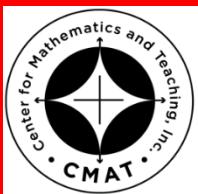


Name \_\_\_\_\_

Period \_\_\_\_\_ Date \_\_\_\_\_



**Mathlinks**

**8-14**  
**STUDENT PACKET**

**MATHLINKS GRADE 8  
STUDENT PACKET 14  
CONGRUENCE AND SIMILARITY**

<b>14.1</b>	<b>Congruence</b>	1
	• Define congruence.	
	• Apply properties of translations, rotations, and reflections to show that figures are congruent.	
	• Explore properties of congruent figures.	
<b>14.2</b>	<b>Dilations</b>	5
	• Perform a dilation experiment.	
	• Find the center point and scale factor of a dilation.	
	• Explore properties of dilations.	
<b>14.3</b>	<b>Similarity</b>	17
	• Define similarity.	
	• Apply properties of translations, rotations, reflections, and dilations to show that figures are similar.	
	• Explore properties of similar figures.	
<b>14.4</b>	<b>Skill Builders, Vocabulary and Review</b>	20

**WORD BANK**

Phrase	Definition or Explanation	Example or Picture
congruent		
deductive reasoning		
dilation		
inductive reasoning		
ratio of proportionality		
scale factor		
similar figures		

# CONGRUENCE

## Summary (Ready)

We will define congruence and explore its properties.

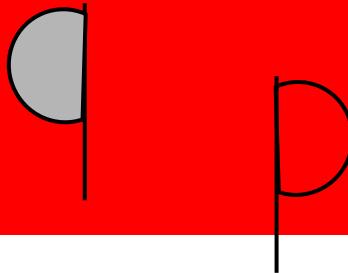
## Goals (Set)

- Define congruence.
- Apply properties of translations, rotations, and reflections to show that figures are congruent.
- Explore properties of congruent figures.

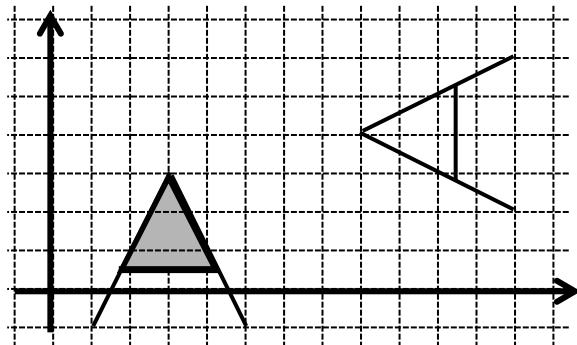
## Warmup (Go)

Write a sequence of steps to describe how each figure (shaded) could be mapped to its image (unshaded). Include notations or diagrams to support your statements. Some sentence frames with notation suggestions are provided below to help you.

1.



2.



Here are some possible sentence frames that may be used to describe a transformation without coordinates.

- Translate using vector  $\vec{v}$ .
- Reflect through line  $m$ .
- Rotate as indicated on the diagram around point  $P$ .

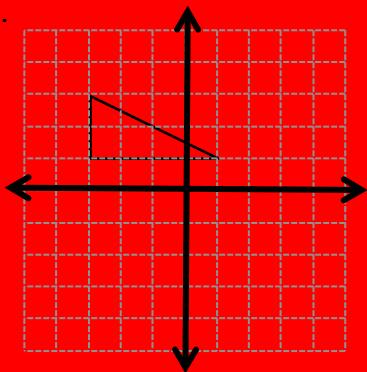
Here are some possible sentence frames to describe a transformation on a coordinate system.

- Translate \_\_\_\_\_ units  
(number) (right/left/up/down).
- Reflect through the line \_\_\_\_\_.  
(write an equation)
- Rotate \_\_\_\_\_ about  
(right/left) (degrees)  
(\_\_\_\_\_, \_\_\_\_\_.)  
(coordinates)
- Map  $(x, y) \rightarrow (_____, _____)$ .

## TALKING TRANSFORMATIONS

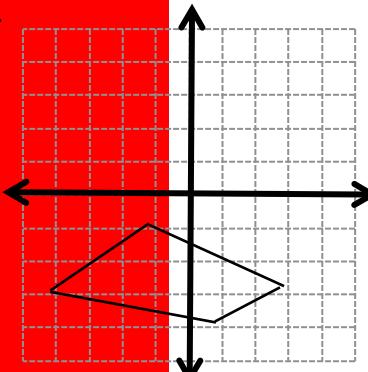
Complete this activity with a partner following the instructions of your teacher.

1a.



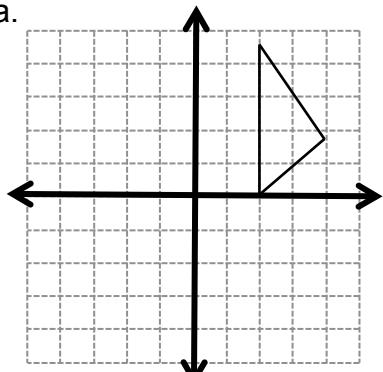
Steps:

2a.



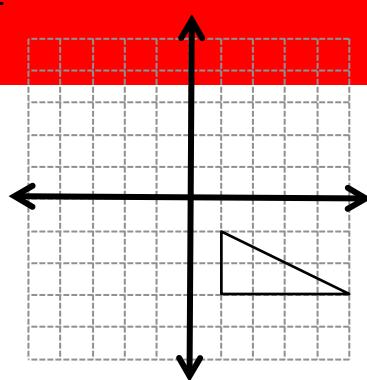
Steps:

3a.



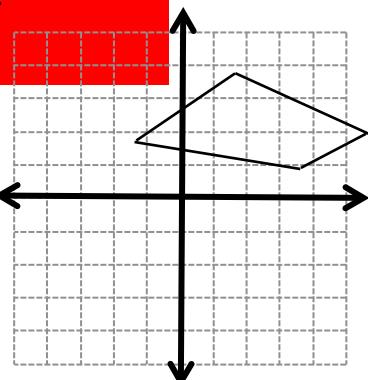
Steps:

1b.



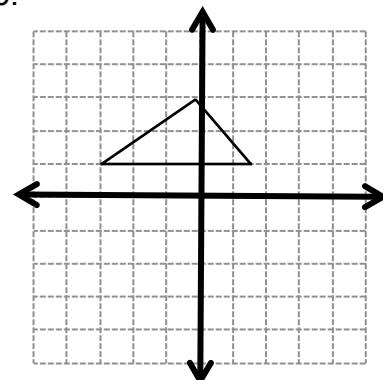
Steps:

2b.



Steps:

3b.



Steps:

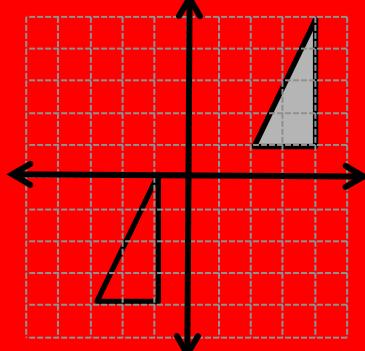
Two figures in the plane are congruent if the second can be obtained from the first by a sequence of translations, rotations, and reflections.

4. How do you know that the figures and images in the “Talking Transformations” activity are congruent?

## PRACTICE WITH CONGRUENT FIGURES

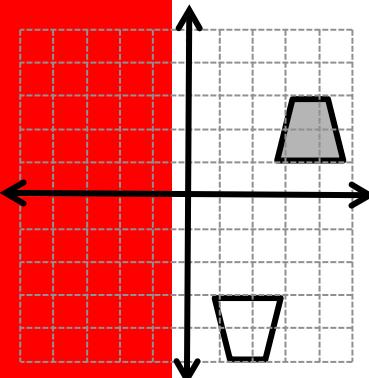
Describe TWO different ways to obtain the image (unshaded figure) from the original figure (shaded) by a sequence of translations, rotations, and reflections. Use patty paper as needed.

1.



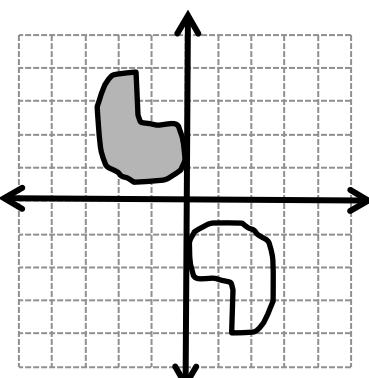
Strategy 1:

2.



Strategy 1:

3.



Strategy 1:

Strategy 2:

Strategy 2:

Strategy 2:

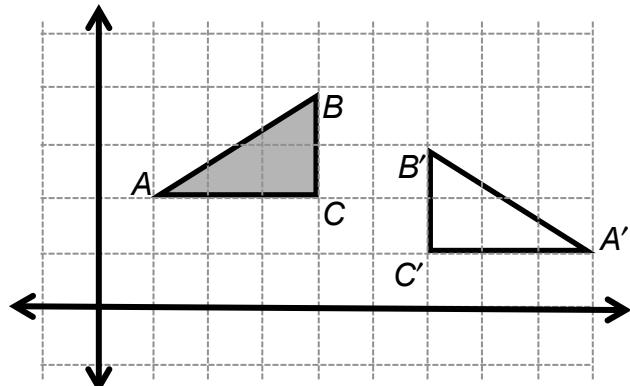
- Explain how translations, rotations, and reflections are related to congruence.

## PROPERTIES OF CONGRUENT FIGURES

1. Describe a sequence of transformations that maps  $\Delta ABC$  to its image  $\Delta A'B'C'$ . Use words, pictures and symbols.

Answer each question using proper notation.

2. Which angles are congruent to each other?
3. When two figures are congruent, what do you know about their corresponding angles?
4. Which sides are congruent to each other?
5. When two figures are congruent, what do you know about their corresponding sides?
6. Rashad says that if all the sides of one triangle have the same length as the corresponding sides in their image, then the triangles will be congruent. Is Rashad correct? Explain.
7. Van says that if all the angles of one triangle have the same length as the corresponding angles in their image, then the triangles will be congruent. Is Van correct? Explain.
8. Brigitta says that congruent figures have the same size and shape. Is Brigitta correct? Explain.



# DILATIONS

## Summary (Ready)

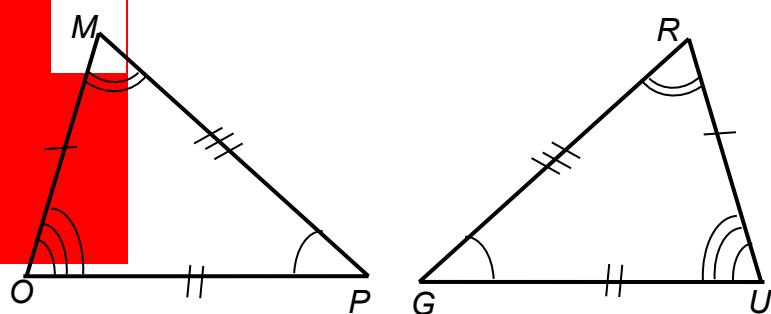
We will experiment with dilations and explore properties of dilations.

## Goals (Set)

- Perform a dilation experiment.
- Find the center point and scale factor of a dilation.
- Explore properties of dilations.

## Warmup (Go)

- These triangles are congruent.
- Congruent angles and congruent sides are marked.



1. Name the corresponding points under a transformation.

$$M \rightarrow \underline{\hspace{2cm}} \quad O \rightarrow \underline{\hspace{2cm}} \quad P \rightarrow \underline{\hspace{2cm}}$$

The symbol “ $\cong$ ” indicates that two objects are congruent.

2. Name the congruent sides.  $\overline{OP} \cong \underline{\hspace{2cm}}$     $\overline{OM} \cong \underline{\hspace{2cm}}$     $\overline{RG} \cong \underline{\hspace{2cm}}$

3. Name the congruent angles.  $\angle M \cong \underline{\hspace{2cm}}$     $\angle O \cong \underline{\hspace{2cm}}$     $\angle P \cong \underline{\hspace{2cm}}$

4. The order of the points in a congruence statement identifies corresponding points. State the triangle congruence in three different ways.

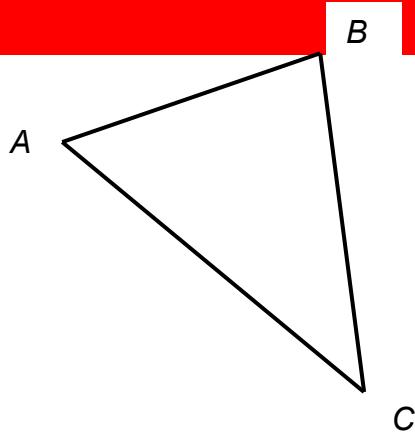
$$\Delta MOP \cong \Delta \underline{\hspace{2cm}} \quad \Delta MPO \cong \Delta \underline{\hspace{2cm}} \quad \Delta POM \cong \Delta \underline{\hspace{2cm}}$$

## RUBBER BAND EXPERIMENT

Use small rubber bands. Link two of them together. Anchor one end of the band at point  $P$  with a pencil. Put another pencil into the other end of the band. Move this pencil so that the knot in the middle of the band traces over the triangle. You will create a new figure that is the image of the first under a transformation.

Shade the original triangle. Do not shade the image. “Clean up” your image by drawing segments formed by the rubber band transformation with a ruler. Label the corresponding vertices  $A'$ ,  $B'$ ,  $C'$ .

What is the shape of the image?



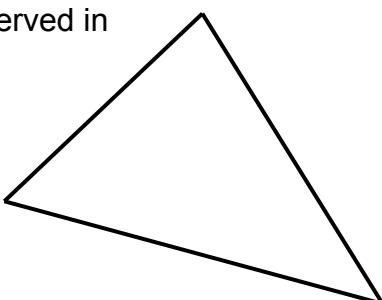
$P$



## RUBBER BAND EXPERIMENT (Continued)

Refer to the diagram you created on the previous page. Make a copy of  $\Delta ABC$  on patty paper or cut out  $\Delta ABC$  at the bottom of this page to help you answer these questions.

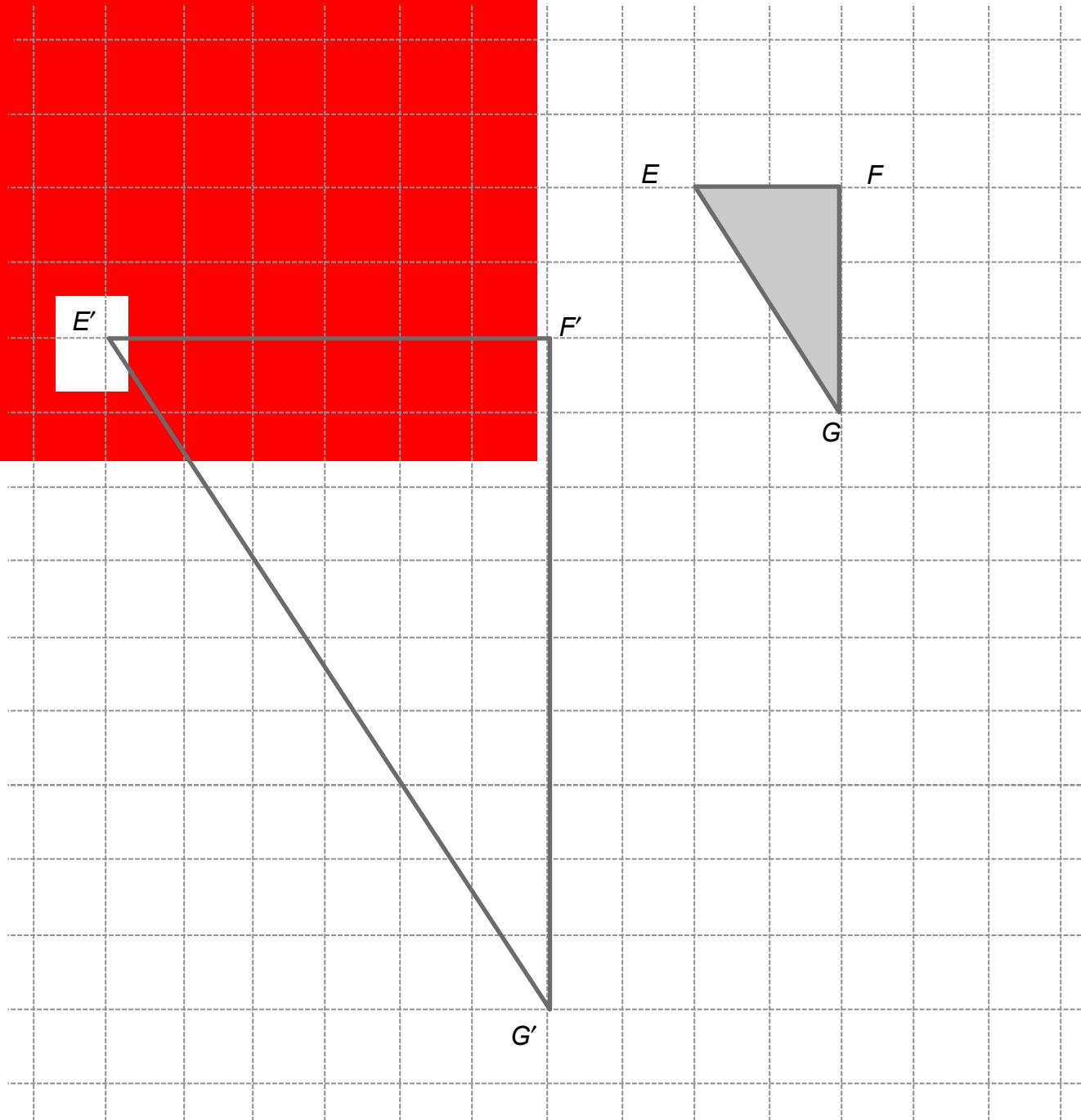
1. Is  $\Delta ABC \cong \Delta A'B'C'$ ? Explain.
2. How are the corresponding angles of  $\Delta ABC$  related to the corresponding angles in  $\Delta A'B'C'$ ? How do you know?
3. The area of  $\Delta A'B'C'$  is about how many times as large as the area of  $\Delta ABC$ ?
4. How are corresponding sides of  $\Delta ABC$  related to the corresponding sides in  $\Delta A'B'C'$ ? How do you know?
5. Find these ratios:  $\frac{|A'B'|}{|AB|} = \underline{\hspace{2cm}}$      $\frac{|B'C'|}{|BC|} = \underline{\hspace{2cm}}$      $\frac{|A'C'|}{|AC|} = \underline{\hspace{2cm}}$
6. How are these ratios related to the rubber band experiment?
7. Draw  $\overrightarrow{PA'}$ . Where is point A in relation to  $\overrightarrow{PA}'$ ? Explain why this happened?
8. Draw  $\overrightarrow{PB'}$  and  $\overrightarrow{PC'}$ . Did the relationships you observed in problem 7 above occur again?



## GRAPH PAPER EXPERIMENT 1

$\triangle EFG$  maps to  $\triangle E'F'G'$ .

1. Find the lengths of the sides of each triangle.
2. Explain why the triangles are not congruent.



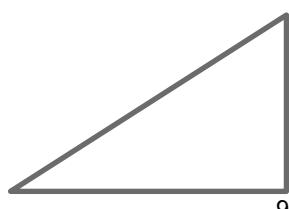
## GRAPH PAPER EXPERIMENT 1 (Continued)

3. Are corresponding angles congruent? Explain.
  
  
  
  
  
  
4. What is the relationship between corresponding sides? Be sure to check all sides and then explain.
  
  
  
  
  
  
5. Draw lines between corresponding vertices in the triangles. Extend the lines so they intersect each other. What do you notice about the point(s) of intersection?
  
  
  
  
  
  
6. Label  $Q$  as the point of intersection. Find the following ratios.

$$\frac{|E'Q|}{|EQ|} = \underline{\hspace{2cm}} \quad \frac{|F'Q|}{|FQ|} = \underline{\hspace{2cm}} \quad \frac{|G'Q|}{|GQ|} = \underline{\hspace{2cm}}$$

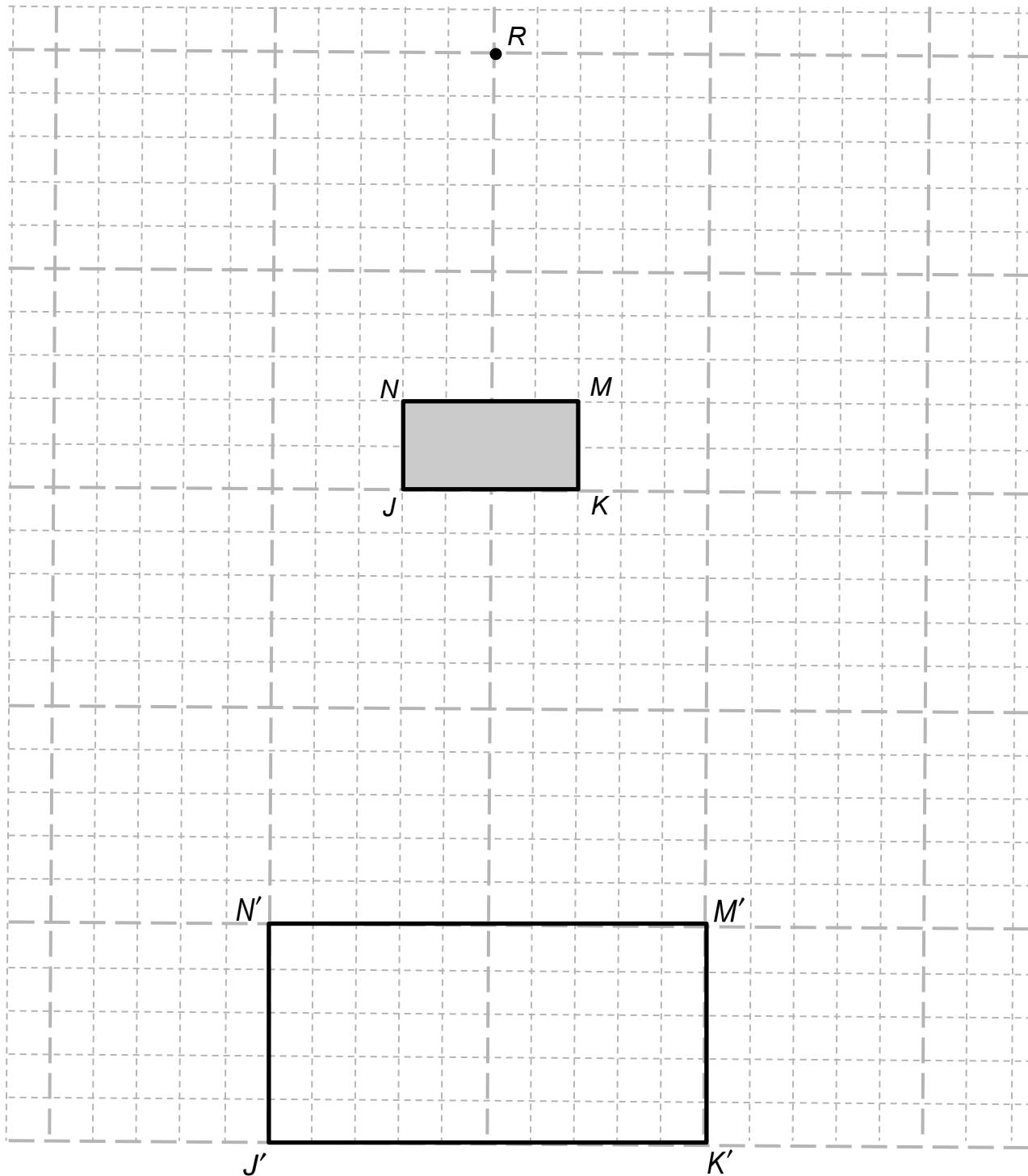
We call this ratio the scale factor.

7. How are the ratios formed from these segments related to the relationship you observed between corresponding sides?
  
  
  
  
  
8. Are corresponding sides in the figure and its image parallel? Explain.



## GRAPH PAPER EXPERIMENT 2

1. Rectangle  $JKMN$  maps to rectangle  $J'K'M'N'$ . Label side lengths of rectangles.
2. Is Rectangle  $JKMN \cong$  Rectangle  $J'K'M'N'$ ? \_\_\_\_\_ Explain.



## GRAPH PAPER EXPERIMENT 2 (Continued)

3. Are corresponding angles in the original rectangle and its image congruent? Explain.
4. What is the relationship between corresponding sides of the original rectangle and its image?
5. Draw lines between corresponding vertices in the original rectangle and its image. Extend the lines so they intersect each other at  $R$ .
6. Use the Pythagorean Theorem to find  $|J'R|$  and  $|JR|$ . Then find  $\frac{|J'R|}{|JR|}$ .
7. Without doing any computation, find  $\frac{|K'R|}{|KR|}$ . \_\_\_\_\_ How did you figure this out?
8. Use the Pythagorean Theorem to find  $|M'R|$  and  $|MR|$ . Then find  $\frac{|M'R|}{|MR|}$ .
9. Why is the following true?
$$\frac{|N'R|}{|NR|} = \frac{|M'R|}{|MR|}$$
10. How are ratios observed in problems 6, 7, 8, and 9 above related to the relationship you observed between corresponding sides in problem 4?

## DILATIONS

The previous experiments illustrate a transformation called a dilation.

A dilation is a transformation that moves each point along the ray through the point emanating from a fixed center, multiplying distances from the center by a common scale factor.

1. Name some features of dilations on the previous pages.

	$\Delta ABC \rightarrow \Delta A'B'C'$	$\Delta EFG \rightarrow \Delta E'F'G'$	$\square JKMN \rightarrow \square J'K'M'N'$
fixed center			
rays through the center			
scale factor			

2. Under the dilations you have observed, are line segments taken to line segments of the same length?
3. Under the dilations you have observed, are angles taken to angles of the same measure?
4. How are dilations different from translations, rotations, and reflections?

Inductive reasoning is a form of reasoning in which the conclusion is supported by the evidence but is not proved.

5. What are some properties you have observed about dilations, based on the evidence of the experiments?

## DILATIONS IN THE COORDINATE PLANE

$\Delta T'R'Y'$  is the image of  $\Delta TRY$  under a dilation.

- Find the lengths of the sides of the triangles.  
Label them on the figures.

- Use a ruler to draw lines  $TT'$ ,  $RR'$  and  $YY'$ .  
Label their intersection as point  $C$ .

Point  $C$  is the \_\_\_\_\_ of the dilation.

- Draw an  $x$ -axis and  $y$ -axis through the center point. What are the coordinates of  $C$ ?  
(\_\_\_\_, \_\_\_\_)

- What are the coordinates of the vertices in the original figure (shaded) and the image (unshaded)?

$$T(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$$

$$R(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$$

$$Y(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$$

$$T'(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$$

$$R'(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$$

$$Y'(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$$

- Describe a relationship between the coordinates in the original figure and coordinates in the image.

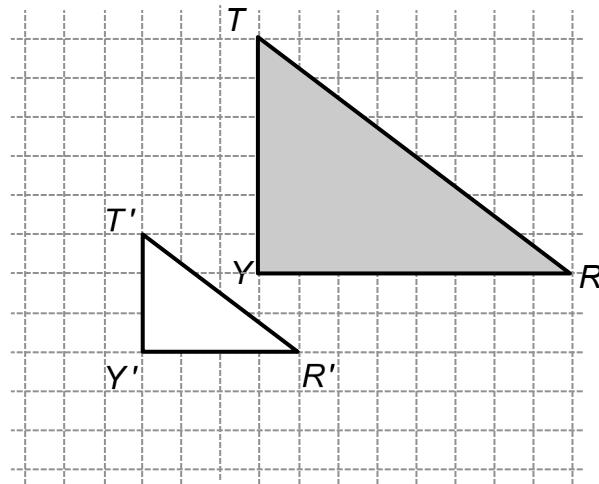
- Use symbols to describe the how the original figure maps to its image.

$$(x, y) \rightarrow (\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$$

- What is the scale factor of the figure.  $\frac{|CT'|}{|CT|} = \underline{\hspace{1cm}}$     $\frac{|CR'|}{|CR|} = \underline{\hspace{1cm}}$     $\frac{|CY'|}{|CY|} = \underline{\hspace{1cm}}$

- How is the scale factor different in this example from those in previous examples?

- What does that mean in terms of the figure and its image?

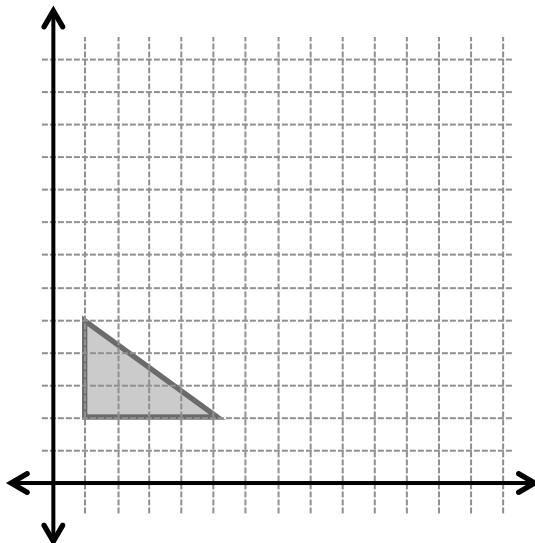


## DILATIONS IN THE COORDINATE PLANE (Continued)

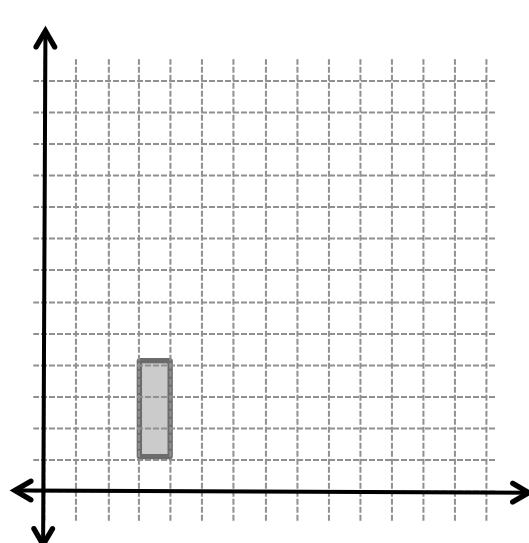
To dilate a figure in the coordinate plane with the center of dilation at the origin, multiply the coordinates of its points by the scale factor.

For each figure, identify the coordinates of the vertices. Then dilate it using the given scale factor. Check that the center of the dilation is the origin by drawing rays from the origin through corresponding points on the figure and its image.

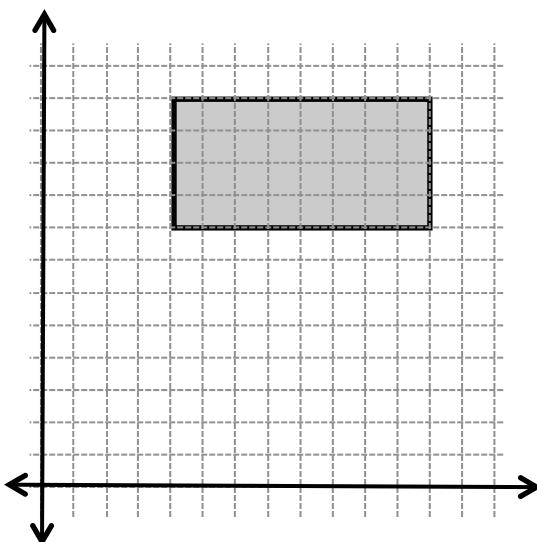
10. scale factor: 2



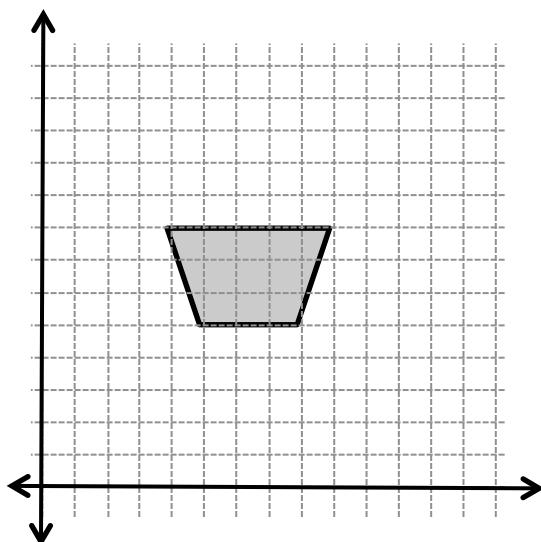
11. scale factor: 3



12. scale factor: 0.25



13. scale factor: 1



14. Make conjectures. What is the effect on a dilation if the scale factor is

greater than 1? \_\_\_\_\_ equal to 1? \_\_\_\_\_ between 0 and 1? \_\_\_\_\_

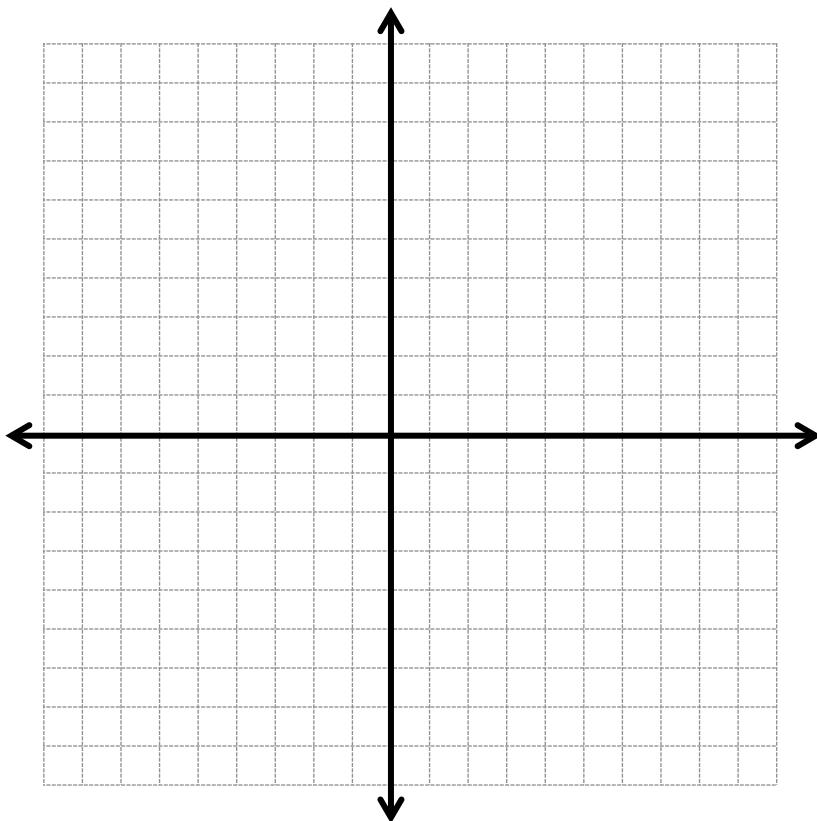
## NAME THAT TRANSFORMATION

1. Draw and shade a right trapezoid with coordinates:

$$L(1, 2), M(1, 3)$$

$$R(4, 3), T(4, 1)$$

2. Draw the images of the trapezoid under the transformations below, and label them. Do not shade the images.



A. $(x, y) \rightarrow (-y, x)$ $(1, 2) \rightarrow$ $(1, 3) \rightarrow$ $(4, 3) \rightarrow$ $(4, 1) \rightarrow$	B. $(x, y) \rightarrow (x, -y)$ $(1, 2) \rightarrow$ $(1, 3) \rightarrow$ $(4, 3) \rightarrow$ $(4, 1) \rightarrow$	C. $(x, y) \rightarrow (2x, 2y)$ $(1, 2) \rightarrow$ $(1, 3) \rightarrow$ $(4, 3) \rightarrow$ $(4, 1) \rightarrow$
---	---	--

Answer “yes” or “no” for each transformation above.

		A	B	C
3.	Is the image of the figure congruent to the original figure?			
4.	Is the transformation a dilation?			
5.	Are segments taken to segments of equal length?			
6.	Are angles taken to angles of equal measure?			
7.	Are parallel lines taken to parallel lines?			

## NAME THAT TRANSFORMATION (Continued)

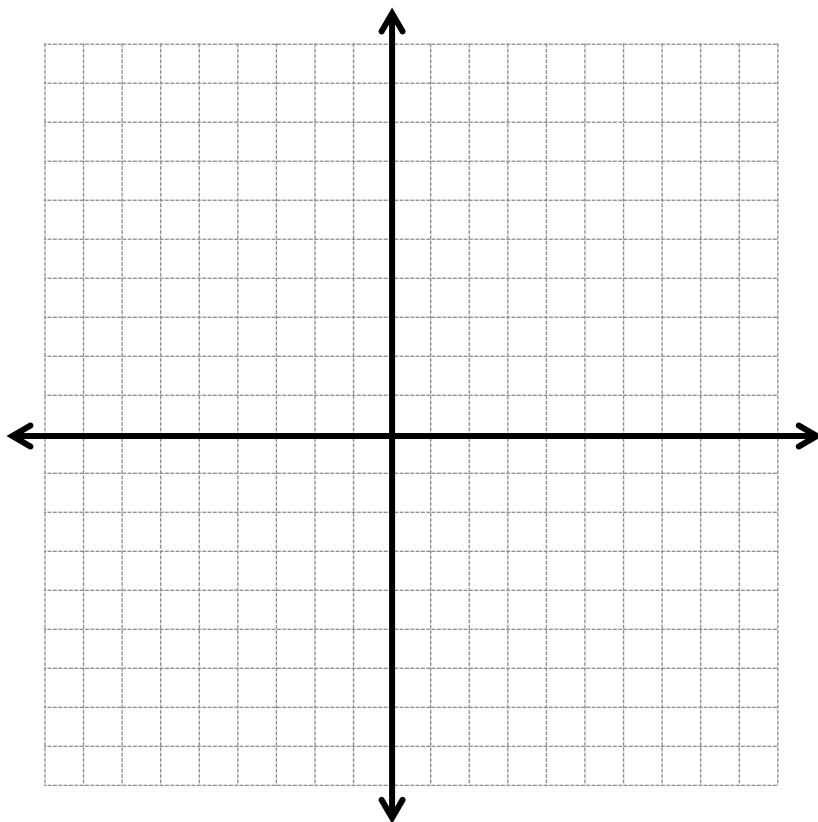
8. Draw three line segments

$\overline{PH}$ :  $P(4, 2)$  to  $H(4, 8)$

$\overline{GJ}$ :  $G(4, 6)$  to  $J(7, 6)$

$\overline{HK}$ :  $H(4, 8)$  to  $K(8, 8)$

9. Perform each transformation in the plane and label it.



D. $(x, y) \rightarrow (x - 8, y - 5)$	E. $(x, y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$	F. $(x, y) \rightarrow (x - y, -y)$
$(4, 2) \rightarrow$	$(4, 2) \rightarrow$	$(4, 2) \rightarrow$
$(4, 6) \rightarrow$	$(4, 6) \rightarrow$	$(4, 6) \rightarrow$
$(4, 8) \rightarrow$	$(4, 8) \rightarrow$	$(4, 8) \rightarrow$
$(7, 6) \rightarrow$	$(7, 6) \rightarrow$	$(7, 6) \rightarrow$
$(8, 8) \rightarrow$	$(8, 8) \rightarrow$	$(8, 8) \rightarrow$

Answer "yes" or "no" for each transformation above.

		D	E	F
10.	Is the image of the figure congruent to the original figure?			
11.	Is the transformation a dilation?			
12.	Are segments taken to segments of equal length?			
13.	Are angles taken to angles of equal measure?			
14.	Are parallel lines taken to parallel lines?			

# SIMILARITY

## Summary (Ready)

We will define similarity and explore its properties. We will compare properties of similarity to properties of congruence.

## Goals (Set)

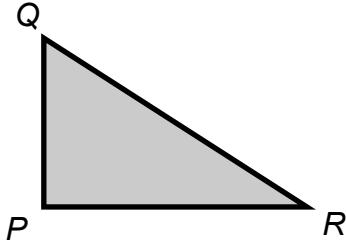
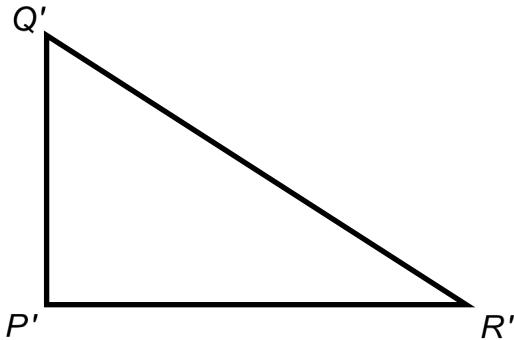
- Define similarity.
- Apply properties of translations, rotations, reflections, and dilations to show that figures are similar.
- Explore properties of similar figures.

## Warmup (Go)

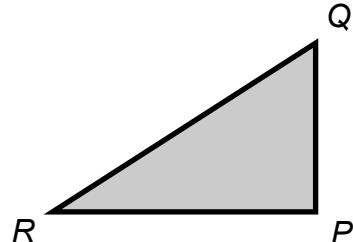
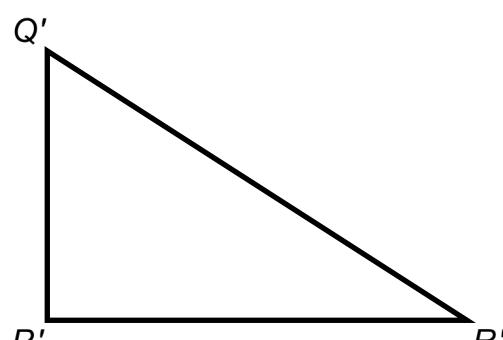
One of these images is a dilation of the original (shaded) figure. The other is not.

- For the dilation, find the center point and scale factor.
- For the pair that is not a dilation, explain a sequence of translations, rotations, and reflections you might perform so that the image could be a dilation of the original (shaded) figure.

1.



2.



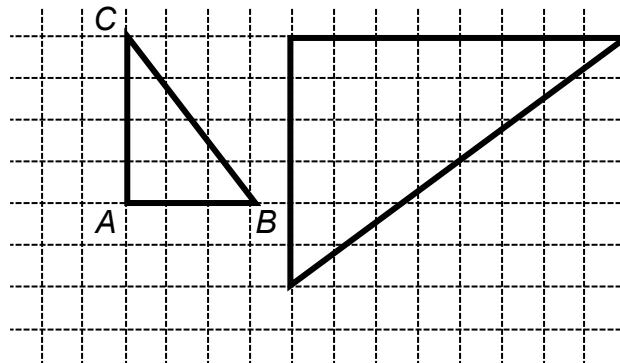
## SIMILARITY

Two figures are similar if one can be obtained from the other by a sequence of translations, rotations, reflections, and dilations.

Here are two similar triangles.

$\Delta ABC$  is labeled.

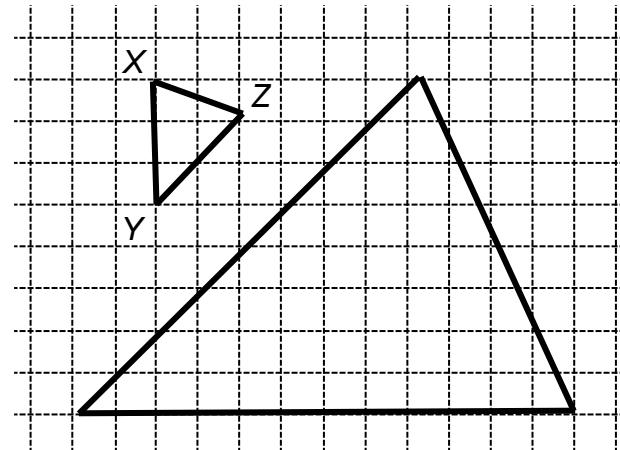
1. Label corresponding vertices of  $\Delta A'B'C'$ .
2. Find the scale factor by computing ratios of corresponding sides.
3. Use symbols, words, and pictures to describe a sequence of translations, rotations, reflections, and dilations that maps  $\Delta ABC$  onto  $\Delta A'B'C'$ . In other words, show that  $\Delta ABC$  is similar to  $\Delta A'B'C'$  (written  $\Delta ABC \sim \Delta A'B'C'$ ).



Here are two similar triangles.

$\Delta XYZ$  is given.

4. Label the corresponding vertices for  $\Delta X'Y'Z'$ .
5. Find the scale factor by computing ratios of corresponding segments.



6. Use symbols, words, and pictures to show that  $\Delta XYZ \sim \Delta X'Y'Z'$ .

7. How are dilations and similarity related?

## COMPARING CONGRUENCE AND SIMILARITY

Deductive reasoning is a form of reasoning in which the conclusion is justified by an argument based on definitions, known facts, and accepted rules of logic.

Decide if each statement is true or false. If it is true, justify your answer with deductive reasoning. If it is false, explain and give a counterexample.

1a. Whenever two figures are congruent, they are similar.	1b. Whenever two figures are similar, they are congruent.
2a. Any dilation of a figure is similar to the figure.	2b. If two figures are similar, then one must be a dilation of the other.
3a. The lengths of corresponding sides of congruent figures are equal.	3b. The lengths of corresponding sides of similar figures are equal.
4a. The measures of corresponding angles of congruent figures are equal.	4b. The measures of corresponding angles of similar figures are equal.
5a. If the measures of corresponding angles of two triangles are equal, then the triangles are congruent.	5b. If the measures of corresponding angles of two quadrilaterals are equal, then the quadrilaterals must be similar.

## SKILL BUILDERS, VOCABULARY, AND REVIEW

### SKILL BUILDER 1

1. Tomas was playing with numerators and denominators in this proportion:  $\frac{1}{3} = \frac{4}{12}$ . When he moved the numerators and denominators around, he found that some of the equations were still true. Circle all of the equations below that are true.

a. $\frac{3}{1} = \frac{12}{4}$	b. $\frac{4}{3} = \frac{12}{1}$	c. $\frac{4}{1} = \frac{12}{3}$
d. $\frac{1}{4} = \frac{3}{12}$	e. $\frac{1}{12} = \frac{3}{4}$	f. $\frac{4}{3} = \frac{1}{12}$

2. Write two more true equations using the numerators and denominators from this proportion:  $\frac{5}{6} = \frac{15}{18}$

Solve each equation using a mental strategy.

3. $-8 = \frac{24}{y}$	4. $\frac{1}{2}(12+z) = 9$
------------------------	----------------------------

Solve each inequality using a mental strategy. Then graph the solutions using open or closed circles and a ray.

5. $4x < -8$  Graph: 	6. $x - 10 \leq 14$  Graph: 
--	--

7. Compute  $\frac{10^3}{10^5}$ .

8. Write  $\frac{3^2 \cdot 3^5}{3^{12}}$  in exponent form:

## SKILL BUILDER 2

The numbers given in the squares in the figure to the right represent their areas in square units. Non-integer values may be left in square root form.

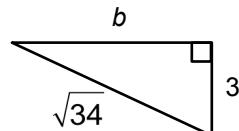
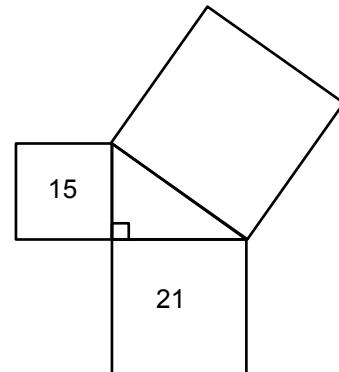
1. Find the area of the third square. \_\_\_\_\_

2. Find the length of the shorter leg. \_\_\_\_\_

3. Find the length of the longer leg. \_\_\_\_\_

4. Find the length of the hypotenuse. \_\_\_\_\_

5. Find the length of the missing side of the triangle. \_\_\_\_\_



6. Explain why a triangle with side lengths 6 cm, 8 cm, and 12 cm cannot be a right triangle.

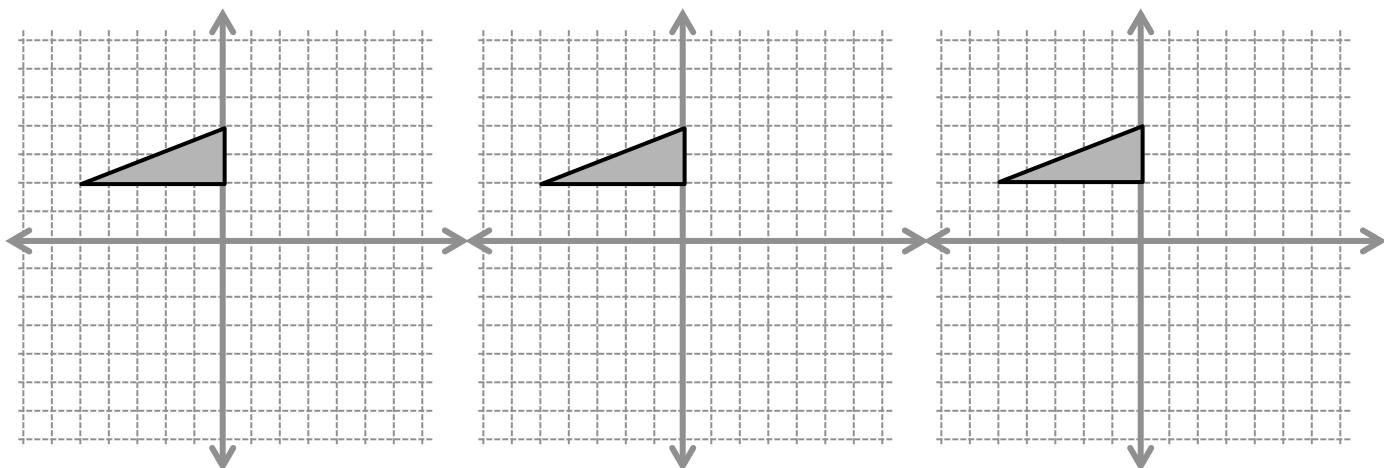
Find the circumference and area of each circle described.

	Given measure	Estimate using $\pi = 3$	Estimate using $\pi = 3.14$	Exact (leave in terms of $\pi$ )
7.	5 cm radius	$C =$ ----- $A =$	$C =$ ----- $A =$	$C =$ ----- $A =$
8.	8 ft. diameter	$C =$ ----- $A =$	$C =$ ----- $A =$	$C =$ ----- $A =$

## SKILL BUILDER 3

- Triangle ABC has vertices at  $A(0, 2)$ ,  $B(0, 4)$ ,  $C(-5, 2)$ .
- Transfer these coordinates to the table at right prior to beginning the problems.
- Explore the effects of translations, rotations, and reflections by observing the effect on the image of the triangle.

Original Figure		
	x	y
A		
B		
C		



1. $(x, y) \rightarrow (-x, y)$		
	-x	y
A'		
B'		
C'		

2. $(x, y) \rightarrow (x + 5, y - 5)$		
A'		
B'		
C'		

3. $(x, y) \rightarrow (-x, -y + 4)$		
A'		
B'		
C'		

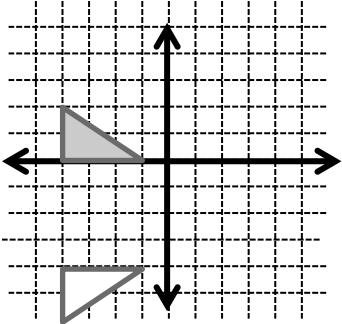
Refer to each of the three problems above (tables and diagrams).

	Describe the transformation	Are segments taken to segments of the same length?	Are angles taken to angles of the same measure?
1.			
2.			
3.			

## SKILL BUILDER 4

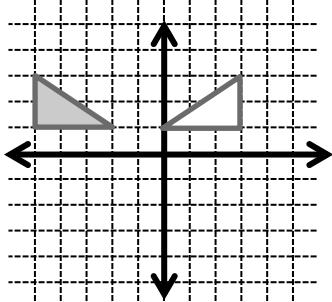
A figure and its image under a reflection are shown. Give the equation for the line of reflection. Use patty paper if needed.

1.



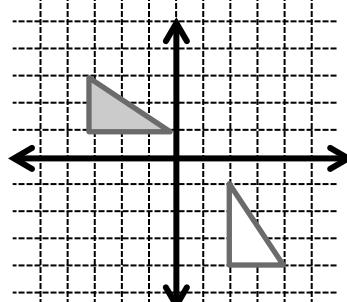
Line of reflection:

2.



Line of reflection:

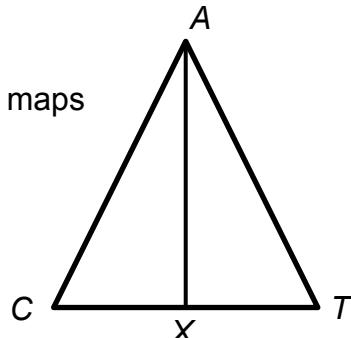
3.



Line of reflection:

In the isosceles triangle at the right,  $\triangle ACX \cong \triangle ATX$ .

4. Name the corresponding vertices under a transformation that maps  $\triangle ACX$  to  $\triangle ATX$ .
5. Name the congruent angles. (Hint: You will need to use three letters to identify some of the angles.)
6. Name congruent sides.
7. Label congruent sides and corresponding angles on the diagram.
8. What is the measure of  $\angle AXC$ ? Explain.
9. Name a pair of supplementary angles.
10. Name a pair of complementary angles.



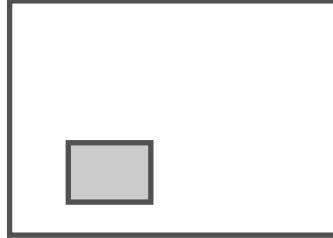
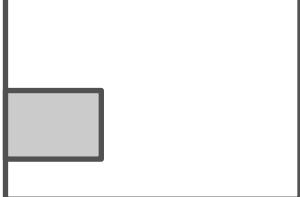
## SKILL BUILDER 5

Each rectangle (shaded) is mapped to an image (unshaded) by a dilation.

- Label vertices of the shaded rectangle  $T, A, N, G$ . Label the corresponding vertices of the image  $T', A', N', G'$ .
- Draw lines that connect corresponding points on the original figure to its image.

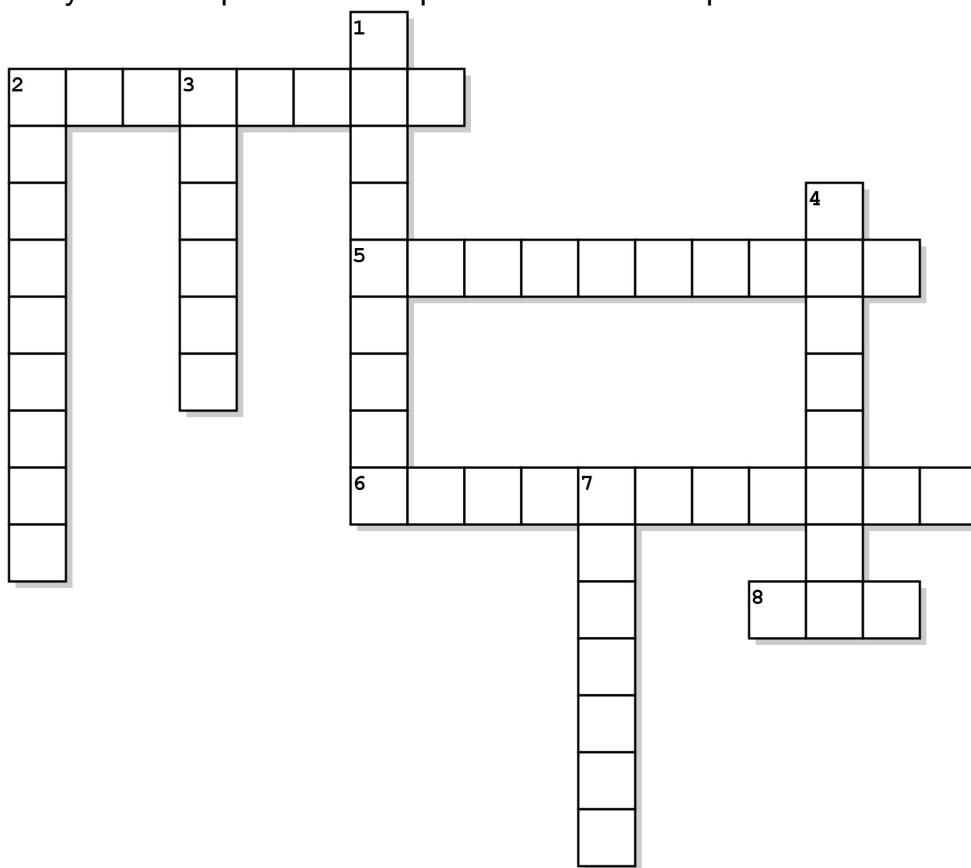
These segments should meet at the \_\_\_\_\_ of the dilation.  
Make this point large (●).

- Measure to find scale factor of the dilation.

<p>1. (original figure inside the image)</p>  <p>scale factor _____</p>	<p>2. (original figure inside the image)</p>  <p>scale factor _____</p>
<p>3.</p>  <p>scale factor _____</p>	<p>4. (original figure inside the image)</p>  <p>scale factor _____</p>
<p>5. Use one of the examples above to show that under a dilation, line segments are not taken to line segments of the same length.</p>	

## FOCUS ON VOCABULARY

Use vocabulary from this packet to complete the crossword puzzle.



<u>Across</u>	<u>Down</u>
2 transformation determined by a center and scale factor	1 characteristic of two figures that are related by a sequence of translations, rotations, and reflections
5 transformation determined by a line of symmetry	2 reasoning based on definitions, properties, and logic
6 transformation determined by a vector	3 ___ are taken to ___ of the same measure by translations, rotations, reflections, and dilations (the same word is used for both blanks)
8 scale factor relating congruent figures	4 transformation determined by a center and an angle
	7 characteristic of two figures related by a sequence of translations, rotations, and reflections, and also a dilation

## SELECTED RESPONSE

Show your work on a separate sheet of paper and choose the best answer(s).

---

1. Circle the letters of the statements below that are true.
  - A. Two figures in the plane are congruent if there is a sequence of translations, rotations, and reflections that maps one figure onto the other.
  - B. Corresponding angles of congruent triangles are congruent.
  - C. Corresponding sides of congruent triangles are congruent.
  - D. Under a transformation, a figure is always congruent to its image.

---
2. Circle the letters of the statements below that are true.
  - A. A dilation is a transformation.
  - B. Under a dilation, segments are taken to segments of the same length.
  - C. Under a dilation, angles are taken to angles of the same measure.
  - D. Under a dilation, a figure and its image are always similar.

---
3. A dilation of scale factor 0.5 with center at the origin transforms a segment with endpoints at  $(0, 10)$  and  $(0, -2)$  to an image with endpoints
  - A.  $(0, 5)$  and  $(0, -2.5)$
  - B.  $(0, 50)$  and  $(0, -10)$
  - C.  $(0, 5)$  and  $(0, -1)$
  - D.  $(0, 20)$  and  $(0, -4)$

---
4. A dilation of scale factor 2 with center at the origin transforms a segment with endpoints at  $(3, 3)$  and  $(0, -1)$  to an image with endpoints
  - A.  $(1.5, 1.5)$  and  $(0, -0.5)$
  - B.  $(6, 6)$  and  $(0, -2)$
  - C.  $(1.5, 1.5)$  and  $(0, -4)$
  - D.  $(6, 6)$  and  $(0, 2)$

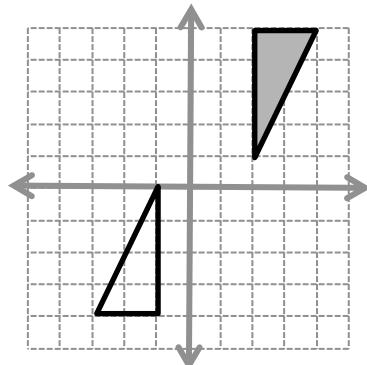
---
5. Circle the letters of the statements below that are true.
  - A. Two figures are similar if the second can be obtained from the first by a sequence of translations, rotations, reflections, and dilations.
  - B. Under a dilation, a figure and its image are similar.
  - C. Similar figures have corresponding angles of equal measure.
  - D. Similar figures have corresponding segments of equal length.

---

## KNOWLEDGE CHECK

### 14.1 Congruence

1. Describe how to obtain the image (unshaded figure) from the original figure (shaded) by a sequence of translations, rotations, and reflections. Use patty paper as needed. Label the original triangle's vertices as  $A, B, C$  and the image  $A', B', C'$ .

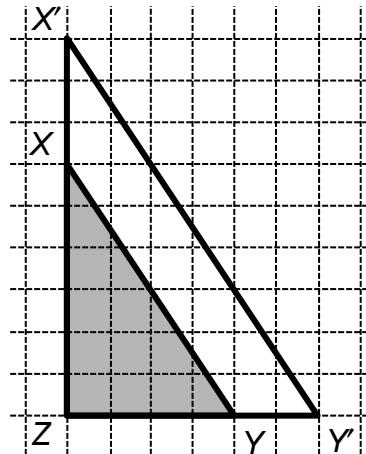


2. List the congruent segments and angles in the figure.

### 14.2 Dilations

$\triangle XYZ$  and its image are pictured.

3. Explain why  $\triangle X'Y'Z$  represents a dilation with center point Z. In your explanation, show that ratios of corresponding side lengths are proportional and name the scale factor.

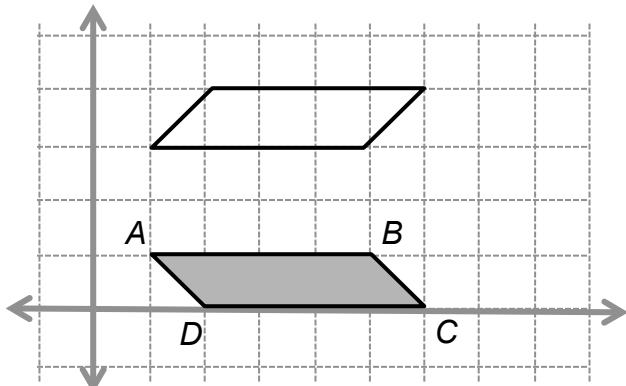


### 14.3 Similarity

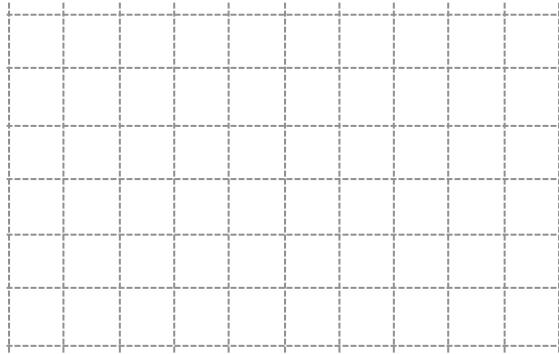
4. Lightly draw in the x- and y-axes so that the origin is at Z. Write a transformation rule that maps  $\triangle XYZ$  to its image.  $(x, y) \rightarrow (\underline{\hspace{2cm}}, \underline{\hspace{2cm}})$ .
5. Explain why  $\triangle XYZ$  and its image are similar.
6. Explain how dilations are related to similarity.

## HOME-SCHOOL CONNECTION

1. Describe a sequence of transformations that maps Figure  $ABCD$  to its image Figure  $A'B'C'D'$ . Use words, pictures and symbols. Be sure to label the vertices of the image appropriately.



2. When two figures are congruent, what do you know about their corresponding angles?
3. When two figures are congruent, what do you know about their corresponding sides?
4. On the grid to the right, draw a figure that is similar to Figure  $ABCD$  with a scale factor of 2. Show ratios of corresponding segments to support this.
5. Explain which of the following hold and which do not under this transformation.
  - a. Line segments are taken to line segments of equal length.
  - b. Angles are taken to angles of equal measure.
  - c. Parallel lines are taken to parallel lines.



## Congruence and Similarity

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## COMMON CORE STATE STANDARDS – MATHEMATICS

### STANDARDS FOR MATHEMATICAL CONTENT

- |       |   |
|-------|---|
| 8.G.2 | Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.                        |
| 8.G.3 | Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.  |
| 8.G.4 | Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two dimensional figures, describe a sequence that exhibits the similarity between them. |
| 8.G.7 | Apply the Pythagorean Theorem to determine the unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.   |

### STANDARDS FOR MATHEMATICAL PRACTICE

- |     |  |
|-----|--|
| MP2 | Reason abstractly and quantitatively.                            |
| MP3 | Construct viable arguments and critique the reasoning of others. |
| MP5 | Use appropriate tools strategically.                             |
| MP6 | Attend to precision.   |
| MP7 | Look for and make use of structure.                              |



9 7 8 1 6 1 4 4 5 2 2 2 5  
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