MathLinks

PROPORTIONAL REASONING 2
STUDENT PACKET

PERCENTS

Parent (or Guardian) signature _____

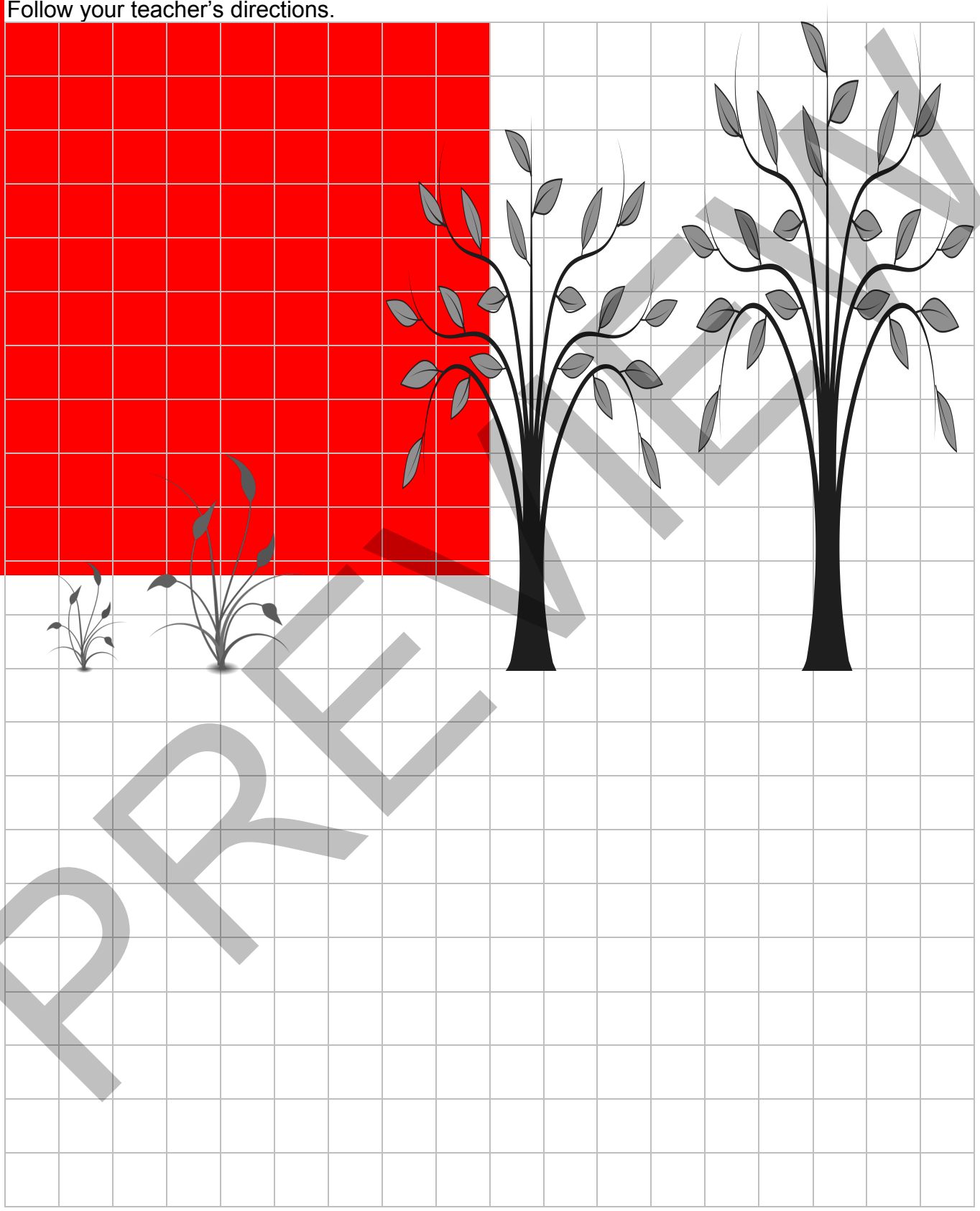
MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. (See section 2.5.) Key mathematical vocabulary is underlined throughout the packet.

discount	markup
percent	percent increase in a quantity
percent decrease in a quantity	percent of a number

GROWTH SPURTS

Follow your teacher's directions.



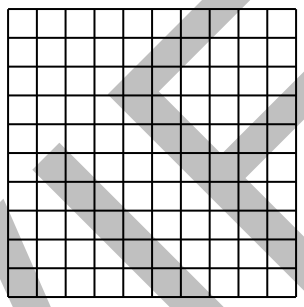
PERCENT AS A NUMBER

We will use an area model, money references, the multiplicative identity (referred to as the Big 1), and division to explore fraction, decimal, and percent concepts.

GETTING STARTED

1. Find percent in section 2.5 and explain what it means in My Word Bank. Make sure to include the percent symbol.

2. Shade $\frac{1}{2}$ of this grid.



The shading shows that $\frac{1}{2} = \frac{\quad}{100}$.

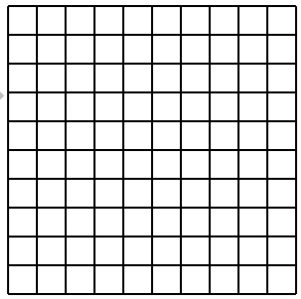
Write $\frac{1}{2}$ of a dollar in cents. _____¢

Write $\frac{1}{2}$ as a percent. _____

Write $\frac{1}{2}$ of a dollar in dollars. $_____

Write $\frac{1}{2}$ as a decimal. _____

3. Shade $\frac{1}{4}$ of this grid.



The shading shows that $\frac{1}{4} = \frac{\quad}{100}$.

Write $\frac{1}{4}$ of a dollar in cents. _____¢

Write $\frac{1}{4}$ as a percent. _____

Write $\frac{1}{4}$ of a dollar in dollars. $_____

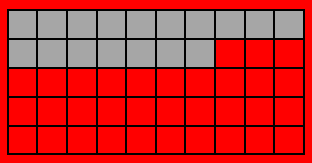
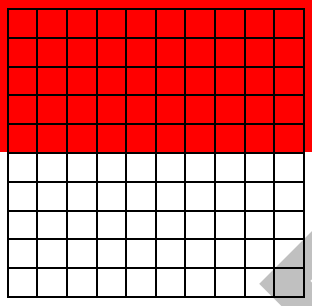
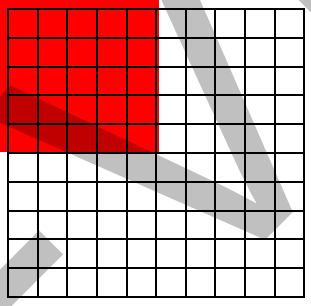
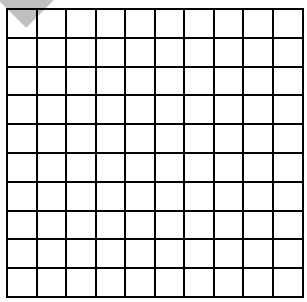
Write $\frac{1}{4}$ as a decimal. _____

4. How is writing fractions in terms of money the same as writing fractions as decimals?

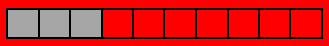
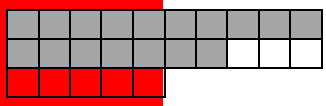
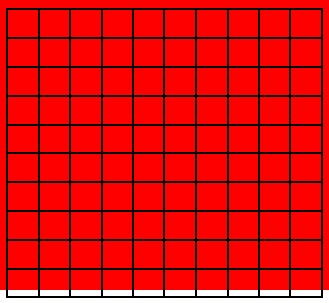
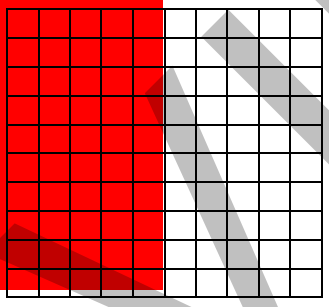
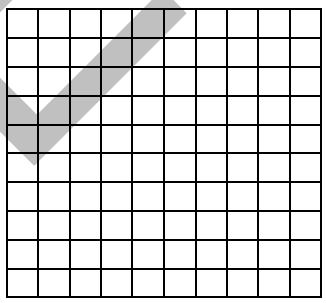
5. How is writing fractions in terms of money the same as writing fractions as percents?

FRACTION GARDENS

We will use an area model to explore percents. Each drawing represents a garden. The shaded part represents the portion that is planted. Answer all problem parts based on the given garden for that problem.

1.	2.	3.
a. What fractional part is planted? 	a. What fractional part is planted? 	a. What fractional part is planted? 
b. Shade this garden so that the same fractional part is planted. 	b. Shade this garden so that the same fractional part is planted. 	b. Shade this garden so that the same fractional part is planted. 
c. The shading shows: $\frac{17}{\quad} = \frac{\quad}{100}$	c. The shading shows: $\frac{\quad}{4} = \frac{\quad}{100}$	c. The shading shows: $\frac{\quad}{\quad} = \frac{\quad}{100}$
d. Complete this equation. $\frac{17}{50} \bullet \frac{\quad}{\quad} = \frac{\quad}{100}$	d. Complete this equation. $\left(\frac{\quad}{4}\right) \bullet \frac{\quad}{\quad} = \frac{\quad}{100}$	d. Complete this equation. $\left(\frac{3}{\quad}\right) \bullet \frac{\quad}{\quad} = \frac{\quad}{100}$
e. Write this fraction as a decimal.	e. Write this fraction as a decimal.	e. Write this fraction as a decimal.
f. Write this fraction as a percent.	f. Write this fraction as a percent.	f. Write this fraction as a percent.

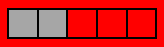
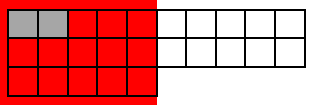
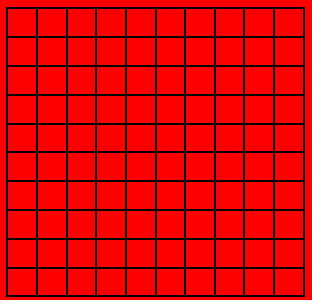
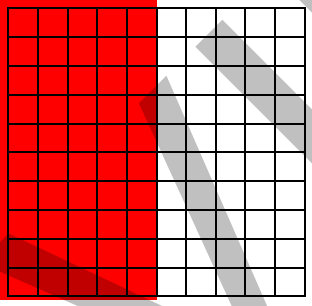
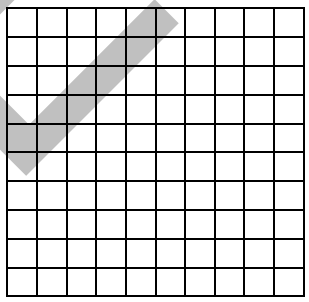
FRACTION GARDENS
(Continued)

4.	5.	6.
a. What fractional part is planted? 	a. What fractional part is planted? 	a. What fractional part is planted? 
b. Shade this garden so that the same fractional part is planted. 	b. Shade this garden so that the same fractional part is planted. 	c. Shade this garden so that the same fractional part is planted. 
c. My shading shows: $\frac{\quad}{100}$	c. My shading shows: $\frac{\quad}{100}$	c. My shading shows: $\frac{\quad}{100}$
d. Complete this equation. $\left(\frac{3}{\quad}\right) \bullet \frac{\quad}{\quad} = \frac{\quad}{100}$	d. Complete this equation. $\left(\frac{\quad}{25}\right) \bullet \frac{\quad}{\quad} = \frac{\quad}{100}$	d. Complete this equation. $\left(\frac{\quad}{20}\right) \bullet \frac{\quad}{\quad} = \frac{\quad}{100}$
e. Write this fraction as a decimal.	e. Write this fraction as a decimal.	e. Write this fraction as a decimal.
f. Write this fraction as a percent.	f. Write this fraction as a percent.	f. Write this fraction as a percent.

7. Refer to section 2.5 (The Big 1). Put a Big 1 around each fraction in “part d” above that represents a form of 1. What happens to a fraction when it is multiplied by a “Big 1” fraction?

PRACTICE 1

Each drawing represents a garden. The shaded part is planted.

1.	2.	3.
a. What fractional part is planted? 	a. What fractional part is planted? 	a. What fractional part is planted? 
b. Shade the hundred square so that the same fractional part is planted. 	c. Shade the hundred square so that the same fractional part is planted. 	b Shade the hundred square so that the same fractional part is planted. 
a. My shading shows: $\frac{\quad}{100} = \frac{\quad}{\quad}$	c. My shading shows: $\frac{\quad}{100} = \frac{\quad}{\quad}$	b. My shading shows: $\frac{\quad}{100} = \frac{\quad}{\quad}$
c. Complete this equation. Show the Big 1. $\frac{2}{\quad} \bullet \frac{\quad}{\quad} = \frac{\quad}{100}$	d. Complete this equation. Show the Big 1. $\frac{2}{\quad} \bullet \frac{\quad}{\quad} = \frac{\quad}{100}$	d. Complete this equation. Show the Big 1. $\frac{4}{\quad} \bullet \frac{\quad}{\quad} = \frac{\quad}{100}$
e. Write this fraction as a decimal.	e. Write this fraction as a decimal.	e. Write this fraction as a decimal.
f. Write this fraction as a percent.	1. Write this fraction as a percent.	f. Write this fraction as a percent.

4. Draw a garden with 10 total parts with no parts planted. Then write this shaded amount as a fraction and as a percent.

PARTS OF A DOLLAR

Follow your teacher's directions for problems 1-5.

1.	2.	3.
4.	5.	

6. Complete the table using these coins.

A

B

C

D

E

F

Coin	Who is on the coin?	Value (in cents) and name of coin	Value (as fraction of a dollar)	Value (as percent of a dollar)	Value (as decimal)
A					
B		1¢ penny			
C					0.05
D	Lead Woman's Suffrage Susan B. Anthony				
E		50¢ half-dollar			
F					

7. Complete the table. Use your knowledge of money to help you. Do not use a calculator.

Fraction*			$\frac{9}{20}$			$\frac{2}{4}$		
Decimal	0.6			0.03			0.15	
Percent		30%			100%			87%

* Equivalent fractions are acceptable.

PRACTICE 2

Complete the table.

	Fraction	Decimal (number)	Decimal (words)	Percent
1	$\frac{1}{4}$		twenty-five hundredths	
2		0.29		
3				70%
4	$\frac{4}{25}$			
5		0.07		
6				65%
7			four tenths (or forty hundredths)	
8				50%

9. Enrique got 17 questions correct out of 20 on his math test. What percent of the test did he get correct?

10. A 25-person dance crew includes 9 boys. What percent of the crew are girls?

11. Li noticed that 8 out of 40 classmates were left-handed. What percent of the class was left-handed? Do you think you would expect to see left-handedness in the general population at this same rate? _____ Support your claim with evidence.

12. Circle methods for renaming fractions, decimals, and percents that make sense to you.

Draw a picture Use the Big 1 Think about money

USING DIVISION TO FIND DECIMALS AND PERCENTS

1. Ronni wanted to rename $\frac{3}{8}$ as a decimal and percent. She divided as shown to the right. Use Ronni's work to complete each equation.

$$\begin{array}{r} .375 \\ 8\overline{)3.000} \\ -24\downarrow\downarrow \\ \quad 60\downarrow \\ \quad -56\downarrow \\ \quad\quad 40 \\ \quad\quad -40 \\ \quad\quad\quad 0 \end{array}$$

$\frac{3}{8} = 0.$_____	$\frac{3}{8} = \frac{\square}{1000}$	$\frac{3}{8} = \frac{\square}{100}$	$\frac{3}{8} =$ _____%

2. Jay divided 3 by 8 on his calculator and got 0.38. Why do you think Jay's result is different than Ronni's?

3. Michael wanted to rename $\frac{1}{3}$ as a decimal and as a percent. He divided as shown to the right. Use Michael's work to complete each statement.

$$\begin{array}{r} .333 \\ 3\overline{)1.000} \\ -9\downarrow\downarrow \\ \quad 10\downarrow \\ \quad -9\downarrow \\ \quad\quad 10 \\ \quad\quad -9 \\ \quad\quad\quad 1 \end{array}$$

$\frac{1}{3} \approx 0.$_____	$\frac{1}{3} \approx \frac{\square}{1000}$	$\frac{1}{3} \approx \frac{\square}{100}$	$\frac{1}{3} \approx$ _____%

4. Susie divided 1 by 3 on her calculator and got 0.3333333. Why do you think Susie's result is different than Michael's?

PRACTICE: Change each fraction to a decimal and a percent. Complete one problem using long division to the right. Use mental math or a calculator for the rest.

5. $\frac{5}{8}$	6. $\frac{9}{40}$	7. $\frac{2}{3}$	8. $\frac{1}{50}$
10. $\frac{1}{6}$	11. $\frac{5}{6}$	12. $\frac{9}{10}$	13. $\frac{7}{25}$

PERCENT OF A NUMBER

We will continue to build concepts about percents as we use chunking, double number lines and computational procedures to find percents of numbers.

GETTING STARTED

Rename each number in different forms.

	Fraction	Decimal (number)		Decimal (words)	Percent
1	$\frac{3}{5}$				
2		0.3			
3	$\frac{1}{4}$				
4				one hundred twenty-five thousandths	

5. Compare the results in problems 1 and 2 above.

a. What is the relationship between the fractions?	b. What is the relationship between the decimals?

6. Compare the results in problems 3 and 4 above.

a. What is the relationship between the fractions?	b. What is the relationship between the percents?

7. Matt's cell phone has a percent in the upper right hand corner. Explain what these mean.

a. 100%	b. 50%	c. 5%

REVIEW: MONEY AND METERS

Complete each sentence.

1. A dollar is _____ cents.

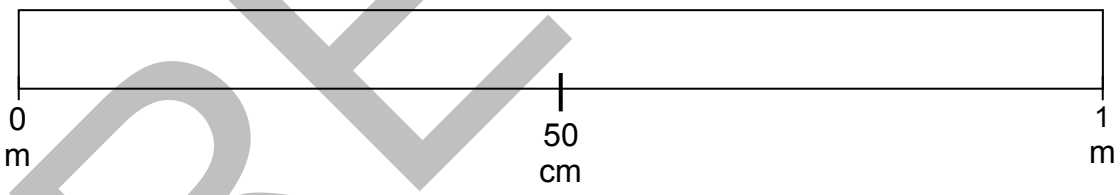
2. Find percent of a number in section 2.5 and explain what it means in My Word Bank.

Find the following percents of a dollar.

3. 100% of a dollar is _____ cents.
4. 50% of a dollar is _____ cents.
5. 25% of a dollar is _____ cents.
6. 75% of a dollar is _____ cents.
7. 0% of a dollar is _____ cents.
8. 10% of a dollar is _____ cents.
9. 20% of a dollar is _____ cents.
10. 5% of a dollar is _____ cents.
11. 34% of a dollar is _____ cents.
12. 76% of a dollar is _____ cents.

12. A meter is _____ centimeters (cm).

13. The rectangle below represents a meter stick. One meter = _____ cm. Make a few more tick marks and label the centimeters.



14. Write the letter on the meter stick above that represents the percent of the way (from zero) for each of the following:

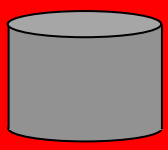
A.	100%	B.	50%	C.	25%
D.	75%	E.	10%	F.	20%
G.	5%	H.	80%	I.	55%

15. How are finding cents in a dollar and finding centimeters on a meter stick related?

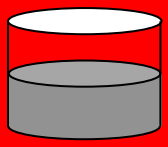
MAKING SENSE OF PERCENTS

Complete the problems below.

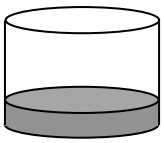
1. Estimate how full each container is using percents.


Container A

A.


Container B

B.


Container C

C.


Container D

D.

2. Suppose that, when full, each container holds 800 ounces of liquid. How many ounces do you estimate are in each container?

A. B. C. D.

3. Now suppose these containers hold 900 ounces of liquid. Now how many ounces do you estimate are in each container?

A. B. C. D.

Fill in the blanks below with appropriate words or numbers to complete each percent statement.

4. Finding 100% of the volume of a container is the same as finding _____ of it.

   100% of $40 is _____. 100% of 300 meters is _____ .

5. Finding 50% of the volume of a container is the same as finding _____ of it.

   This is the same as multiplying by _____ or dividing by _____.

   50% of $40 is _____. 50% of 300 meters is _____.

6. Finding 25% of the volume of a container is the same as finding _____ of it.

   This is the same as multiplying by _____ or dividing by _____.

   25% of $40 is _____. 25% of 300 meters is _____.

7. Finding 10% of something is the same as finding _____ of it.

   This is the same as multiplying by _____ or dividing by _____.

   10% of $40 is _____. 10% of 300 meters is _____.

PRACTICE 3

For 1 and 2, answer the questions. Then for parts a and b, show your work or explain your reasoning.

1. What is 10% of $50?

   a. Use this result to find 20% of $50.

   b. Use this result to find 5% of $50.

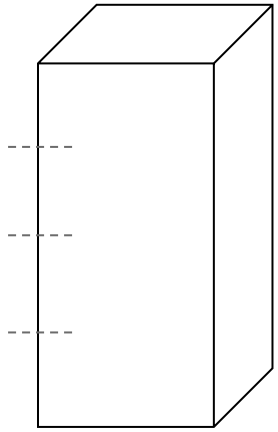
2. What is 10% of 200 meters?

   a. Use this result to find 30% of 200 m.

   b. Use this result to find 1% of 200 m.

A rectangular container holds 600 ounces of liquid when full. Use the picture if it is helpful, and determine how much liquid makes the container:

3. 50% full
4. 25% full
5. 75% full
6. 10% full
7. 20% full
8. 30% full
9. 5% full
10. 15% full
11. 1% full
12. 2% full
13. 0.5% full
14. 1.5% full



15. What would it mean for the container to be 150% full?

    How much liquid would it contain if it was 150% full?

    Do you think this is possible? Explain.

FINDING PERCENTS OF QUANTITIES USING CHUNKING STRATEGIES

We will use the word “chunking” to describe a mental math process of composing and decomposing numbers to make calculations easier. Complete the tables below. Use division and chunking to compute. Be ready to explain your strategies.

	Amount of Money	Find 10%	Find 5%	Find 15%	Find 20%
1.	$30	$3			
2.	$150				
3.	$500				
4.	$16				
5.	$42				

	Item	Find 10%	Find 5%	Find 50%	Find 25%
6.	80 games				
7.	60 apples				
8.	120 students				
9.	20 cars				

	Some Challenges	Find 1%	Find 0.5%	Find 1.5%	Find 150%
10.	$200				
11.	$7,000				
12.	$150				

13. Describe a situation for which finding a percent greater than 100% makes sense.

PRACTICE 4

Complete the table below. Use division and chunking to compute.

	Quantity	Find 10%	Find 5%	Find 15%	Find 30%	Find 1%
1.	60 minutes					
2.	$1000					
3.	30 miles					
4.	$3.60					
5.	50 cookies					

Solve each percent problem below by chunking. Show your work clearly.

6. The local sales tax in Los Angeles in the beginning of 2017 was 9%. How much was tax on a $60 jacket?

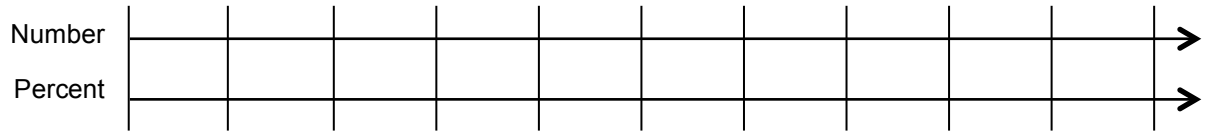
   What was the total cost of the jacket, including tax?

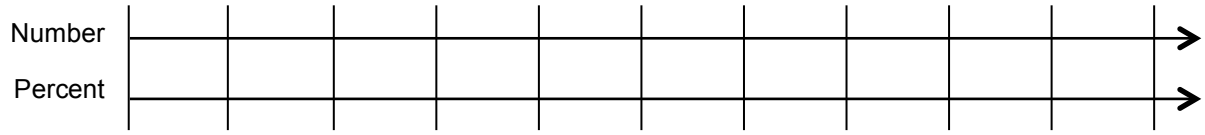
7. Suppose a pair of jeans cost $40. If there is a 35% off sale, how much will you save?

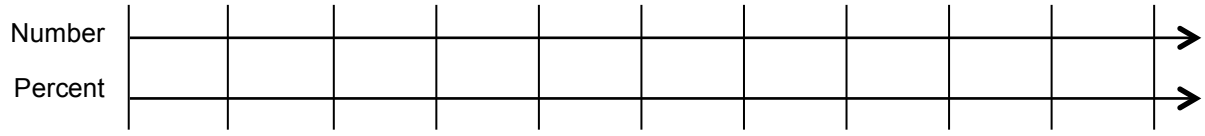
   What will be the cost of the jeans after the discount?

PERCENTS AND DOUBLE NUMBER LINES

Follow your teacher's directions to learn about percents and double number lines.

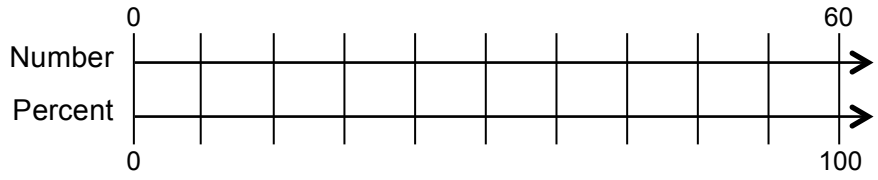


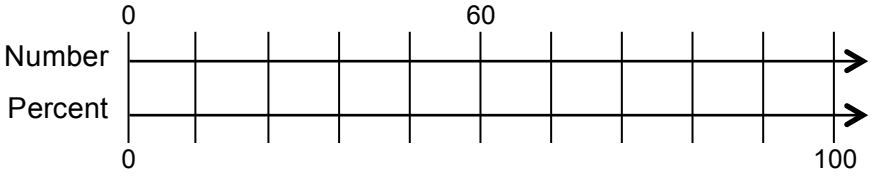




PRACTICE 5

1. Complete the double number lines with the given information.





For problems 2-7, use the double number lines above to help you answer questions and write equivalent fractions.

	Question	Answer	Equivalent Fractions
2.	What is 20% of 60?	12 is 20% of 60.	$\frac{\quad}{\quad} = \frac{20}{100}$
3.	A dress that cost $120 was discounted 40%. What was the discount?		
4.	What is 25% of 60?		
5.	48 is 40% of what?		
6.	Jabari answered 54 out of 60 questions. What percent did he answer?		
7.	Angela decorates 9 cupcakes. She is 15% done. How many total cupcakes are there?		

USING MULTIPLICATION TO FIND PERCENT OF A NUMBER

Try these computational strategies for finding 20% of 50.

1.	2.	3.
Multiply 50 by $\frac{20}{100}$.	Multiply 50 by 0.2.	Use a calculator to compute 20% of 50.

Find 5% of 320.

4.	5.	6.
Multiply 320 by $\frac{\square}{100}$.	Multiply 320 by 0._____	Use a calculator to compute 5% of 320.

PRACTICE: Find the percent of each number. Complete one problem to the right using multiplication. Use a calculator for the rest.

7. 9% of $60	8. 28% of $60
9. 87% of $110	10. 2% of $110
11. Sales tax in Los Angeles rose to 9.25% in July 2017. What was the sales tax on a phone that costs $325.00?	

13. Circle all strategies you understand for finding percent of a number. Put stars by the ones you prefer.

double number line	chunking	multiply by a fraction	multiply by a decimal	calculator

PERCENT APPLICATIONS

We will solve percent problems that involve gratuities, markups, discounts, and sales taxes.

GETTING STARTED

Use sense-making strategies to compute. See section 2.5 for ideas.

1. 50% of $240	2. 150% of $240
3. 15% of $240	4. 1% of $240
5. 17% of $240	6. 0.5% of $240

7. Janie got 24 out of 40 problems correct on a quiz. What percent of the problems did she get correct?

GRATUITIES

1. Many people provide services for which they receive gratuities (or tips). Better service frequently gets the worker a better tip. List three jobs for which workers might receive tips.

Complete the table.

Amount	Percent for Gratuity	Amount of Gratuity	Total Amount
2. $40	10%		
3. $60		$12	
4. $90			$99
5. $28		$2.80	
6. $100			$101
7. $50	20%		
8. $80			$92
9. $25	5%		

MARKUPS AND DISCOUNTS

1. The following terms are in section 2.5. Discuss them in class and record in My Word Bank.

Word List			
discount	markup	percent decrease in a quantity	percent increase in a quantity

Match each description to all appropriate entries in the word list above.

2. Woohoo! Your favorite clothing store is having a big 20% off sale.

3. Oh no! Your favorite music site is increasing all of its song prices by 5%.

4. Name three situations where a percent increase or percent decrease might occur. Come up with different contexts than in problems 2 and 3 above.

Complete the table.

Original Amount	% of Change	Amount of Change	New Amount
5. $100	10% markup		
6. $50	100% markup		
7. $50	10% discount		
8. $20	20% discount		
9. $20		$1 increase	
10. $20		$1 decrease	
11. $65		$13.00 increase	
12. $22		$0.22 decrease	

SALES TAXES

1. In your own words, describe sales tax.

Sales tax is a markup. Sales taxes vary from city to city because they include a state tax rate as well as local city taxes. Sales taxes are added to the basic cost (subtotal) of items.

Complete each cash register receipt. Use a calculator and round appropriately.

2.

Bulls-Eye Department Store
Chicago, Illinois

Item	Cost
craft glue	$8.99
shirt	$25.99
Subtotal	
Sales Tax (10.25%)	
TOTAL	

3.

Super Sales Electronics
Sacramento, California

Item	Cost
ear buds	$29.99
flash drive	$12.49
Subtotal	
Sales Tax (8.25%)	
TOTAL	

Sometimes receipts show unit prices (cost for one item) and quantities. Complete this cash register receipt for a restaurant in Washington, D.C., where sales tax is 5.75%. Use a calculator.

4.

McDucky's Restaurant
Washington, D.C.

Item	Unit price	Quantity	Cost
McDucky burger	$5.49	2	
Cluck cluck chicken salad	$7.50	1	
Drink (small)	$1.89	3	
		Subtotal	
		Sales Tax (_____)	
		TOTAL	

PRACTICE 6

Use a calculator as needed and round appropriately.

1. A department store is having a sale on jackets. The original cost of a jacket that Marika wants to buy is $90.

a. If the jacket was marked down to $72, what was the percent discount?	b. Sales tax in Seattle, Washington is 9.6%. What is the sales tax amount for the discounted jacket?
c. Marika has $75 to spend on the jacket. Will this be enough money? Explain.	

2. Marcus buys 2 pairs of baseball pants ($29.95 each), 1 baseball glove ($49.50), and 3 baseballs ($2.95 each) at Giant Sports in Vancouver, Washington. Sales tax is 8.4%.

   a. Prepare a receipt for these purchases.

Giant Sports
Vancouver, Washington

Item	Unit Price	Quantity	Cost
		Subtotal	
		Sales Tax (_____)	
		TOTAL	

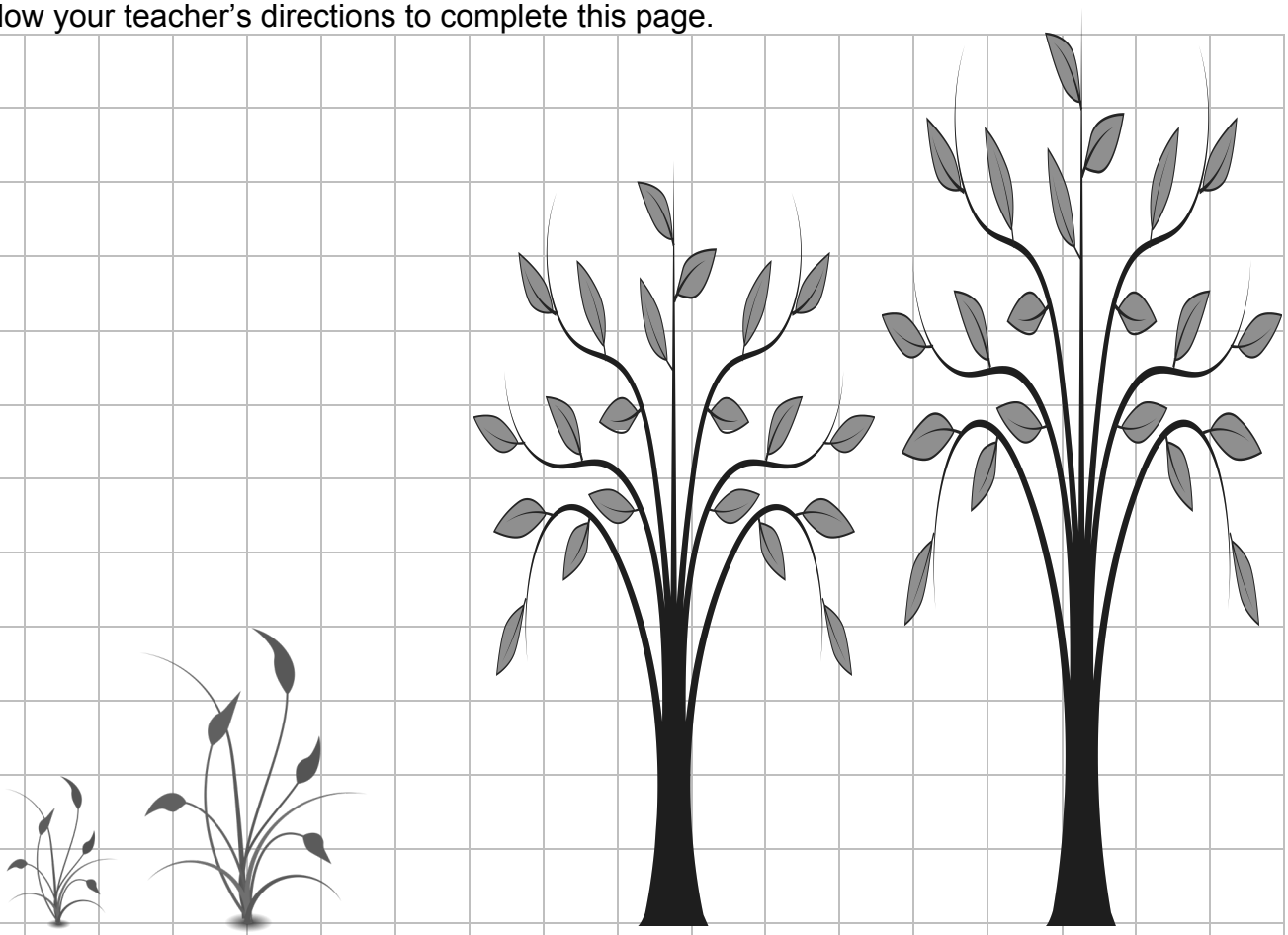
   b. Suppose Marcus drives to Portland, Oregon (30 miles away) and uses a “10% off” coupon at the local Giant Sports. There is no sales tax in Oregon. How much will he save?

   c. Do you think it is worth it to go to Portland for this purchase? Explain.

REVIEW

GROWTH SPURTS – REVISITED

Follow your teacher's directions to complete this page.



GAME: FRACTION, DECIMAL, AND PERCENT RUMMY

This game is for 2-4 players. Each group will need 40-48 blank cards (at least 2.5" by 3"). Before the game begins, each group creates its own set of Rummy Cards.

1. Groups create 10-12 sets of 4 equivalent fractions, decimals, and percents cards. Two examples are:

$\frac{1}{2}$	0.5	50%	$\frac{4}{8}$

$\frac{3}{4}$	0.75	75%	$\frac{75}{100}$

2. Record two of the sets of equivalent fractions, decimals, and percents cards that you made.

3. Establish the game rules and play. One variation is:

   - The dealer shuffles the deck and deals seven cards to each player. The next card is turned face-up in the center of the table and the rest of the deck is stacked face-down next to it. Each player builds sets of three or four matching cards from his/her hand. Matching sets are cards with equivalent fractions, decimals, and percents.

   - The play moves in a clockwise direction starting with the player on the dealer's left. Each player's turn starts by drawing a card, either the top card of the deck or the top card of the discard pile. Then, if the player has any sets, s/he may (but is not required to) lay them down for everyone to see. If there is one card that matches a set that someone else has played, the player may also lay it down. Finally, the player must discard one card face-up on the top of the discard pile.

   - If all of the cards in the deck are used before a player goes out, the discard pile, except for the top card, can be shuffled and used as the deck.

   - Play ends when a player discards his/her last card. At this time, each player scores one point for each card they have laid down and loses one point for each card they still hold. The player who goes out earns seven extra points.

   - Play continues until one player earns 50 points.

4. Challenge: Create another game that can be played with your cards. Write the rules and play with your classmates.

POSTER PROBLEM: EATING OUT

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _____.
- Each group will have a different colored marker. Our group marker is _____.

Part 2: Do the problems on the posters by following your teacher's directions.

Poster 1 (or 5)	Poster 2 (or 6)	Poster 3 (or 7)	Poster 4 (or 8)
You and three friends go out to lunch. The bill is $47.75.	You and one friend go out to dinner. The bill is $47.75.	You and two friends go out to lunch. The bill is $47.75.	You and four friends go out to breakfast. The bill is $47.75.

A. Copy the fact statement. Estimate the tip at 20%. Explain or show your work.

B. Estimate the tip at 15%. Explain or show your work.

C. How much should each person pay, if you want to share the bill, leave a tip between 15% and 20%, and round each person's part to the nearest dollar? Explain or show your work.

D. Use a calculator to determine what each person's share would be if you left a 17% tip and you shared the bill equally, to the penny. Explain or show your work.

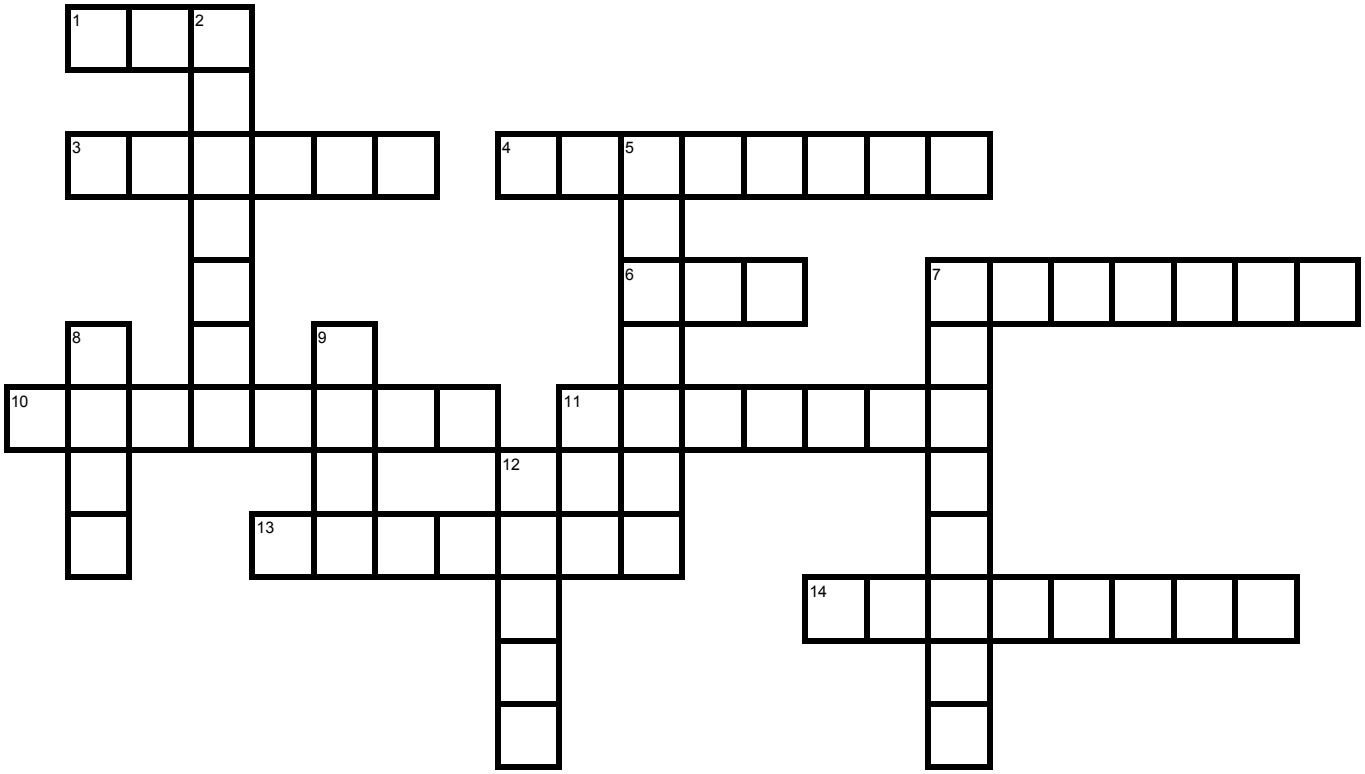
Part 3: Return to your seats. Work with your group.

1. How were the problems and their solutions similar?

2. How were the problems and their solutions different?

3. Do you think it is better to estimate how much each of you will pay at a restaurant (like in part 2C) or compute exactly (like part 2D)? Explain.

VOCABULARY REVIEW



Across

1 Another name for gratuity.

3 Increase in a quantity.

4 $\frac{3}{4}$ is an example of _____.

6 A fee charged by government for goods and services.

7 0.25 is an example of _____.

10 Money given for goods or service.

11 A money model for decimals.

13 The president on the half-dollar coin.

14 A mental math technique for finding percent of a number.

Down

2 Parts per hundred.

5 The woman on a $1 coin (last name).

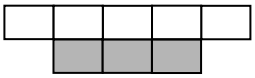
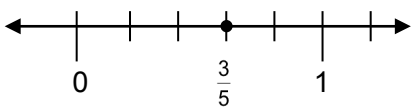
7 Decrease in a quantity.

8 A model where grids are shaded to show parts of a whole (as in fraction gardens).

9 10% of a dollar.

12 A money model for percents.

DEFINITIONS, EXPLANATIONS, AND EXAMPLES

Word or Phrase	Definition
area model for fractions	An area model for fractions represents fractions pictorially using figures in the plane. In this model, a figure is divided into pieces of equal area, and some of the pieces may be shaded. $\frac{3}{8}$ of this figure is shaded. 
decrease in a quantity	The decrease in a quantity is the original value minus the new value. The percent decrease in a quantity is the value of the ratio of the decrease to the original quantity, expressed as a percent. Last year, there were 200 students in the school. This year, there are 178 students in the school. The decrease in the number of students is 200 – 178 = 22. Since $\frac{22}{200} = \frac{11}{100}$, the percent decrease is 11%.
decimal	A decimal is an expression of the form *n.abc* … , where *n* is a whole number written in standard form, and *a, b, c,* … are digits. The decimal expansion of $\frac{4}{5}$ = 0.8 or 0.800000…. The decimal expansion of $\frac{4}{3}$ is 1.333333… .
discount	The discount (or markdown) of an item is the decrease in the price of the item; that is, the original price of the item minus the new price. The percent discount is the percent decrease in the price of the item; that is, the value of the ratio of the decrease to the original value, expressed as a percent. Last week, the price of an MP3 player was \$200. This week, the price is \$178. The discount is 200 – 178 = 22. Since $\frac{22}{200} = \frac{11}{100}$, the percent discount is 11%.
fraction	A fraction is a number expressible in the form $\frac{a}{b}$ where *a* is a whole number and *b* is a positive whole number.  The fraction $\frac{3}{5}$ is represented by the dot on the number line above.

Word or Phrase	Definition
increase in a quantity	The increase in a quantity is the new value minus the original value. The percent increase in a quantity is the value of the ratio of the increase to the original quantity, expressed as a percent. Last year there were 200 students in school. This year, there are 208 students. The increase in the number of students is 208 – 200 = 8. Since $\frac{8}{200} = \frac{4}{100}$, the percent increase is 4%.
markup	The markup on an item is the increase in the price of the item, that is, the new price of the item minus the original price. The percent markup is the percent increase in the price of the item. Last week, the price of an MP3 player was \$200. This week, the price is \$208. The markup is 208 – 200 = 8. Since $\frac{8}{200} = \frac{4}{100}$, the percent markup is 4%.
percent	A percent is a number expressed in terms of the unit 1% = $\frac{1}{100}$. To convert a positive number to a percent, multiply the number by 100. To convert a percent to a number, divide the percent by 100. $4 = 4 \times 100\% = 400\%$. Fifteen percent = 15% = $\frac{15}{100}$ = 0.15.
percent decrease in a quantity	See decrease in a quantity.
percent increase in a quantity	See increase in a quantity.
percent of a number	A percent of a number is the product of the percent and the number. It represents the number of parts per 100 parts. 15% of 300 is $\frac{15}{100} \bullet 300 = 45$. If 45 out of 300 students are boys, then 15 out of every 100 students are boys, and 15% of the students are boys.

Equivalent Fractions: The Big 1

The number 1 is called the multiplicative identity. Multiplying a fraction by any form of 1 does not change its value.

The Big 1 is a notation for 1 in the form of a fraction $\frac{n}{n}$ ($n \neq 0$). For example,

$$1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \ldots$$

We can use the following picture to help remind us that these fractions are equivalent to 1:

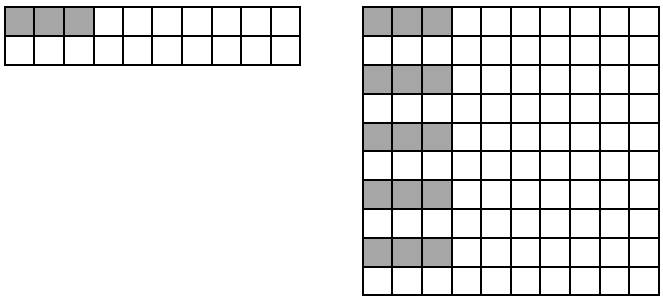
$$1 = \boxed{\frac{8}{8}}$$

The Big 1 can be used to show equivalence of fractions. For example,

$$\frac{2}{5} \times \boxed{\frac{10}{10}} = \frac{20}{50} \qquad \text{or} \qquad \frac{20}{50} \div \boxed{\frac{10}{10}} = \frac{2}{5}.$$

"Replicating Diagrams" and Equivalent Fractions

"Replicating patterns" visually illustrate equivalent fractions that have the same fractional amount shaded. For example, to show that $\frac{3}{20} = \frac{15}{100}$ we replicate this 20-square pattern to obtain a 100-square grid.



$$\frac{3}{20} = \frac{15}{100}$$

Using the "Big 1," this equivalence can be written:

$$\frac{3}{20} \bullet \boxed{\frac{5}{5}} = \frac{15}{100}.$$

Visually, multiplying the numerator by 5 represents replicating the shaded parts five times, and multiplying the denominator by 5 represents replicating the total number of parts in the denominator five times.

In a "replicating diagram," the size of the part does not change.

Some Fraction-Decimal-Percent Equivalents

$\frac{1}{2} = \frac{50}{100} = 0.5 = 50\%$

$\frac{1}{4} = \frac{25}{100} = 0.25 = 25\%$

$\frac{3}{4} = \frac{75}{100} = 0.75 = 75\%$

$\frac{5}{4} = \frac{125}{100} = 1.25 = 125\%$

Conversion strategy:

Think: $\frac{3}{4}\left(\frac{25}{25}\right) = \frac{75}{100} = 75\%$

$\frac{1}{10} = \frac{10}{100} = 0.1 = 10\%$

$\frac{3}{10} = \frac{30}{100} = 0.3 = 30\%$

$\frac{5}{10} = \frac{50}{100} = 0.5 = 50\%$

Conversion strategy:

Think: $\frac{3}{10} = \frac{30}{100}$, so

$0.3 = 0.30 = 30\%$

$\frac{1}{25} = \frac{4}{100} = 0.04 = 4\%$

$\frac{16}{25} = \frac{64}{100} = 0.64 = 64\%$

$\frac{9}{50} = \frac{18}{100} = 0.18 = 18\%$

Conversion strategy:

Think: $25(4) = 100$, so

$\frac{16}{25}\left(\frac{4}{4}\right) = \frac{64}{100} = 64\%$

$\frac{3}{20} = \frac{15}{100} = 0.15 = 15\%$

$\frac{13}{20} = \frac{65}{100} = 0.65 = 65\%$

$\frac{19}{20} = \frac{95}{100} = 0.95 = 95\%$

Conversion strategy:

Think: 20 nickels in a dollar

$\frac{1}{20}$ of a dollar is \$0.05

$\frac{1}{5} = \frac{2}{10} = 0.2 = 20\%$

$\frac{2}{5} = \frac{4}{10} = 0.4 = 40\%$

$\frac{3}{5} = \frac{6}{10} = 0.6 = 60\%$

$\frac{4}{5} = \frac{8}{10} = 0.8 = 80\%$

Conversion strategy:

Think: If I know tenths, I can easily convert to hundredths.

$\frac{1}{8} = \frac{12.5}{100} = 0.125 = 12.5\%$

$\frac{3}{8} = \frac{37.5}{100} = 0.375 = 37.5\%$

$\frac{5}{8} = \frac{62.5}{100} = 0.625 = 62.5\%$

$\frac{7}{8} = \frac{87.5}{100} = 0.875 = 87.5\%$

Conversion strategy:

Think: $\frac{1}{4} = \frac{25}{100}$, so

half of $\frac{1}{4}$ is $\frac{1}{8} = \frac{12.5}{100}$

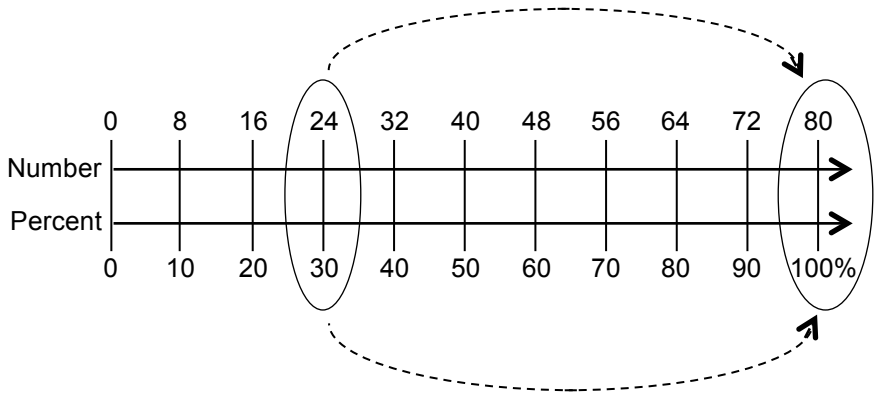
$= 12.5\%$

Using Double Number Lines to Solve Percent Problems

Strategy 1: Solve on the double number line

30% of 80 is what amount?

Create a double number line with percents represented in increments of 10% on the bottom line, and the whole number represented in increments on the top. Since the whole is 80 (in this case), count by 8s for the increments (80 ÷ 10 = 8).



Since 30% corresponds to 24 on the double number line, 30% of 80 is 24.

Strategy 2: Identify equivalent ratios on the double number line.

Create equal ratios based on the part to whole relationships.

$$\frac{\text{part}_{number}}{\text{whole}_{number}} = \frac{\text{part}_{percent}}{\text{whole}_{percent}}$$

$$\frac{\boxed{24}}{80} = \frac{30}{100}$$

This equivalence is based on the dotted arrows above.

Create equal ratios based on the part to part relationships.

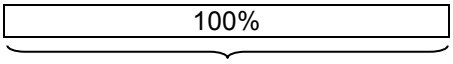
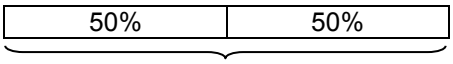
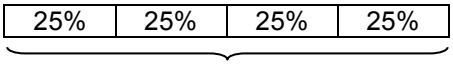
$$\frac{\text{part}_{number}}{\text{part}_{percent}} = \frac{\text{whole}_{number}}{\text{whole}_{percent}}$$

$$\frac{\boxed{24}}{30} = \frac{80}{100}$$

This equivalence is based on the circles above.

Using "Chunking" to Find Percents of Numbers

We use the word "chunking" to describe a process of decomposing and composing numbers to make calculations easier, especially when done mentally. Another way to describe this is "taking numbers apart and putting them back together." For example, if adding 17 and 26, we might decompose each number into tens and ones, adding 10 + 20 = 30, and 7 + 6 = 13, and finalizing the sum by adding 30 + 13 = 43.

Think	Example
Finding 100% of something is the same as finding all of it.	100% of \$80 = \$80  \$80
Finding 50% of something is the same as finding one-half of it. This is the same as multiplying by $\frac{1}{2}$ or dividing by 2.	50% of \$80 = $\frac{1}{2}$(\$80) = \$40 \$80 ÷ 2 = \$40  \$80
Finding 25% of something is the same as finding one-fourth of it. This is the same as multiplying by $\frac{1}{4}$ or dividing by 4.	25% of \$80 = $\frac{1}{4}$(\$80) = \$20 \$80 ÷ 4 = \$20  \$80
Finding 10% of something is the same as finding one-tenth of it. This is the same as multiplying by $\frac{1}{10}$ or dividing by 10.	10% of \$80 = $\frac{1}{10}$(\$80) = \$8 \$80 ÷ 10 = \$8
Finding 1% of something is the same as finding one-hundredth of it. This is the same as multiplying by $\frac{1}{100}$ or dividing by 100.	1% of \$80 = $\frac{1}{100}$(\$80) = \$0.80 \$80 ÷ 100 = \$0.80
Finding 20% of something is the same as doubling 10% of it.	20% of \$80 = 2(\$8) = \$16
Finding 5% of something is the same halving 10% of it.	5% of \$80 = $\frac{1}{2}$(\$8) = \$4
Finding 15% of something is the same as adding 10% of it and 5% of it.	15% of \$80 = \$8 + \$4 = \$12

Using Multiplication to Find Percents of Numbers

Some percents are hard to find mentally. For example, finding 17% of something is the same as finding $\frac{17}{100} = 0.17$ of it. In this case, it may be easier to find the percent by using the definition of a percent of a number:

> A percent of a number is the product of the percent and the number.

Find 17% of $80.

Strategy 1: Use fractions

$$\frac{17}{100} \bullet 80 = \frac{17 \bullet 80}{100} = \frac{1360}{100} = 13.60$$

So 17% of $80 is $13.60.

Strategy 2: Use decimals

$(0.17) \bullet (80) = 13.6$ or 13.60

So 17% of $80 is $13.60.

Percent Increase

Percent increases occur frequently as tips, taxes, and price markups. To find a percent increase, find the amount of the increase and add it to the original quantity.

Example	Original amount	Percent increase	Amount of increase	New amount (original + increase)
Leave a **tip** on a restaurant bill.	$40	20%	20% of $40 = $8	$40 + $8 = $48
Pay **tax** on a clothes purchase.	$50	8%	8% of $50 = $4	$50 + $4 = $54
Pay a **markup** on a video game.	$75	10%	10% of $75 = $7.50	$75 + $7.50 = $82.50

Percent Decrease

Sales and discounts may be described as percent decreases. To find a percent decrease, find the amount of the decrease and subtract it from the original quantity.

Example	Original amount	Percent decrease	Amount of decrease	New amount (original – decrease)
Sale on shoes purchase	$50	25%	25% of $50 = $12.50	$50 – $12.50 = $37.50
Discount on a dress	$90	40%	40% of 90 = $36.00	$90 – $36 = $54

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