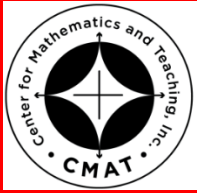


Name \_\_\_\_\_

Period \_\_\_\_\_

Date \_\_\_\_\_



**MathLinks**

**8-1**

STUDENT PACKET

**MATHLINKS GRADE 8  
STUDENT PACKET 1  
INTEGERS REVIEW**

<b>1.1</b>	<b>Integer Operations: Patterns</b> <ul style="list-style-type: none"><li>• Explore the meaning of integer addition, subtraction, multiplication, and division.</li><li>• Review rules for integer addition, subtraction, multiplication, and division.</li></ul>	<b>1</b>
<b>1.2</b>	<b>Integer Operations: A Counter Model</b> <ul style="list-style-type: none"><li>• Explore the meaning of integer addition, subtraction, multiplication, and division.</li><li>• Review rules for integer addition, subtraction, multiplication, and division.</li></ul>	<b>10</b>
<b>1.3</b>	<b>Order of Operations</b> <ul style="list-style-type: none"><li>• Understand the convention for order of operations.</li><li>• Use order of operations to simplify expressions.</li></ul>	<b>19</b>
<b>1.4</b>	<b>Skill Builders, Vocabulary, and Review</b>	<b>23</b>

# WORD BANK

Word or Phrase	Definition or Description	Example or Picture
absolute value		
difference		
factors		
integers		
opposite		
product		
quotient		
sum		

# INTEGER OPERATIONS: PATTERNS

## Summary (Ready)

We will use patterns to revisit the rules for adding, subtracting, multiplying, and dividing integers.

## Goals (Set)

- Explore the meaning of integer addition, subtraction, multiplication, and division.
- Review rules for integer addition, subtraction, multiplication, and division.

## Warmup (Go)

Fill in the next three entries for each pattern.

1. 12, 14, 16, 18, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_
2. -10, -9, -8, -7, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_
3.  $7, 7\frac{1}{3}, 7\frac{2}{3}, 8, 8\frac{1}{3},$  \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_
4. -4, -8, -12, -16, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_

Write the opposite of each number.

5.  $7 \rightarrow$  \_\_\_\_\_
6.  $-12 \rightarrow$  \_\_\_\_\_
7.  $0 \rightarrow$  \_\_\_\_\_
8. In your own words, explain what it means for a number to be an opposite of another number. Use a number line example to support your explanation.

Simplify each absolute value expression.

9.  $|7| =$  \_\_\_\_\_
10.  $|-12| =$  \_\_\_\_\_
11.  $|0| =$  \_\_\_\_\_
12. In your own words, explain what it means to find the absolute value of a number. Use a number line example to support your explanation.

## COMPLETE THE TABLE

1. Complete the pattern started below by filling numbers in the blank spaces.

	<b>II</b>											<b>I</b>
					5							
					4							
	-2			1		3		5	6			
					2	3			5	6	7	
				-1	0	1	2	3				
	-5	-4	-3	-2	-1	0	1	2	3	4	5	
				-3	-2	-1	0	1				
	-7				-3	-2	-1					
					-3							
	-9	-8				-4		-2				
<b>III</b>					-5							<b>IV</b>

2. Describe at least two patterns that you notice in the entire table.

3. Describe at least two patterns that you notice in any of the regions I, II, III, and IV.

## USING AN ADDITION TABLE

Did you notice that the grid on the previous page can be read as an addition table? Use it to find these sums:

1. $(2) + (3) = \underline{\quad}$ This is located in region $\underline{\quad}$ .	2. $(-2) + (-3) = \underline{\quad}$ This is located in region $\underline{\quad}$ .
3. $(2) + (-3) = \underline{\quad}$ This is located in region $\underline{\quad}$ .	4. $-2 + (3) = \underline{\quad}$ This is located in region $\underline{\quad}$ .
5. $(-3) + (-3) = \underline{\quad}$ This is located in region $\underline{\quad}$ .	6. $(3) + (-3) = \underline{\quad}$ This is located in region $\underline{\quad}$ .

Refer to the table on the previous page. Look for patterns for adding integers:

7. In region  $\underline{\quad}$ , the sum of a positive number and a positive number is  $\underline{\quad}$ .

That is,  $(+)$  plus  $(+)$  is  $(\underline{\quad})$ .

8. In region  $\underline{\quad}$ , the sum of a negative number and a negative number is  $\underline{\quad}$ .

That is,  $(-)$  plus  $(-)$  is  $(\underline{\quad})$ .

9. The sum of a positive number and a negative number is

positive when

negative when

zero when

So,  $(+)$  plus  $(-)$  can be  $(\underline{\quad})$ ,  $(\underline{\quad})$ , or  $(\underline{\quad})$

Compute. Use the table if needed.

10.  $4 + 5 = \underline{\quad}$       11.  $-4 + -5 = \underline{\quad}$       12.  $-4 + 5 = \underline{\quad}$       13.  $4 + -5 = \underline{\quad}$

## RELATING SUBTRACTION TO ADDITION

- How can the addition table be used to subtract?
- Complete the patterns in columns 1 and 2 based on the fact that addition and subtraction are inverse operations.

<u>Column 1</u> Addition Facts (from previous page)		<u>Column 2</u> Corresponding Subtraction Facts	<u>Column 3</u>
a. $(2) + (3) = 5$		$(5) - (2) = \underline{\quad}$	_____
		$(5) - (3) = \underline{\quad}$	_____
b. $(-2) + (-3) = \underline{\quad}$		$(\underline{\quad}) - (-2) = -3$	_____
		$(\underline{\quad}) - (-3) = -2$	_____
c. $(2) + (-3) = \underline{\quad}$		$(\underline{\quad}) - (2) = -3$	_____
		$(\underline{\quad}) - (-3) = 2$	_____
d. $(-2) + (3) = \underline{\quad}$		$(\underline{\quad}) - (-2) = 3$	_____
		$(\underline{\quad}) - (3) = -2$	_____

- In column 3 above, write “smaller” next to all of the subtraction facts for which the result (difference) is less than the starting value (minuend). What types of numbers are being subtracted here (positive or negative)?
- In column 3 above, write “larger” next to all of the subtraction facts for which the result (difference) is more than the starting value (minuend). What types of numbers are being subtracted here (positive or negative)?
- What would you tell a classmate who said, “Subtraction makes numbers smaller”?
- Shade a number in region II in the addition table. Then write an addition fact and two related subtraction facts based on the shaded number.
- Shade a number in region III in the addition table. Then write an addition fact and two related subtraction facts based on the shaded number.

## ADDITION AND SUBTRACTION PATTERNS

Complete the tables based on patterns you observe.

1.

Expression	Difference
$5 - (\underline{\quad})$	
$5 - (\underline{\quad})$	
$5 - (\underline{\quad})$	
$5 - (-1)$	
$5 - (0)$	5
$5 - (1)$	4
$5 - (2)$	
$5 - (3)$	
$5 - (\underline{\quad})$	
$5 - (\underline{\quad})$	
$5 - (\underline{\quad})$	

2.

Expression	Sum
$5 + (\underline{\quad})$	
$5 + (\underline{\quad})$	
$5 + (\underline{\quad})$	
$5 + (1)$	
$5 + (0)$	5
$5 + (-1)$	4
$5 + (-2)$	
$5 + (\underline{\quad})$	
$5 + (\underline{\quad})$	
$5 + (\underline{\quad})$	
$5 + (\underline{\quad})$	

Consider the numbers in the tables above (remember, all of these expressions begin with 5).

3. Under what circumstances are the results **less than 5**?

Subtracting a \_\_\_\_\_ number or adding a \_\_\_\_\_ number.

4. Under what circumstances are the results **greater than 5**?

Adding a \_\_\_\_\_ number or subtracting a \_\_\_\_\_ number.

Focus your attention on the results above.

5. What two expressions give a result of 4? \_\_\_\_\_

6. What two expressions give a result of 8? \_\_\_\_\_

7. What two expressions give a result of -1? \_\_\_\_\_

8. What pattern do you notice in expressions with the same results?

-----

## COMPLETE A DIFFERENT TABLE

Complete the patterns started below by filling numbers in the blank spaces.

<b>II</b>												<b>I</b>
						3	6	9		15		
				-2	0	2		6	8	10		
			-2	-1	0	1	2	3	4			
						0	0					
						-1						
			4									
							-8					
<b>III</b>												<b>IV</b>

Cut the strips below on the thick lines only. Lay them over the zeroes to create horizontal and vertical number lines. Describe at least one pattern that you notice in the overall table and at least one in any of the regions I, II, III, and IV.

---

-5	-4	-3	-2	-1	0	1	2	3	4	5
5	4	3	2	1	0	-1	-2	-3	-4	-5



## USING A MULTIPLICATION TABLE

Did you notice that with the number lines in place, the grid now serves as a multiplication table? Use it to find these products:

1. $(5) \cdot (3) = \underline{\quad}$ located in region $\underline{\quad}$	2. $(2) \cdot (-1) = \underline{\quad}$ located in region $\underline{\quad}$
3. $(-3) \cdot (4) = \underline{\quad}$ located in region $\underline{\quad}$	4. $(-4) \cdot (-3) = \underline{\quad}$ located in region $\underline{\quad}$
5. $(-5) \cdot (-3) = \underline{\quad}$ located in region $\underline{\quad}$	6. $(4) \cdot (-3) = \underline{\quad}$ located in region $\underline{\quad}$

Patterns for Multiplying Integers:

7. The product of a positive number and a positive number is $\underline{\hspace{2cm}}$ . (See region $\underline{\hspace{1cm}}$ .) $(+) \cdot (+) = (\underline{\hspace{1cm}})$	8. The product of a positive number and a negative number is $\underline{\hspace{2cm}}$ . (See regions $\underline{\hspace{1cm}}$ , $\underline{\hspace{1cm}}$ .) $(+) \cdot (-) = (\underline{\hspace{1cm}})$ $(-) \cdot (+) = (\underline{\hspace{1cm}})$	9. The product of a negative number and a negative number is $\underline{\hspace{2cm}}$ . (See region $\underline{\hspace{1cm}}$ .) $(-) \cdot (-) = (\underline{\hspace{1cm}})$
--	--	--

Compute.

10. $(-8) \cdot (3) = \underline{\quad}$	11. $(-9) \cdot (-4) = \underline{\quad}$	12. $(-6) \cdot (5) = \underline{\quad}$
13. $(-20) \cdot (3) = \underline{\quad}$	14. $(-30) \cdot (-40) = \underline{\quad}$	15. $(-60) \cdot (8) = \underline{\quad}$

Challenge.

16. $(-2) \cdot (-3) \cdot (-6) = \underline{\quad}$	17. $(-20) \cdot (50) \cdot (-7) = \underline{\quad}$
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## RELATING DIVISION TO MULTIPLICATION

- How can the multiplication table be used to divide?
- Complete the patterns below based on the fact that multiplication and division are inverse operations.

	<b>Multiplication Facts (from previous page)</b>		<b>Corresponding Division Facts</b>
a.	$(4) \cdot (5) = 20$		$(20) \div (4) = \underline{\quad}$ $(20) \div (\underline{\quad}) = 4$
b.	$(3) \cdot (-2) = \underline{\quad}$		$(\underline{\quad}) \div (3) = -2$ $(\underline{\quad}) \div (-2) = 3$
c.	$(-3) \cdot (4) = \underline{\quad}$		$(\underline{\quad}) \div (-3) = 4$ $(\underline{\quad}) \div (4) = -3$
d.	$(-2) \cdot (-3) = \underline{\quad}$		$(\underline{\quad}) \div (-2) = -3$ $(\underline{\quad}) \div (-3) = -2$

### Patterns for Dividing Integers:

3. The quotient of a positive number and a positive number is _____. $(+) \div (+) = \underline{\quad}$	4. The quotient of a positive number and a negative number is _____. $(+) \div (-) = \underline{\quad}$
5. The quotient of a negative number and a positive number is _____. $(-) \div (+) = \underline{\quad}$	6. The quotient of a negative number and a negative number is _____. $(-) \div (-) = \underline{\quad}$

Compute.

7.  $(-14) \div (7) = \underline{\quad}$       8.  $(27) \div (-3) = \underline{\quad}$       9.  $\frac{-28}{-4} = \underline{\quad}$

**PRACTICE**

Compute. If needed, refer to a table or patterns and rules from previous pages

1. $-5 + 4$	2. $4 + -5$	3. $-4 + -5$	4. $-3 + 3$
5. $-1 + 3$	6. $1 + -3$	7. $-1 + -3$	8. $2 + -2$
9. $3 - 5$	10. $3 - (-2)$	11. $-2 - 1$	12. $-2 - (-3)$
13. $-2 - 3$	14. $-2 - (-5)$	15. $-8 - (-5)$	16. $-8 - (-3)$
17. $6 - (-1)$	18. $0 - 5$	19. $0 - (-5)$	20. $-4 - (-4)$
21. $2 \cdot 4$	22. $2 \cdot -4$	23. $-2 \cdot -4$	24. $-2 \cdot 4$
25. $-3 \cdot -5$	26. $5 \cdot -3$	27. $12 \div -6$	28. $-12 \div -6$
29. $-12 \div 6$	30. $\frac{18}{-3}$	31. $\frac{-18}{-3}$	32. $\frac{-24}{2}$

33. In Alaska, the temperature was  $-14^{\circ}\text{F}$  one morning. By noon, the temperature had dropped  $7^{\circ}\text{F}$ . What was the temperature at noon? \_\_\_\_\_
34. A fish is 80 feet below sea level. If it ascends 25 feet, what is its new position? \_\_\_\_\_
35. Mount Everest (the highest elevation on earth) is 29,028 feet above sea level. The Dead Sea (the lowest elevation on earth) is 1,312 feet below sea level. What is the difference between these two elevations? \_\_\_\_\_

**INTEGER OPERATIONS: A COUNTER MODEL****Summary (Ready)**

We will use a model to revisit the rules for adding, subtracting, multiplying, and dividing integers.

**Goals (Set)**

- Explore the meaning of integer addition, subtraction, multiplication, and division.
- Review rules for integer addition, subtraction, multiplication, and division.

**Warmup (Go)**

Write the opposite of each number.

1.  $9 \rightarrow$  \_\_\_\_\_      2.  $-3 \rightarrow$  \_\_\_\_\_      3.  $0 \rightarrow$  \_\_\_\_\_

4. In your own words, explain what it means for a number to be an opposite of another number. Use a number line example to support your explanation.

Simplify each absolute value expression.

5.  $|8| =$  \_\_\_\_\_      6.  $|-5| =$  \_\_\_\_\_      7.  $|0| =$  \_\_\_\_\_

8. In your own words, explain what it means to find the absolute value of a number. Use a number line example to support your explanation.

9. From a standing position, where do you end up if you move one yard backward and then one yard forward?

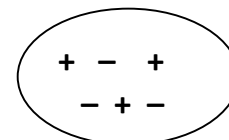
10. Given a certain amount of money, how much do you have if you gain one dollar and then lose one dollar?

# A COUNTER MODEL

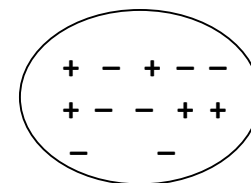
We can use different colored counters to represent positive numbers and negative numbers.

A positive counter is represented by a plus (+).  
 A negative counter is represented by a minus (-).  
 A combination of one positive counter and one negative counter is a “zero pair.” (+ -)

1. What is the value of this collection? \_\_\_\_\_ Explain.



2. What is the value of this collection? \_\_\_\_\_ Explain.



3. Use combinations of counters to draw a value of 3 in three different ways.

a. Using 5 counters:	b. Using 7 counters:	c. Using 9 counters:
How many zero pairs are in your collection? _____	How many zero pairs are in your collection? _____	How many zero pairs are in your collection? _____

4. Use combinations of counters to draw a value of -2 in three different ways.

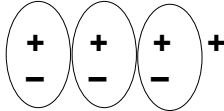
a. Using 4 counters:	b. Using 6 counters:	c. Using 10 counters:
How many zero pairs are in your collection? _____	How many zero pairs are in your collection? _____	How many zero pairs are in your collection? _____

5. Does adding a zero pair to a number change the value of the number? Explain. Name the property that justifies your explanation.

6. Explain the meaning of zero pairs in your own words.

# ADDITION

Compute each sum. Show your work by drawing positive (+) and negative (-) counters.

<p>Example A: <math>(-3) + (-2) = -5</math></p> <p style="text-align: center;">- - - - -</p> <ul style="list-style-type: none"> <li>Start with a value of zero on the work space;</li> <li>Create a value of -3;</li> <li>Then place 2 (-) counters</li> </ul>	<p>Example B: <math>(4) + (-3) = 1</math></p> <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> <ul style="list-style-type: none"> <li>Start with a value of zero on the work space;</li> <li>Create a value of 4;</li> <li>Then place 3 (-) counters</li> </ul> </div>  </div>
1. $(3) + (4) = \underline{\hspace{2cm}}$	2. $(-3) + (-4) = \underline{\hspace{2cm}}$
3. $(-2) + (-3) = \underline{\hspace{2cm}}$	4. $(-3) + (7) = \underline{\hspace{2cm}}$
5. $(-5) + (3) = \underline{\hspace{2cm}}$	6. $(5) + (-8) = \underline{\hspace{2cm}}$
7. $(-5) + (5) = \underline{\hspace{2cm}}$	8. $(4) + (-4) = \underline{\hspace{2cm}}$

For addition: addend + addend = sum

Generalize the rules for adding integers. Explain your thinking in terms of the counter model.

9. The sum of a positive addend and a positive addend is \_\_\_\_\_.

If you start with positive counters and then add more positive counters, then...

10. The sum of a negative addend and a negative addend is \_\_\_\_\_.

If you start with negative counters and then add more negative counters, then...

11. The sum of a positive number and a negative number is

Positive when...

Negative when...

Zero when....

**MORE ADDITION**

Without computing, determine whether each sum is positive, negative, or zero.

Example A: $-4 + (-4)$ <b>negative</b>	Example B: $5 + (-4)$ <b>positive</b>	Example C: $-8 + 8$ <b>zero</b>
1. $-3 + (-11)$	2. $8 + (-3)$	3. $-2 + (-7)$
4. $9 + 5$	5. $-7 + 4$	6. $12 + (-4)$
7. $-6 + (-2)$	8. $-5 + 2$	9. $8 + (-8)$

Compute each sum. Use positive (+) and negative (-) counters if needed.

10. $7 + (-3) = \underline{\hspace{2cm}}$	11. $-9 + 9 = \underline{\hspace{2cm}}$	12. $-2 + (-5) = \underline{\hspace{2cm}}$
13. $9 + 3 = \underline{\hspace{2cm}}$	14. $5 + (-8) = \underline{\hspace{2cm}}$	15. $-5 + 7 = \underline{\hspace{2cm}}$
16. $6 + (-6) = \underline{\hspace{2cm}}$	17. $-3 + (-7) = \underline{\hspace{2cm}}$	18. $12 + (-3) = \underline{\hspace{2cm}}$

Compute each sum without positive or negative counters.

19. $-25 + (-75) = \underline{\hspace{2cm}}$	20. $-100 + 1 = \underline{\hspace{2cm}}$	21. $70 + (-30) = \underline{\hspace{2cm}}$
22. $-38 + (-26) = \underline{\hspace{2cm}}$	23. $-64 + 22 = \underline{\hspace{2cm}}$	24. $53 + (-27) = \underline{\hspace{2cm}}$

# SUBTRACTION

Compute each difference. Show your work by drawing positive (+) and negative (-) counters.

<p><u>Example A</u></p> <p style="text-align: center;"><math>(-3) - (-2) = -1</math></p> <div style="text-align: center;"> </div> <p>Place 3 (-) counters and remove 2 (-) counters.</p>	<p><u>Example B</u></p> <p style="text-align: center;"><math>(4) - (-3) = 7</math></p> <p>Place 4 (+) counters and then remove 3 (-) counters. Since there are no (-) counters to remove, add <b>zero pairs</b> first.</p> <div style="text-align: center;"> </div>
1. $(4) - (1) = \underline{\hspace{2cm}}$	2. $(-3) - (-3) = \underline{\hspace{2cm}}$
3. $(-2) - (-1) = \underline{\hspace{2cm}}$	4. $(-6) - (-2) = \underline{\hspace{2cm}}$
5. $(1) - (4) = \underline{\hspace{2cm}}$	6. $(2) - (6) = \underline{\hspace{2cm}}$
7. $(-2) - (-3) = \underline{\hspace{2cm}}$	8. $(-2) - (-4) = \underline{\hspace{2cm}}$
9. $(-3) - (2) = \underline{\hspace{2cm}}$	10. $(-5) - (3) = \underline{\hspace{2cm}}$
11. $(4) - (-1) = \underline{\hspace{2cm}}$	12. $(-4) - (-2) = \underline{\hspace{2cm}}$

For subtraction:  $\text{minuend} - \text{subtrahend} = \text{difference}$

13. Write “smaller” next to all of the subtraction equations for which the result (difference) is less than the starting value (minuend). Is the number subtracted (subtrahend) positive or negative in these cases?
14. Write “larger” next to all of the subtraction equations for which the difference is more than minuend. Is the subtrahend positive or negative in these cases?
15. What would you tell a classmate who said, “Subtraction makes numbers smaller”?



## COMPARING ADDITION AND SUBTRACTION

Compute each difference. Use positive (+) and negative (-) counters if needed.

1a.  $8 - 4 = \underline{\quad}$

1b.  $8 + (-4) = \underline{\quad}$

2a.  $-7 - 4 = \underline{\quad}$

2b.  $-7 + (-4) = \underline{\quad}$

3a.  $3 - (-1) = \underline{\quad}$

3b.  $3 + 1 = \underline{\quad}$

4a.  $-6 - (-2) = \underline{\quad}$

4b.  $-6 + 2 = \underline{\quad}$

Compare parts (a) and (b) for each problem.

- Subtracting 4 gives the same result as adding \_\_\_\_\_.
- Subtracting -1 gives the same result as adding \_\_\_\_\_.
- Subtracting a number gives the same result as adding \_\_\_\_\_.
- Write an addition expression that is equivalent to  $10 - 5$ . \_\_\_\_\_
- Write an addition expression that is equivalent to  $6 - (-3)$ . \_\_\_\_\_

Algebraically, we can write the integer subtraction rule as:

$$a - b = a + (-b), \quad \text{or} \quad a - (-b) = a + b$$

for all integers  $a$  and  $b$

## MORE SUBTRACTION

Without computing, determine whether each difference is positive, negative, or zero.

<u>Example A</u> $-7 - (-7)$ zero	<u>Example B</u> $8 - (-2)$ positive	<u>Example C</u> $-9 - 4$ negative
1. $-5 - (-12)$	2. $12 - (-6)$	3. $-7 - (-10)$
4. $13 - 3$	5. $-10 - 2$	6. $14 - (-3)$
7. $-10 - (-1)$	8. $-8 - 3$	9. $16 - (-16)$

Compute each difference. Use positive (+) and negative (-) counters if needed.

10. $10 - (-4) = \underline{\hspace{2cm}}$	11. $-12 - 12 = \underline{\hspace{2cm}}$	12. $-6 - (-8) = \underline{\hspace{2cm}}$
13. $8 - 5 = \underline{\hspace{2cm}}$	14. $5 - (-9) = \underline{\hspace{2cm}}$	15. $-7 - 8 = \underline{\hspace{2cm}}$
16. $7 - (-7) = \underline{\hspace{2cm}}$	17. $-6 - (-7) = \underline{\hspace{2cm}}$	18. $15 - (-3) = \underline{\hspace{2cm}}$

Compute each difference without drawing positive or negative symbols.

19. $-80 - (-40) = \underline{\hspace{2cm}}$	20. $-70 - 5 = \underline{\hspace{2cm}}$	21. $75 - (-25) = \underline{\hspace{2cm}}$
22. $-22 - (-35) = \underline{\hspace{2cm}}$	23. $-53 - 38 = \underline{\hspace{2cm}}$	24. $89 - (-73) = \underline{\hspace{2cm}}$

# MULTIPLICATION

Compute each product. Show your work by drawing positive (+) and negative (-) counters.

<p><b>Example A</b></p> $(2) \cdot (5) = 10$ <p>Place 2 groups of 5 positive counters</p> <pre> + + + + + + + + + +                     </pre>	<p><b>Example B</b></p> $(-2) \cdot (5) = -10$ <p>Remove 2 groups of 5 positive counters</p> <p>First create zero pairs:</p> <pre> + + + + + - - - - - + + + + + - - - - -                     </pre>	<p>1. <math>(2) \cdot (-5) = \underline{\hspace{2cm}}</math></p>	<p>2. <math>(-2) \cdot (-5) = \underline{\hspace{2cm}}</math></p>
<p>3. <math>(3) \cdot (4) = \underline{\hspace{2cm}}</math></p>	<p>4. <math>(3) \cdot (-4) = \underline{\hspace{2cm}}</math></p>	<p>5. <math>(-3) \cdot (4) = \underline{\hspace{2cm}}</math></p>	<p>6. <math>(-3) \cdot (-4) = \underline{\hspace{2cm}}</math></p>

- The product of a positive number and a positive number is a \_\_\_\_\_ number.
- The product of a positive number and a negative number is a \_\_\_\_\_ number.
- The product of a negative number and a positive number is a \_\_\_\_\_ number.
- The product of a negative number and a negative number is a \_\_\_\_\_ number.

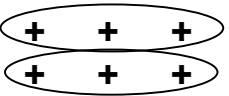
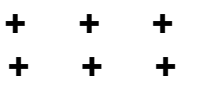
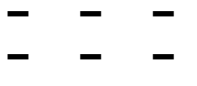
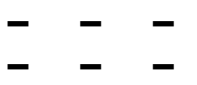
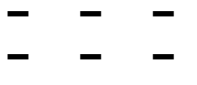
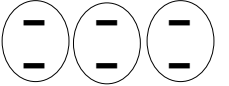


Compute the products.

11. $(-6) \cdot (7)$	12. $(-8) \cdot (-3)$	13. $(11) \cdot (-4)$	14. $(20) \cdot (-7)$
15. $(-20) \cdot (-60)$	16. $(2) \cdot (-5) \cdot (-3)$	17. $(-3) \cdot (6) \cdot (-10)$	18. $(-6) \cdot (40) \cdot (2)$

## DIVISION AND THE COUNTER MODEL

We used the counter model to explain the meaning of integer addition, subtraction, and multiplication, and established the rules for these operations. We will now explore how the counter model illustrates division of integers.

1. Use the concept of grouping and the counter model to investigate the four cases for integer division. Some are done for you.

	Numbers	Grouping Concept	Use Counter Model	Can the case be modeled with counters?
Case 1	$6 \div 2 = 3$	Divide 6 into 2 equal groups		YES
		Divide 6 into groups of 2 (positive counters)		
Case 2	$-6 \div 2 = -3$	Divide -6 into 2 equal groups		
		Divide -6 into groups of 2 (positive counters)		
Case 3	$-6 \div (-2) = 3$	Divide -6 into -2 equal groups		NO
		Divide -6 into groups of -2 (negative counters)		YES
Case 4	$6 \div (-2) = -3$	Divide 6 into -2 equal groups		
		Divide 6 into groups of -2 (negative counters)		

2. Neither part of case \_\_\_\_\_ above can be performed using the counter model. Therefore, which integer division rule cannot be established using the counter model?

\_\_\_\_\_  $\div$  \_\_\_\_\_ = \_\_\_\_\_

3. Is the integer division rule  $(+) \div (-) = (-)$  still true? \_\_\_\_\_

# ORDER OF OPERATIONS

## Summary (Ready)

We will review the conventions for the order of operations and apply them to evaluate expressions.

## Goals (Set)

- Understand the convention for order of operations.
- Use order of operations to simplify expressions.

## Warmup (Go)

1. A local food bank had 150 cans of food.

The food bank gave 25 cans of food each week to needy families for three weeks.

- a. How many cans of food did the food bank have left at the end of three weeks?

- b. Write an expression to describe this situation.

2. A different food bank also had 150 cans of food.

This food bank gave 25 cans of food to needy families.

Then the food bank received a donation of food that tripled the number of cans that were left.

- a. How many cans of food do they have now?

- b. Write an expression to describe this situation.

## CONVENTIONS FOR ORDER OF OPERATIONS

The order in which we perform mathematical calculations is determined by agreed-upon rules. Order of operations is a mathematical convention.

### Order of Operations

1. Simplify expressions that are grouped.
2. Compute expressions with exponents.
3. Perform multiplication and division from left to right.
4. Perform addition and subtraction from left to right.

Evaluate each expression.	List the operations in order from first to last
1. $75 \div (2+3)^2 \cdot 4$ $= 75 \div (5)^2 \cdot 4$ $= 75 \div 25 \cdot 4$ $= 3 \cdot 4$ $= 12$	1. Grouping (add 2 + 3 in parenthesis) 2. Exponent (compute 5 to the 2 <sup>nd</sup> power) 3a. Multiplication and division from left to right (divide 75÷25) 3b. Multiplication and division from left to right (multiply 3•4)
2. $3(-2+7)$	
3. $16 \div 8 \cdot 2^3$	
4. $\frac{-8}{2 - (-4) + 2}$	
5. $(10-8) \cdot 3 + (-7)$	

## WHERE DO THE PARENTHESES GO?

Place parentheses in the equations below so that each becomes a true statement. You may use additional parentheses to make order of operations clear if you like. Write "none needed" if the equation is already true.

1a. $5 \cdot 4 - 3 + 2(-1) = 3$	1b. $5 \cdot 4 - 3 + 2(-1) = -5$
2a. $6 + 3 \cdot 6 \div 3 = 18$	2b. $6 + 3 \cdot 6 \div 3 = 8$
3a. $\frac{6 - 4 \cdot 3}{-3} = 2$	3b. $\frac{6 - 4 \cdot 3}{-3} = -2$
<p>4. Alejandra says that both sets of parentheses for the problem below are necessary to make the equation true.</p> $3 + (5 \cdot 2) \div (7 - 5) = 8$ <p>Is Alejandra correct? Explain.</p>	

## PRACTICE WITH ORDER OF OPERATIONS

Simplify each expression.

1. $25 - (6 - 4)$	2. $48 \div 8 - 1$	3. $6^2 - 12 \div 2 \div 3$
4. $60 \div 3 - 5 \cdot 2^3$	5. $18 \div (5 - 2)$	6. $4 - 2 - 6 \cdot 2$
7. $(36 - 8) \div 14 + 6 \div 2$	8. $\frac{7 + (21 \div 7)}{5}$	9. $\frac{(-24 \div 8) + (-13)}{-8 + 2^4}$

Use the symbols  $>$ ,  $<$ , or  $=$  to make each statement true.

10. $15 \cdot 3 - 2$ _____ $15 \cdot (3 - 2)$	11. $8 + 12 \div 4$ _____ $(8 + 12) \div 4$
12. $12 \div 3 + 9 \cdot 4$ _____ $12 \div (3 + 9) \cdot 4$	13. $(7 \cdot 3) - (4 \cdot 2)$ _____ $7 \cdot 3 - 4 \cdot 2$
14. $11 \cdot 3 - (-2)$ _____ $11 \cdot [3 - (-2)]$	15. $-3 \cdot (4 - 2) \cdot 5$ _____ $-3 \cdot 4 - 2 \cdot 5$



# SKILL BUILDERS, VOCABULARY, AND REVIEW

## SKILL BUILDER 1

### Selected Properties of Arithmetic

Commutative property of multiplication:

For any two numbers  $a$  and  $b$ ,  $a \cdot b = b \cdot a$ . In other words, changing the order of the factors does not change the product.

Associative property of multiplication:

For any three numbers,  $a$ ,  $b$ , and  $c$ ,  $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ . In other words, the product does not depend on the grouping of the factors.

Distributive property:

For any three numbers  $a$ ,  $b$ , and  $c$ ,  $a(b + c) = ab + ac$  and  $(b + c)a = ba + ca$ .

Additive inverse property:

For every number  $a$ ,  $a + (-a) = 0$  and  $-a + a = 0$ .

Additive identity property:

For every number  $a$ ,  $a + 0 = a$  and  $0 + a = a$ .

Write the property of arithmetic illustrated by each equation.

1. $(8 \cdot 9) 5 = 8 (5) \cdot 9 (5)$	2. $(12 \cdot 8) \cdot 9 = 12 \cdot (8 \cdot 9)$
3. $(15)(24) = (24)(15)$	4. $14 = 0 + 14$

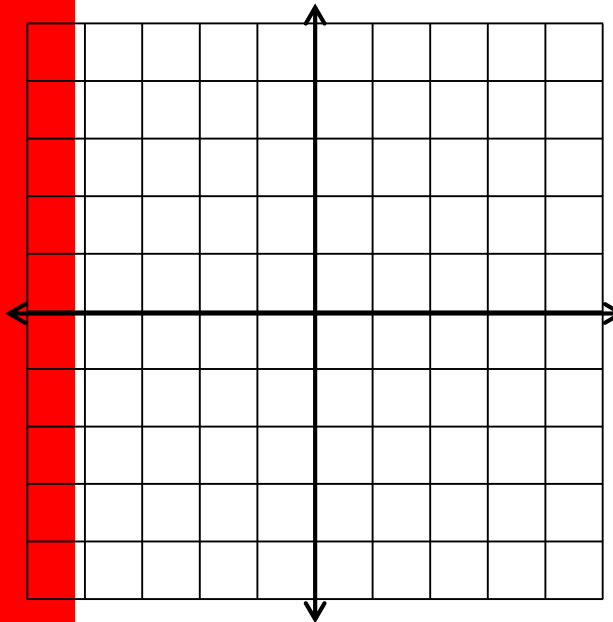
Write the property of arithmetic illustrated by each verbal statement.

5. A number plus zero is always the number.	6. Two numbers can be multiplied in any order.
7. Three numbers can be multiplied in any order.	8. A number plus its opposite is always zero.

## SKILL BUILDER 2

1. Graph and label each point on the coordinate plane.

$A(5, -2)$	$E(-3, -3)$
$B(3, 5)$	$F(-5, 1)$
$C(-3, 0)$	$G(2, -1)$
$D(-3, 4)$	$H(0, 4)$

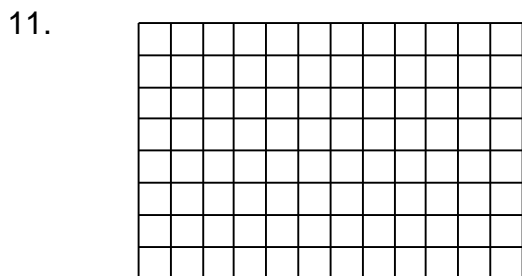


2. In what quadrant are points G and A located?
3. Name the point that lies on the x-axis.
4. Name the point that lies on the y-axis.

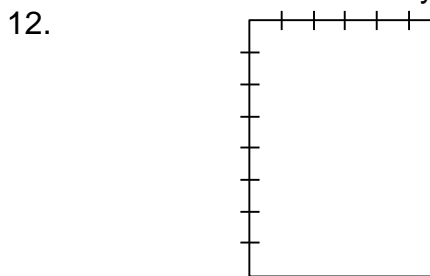
Evaluate each expression for  $x = 3$  and  $y = 6$ .

5. $4x + 5y$	6. $\frac{27}{x+y}$	7. $27 \div x+y$
8. $(x+y)^2$	9. $x+y^2$	10. $(y-x)^2$

Find the perimeter and area of each rectangle. Each square is a square unit and the distance between tic marks indicates one unit. Be sure to indicate the units in your answer.



$P =$  \_\_\_\_\_  $A =$  \_\_\_\_\_

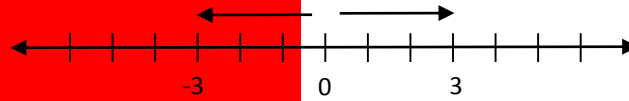


$P =$  \_\_\_\_\_  $A =$  \_\_\_\_\_

### SKILL BUILDER 3

The absolute value  $|x|$  of a number  $x$  is the distance from  $x$  to 0 on the number line.

Example:  $|3| = 3$  and  $|-3| = 3$  because 3 and -3 are both 3 units from 0 on the number line.



Simplify each expression.

1.  $|21|$

2.  $|-8|$

3.  $-|17 - 10|$

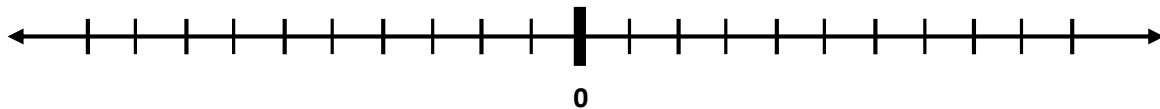
4.  $|3.4|$

5.  $|-8\frac{1}{2}|$

6.  $-|10\frac{1}{2} - 8|$

7. Locate the following numbers and their opposites on the number line.

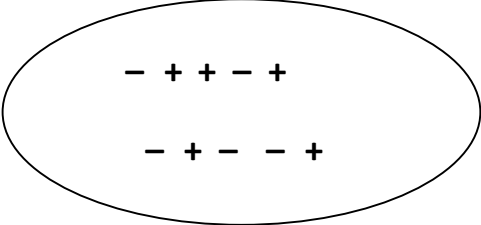
9                      -4                      0                      -10



Use the symbols  $<$ ,  $>$ ,  $=$  to write three number sentences using some of the numbers from the number line.

\_\_\_\_\_

8. What is the value of this collection of counters?



9. Explain the meaning of “zero pairs” in your own words.

## SKILL BUILDER 4

Without computing, determine whether each sum is **positive**, **negative**, or **zero**.

1. $-72 + (-53)$	2. $-81 + 105$	3. $57 + 39$
4. $-43 + 57$	5. $29 + (-29)$	6. $-28 + 42$

Compute. Show your work using positive (+) and negative (–) symbols if needed.

7. $(-2) + (5) = \underline{\hspace{2cm}}$	8. $(4) + (-1) = \underline{\hspace{2cm}}$	9. $(-2) + (-2) = \underline{\hspace{2cm}}$
10. $(-2) + (2) = \underline{\hspace{2cm}}$	11. $(-1) + (0) = \underline{\hspace{2cm}}$	12. $(-9) + (2) = \underline{\hspace{2cm}}$
13. $(2) + (5) = \underline{\hspace{2cm}}$	14. $(-4) + (-6) = \underline{\hspace{2cm}}$	15. $(3) + (-3) = \underline{\hspace{2cm}}$
16. $(3) + (-7) = \underline{\hspace{2cm}}$	17. $(-8) + (5) = \underline{\hspace{2cm}}$	18. $(-6) + (-7) = \underline{\hspace{2cm}}$

The sum of adding two integers is:

19. **Positive** when both addends are positive or when \_\_\_\_\_

\_\_\_\_\_.

20. **Negative** when both addends are \_\_\_\_\_ or when \_\_\_\_\_

\_\_\_\_\_.

21. **Zero** when \_\_\_\_\_.

**SKILL BUILDER 5**

Compute. Show your work using positive (+) and negative (-) symbols if needed.

1. $(-5) - (-2) = \underline{\hspace{2cm}}$	2. $(5) - (-2) = \underline{\hspace{2cm}}$	3. $(1) - (6) = \underline{\hspace{2cm}}$
4. $(1) - (-6) = \underline{\hspace{2cm}}$	5. $(6) - (-1) = \underline{\hspace{2cm}}$	6. $(0) - (8) = \underline{\hspace{2cm}}$
7. $(-4) - (2) = \underline{\hspace{2cm}}$	8. $(-7) - (-5) = \underline{\hspace{2cm}}$	9. $(4) - (-2) = \underline{\hspace{2cm}}$
10. $(-10) - (-80) = \underline{\hspace{2cm}}$	11. $(50) - (-40) = \underline{\hspace{2cm}}$	12. $(20) - (60) = \underline{\hspace{2cm}}$
13. $(-100) - (300) = \underline{\hspace{2cm}}$	14. $(150) - (30) = \underline{\hspace{2cm}}$	15. $(200) - (-400) = \underline{\hspace{2cm}}$

16. Consider a number,  $n$ . Does subtracting a negative number from  $n$  result in a greater number or lesser number compared to  $n$ ? Which examples above support your answer?
17. One day, the Mojave Desert was  $136^{\circ}\text{F}$  and the Gobi Desert was  $-50^{\circ}\text{F}$ . What is the difference in these two temperatures?
18. The melting point of mercury is  $-39^{\circ}\text{C}$ . The freezing point of alcohol is  $-114^{\circ}\text{C}$ . How much warmer is the melting point of mercury than the freezing point of alcohol?

**SKILL BUILDER 6**

Compute. Show your work using positive (+) and negative (–) symbols if needed.

1. $8 \cdot (-5)$	2. $-42 \div 7$	3. $-6 \cdot 12$
4. $(-10) \cdot (-10)$	5. $(-45) \div (-15)$	6. $\frac{48}{-12}$
7. $5 \cdot (-12)$	8. $7 \cdot (-3) \cdot (-2)$	9. $\frac{-26}{-13}$
10. $(-88) \div (22)$	11. $(-2) \cdot (-16) \cdot (-10)$	12. $100 \div (-5)$
13. The quotient of a positive number divided by a negative number is _____.		
14. The product of a negative number and a negative number is _____.		

15. DeJon says, “a negative and a negative make a positive.” He applies this rule and writes the following:

$$-5 + (-20) = 25$$

Is DeJon correct? Explain.

## SKILL BUILDER 7

Place parentheses in the equations below so that each becomes a true statement. You may use additional parentheses to make order of operations clear if you like. Write “none needed” if the equation is already true.

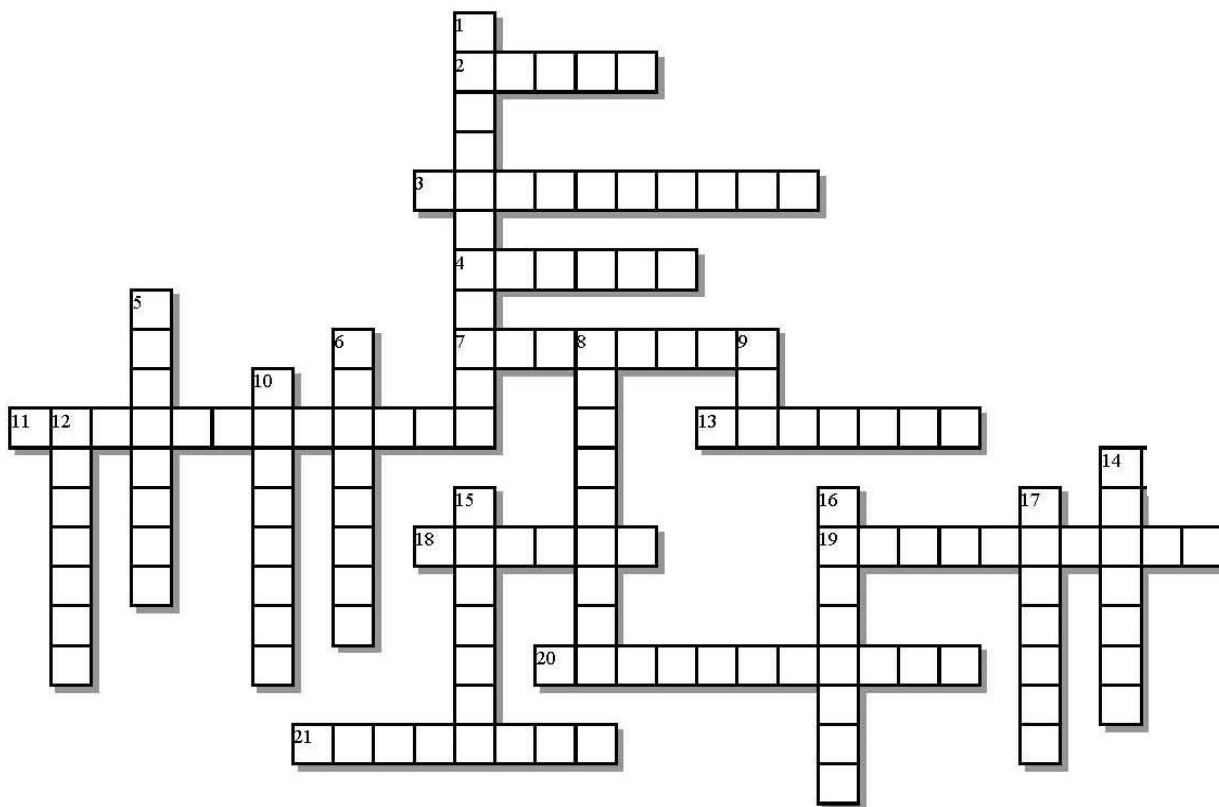
1. $12 - 4 \div 2 = 4$	2. $-12 \div 2 \cdot 3 = -18$
3. $8 \cdot 16 - 8 = 64$	4. $-3 + 4 \cdot (-1) = -1$
5. $\frac{12 + (-6)}{-5 + 2} = -2$	6. $12 + (-6) \div -5 + 2 = -2$

Simplify.

7. $25 - 3 \cdot 6 + 7^2$	8. $-4 \cdot 5 - (4 - 6)^2$
9. $\frac{-12 - 6 \cdot 3}{-9 - 1}$	10. $-12 - 6 \cdot 3 \div (-9) - 1$

11. Angelica spent \$20 per month for 7 months on a gym membership. If she started with \$200 in her gym budget, how much money does she have left now? How many more months can she pay for a gym membership at this rate?

## FOCUS ON VOCABULARY



### Across

- 2 \_\_\_\_ of operations (conventions)
- 3 “4” in the subtraction statement  $12 - 4 = 8$
- 4 “-5” in the addition statement  $-5 + 3 = 2$
- 7  $\{\dots-3, -2, -1, 0, 1, 2, 3\dots\}$
- 11 The \_\_\_\_\_ property:  $a(b + c) = ab + ac$  for any three numbers  $a$ ,  $b$  and  $c$ .
- 13 2 is \_\_\_\_\_ than 1000
- 18 50 is \_\_\_\_\_ than 1
- 19 the answer to a subtraction problem
- 20 a property:  $a + (b + c) = (a + b) + c$
- 21 an interpretation of the minus sign

### Down

- 1 a property:  $-17(4) = 4(-17)$
- 5 the result of division
- 6 repeated arrangements
- 8 “3” in  $-5^3$  and “2” in  $x^2$
- 9 the result of addition
- 10 the \_\_\_\_\_ value of  $|-5|$  is 5
- 12 an operation that reverses the effect of another
- 14 “12” in the subtraction statement  $12 - 4 = 8$
- 15 numbers that are multiplied together
- 16 Zero is the additive \_\_\_\_\_
- 17 the result of multiplication

(For word hints, see the word bank and other vocabulary used in this packet.)



**SELECTED RESPONSE**

Show your work on a separate sheet of paper and choose the best answer(s).

---

1. Choose ALL that are equivalent to  $17 + (-13)$ .

A.  $-(-4)$

B.  $-4$

C.  $4$

D.  $-(4 - 8)$ 

---

2. The product of a positive number and a negative number is:

A. Always positive

B. Always negative

C. Zero

D. Sometimes negative

---

3. For the multiplication fact  $8 \cdot (-9) = -72$ , choose ALL division statements that illustrate that multiplication and division are inverse operations.

A.  $8 \div (-9) = -72$

B.  $(-8) \div (-72) = 9$

C.  $(-72) \div (-9) = 8$

D.  $(-72) \div 8 = -9$ 

---

4. Choose ALL that are equivalent to  $-8 - (-9)$

A.  $-1$

B.  $1$

C.  $9 - 8$

D.  $8 - 9$ 

---

5. Which expression does not have a value of 18?

A.  $36 \div 4 \cdot 2$

B.  $(18 + 10) \div 4 + 11$

C.  $8 \cdot 2 + 5 - 1 \cdot 3$

D.  $36 \div (4 \cdot 2)$ 

---

6. Dawson bought three DVDs at \$15 each and four CDs at \$8 each online. The shipping charge was \$5. Which expression shows the total cost of the order?

A.  $3(15 + 5) + 4(8 + 5)$

B.  $5 + 3(15) + 4(8)$

C.  $(3 + 15) + (4 + 8) + 5$

D.  $3(15) + 4(8)$ 

---

## KNOWLEDGE CHECK

Show your work on a separate sheet of paper and write your answers on this page.

### 1.1 Integer Operations: Patterns

1. Write two subtraction facts related to the equation  $(-10) + (-20) = (-30)$
2. Write two division facts related to the equation  $(-7) \cdot (-9) = (63)$

### 1.2 Integer Operations: A Counter Model

Compute. Show your work using positive (+) and negative (−) symbols if needed.

3.  $(-7) + (-12)$
4.  $(-29) + (15)$
5.  $(9) - (-5)$
6.  $(-5) - (9)$
7.  $(-4)(-8)$
8.  $-7 \cdot 3$
9.  $-42 \div (-7)$
10.  $\frac{60}{-5}$

### 1.3 Order of Operations

Insert parentheses in the equation to make each statement true.

11.  $7 \cdot 8 - 6 + 3 = 47$
12.  $3 + 8 - 2 \cdot 5 = 45$

Simplify each expression.

13.  $12 \div 4 - 6 \div 3$
14.  $\frac{2+8 \div 4}{5}$

## HOME-SCHOOL CONNECTION

Here are some questions to review with your young mathematician.

1. Compute the sum:  $(-16) + (9)$
2. Compute the difference:  $(5) - (-7)$
3. Compute the product:  $(3) \cdot (-12)$
4. Compute the quotient:  $(-36) \div (-6)$
5. Use  $>$ ,  $<$ , or  $=$  to make the statement true.

$$20 \div 2 + 8 \cdot 2 \quad \underline{\hspace{1cm}} \quad 20 \div (2 + 8) \cdot 2$$

Dear Parent (or Guardian),

This year, your student will be taking a mathematics course that is fully aligned with the Common Core State Standards in Mathematics for Grade 8. A primary focus of this course will be algebra, as students study linear functions and equations, and they connect their knowledge to other areas such as statistics and geometry.

We encourage you to join your student's teacher and school as a partner in your young mathematician's progress. A major portion of the work in this course will be the completion of sixteen packets (such as this one) throughout the year. Each packet will take about two weeks to complete. Some of this work will be done in class, and some will be assigned for homework. You can take an active role by reviewing the packet and asking your student to explain some problems to you. This will allow your student to practice communicating about mathematics, and give you an opportunity to find out what is being taught in the classroom. Your signature will indicate to the teacher that you have reviewed the work together.

If you see that your student does not fully understand a concept, please encourage your student to use the Resource Guide that accompanies the program as a reference and to ask the teacher for additional help.

Thank you in advance for your support. We hope you enjoy watching your student grow mathematically this year.

Sincerely,

The Writing Team at the Center for Mathematics and Teaching

Parent (or Guardian) Signature \_\_\_\_\_

# COMMON CORE STATE STANDARDS – MATHEMATICS

## STANDARDS FOR MATHEMATICAL CONTENT

- 6.NS.5\* Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
- 6.NS.7c\* Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.
- 6.EE.2c\* ~~Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems.~~ Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas  $V = s^3$  and  $A = 6s^2$  to find the volume and surface area of a cube with sides of length  $s = 1/2$ .*
- 7.NS.1a\* Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*
- 7.NS.1c\* ~~Understand subtraction of rational numbers as adding the additive inverse,  $p - q = p + (-q)$ . Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.~~
- 7.NS.2c\* Apply properties of operations as strategies to multiply and divide rational numbers.

\*Review of content essential for success in 8<sup>th</sup> grade.

## STANDARDS FOR MATHEMATICAL PRACTICE

- MP2 Reason abstractly and quantitatively.
- MP3 Construct viable arguments and critique the reasoning of others.
- MP5 Use appropriate tools strategically.
- MP6 Attend to precision.
- MP8 Look for and express regularity in repeated reasoning.



9 7 8 1 6 1 4 4 5 2 0 9 6

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