ALGEBRAIC PUZZLES PROMOTE CURIOSITY, ANALYSIS, AND CREATIVE SOLUTIONS

Carole Greenes and Shelley Kriegler
cgreenes@asu.edu , shelley@mathandteaching.org

The *Los Angeles Daily News* has four pages of puzzles in every issue! Why? Because people like puzzles (and therefore newspapers increase subscriptions). Educators are tapping into our innate desire to solve puzzles and play games with "Game-Based Learning" (GBL), which is now one of the major learning trends (Ersen & Ergul, 2022) Why? GBL has several advantages including: piquing student interest, engaging students in conversations about known and needed information, and making decisions about what needs to be done to solve the problem (they like being detectives!).

GBL requires providing students with sufficient time to chat with friends to "work it out." We found that 10-15 minutes/puzzle with groups of 3 students, works best. And through this process, students reveal their talents!

What follows are two such puzzles: *Alge-Grid: What's the a?* (Greenes and Wolfram, 2021), and *READY-X* (Greenes and Wolfram, 2022), developed for students in grades 6 and up. (You can download these books for free at www.mathandteaching.org/publications.) In the discussion below, we describe the thinking of groups of Grade 6 students who have not taken a formal course in Algebra.

Alge-Grid: What's the *a*?

Goal: Reason algebraically and deductively. Specifically, solvers (1) identify numbers in the rectangles of the Clue, (2) test possible locations for the Clue on the Grid, (3) determine the value of *a* that will produce those numbers, and then (4) evaluate the other algebraic expressions in the Grid using that same value for *a*. Clue information may be mathematical, or related to sports, the sciences, history, or the arts. Note that the Clue cannot be rotated or flipped.

Alge-Grid Puzzle 1

The clue:

Fourth prime number	Factor of all even numbers	Roman numeral is V

The "Grid": Use the clue above to fill in the numbers from 1 – 9 in the grid below. The letter a represents the same number.

$\sqrt{a}-1$	$a \div 2 + 6$	$a+5$
$(a^3 - a^2) \div 12 + 2$	$a^2 \div 4 - 1$	$a^2 - 3a$
$2a-1$	$\sqrt{a}$	$a+1$

Before students began, we pointed out that their first job was to figure out the numbers in the Clue. With this Grid, Clue numbers were easily determined**: 7, 2, 5**. Next, students had to identify the row where the same value for ***a*** will produce **7, 2, 5**. Students tested rows and determined that the bottom row matched the Clue when $a = 4$. They completed the Grid by writing the value of each expression in the Grid. Some students asked for help with the meaning of some symbols and ways to evaluate expressions, and the puzzle provided them with a "need to know." Once solved, students asked for more puzzles!

Alge-Grid Puzzle 2

The clue:

Base of Binary System	Number of sharps in A Major
	Sum of two different prime numbers

The "Grid": Use the clue to fill in the numbers 1 – 9 in the grid below. The letter a represents the same number.

$(\frac{1}{3}a)^3$	$a \div a$	$2(a+1)-a-1$
$\frac{1}{3}a$	$\frac{1}{2}a$	$a+\frac{1}{2}a$
$2\times\frac{1}{3}a$	$2a-7$	$a\times a^0$

In this Alge-Grid, students talked about the Clue. They knew that **2** is the base of the binary system. Fortunately, one of the student musicians identified **3** sharps in A major. (They confirmed this with their phones!) The Clue, "sum of two different prime numbers," provoked discussion. We teachers (T), questioned one group of students (S).

T: "What are the possibilities?"

S: There are lots of sums. But the sum can't be more than 9. We need to figure out *a* first.

T: Good idea.

Students tested various locations and concluded that the Clue overlaid the first two rectangles at the start of the second row, and that a had to be 6. Once they determined the value of a, they computed $2a - 7$. That confirmed that the sum of the two different primes was 2 x 6 – 7, or 5. They knew this would work because the sum of 2 and 3 is 5. With that value for *a,* they completed the Grid.

READY-X

Goal: Determine the values of the letters: R, E, A, D, Y, X. At the end of each Row and each Column is the sum of the numbers in that row and column. To identify the values, students solve equations and systems of equations. As a warm-up, we used this problem.

READY-X Puzzle 1

Solve for the values of R, E, A, D, Y, X.

Rows \ Columns	1	2	3	
1	**X**	**X**	**D**	**17**
2	**A**	**Y**	**D**	**16**
3	**Y**	**X**	**D**	**14**
4	**E**	**R**	**D**	**17**
	14	**14**	**36**	

Before students began, we stated that: "Same letters have same values. The number at the end of each row is the sum of the numbers in that Row. The number at the bottom of each Column is the sum of the numbers in that Column. Once you figure out the number for each letter, write that number next to each letter in the Grid." We gave students 15 minutes to consult with other students and solve the puzzle. We teachers (T) then interviewed one group of students (S).

T: What did you do first?

S: Column 3 has all D's. So, 4 Ds are 36 and one D is 9. Put 9 with all Ds.

T: Then?

S: Went to Row 1: $2X + 9 = 17$. So, $2X = 8$, and $X = 4$. Put 4 where other X's are.

T: Then?

S: Row 3: $Y + 4 + 9 = 14$. So, $Y + 13 = 14$, and $Y = 1$. Put 1 for the other Y.

(Students continued talking and solving, mimicking our "then?" prompt.)

S: Then, Row 2: $A + 1 + 9 = 16$ and $A = 6$. Then, Column 2: $4 + 1 + 4 + R = 14$. So, $9 + R = 14$, and $R = 5$. Then Row 4: $E + 5 + 9 = 17$, and $E = 3$.

After finishing this warm-up problem, students asked for another problem.

READY-X Puzzle 2

Solve for the values of R, E, A, D, Y, X.

Rows \ Columns	1	2	3	
1	**R**	**X**	**R**	**23**
2	**D**	**Y**	**Y**	**9**
3	**R**	**X**	**D**	**21**
4	**D**	**E**	**A**	**14**
	24	**23**	**20**	

For this puzzle, we prompted students as they worked.

T: What will you do first? (After seeing that there were no rows or columns with the same letter, students pondered and chatted for a few minutes before responding.)

S: Column 1 has two Rs and Ds. So, R + D = 12. Row 3 has R + D. So, 12 + X = 21 and X = 9. Put 9 with all Xs.

T: What will you do next?"

S: Row 1 has an X and two Rs. So. 2R + 9 = 23. So, 2R = 14, and R = 7.

T: Now what?

S: In Column 1, R + D = 12. Because R is 7, then D is 5. Row 2 has D and two Ys. So, 5 + 2Y= 9, and Y = 2. I'll do Column 2 next: 9 + 2 + 9 + E = 23, so E= 3. Next Row 4: 5 + 3 + A = 14. So, A = 6.

Final Observations and Recommendations

While using **What's the *a*?** and **READY-X,** we observed students enjoying the activities, contributing various approaches when stumped, and not giving up until solutions were obtained. We also learned how much students know before being taught, and the value of good questioning. These questions were most productive in revealing student talent that we never anticipated.

1.What do you need to know to solve this puzzle/problem?

2. With "clues", which clue will you start with?

3. Is there a different way to solve this puzzle/problem. Tell us about it.

After completing 5 or more puzzles, we asked students to study their completed puzzles, order them by difficulty, and provide a rationale for that order. For us, this was an excellent assessment technique. Try it!

References and Free Resources

Erşen, Z. B. & Ergül, E. (2022). Trends of game-based learning in mathematics education: A systematic review. International Journal of Contemporary Educational Research 9(3), 603- 623

Greenes, C. & Wolfram, T. (2022) *Alge-Grids: What's the a?* Redondo Beach, CA: Center for Mathematics and Teaching.

Greenes, C. & Wolfram, T. (2021). *READY-X*. Redondo Beach, CA: Center for Mathematics and Teaching.

Goldstein, M., Kriegler, S., Moffett J., Gamelin, T. (2023) *MathLinks: Grade 8 (2nd ed).* Redondo Beach, CA: Center for Mathematics and Teaching.