6-11
TEACHER PACKET

MATHLINKS: GRADE 6
TEACHER PACKET 11
RATIOS AND UNIT RATES

Table of Contents

Commentary on the packet will be in red in text boxes along the way.

Welcome to a *MathLinks* Teacher Packet (TP). This packet is from *MathLinks*: Grade 6 and is TP11, meaning it is the 11th packet out of 16. All TPs can be found within the Teacher Guide.

On the cover sheet you will find the titles, goals, and page numbers (for the Student Packet (SP) and Teacher Packet (TP) of the three concept lessons as well the location of the fourth lesson which is always the Skill Builder, Vocabulary, and Review.

GENERAL INFORMATION

PACING PLAN SUGGESTIONS

TRADITIONAL MATH SCHEDULE

Days-Modified	Days-Basic	Days-Enriched	Lesson	Review/Practice
3	4	4	[11.1] Pages 0, 1-8	Pages 25-27
4	5	5	[11.2] Pages 0, 9-16	Pages 28-30
3	4	4	[11.3] Pages 0, 17-24	Pages 31-36
3	4	4	Catch up, Tasks, Assessment	

BLOCK SCHEDULE

Days-Modified	Days-Basic	Days-Enriched	Lesson	Review/Practice
2	3	3	[11.1] Pages 0, 1-8	Pages 25-27
3	3	3	[11.2] Pages 0, 9-16	Pages 28-30
2	2	2	[11.3] Pages 0, 17-24	Pages 31-36
2	2	3	Catch up, Tasks, Assessment	

This is suggested pacing for different groups of students. All teachers should decide what pacing works best for them and their students.

- Lesson pages are not intended to be used only as class wor nded to be used only as homework. How they are used is up to the t
- The number of days estimated for each lesson will vary depe tyles, and student proficiency.
- Although they are listed at the end of the tables, use catch up days when needed.
- Tasks may be assigned at any time after students have completed the prerequisite content work.
- Multiple assessment measures are encouraged, including (but not limited to) quizzes, tasks, proficiency challenges, strategically selected student pages, skill builders, selected response page, knowledge check, etc.
- Consider requiring a math journal, to be collected and checked periodically, or collecting an “exit slip” at the end of selected class periods. Journals and exits slips may include short skills review, explanations of concepts, or anything else the instructor may want to assess.
- As part of a modified program, consider omitting the following, depending upon time constraints:
  Student Packet 11: Pages 18-32 (select problems)

COMMON CORE STATE STANDARDS – MATHEMATICS

STANDARDS FOR MATHEMATICAL CONTENT

6.RP.A	**Understand ratio concepts and use ratio reasoning to solve problems.**[1]
6.RP.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every v* *te C received nearly three votes."*
6.RP.2	Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*
6.RP.3a	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations: Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, ~~and plot the pairs of values on the coordinate plane~~. Use tables to compare ratios.
6.RP.3b	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations: Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

Major clusters identified as well as standards

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[1] A major cluster for the grade level.

STANDARDS FOR MATHEMATICAL PRACTICE

MP1	Make sense of problems and persevere in solving them.
MP3	Construct viable arguments and critique the reasoning of others.
MP4	Model with mathematics.
MP7	Look for and make use of structure.

PACKET PLANNING INFORMATION

Assessments*, Reproducibles**, and Tasks**

Quiz 11A, 11B
Proficiency Challenge 11
Test Part 11 (See Assessment Tab, page iv)

Reproducible 26: Grape Juice Mixture Cards (1/pair or group) [11.1]
Reproducible 27: Blank Cards (1/group) [11.1]
Reproducible 28: Poster Problems 3 (1/group) [11.2]

Task, Page 14: Grapey Mixtures [11.1]

*Located in the assessment envelope and on the secure website
**Located in the back of the Teacher Guide

Materials

- Scissors (1/student or pair) [11.1]
- 11 X 17 (or larger) poster-size paper (1/student or
- Ma
- Ta
- Str
- Ch

The materials listed lets you know what you need for each lesson and how much you need to prepare. A complete list of materials for the entire year is located before the first tab in the Teacher Guide.

MathLinks: Grade 6 Resource Guide (Part 2)

Key vocabulary in the Word Bank:

- double number line diagram
- equivalent ratios
- ratio
- tape diagram
- unit price
- unit rate
- value of a ratio

Explanations and examples:

- Ratios and Proportional Relationships

Prepare Ahead

Go to www.mathandteaching.org for additional resources.

Lessons 11.1, 11.2, 11.3:

- The approach to ratios and rates taken follows recommendations made in the "Progressions" documents that accompany CCSS-M. Read the math notes, teacher notes, and complete the lessons ahead of time to become familiar with a new approach to this topic.

Technology Resources

Have students play "Ratio Rumble" at www.MathSnacks.org. This engaging game requires students to identify equivalent fractions.

Show the video "Math Snacks: Bad Date," which was created by the MathSnacks team and is available on their website and on YouTube at https://www.youtube.com/watch?v=BZ1M01YBKhk

This applet allows you to vary the gear ratio of a bike http://illuminations.nctm.org/Activity.aspx?id=3549

A good source for simple games students can play for free can be found at the link below. The ratio games are near the bottom of the web page. http://www.sheppardsoftware.com/math.htm

Options for a Substitute

Any time:	Pages 25-27
After 11.1:	Pages 5, 8, 28-29
After 11.2:	Pages 15-16, 30
After 11.3:	Pages 31-36

This page lets you know what you need for this packet and where to find it.

Technology enhancements are an ongoing project for us. We have found some good resources, but want more options and opportunities for teachers and students. Check out "Bad Date" on the MathSnacks website or YouTube. It's very good.

TEACHER CONTENT INFORMATION

MATH NOTES

MN1: Ratios Are Everywhere [11.1, 11.2, 11.3]

Under every rug there is a ratio.

In mathematics:

- the ratio of the circumference of a circle to its diameter (π)
- the ratio of lengths of corresponding sides of similar triangles
- the ratios of side lengths of right triangles (trigonometric ratios)
- the ratio of the "increase in the *y*-variable" to the "increase in the *x*-variable" (slope of a line)

In science:

- laws of physics, such as the ratio of momentum to velocity of falling objects
- conversion rates, such as feet to meters or minutes to hours
- comparisons, such as nineteen out of twenty glaciers are receding

In daily activities:

- two cups water for every cup oatmeal (recipe)
- a dozen almonds per serving
- thirty miles per hour (a speed limit)
- twenty-seven miles per gallon (fuel consumption)

In pricing:

- cheese at $5 per pound
- farmland at $8000 per acre

In sports and exercise:

- odds of Boston winning the World Series
- calories burned in fifteen minutes jogging

Whenever we refer to percentages, we are using ratios. The battery life of our electronic device, the sales tax on our pizza, and the discount on sale items are given as a percentage.

Math Notes (MN) were written by our mathematicians, and provide information to help understand, in depth, the topics in this packet. Each MN lets you know what lesson(s) is being addressed.

MATH NOTES (Continued)

MN2: Ratio, Rate, Unit Rate, and Value [11.1, 11.2, 11.3]

The words “ratio” and “rate” have various shades of meaning in common language. The definitions in school mathematics textbooks vary. The Common Core State Standards for Mathematics (CCSS-M) and Progressions prescribe a formal definition of “ratio,” and at least implicitly a definition of “unit rate.” On the other hand, “rate” is treated as a term in common language. No formal definition of “rate” appears in the documents.

- A ratio is an ordered pair of nonnegative numbers, not both zero. The ratio of a to b is denoted by $a : b$ (read “a to b,” or “a for every b”).

  Examples of ratios: $3 : 2$, $\frac{3}{2} : 2$, $3.14 : 10$, $8 : 0$, $0 : 8$.

  These are NOT ratios: $0 : 0$, $2 : -3$.

- Unit rate associated with a ratio: Suppose $a : b$ is a ratio, and $b \neq 0$. The unit rate associated to $a : b$ is the number $a \div b$. which may have units attached to it. If a and b have units attached to them, say “a-units” and “b-units,” the appropriate unit of measure for the unit rate is “a-units per b-unit.”

  Example: The ratio “400 miles every 8 hours” has unit rate “50 miles per hour.” There is a convenient calculation device that leads to the unit for the unit rate:

  $$\frac{400 \text{ miles}}{8 \text{ hours}} = \frac{400}{8} \frac{\text{miles}}{\text{hours}} = 50 \frac{\text{miles}}{\text{hours}} = 50 \text{ m}$$

- Value of a ratio: The value of a ratio $a : b$, $b \neq 0$, is the quotien

  Example: The value of the ratio $6 : 3$ is $6 \div 3 = 2$. The v

  The value of the ratio “400 miles every 8 hours” is $\frac{400}{8} = 50$.

The approach to ratios is different in CCSS-M than in previous years, so we have several MN to explain.

Both terms “value” and “unit rate” are based on the same numerical value, the quotient number $a \div b$. The difference between the terms is that *all* ratios $a : b$ with $b \neq 0$ have a value, whereas we generally talk about unit rates only for ratios that have units attached to them. In the latter case, the unit rate is equal to the value of the ratio with “something per something” attached.

MN3: Dual Definitions and Common Language [11.1, 11.2, 11.3]

It is common in mathematics to use the same term to refer to two concepts that are closely related though technically different. Examples include “fraction” and “ratio.” When this occurs, it is generally easy to determine the appropriate interpretation from context.

There are two interpretations of “fraction.” We can view a fraction as a division problem, or we can view a fraction as the number that is the result of the division. The fractions $\frac{3}{2}$ and $\frac{6}{4}$ are different when they are viewed as division problems. Dividing 3 cookies among 2 children has a different meaning than dividing 6 cookies among 4 children. However, both situations result in the same number of 1.5 cookies per child. It is easy to determine which interpretation is appropriate from context.

Similarly, there are two interpretations of “ratio.” We can interpret a ratio as a number pair, such as $3 : 2$. However, in common language the word “ratio” is often used to refer to the value of the ratio. We may say for instance that “the ratio of 3 to 2 is 1.5.” When we refer to π as being the ratio of circumference to diameter, we have in mind the numerical value of the ratio. When we refer to the slope of a line as the ratio of “rise” to “run,” we have in mind the numerical value of the ratio. It is easy to determine from context the specific interpretation of “ratio.”

MATH NOTES (Continued)

MN4: Attaching Units of Measure to Quantities [11.1, 11.2, 11.3]

Ratios usually arise from concrete situations in which the numbers are measures of things. When we say that the ratio of girls to boys in the class is 6 : 5, it is usually clear from context what we are measuring, and we focus on the pure numbers.

In many applications, it is important to keep track of exactly what we are measuring, and in particular what units we are using for measurement. In this case we attach units to the numbers to make precise what the numbers mean. This attachment of units is not part of the formal math structure – it is done unofficially to clarify the meaning of the numbers in our minds. It can be a wonderfully useful and effective device for making sense of the mathematics.

The ratio for converting centimeters to inches is "2.54 centimeters per inch." Stripped of units, this ratio becomes simply 2.54 : 1, which by itself lacks meaning. We attach the units of measurement "cm," "in," and "cm per in" to remind ourselves what we are measuring and to clarify its meaning in context. The corresponding unit rate (conversion rate) is expressed with units attached as

$$\frac{2.54\text{ cm}}{1\text{ in}} = \frac{2.54}{1}\,\frac{\text{cm}}{\text{in}} = 2.54\,\frac{\text{cm}}{\text{in}} = 2.54\text{ cm per in.}$$

Theoretically, it is possible to attach units to any ratio, even ratios of pure numbers. In the ratio 12 : 6 of pure numbers, we may think of the number "one" as being the unit, and the ratio becomes "12 ones to 6 ones."

MN5: Geometric Interpretation of Equivalent Ratios [11.1]

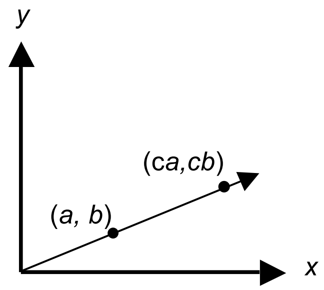
Two ratios are equivalent if each number in one ratio is a multiple of the corresponding number in the other ratio by the same positive number. Thus the ratio $a : b$ is equivalent to the ratio $ca : cb$ for all numbers $c > 0$.

When $b \neq 0$, the value of a ratio $a : b$ is the quotient number $a \div b$. We extend the definition of value to ratios $a : b$ with $b = 0$ by declaring that the value of the ratio $a : 0$ is $+\infty$. This is analogous to thinking of a vertical line in the plane as having slope $+\infty$. Though $+\infty$ this is not a number, it is a perfectly legitimate value for a function.

Now that we have extended the definition of value of a ratio to cover the case when $b = 0$, we can give a simple geometrical characterization of equivalent ratios in terms of rays in the plane.

Each ratio $a : b$ determines a point (a, b) in the first quadrant of the coordinate plane. This correspondence $a : b \rightarrow (a, b)$ maps ratios to the first quadrant, including the positive x-axis and y-axis but omitting the origin. Ratios $0 : b$ with value 0 are mapped to points $(0, b)$ on the positive x-axis, and ratios $a : 0$ with value $+\infty$ are mapped to points $(a, 0)$ on the positive y-axis. Under this correspondence, the ratios $ca : cb$ equivalent to $a : b$ correspond to the points (ca, cb) on the ray through (a, b) emanating from the origin. In fact, if we assign a slope of $+\infty$ to a vertical line, then the following statements are valid for all ratios:

- The ratios equivalent to $a : b$ correspond to the ray (half-line) issuing from the origin through (a, b).
- The slope of the ray through (a, b) is the value of the ratio $a : b$.
- Two ratios are equivalent if, and only if, they have the same value.



MATH NOTES (Continued)

MN6: Ratios and Rates – Then and Now [11.1, 11.2, 11.3]

Many definitions in mathematics have metamorphosed over time. Originally, the definition of "rectangles" did not include "squares," but it has become standard to include square as a subset of the rectangle family because it makes many properties easier to explain. To complicate the picture even more, some mathematical terms are defined differently in different textbooks and in different parts of the world. For example, some books define a trapezoid as a quadrilateral with exactly one pair of parallel sides, while others define it as a quadrilateral with at least one pair of parallel sides.

One of our favorite MN.

Here we observe changes in the definitions of ratio and rate, as a result of the Common Core State Standards in Mathematics (CCSS-M). In the first column are samples of definitions that have been used in the past.

Before CCSS-M (Some Examples)	Definitions Based on CCSS-M
A <u>ratio</u> is a comparison of two numbers by division. The ratio of a to b is denoted by $a : b$ (read "a to b"), or by $\frac{a}{b}$, where $b \neq 0$. Example: The ratio of 3 to 2 may be denoted by $3 : 2$ or by $\frac{3}{2} = 1.5$.	A <u>ratio</u> is a pair of nonnegative numbers, not both zero, in a specific order. The ratio of a to b is denoted by $a : b$ (read "a to b," or "a for every b"). Example: If there are 3 coins and 2 paperclips in your pocket, then the ratio of coins to paperclips may be denoted 3 to 2 or $3 : 2$.
	The <u>value of a ratio</u> $a : b$ is the number $\frac{a}{b}$, $b \neq 0$. Example: The value of the ratio of $3 : 2$ is $\frac{3}{2} = 1.5$.
Two ratios are <u>equivalent</u> if they have the same value. Example: The ratios $3 : 2$ and $9 : 6$ are equivalent because $\frac{3}{2} = \frac{9}{6}$.	Two ratios are <u>equivalent</u> if each number in one ratio is a multiple of the corresponding number in the other ratio by the same positive number. Example: This arrow diagram shows that the ratios $3 : 2$ and $9 : 6$ are equivalent. $\begin{tikzcd} 3 \arrow[d, bend right, "\times 3"'] & : & 2 \arrow[d, bend left, "\times 3"] \\ 9 & : & 6 \end{tikzcd}$
A <u>rate</u> is a ratio in which the numbers have units attached to them. Example: $\frac{\text{20 miles}}{\text{10 minutes}}$ is a rate.	There is no formal definition of "rate." It is treated as a word in common language. Such phrases as "at that rate" or "at the same rate" are used. Example: Sally runs 10 miles in 50 minutes. If she runs the entire marathon at that rate, what will be her marathon time?
A <u>unit rate</u> is a rate for one unit of measure. Example: 80 miles per hour may be written $\frac{\text{80 miles}}{\text{1 hour}}$ or $80\ \frac{\text{miles}}{\text{hour}}$.	The <u>unit rate</u> associated to a ratio $a : b$, where a and b have units attached, is the number $\frac{a}{b}$, with the units "a-units per b-unit" attached. Example: The ratio of 400 miles for every 8 hours corresponds to the unit rate 50 miles per hour.

TEACHING NOTES

TN1: Select Standards for Mathematical Practice Examples [11.1, 11.2, 11.3]

Here are a few examples of how the Standards for Mathematical Practice are applied in these lessons.

MP1 Students make sense of problems and persevere in solving them. [11.3] Students work in expert groups to solve proportional reasoning problems and then return to home groups to help solve them.

MP3 Construct viable arguments and critique the reasoning of others. [11.2] Students analyze student reasoning about grape mixtures.

MP4 Model with mathematics. [11.1, 11.2, 11.3] Students use strategie ne diagrams, tape diagrams, and equations to represent proportiona .

MP7 Look for and make use of structure. [11.1, 11.2] Students explore ing for solving proportion problems.

TN2: Strategies for English Learners [11.1,

Lesson Preparation

(Write and execute clear content objectives.) [11.1, 11.2,11.3] As a remin goals at the beginning of the lesson. Ask students to read goals and use within a context and to make sure students know what they are expected

Instructional Strategies

(Make concepts clear with visuals.) [11.1, 11.2, 11.3] Encourage students pe diagrams and double number lines to represent ratios and rates. The use ps students make sense of rates and ratios by giving them a variety of entry

Practice/Review

(Provide student feedback on their output.) [11.1,11.2,11.3] The three spe uice Mixtures in Lesson 11.1, Poster Problems in Lesson 11.2, and Jigsaw in L es for students to actively engage in their learning and to explain math ideas d provide feedback. This process allows for the development of their listenir skills.

All Teaching Notes (TN) sections include:

TN1 – Standards for Mathematical Practice

TN2 – Strategies for English Learners

TN3 – Strategies for Special Learners

TN4 – Strategies for Enrichment

TN5 – Creating an Itch (This note is currently in grades 6 and 7 only. Grade 8 will be added soon.)

Other TNs include good questioning strategies, grouping strategies, and other teaching tips that are

TN3: Strategies for Special Learners [11.1, 11.2, 11.3]

Create a positive classroom culture

(Teach students to compliment each other. Deemphasize goals that foster competition.) [11.1, 11.2, 11.3] Specialized group activities (Grape Juice Mixtures in Lesson 11.1, Poster Problems in Lesson 11.2, and Jigsaw in Lesson 11.3) provide opportunities to help students work together towards the common goal of learning.

Increase communication and participation

(Monitor to ensure all students are benefiting from interactions.) [11.1,11.2,.11.3] Specialized group activities also give the teacher the opportunity to watch student interactions carefully and encourage participation.

TEACHING NOTES (Continued)

TN4: Strategies for Enrichment [11.1, 11.2, 11.3]

[11.1, 11.2, 11.3] Challenge students to solve proportional reasoning problems using multiple representations, including tables, diagrams, and equations.

TN5: Creating an Itch [11.1]

[11.1] ***Where do we use ratios in everyday life***? See Math Note 1 for some ideas.

[11.2] Ask students how far they think it is to a local theme park, and how long it takes to get there. After agreeing on a typical time and distance, use this information as the example for the introduction of rates.

[11.3] This lesson contains seven different contexts for using ratios and rates. Hopefully, students will see the utility of learning this mathematics from these examples.

TN6: Equivalent Ratios and Tables [11.1]

The Common Core State Standards in Mathematics refer to “tables of equivalent ratios.” The “progressions” document on the 6th and 7th grade Ratios and Proportional Relationships domain (written by authors of CCSS-M) refer to “ratio tables”; however, these tables do not have ratios as entries. Rather, they are tables of numbers that are pedagogical recording devices used for teaching about ratios.

The table to the right has two variables in rows, and the columns have entries that indicate equivalent ratios. For example, 2 stars : 3 circles is equivalent to 4 stars : 6 circles and 6 stars : 9 circles.

# of Stars	2	4	6
# of Circles	3	6	9

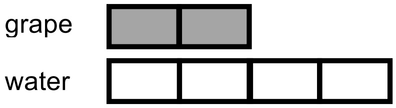
These tables may also have variables as column heads. In this case, the rows would have entries that indicate equivalent ratios.

Tables may have more than two rows or columns, corresponding to more than two variables. In this case, any two columns or rows determine number pairs that form equivalent ratios.

TN7: Tape Diagrams [11.1, 11.2, 11.3]

A <u>tape diagram</u> is a visual model consisting of strips divided into rectangular segments whose areas represent relative sizes of quantities. Tape diagrams are typically used when quantities have the same units.

Example: Here are two tape diagrams that show the ratio of grape concentrate to water is 2:4.

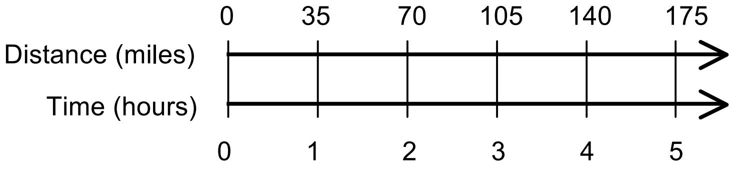


TEACHING NOTES (Continued)

TN8: Double Number Line Diagrams [11.1, 11.2]

A double number line diagram is a graphical representation of two variables, in which the corresponding values are placed on two parallel number lines for easy comparison. Double number lines are often used to compare two quantities that have different units.

Example: This double number line shows corresponding ratios if a car goes 70 miles for every 2 hours.



According to the Progressions for the Common Core State Standards in Mathematics document for proportional reasoning, a ratio is a pair of non-negative numbers $A : B$, which are not both zero. Thus we do not show negative numbers on our number lines that correspond to ratios.

RATIOS

Summary

Students define ratio and explore when ratios are equivalent. Students represent ratios using tables and diagrams, and solve problems involving ratios.

The black strip along the top of the page, along with the Summary, Goals, and Warmup, lets you know you are starting a lesson.

Goals

- Define ratio terminology.
- Explore equivalent ratios.
- Represent ratios using symbols, words, tables, and tape diagrams.
- Solve problems using tables and tape diagrams.

PREVIEW / WARMUP

Whole Class / Partners

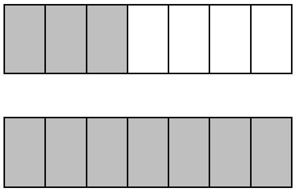
Page 0
Word Bank

Page 1
Ratios

The side bar provides information on how the class should be set up, what student pages (SP) are being used, and if any materials or reproducibles needed.

- Introduce the goals and standards of the lesson. Discuss important vocabulary as relevant.
- Students analyze Gretchen's work and identify her error.

Create an "itch" here. See Teaching Note 5.



What is Gretchen's error? Gretchen "added across." When adding fractions, we must use a common denominator d equal size parts. Looking closely at the addends and also shows that the results make no sense. For ple, $\frac{3}{7}+\frac{2}{2}=\frac{3}{7}+1=1\frac{3}{7}$.

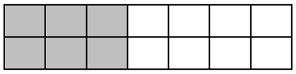
dents struggle with an explanation for the d.

is this equation different than Gretche sentence. Multiplying by $\frac{2}{2}$ (this "big one") doe fraction. If we multiply across, the result is $\frac{6}{14}$.

The body of the lesson helps you prepare ahead of time. It is not intended to be a script.

Notice the questions and explanations here to help teachers facilitate a discussion about the warmup page.

Why is this a correct procedure? With an ar shaded parts and the number of parts in the wh remains the same. In the diagram below, the size of the whole remains the same and the size of each part is half as large.



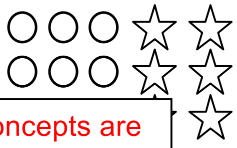
INTRODUCE

Whole Class

- Draw three circles and two stars on the board.

What are some ways to describe the relationship between circles and stars? Students may give additive responses, such as “there are three circles and two stars.” Record student responses, and listen closely for multiplicative thinking. If students do not volunteer it, repeat the pattern two more times, and provide sentence starters to help them. For example:

For every ____ circles, there are ____ stars.
The ratio of circles to stars is ____ to ____.



- Show students different ways to record ratios. Some possibilities are: 3 to 2, 3 : 2, and 3 circles for every 2 stars.

Some special concepts are boxed, as below, to bring attention to the idea.

As suggested in the CCSS-M Progressions Document, written by authors of CCSS-M to add clarity, we do not use fraction notation for ratios here. If students offer a fraction as an option, simply identify it as the value of the ratio. This will be explored in the next lesson. See Math Notes for more details.

What are some other ratios that can be created from this diagram? Record student responses. Some possibilities include: 9 circles for every 6 stars, (written 9:6), or the ratio of stars to total objects is 2 to 5.

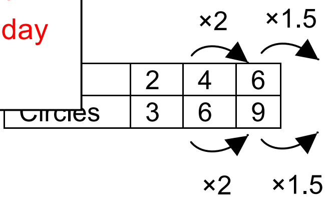
- Explain to students that two ratios are equivalent if each number in one ratio is a multiple of the corresponding number in the other ratio.

$$\begin{tikzcd} 3 \arrow[d, bend right, "\times 3"'] & : & 2 \arrow[d, bend left, "\times 3"] \\ 9 & : & 6 \end{tikzcd}$$

What is the multiplier t ***rs for every 3 circles is the same as the ratio o*** Show students how to record this with an arrow

- Finally, show students h .

Do columns in this a ta ratios? Yes. ***How do y*** 4 : 6, and 6 : 9 are all equivalent. Have students determine some of the common multipliers. Notice that multiplier need not be a whole number.



	2	4	6
Circles	3	6	9

Lessons generally include one or more cycles of Introduce, Explore, Summarize, Practice, and Extend. Lessons vary from 1-5 days in length. Most are probably in the 2-3 day range.

Page 2
Introduction to Ratios

Page 3
Equivalent Ratios in Tables

- Ask students to read the definitions and explanations in the boxes on pages 2 and 3.

What are some ways to write ratios? *a* : *b*, *a* to *b*, a for every *b*.

How can we check if two ratios are equivalent? Look for a common multiplier. Students may also notice that if ratios are written as fractions, those fractions are equivalent. Identify this as the value of the ratio, which will be explored later.

How might we keep track of equivalent ratios? One way is in a table.

EXPLORE 1 / SUMMARIZE 1

Whole Class/ Partners

Page 2
Introduction to Ratios

Page 3
Equivalent Ratios in Tables

- Students complete problems that focus on the language and notation of ratios, and determine if ratios are equivalent. Discuss.

  (Page 3, problem 1) ***How many frogs are there if there are 10 fish?*** 2. ***How did you get that?*** Students may observe that 5 + 5 = 10 and 1 + 1 = 2. This is additive thinking. Encourage students to rephrase using multiplicative thinking (e.g., “we double the number of fish, so we double the number of frogs”).

  What is the common multiplier for this ratio? The multiplier is 2.

  What multiplier can you use to determine the number of fish and frogs when the total is 600? 100 because $6 \times 100 = 600$. Therefore, $5 \times 100 = 500$ fish and $1 \times 100 = 100$ frogs.

  (Page 3, problem 2b) Create a table and record some of the student values. ***How can we verify that these are equivalent ratios?*** Look for common multipliers, which may be fractions.

Page 4
Exploring Ratios

  (Page 4, problem 4) ***How do you know that the numbers in rows in this table do not represent equivalent ratios?*** None of the ratios of numbers of feet to numbers of eyes are equivalent. There is no common multiplier for any of these pairs of numbers.

PRACTICE 1

Individuals

Page 5
Practice 1

- This page is appropriate for classwork or homework.

EXPLORE 2A / SUMMARIZE 2A

Whole Class / Partners

Page 6
Grape Juice Mixtures

Reproducible 26
Grape Juice Mixture Cards

Materials
- Scissors
- Strips of paper (optional)

- Distribute one set of cards to each pair and ask students to cut them up.
- Students organize the cards from “least grap . As students work, circulate and ask for explana representations on the cards in different way g decisions.
- If desired, give each partner a strip of paper, r. Post them for discussion. (The correct order is H
- Compare ordering and discuss. Choose a c scuss the ordering of the pair with their partners. T he board. Encourage students to challenge eac

Grape Mixtures starts here with directions and discussion ideas.

There is a reproducible that contains cards to cut out for this activity. It is located in the Reproducible tab of the Teacher Guide.

  Student 1: “I think D and F represent the same grapeyness because they are both $\frac{2}{3}$ grape concentrate.”

  Student 2: “ I agree that F is $\frac{2}{3}$ grape concentrate, but D is $\frac{2}{3}$ water, so it is only $\frac{1}{3}$ grape concentrate.

EXPLORE 2B / SUMMARIZE 2B

Whole Class /
Partners

Page 6
Grape Juice Mixtures

Reproducible 26
Grape Juice Mixture Cards

- Students discuss and critique the reasoning of the four students' statements. Then they compare the "grapeyness" of mixture J to mixtures with different units of measure. Possible true statements for problem 3 are:

  (1) J is more grapey than A because though they have the same amount of grape concentrate, A has much more water.

  (2) J is more grapey because it is $\frac{2}{3}$ cups of grape concentrate and A is only $\frac{1}{3}$ cups of grape.

  (3) If the cups in J are doubled (use 2 as the multiplier), there are 4 cups grape concentrate for every 2 cups water. A has 2 cups grape for every 4 cups water.

- Challenge students with questions related to changing of units of measure. Generalize comparisons of specific units (such as cups) to any equal units of measure (called "parts").

  (Problem 7) ***Last period some students said that*** ***because ounces are less than cups. Were they c*** are consistent (cups : cups, ounces : ounces, or in general parts : parts), the ratios are equivalent.

  Questions are bold and italic, and are provided to help students get started.

  (Problem 9) ***Will this mixture taste the same as J ?*** No. J has twice as much grape as water. 2 ounces of grape is much less than 1 gallon of water, so it will taste much more watery. This is no longer the same ratio of parts to parts.

EXPLORE 2C / SUMMARIZE 2C

Whole class /
Partners

Page 7
Tape Diagrams

Reproducible 26
Grape Juice Mixture Cards

- Read the description of tape diagrams and discuss the examples. Students draw tape diagrams for four more cards.

  (Problem 3) ***How can you draw the tape diagram for F?*** Answers may vary. One way is to use a table (or repeated addition) to arrive at the fact that this ratio is equivalent to 2 cups grape concentrate : 3 cups water . Some students may see that using 3 as a multiplier leads to the same conclusion.

× 3

cups grape	$\frac{2}{3}$	$\frac{4}{3}$	$\frac{6}{3}$
cups water	1	2	3

× 3

(Problem 4) ***How can you draw the tape diagram for H?*** Using a variety of multipliers, one can determine that all of the grape concentrate : water ratios in the table are equivalent. Draw one rectangle for grape concentrate and three for water.

EXPLORE 2C / SUMMARIZE 2C (Continued)

Whole Class

Page 0
Word Bank

Page 1
Ratios

- Discuss two fundamentally different methods for using tape diagrams.

  How do Alex's and Andrea's methods differ? Card A represents 2 parts grape concentrate and 4 parts water, for a total of 6 parts mixture. The students are asked to make 12 gallons. Alex notices that he can double the number of parts to make 12, so his picture represents doubling parts. Andrea notices that each part can represent 2 gallons, so rather than doubling the parts, she doubles how much the parts each represent.

- Students use the different methods to solve two problems.

  (Problem 7) ***Why might Alex's method be difficult to use here?*** We would have to draw a lot of boxes to make our tape diagram represent 72 parts. Anderea's method is much more efficient.

EXTEND

Whole Class

Task, Page 14
Grapey Mixtures

Reproducible 26
Grape Mixture Cards

Reproducible 27
Blank Cards

Materials
- poster-sized paper
- markers
- tape or glue

- For this project, provide clean copies of grape mixture cards and other supplies to students. Students create a poster that shows mixtures from least grapey to most grapey using different representations. They also include additional mixture(s) and representations of their own.

  What are some representations we might use to illustrate grape mixtures? Pictures, tape diagrams, *a* : *b* notation, "for every…" statement, etc.

  How might we organize cards on a poster? Answers will vary. One possibility is:

	least grapey <- - - - - - - - - - - - - -> most grapey					
picture						
tape diagram						
a : *b* notation						
verbal statement ("for every…")						
table						

PRACTICE 2

Whole Class

Page 8
Practice 2

- This page is appropriate for classwork or homework.

CLOSURE

Whole Class

Page 0
Word Bank

Page 1
Ratios

- Review the goals, standards, and vocabulary

No matter how many cycles of Introduce, Explore, Summarize, Practice and Extend a lesson has, it always ends with one Closure.

UNIT RATES

Summary

Students explore the relationship between ratios and rates, and unit rate is defined. Students represent ratios and rates with tables and double number line diagrams, and solve problems using rates.

6.RP.A

Goals

- Relate unit rate to ratio.
- Represent rates using symbols, words, tables, and double number line diagrams.
- Solve problems using rates, tables and double number line diagrams.

PREVIEW / WARMUP

Whole Class / Individuals

Page 0
Word Bank

Page 9
Rates

- Introduce the goals and standards of th ulary as relevant.
- Students choose statements that descri (Statements 1, 2, 3, 4, and 8 are true.)

The second lesson starts again with the black strip, Summary, Goals, and Warmup.

INTRODUCE

Whole Class

- Gather time and distance data from students by asking them a question about the time it takes to drive a local distance. Use student data for this introduction.

Create an “itch” here. See Teaching Note 5.

(Example) ***How long does it typically take to drive to our closest theme park?*** 2 hours. ***How far is it?*** 70 miles.

- Begin a table that shows this distance and time.

Distance (miles)	70		
Time (hours)	2		

If we drive for 4 hours at that rate, how far will we go? 140 miles. Record in the table.

If we drive 35 miles at this rate, how long will it take? 1 hour. Record in the table.

State this relationship as a ratio statement. We drive 2 hours for every 70 miles. Some students may say “35 miles per hour.” Use this statement to introduce unit rate.

Are the ratios in our table equivalent? Yes. Identify multipliers to support this conclusion.

- Introduce the concept of unit rate (the value of the ratio, or the ratio $a : b$ written as the fraction $\frac{a}{b}$ with units attached). In the example above, the unit rate is $\frac{70}{2}$ = 35 miles per hour. The value of the ratio is 35, and the unit rate is measured in units of miles per hour. The unit rate is then 35 miles per hour.

Which value in our table indicates the unit rate? 35 miles in 1 hour.

EL Tip

Use color, circle the value, and label it “unit rate.”

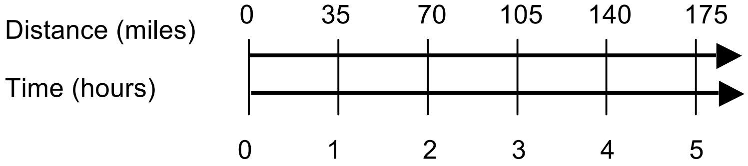
INTRODUCE (Continued)

Whole Class

Page 0
Word Bank

Page 10
Unit Rates Associated with Ratios

- Explain to students that when we have equivalent ratios that involve different units (say miles and hours), we can represent them with a double number line diagram. Show students how to display this information on a double number line. Include other values that fit on the line as well.



- Compare the double number line to the table we created.

  How is information on the double number line different than the table? The numbers are scaled and in order on a double number line. Tables may have values in any order.

  Where do we identify the unit rate on this double number line? Look for the number of miles traveled for one unit of time (one hour).

- Ask students to read the definition of unit rate and explain it to each other. Then have them complete the questions on the page and discuss as needed.

EXPLORE

Partners

Page 11
Rates and Tables

Pages 12-13
Ratios and Double Number Lines

- Students work together to answer questions related to rates, tables, and double number lines. Circulate to identify misconceptions or sticking points. Encourage students to ask each other for help as needed.

SUMMARIZE

Partners

Page 11
Rates and Tables

Pages 12-13
Ratios and Double Number Lines

- Discuss problems where students had difficulty.

  (Page 11, Problem 4) ***How do you know that prices and quantities in columns in Table III represent equivalent ratios?*** One way is to calculate their unit rate. In all cases, the unit rate is 0.10.

  (Page 11, Problem 4) ***How can we verify that these prices and quantities represent equivalent ratios using a constant multiplier?*** We can find a constant multiplier for any pair of bags. For example, the multiplier for prices and quantities for B and D is 3. The multiplier for prices and quantities for C and D is 0.75.

  (Pages 12 and 13) ***What table values were difficult to find?*** Answers will vary. ***How did you find them***? Encourage students to explain how they found values using unit rates, multipliers, or other strategies.

  Do you prefer to find unit rates or multipliers to determine if ratios are equivalent? Answers will vary. It probably depends on the numbers in the ratios.

PRACTICE 1

Groups of 4

Page 14
Poster Problems

Reproducible 28
Poster Problems 3

Materials
- Chart Paper
- Markers

"Poster Problems" allow students to practice math in groups as they move around the room. To prepare, print one copy of the reproducible for each group, and create numbered posters (chart paper) equal to the number of groups you have in your classroom. Put chart paper around the room.

Poster Problems are a collaborative activity designed to make practice more engaging. There is a reproducible, directions in the teacher pages (here), and some directions and followup in student pages. This kind of activity recurs in the program.

- (For Poster Problems) Arrange students into gr s identify themselves as A, B, C, or D. Give each accountability purposes.
- Each group begins at their numbered poster. (part A of the poster problem with the group's colored and help. (2) After about 3-4 minutes, say "Move to move to the next poster. The entire group checks the at problem. Then Student "B" writes the answer to group's colored marker. Other members suppo more times to complete parts C and D.
- After problems are completed, ask students to return to their table groups. Ask students to complete the problems in their packets and discuss all problems. Encourage students to share constructive feedback that would improve explanations for placement of fractions using conceptual strategies.

  What are the "twists" in each problem? For problem 1, you can purchase 1 marker for 60¢ and there will be money leftover. For problem 2, the team will not play $2\frac{2}{3}$ games in a week. They will play more games in one week and fewer games another. For problem 3, when milk and flour are mixed together, the milk absorbs the flour so there are not really 7 cups of mixture. Furthermore, For problem 4, people do not grow at a constant rate forever. If that happened, we would all be REALLY tall!

PRACTICE 2

Groups of 4

Page 15-16
Practice

- These pages are appropriate for classwork or homework.

CLOSURE

Whole Class

Page 0
Word Bank

Page 9
Rates

- Review the goals, standards, and vocabulary of the lesson.

RATIO AND UNIT RATE PROBLEMS

Summary	Goals
Students solve ratio and rate problems using tables, diagrams, and equations. *6.RP.A*	- Solve ratio and unit rate problems using a variety of strategies.

PREVIEW / WARMUP

Whole Class

Page 0
Word Bank

Page 17
Ratio and Rate
Problems

- Introduce the goals and standards of the lesson. Discuss important vocabulary as relevant.

Create an "itch" here. See Teaching Note 5.

- Students review ratios and unit rates.

INTRODUCE 1

Whole Class

This lesson contains seven situations where students apply proportional reasoning to solve problems. Consider assigning the first three together.

Page 18
The Green Grocer

- (The Green Grocer) This problem requires students to identify the unit rate within different representations and then solve a problem.

  What information is given? A ratio: It costs $3.50 for every 2 melons.

  What are you asked to do? Create a unit rate, a table, a double number line, and solve two problems.

Page 19
The Toothpaste
Problem

- (The Toothpaste Problem) This problem asks students to solve a problem using tables to represent equivalent ratios. They need to determine appropriate multipliers that will lead them to a fractional solution.

  What information is given? A ratio: Pippy uses 3 tubes for every 5 months.

  What are you asked to do? Determine the number of tubes used in a year.

  Explain how to estimate the number of tubes needed for one year. From the first table, 6 is too few and 9 is too many, so it is somewhere in between.

Page 20
Apples, Apples,
Apples

- (Apples, Apples, Apples) Students determine the best buy given costs for apples at different stores.

  What information is given? The amount of apples you want to buy. The cost for apples at different stores.

  What are you asked to do? Determine the best buy. In other words, get the most pounds of apples for some amount of money, or spending the least amount of money for some number of pounds of apples.

EXPLORE 1 / SUMMARIZE 1

Partners

- Students work together to solve the problems. As they work, circulate and encourage students to explain their thinking in different ways and to ask questions of each other. Some group discussion questions are included here.

Page 18
The Green Grocer

(The Green Grocer) ***Explain how to find the unit rate in the different representations?*** Problem 1 asked for it explicitly. For problem 2, it was the last entry in the table. For problem 3, it is the dollars for 1 melon. For problem 4, it is the coefficient of M. Follow up with student explanations for how they used unit rates or another strategy to solve problems 6 and 7.

Page 19
The Toothpaste Problem

(The Toothpaste Problem) ***Why was this problem not solved quickly with a table?*** The result is not a whole number.

How is Zippy's strategy different than Tippy's strategy? Tippy recorded the unit rate in the table by dividing the first entry by 5. Then she multiplied entries by 2 and then by 6. Instead of dividing initial table entries by 5 and then multiplying by 12 in several steps, Zippy multiplied both numbers in the initial ratio by $\frac{12}{5}$.

Page 20
Apples, Apples, Apples

(Apples, Apples, Apples) ***How did you start this problem?*** Most likely with Store A. At this store, 5 pounds will cost $10.00.

What did you do then? Students may have determined the unit price for each store, but if they compare to Store A, they can eliminate some options.

Which calculations were difficult? Some students may have difficulties with Stores C and D if they are trying to convert the given bags to their 5 lb equivalent prices. Some students may be challenged by the fractional pound savings statements for Stores E and F.

- Invite students to present solutions to problems. ng questions when explanations are unclear.

We suggest a jigsaw structure for some problems. That holds students more individually accountable when doing challenging work.

PRACTICE 1

Individuals

Page 21
Building a Deck

- This is appropriate for classwork or homework. lem.

INTRODUCE 2

Groups

Page 22
The Grain Grocer

Page 23
The Assembly

Page 24
The Paint Mistake

For the final three situations, consider a jigsaw structure. In "home groups," have students number off 1, 2, 3, 1, 2, 3 … Then ask all 1s to work with other 1s to solve The Grain Grocer problem. Ask all 2s to work with other 2s to solve The Assembly problem. Ask all 3s to work with other 3s to solve The Paint Mistake problem. Experts then return to their "home groups" and help each other solve all three problems.

- Explain the jigsaw structure to students and have them number off (1, 2, 3, 1, 2, 3, etc) in their home groups. Then have students regroup themselves (all 1s will work in the northeast corner of the room, etc) and create three working groups. Assign one problem to each working group.

EXPLORE 2A

Groups

Page 22
The Grain Grocer

Page 23
The Assembly

Page 24
The Paint Mistake

- Once in working groups, students may want to work individually or with a partner to solve problems, and then compare and discuss solutions. Allow students to struggle with representations and explanations. Encourage students who are creating double number lines or tape diagrams to share with others. Once they have solved the problems, they will be “experts” for that problem.

  What representation might be useful for the Grain Grocer? Double number line.

  What representation might be useful for the Assembly? Tape diagram.

  What representation might be useful for the Paint Mistake? Tape diagram.

EXPLORE 2B

Groups

Page 22
The Grain Grocer

Page 23
The Assembly

Page 24
The Paint Mistake

- Students return to their home groups and continue to work on the problems. Students should be able to complete all three problems with the help of the “expert,” who returned from the working group. Encourage experts to ask good questions rather than just give partners answers.

SUMMARIZE 2

Whole Class

Page 22
The Grain Grocer

Page 23
The Assembly

Page 24
The Paint Mistake

- Discuss the process and the problems.

  Which representations were useful for the problems? Hopefully students recognize the utility of double number line diagrams or tape diagrams for solving these problems.

  If you were an “expert,” where did people in your group get stuck on your problem? How did you help them get unstuck? Answers will vary.

CLOSURE

Whole Class

Page 0
Word Bank

Page 17
Ratio and Rate
Problems

- Review the goals, standards, and vocabulary of the lesson.