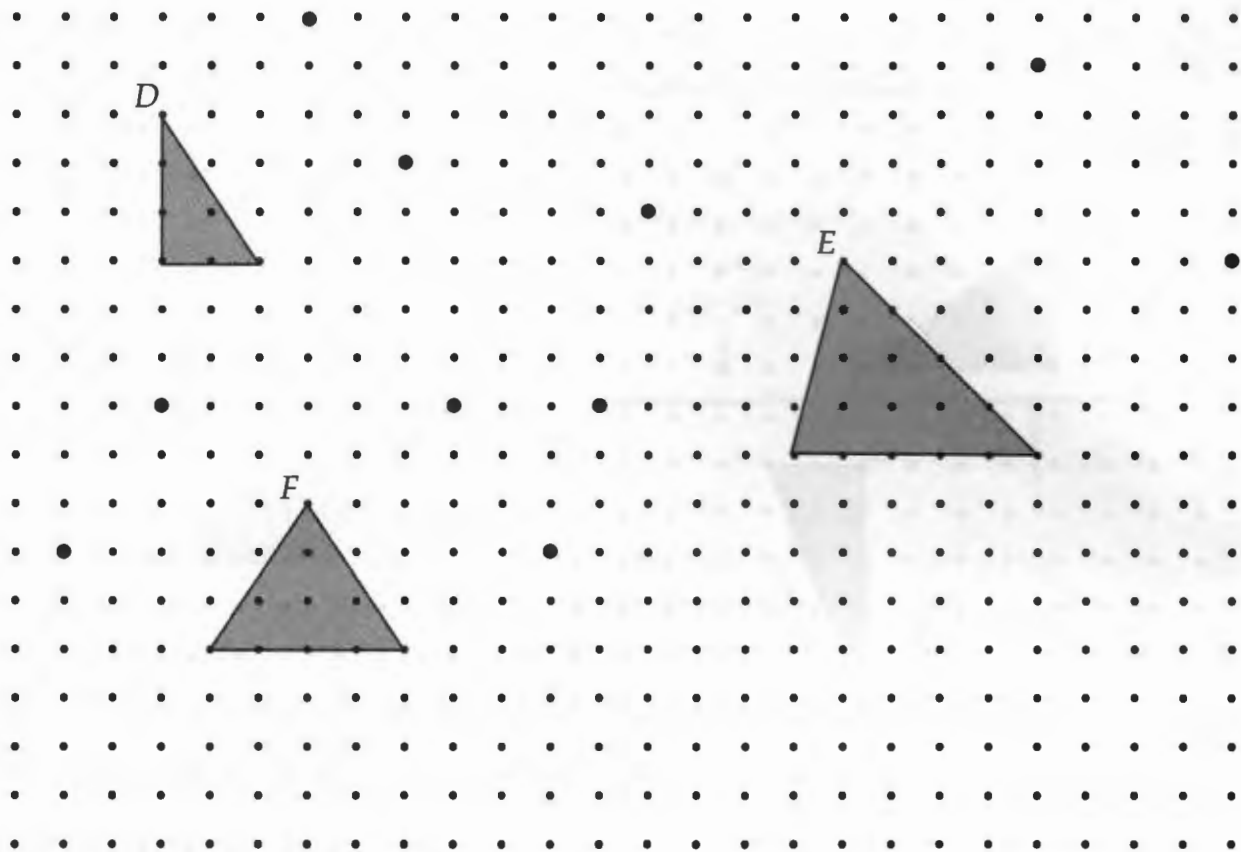
2. Complete the table by finding the following measurements.

	Triangle A	Triangle B	Triangle C
Length of the shorter leg			
Length of the longer leg			
Area of the square on the shorter leg			
Area of the square on the longer leg			
Area of the square on the hypotenuse			
Length of the hypotenuse			

3. Write a conjecture about the relationship between the area of the square on the hypotenuse and the area of the squares on the legs of a right triangle.

The Pythagorean Theorem Activity 2

1. Draw the squares on the sides (and hypotenuse for triangle D) for each triangle D, E, and F below.



2. Find the area of each square on triangle D's legs and hypotenuse and fill in the blanks for the area equations.	Area equation: $(\quad) + (\quad) = (\quad)$
3. Find the length of the legs and hypotenuse of each triangle and fill in the blanks for the side lengths equation.	Side length equation: $(\quad)^2 + (\quad)^2 = (\quad)$
4. Draw squares on the sides of triangle E, find the areas of the squares, and demonstrate that the relationship in problems 2 and 3 does not hold for this triangle (which is not a right triangle).	
5. Draw squares on the sides of triangle F, find the areas of the squares, and demonstrate that the relationship in problems 2 and 3 does not hold for this triangle (which is not a right triangle).	