FUNCTIONS 3
STUDENT PACKET

SYSTEMS OF LINEAR EQUATIONS

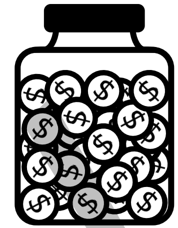
Parent (or Guardian) signature ___________________________________

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. (See section 1.5.) Key mathematical vocabulary is underlined throughout the packet.

slope-intercept form	standard form
substitution	system of linear equations

TAFFY'S COIN JAR



Taffy's friends, Sadie and Karen, are looking at Taffy's coin jar. "Try to guess how much money is in the jar," says Taffy. Sadie guesses that there is $6.75 in the jar. Karen guesses there is $6.87 in the jar.

Follow your teacher's directions to explore this situation.

SOLVING LINEAR SYSTEMS BY GRAPHING

We will solve systems of linear equations by graphing. We will write linear equations in equivalent forms.

GETTING STARTED

Here are two ways to write the equation of a line:

Slope-intercept form	Standard form
$y = mx + b$	$ax + by = c$
• y is written in terms of x • m and b are real numbers • m is the slope • b is the y-intercept	• variables (x, y) are on one side of equation • a, b, and c are real numbers • a and b are not both zero

Write each equation below in standard form and slope-intercept form. Complete the tables. Graph the equations.

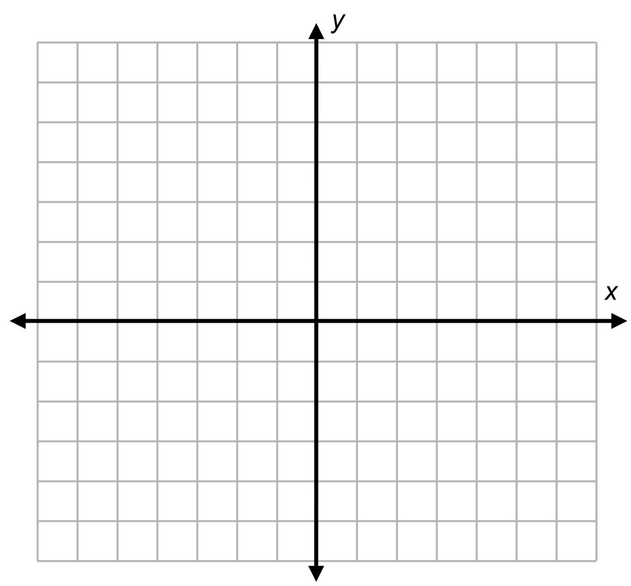
1. $2x + y - 3 = 0$

x	y
3	
2	
1	
0	
-1	

2. $3x = 4 - y$

x	y
3	
2	
1	
0	
-1	

3. Circle the ordered pairs in the tables that are the same. What is special about the point of intersection on the graph?



SAVING FOR A CAMERA

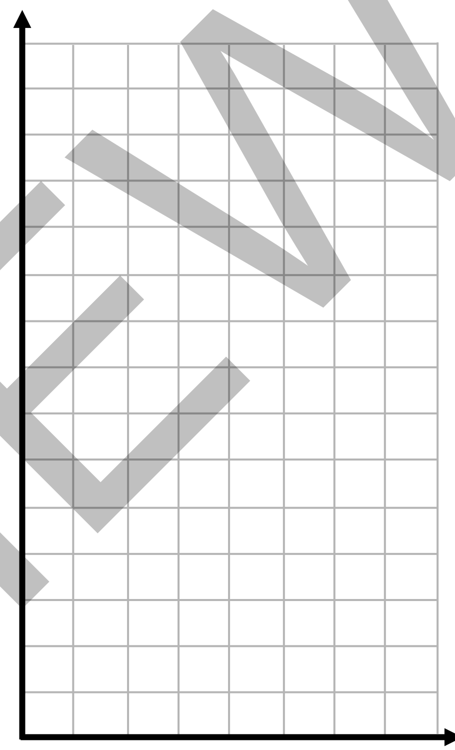
Bennie and Manny are saving for a camera. Both have some money in the bank and will save at a constant rate each month. Follow your teacher's directions to complete this page.

PRACTICE 1

Jorge and Zhang are saving for a phone. Jorge has $100 in the bank and will save $30 each month. Zhang has $40 in the bank and will save $45 each month.

1. How much does Zhang have at month 0?

2. Fill in the table for several months. Then graph the data. Use a different color for each person's table values and graphs. Label and scale the axes appropriately.



Jorge		**Zhang**	
Month # (*x*)	**Total saved in $ (*y*)**	**Month # (*x*)**	**Total saved in $ (*y*)**
0		0	
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	
7		7	

3. Write an equation that represents Jorge's savings. $y =$ __________

4. Write an equation that represents Zhang's savings. $y =$ __________

5. Which of them is saving at a faster rate? __________ Justify your answer by referring to the table, the graphs, and the equations.

6. During which months does Jorge have more money?

7. During which months does Zhang have more money?

8. At which month do they have the same amount of money?

   What do you notice about the tables at this month?

   What do you notice about the graphs at this month?

WHAT ARE SYSTEMS OF LINEAR EQUATIONS?

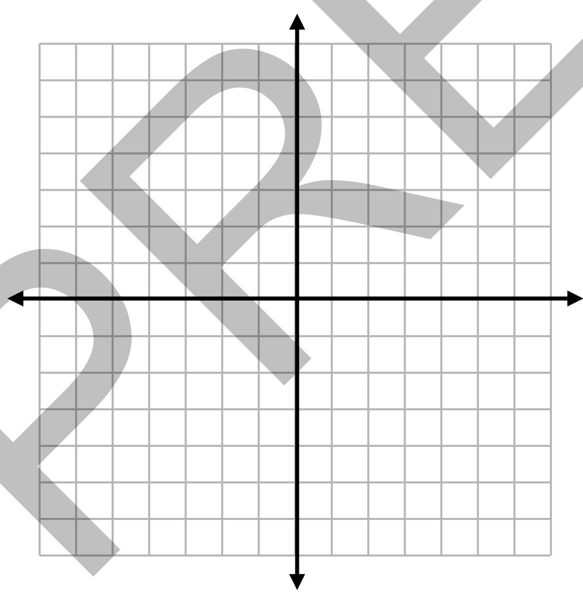
1. When a linear equation in two variables is graphed, what does the line represent?

2. When two linear equations are graphed and they intersect in one point, what does that point represent?

3. Discuss with your teacher and the class the definition of a <u>system of linear equations</u> in section 3.5 and write it in My Word Bank.

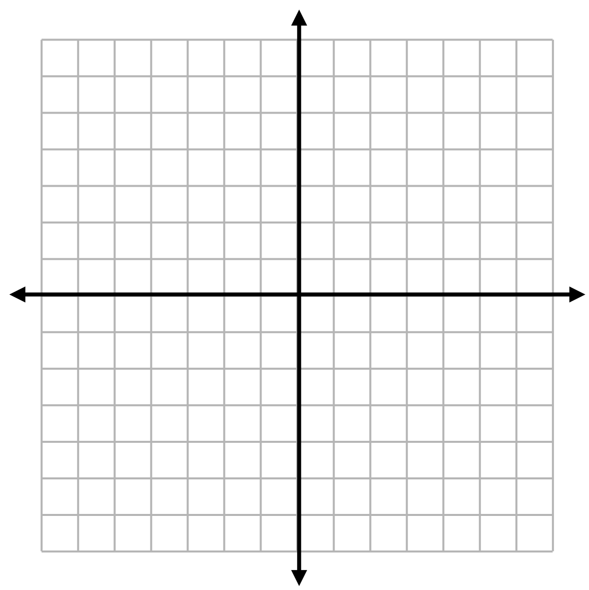
For each system, graph the lines, and then write the solutions.

1. $\begin{cases} y = -2x + 4 \\ y = 2x \end{cases}$



2. $\begin{cases} y + 1 = 4x \\ 6x - 3y = -3 \end{cases}$

Hint: change these to slope-intercept form first.



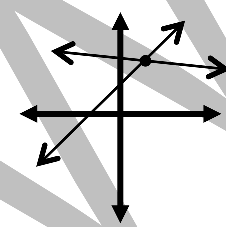
WHAT ARE SYSTEMS OF LINEAR EQUATIONS?
(Continued)

A system of two linear equations in two variables has **one solution**, **no solution**, or **infinitely many solutions**.

6. This system of equation has **exactly one solution**.

   a. Circle the solution.

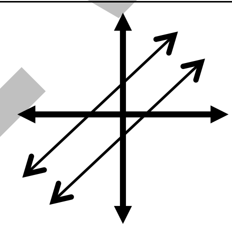
   b. What can you say about the slopes of these lines?



7. This system has **no solutions**.

   a. Describe the lines and how they're slopes relate to one another.

   b. Why do you think the graph of the system on the right has no solutions?



8. Here is a system of equations.

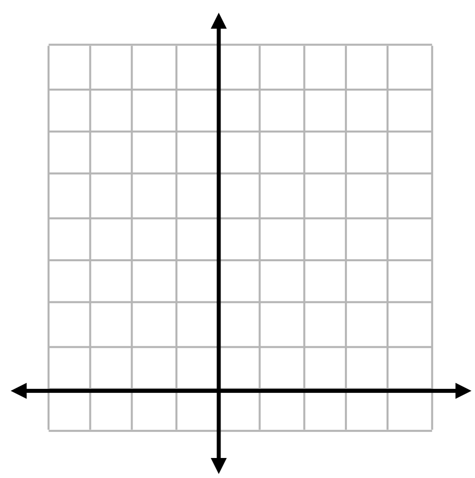
$$\begin{cases} y = 2x + 3 \\ 2y = 4x + 6 \rightarrow \underline{\hspace{3cm}} \end{cases}$$

   a. Change the second equation to slope-intercept form.

   b. What do you notice about these two equations?

   c. Graph both lines and describe what this graph looks like.

   d. Why do you think we say that this system has **infinitely many solutions**?



9. Study the system of equations representing Jorge and Zhang's savings (Practice 1). Does this system of equations have one solution, no solutions, or infinitely many solutions?

PRACTICE 2

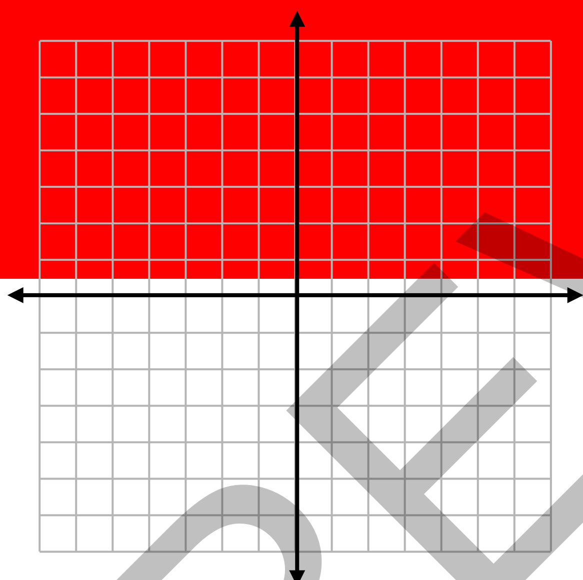
1. A system of equations represented by two parallel lines must have _____ solution(s).

2. A system of equations represented by lines intersecting in one point must have _____ solution(s).

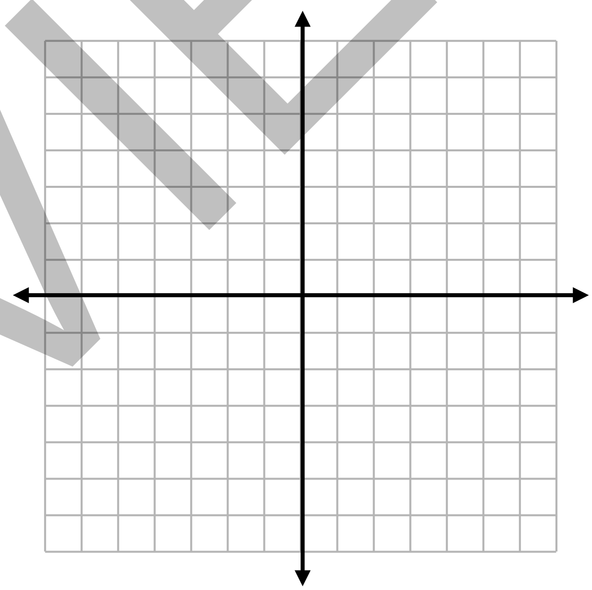
3. A system of equations represented by lines that coincide (equivalent equations) must have _______________ solution(s).

For each system of equations, change the equations to slope-intercept form when needed, graph the lines, and then write the solutions.

4. $\begin{cases} 3y = 9x + 3 \\ y = 3x - 5 \end{cases}$



5. $\begin{cases} y - 5 = x \\ 2x + y = -4 \end{cases}$



6. Consider the systems below. Without graphing or manipulating either system, explain how you know that there can be no solution to either system.

a. $\begin{cases} y = -x + 4 \\ y = -x + 1 \end{cases}$

b. $\begin{cases} 3x + 2y = 5 \\ 3x + 2y = 6 \end{cases}$

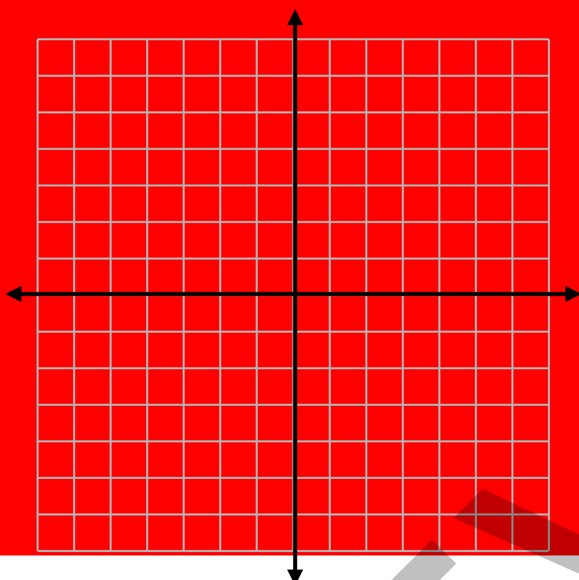
7. Without graphing, how do you know that the following system must have infinitely many solutions? Hint: change the second equation to slope-intercept form first.

$$\begin{cases} y = 2x - 9 \\ 3y = 6x - 27 \end{cases}$$

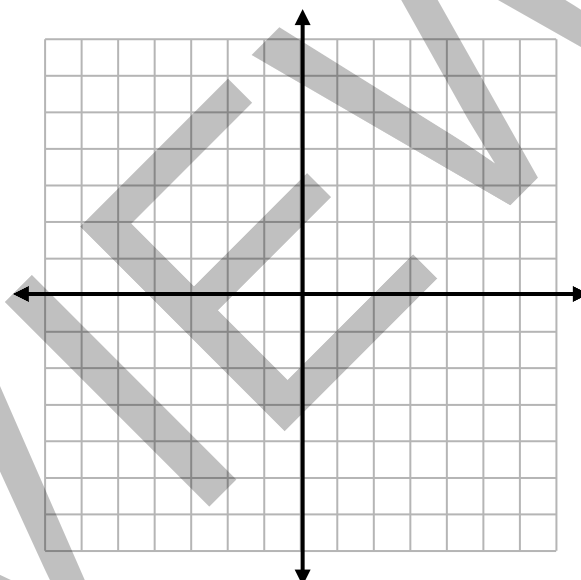
PRACTICE 3

Graph two lines that fit the criteria for each problem. Then describe the solution(s) for each system.

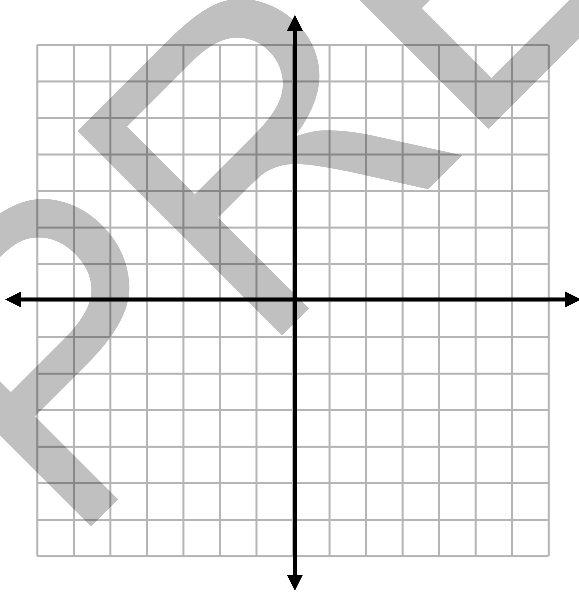
1. Two lines with different slopes and different *y*-intercepts.



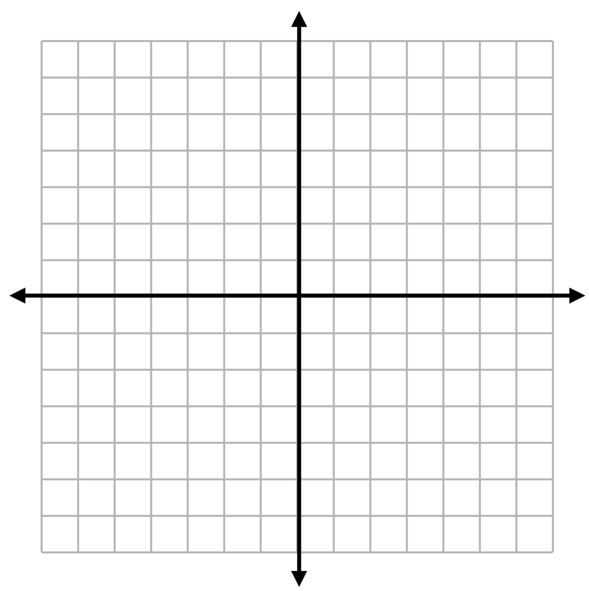
2. Two lines with different slopes and the same *y*-intercepts.



3. Two lines with the same slopes and different *y*-intercepts.



3. Two lines with the same slopes and the same *y*-intercepts.



SOLVING LINEAR SYSTEMS BY SUBSTITUTION

We will solve systems of linear equations by substitution.

GETTING STARTED

The transitive property of equality states that if $a = b$ and $b = c$, then $a = c$.

1. Andre is the same height as Betty. Betty is the same height as Cleo. What can we conclude about the relative heights of Andre and Cleo?

2. If x has the same value as $2y + 3$ and x is also has the same value as $5y – 6$, what can we conclude about the relationship between $2y + 3$ and $5y – 6$?

The substitution property of equality states that if $a = b$, then a can replace b in any equation or inequality.

3. Given: ♡ = 2 ☁

   "The value of one heart is equal to the value of two clouds."

   For the expression below, use <u>substitution</u> to create an equivalent expression involving only clouds and numbers.

   3♡ + 5☁ = 3(______) + 5☁

   Simplify the right side of the equation:

4. Given: $y = 4x$

   "The value of one y is equal to the value of 4 x's."

   For the expression below, use substitution to create an equivalent expression involving only x's and numbers

   $6y + 5x =$ 6(______) $+ 5x$

   Simplify the right side of the equation:

GOING TO THE PARK

Malcolm and Zeke are going to the park near their school. They travel down the same street. They leave at the same time from different locations. Someone is taking pictures from a window every 6 seconds so that they can keep track of the distance they travel over time. Follow your teacher's directions to complete this page.

INTRODUCTION TO SUBSTITUTION

1. Write information about the graphs of Malcolm and Zeke walking to the park.

	Equation in slope-intercept form Let x = the number of seconds Let y = the distance from school	**Slope**	**Intercept**
Malcolm	y = __________		
Zeke	y = __________		

2. Find the value of x that that makes both equations true.

3. What does this solution for x mean in the context of the problem?

4. From above, since x = _____, use substitution to solve for y in the equations for Malcolm and Zeke and simplify:

   Malcolm: y = _________ Zeke: y = _________

5. What do the solutions for y mean in these equations?

6. After how many seconds are the boys the same distance from school?

   What is this distance?

7. Looking at the algebraic substitution process above, does the number of seconds when the boys are the same distance from school match what is in the table and the graphs on the previous page?

PRACTICE 4

Find the solutions for these systems of equations using substitution.

1. $\begin{cases} y = 2x \\ y = x + 2 \end{cases}$	2. $\begin{cases} x = 5y \\ x = y + 20 \end{cases}$
3. $\begin{cases} y = 4x \\ x + 12 = y \end{cases}$	4. $\begin{cases} x = 2y \\ x - y = 4 \end{cases}$
5. $\begin{cases} y = 2x + 12 \\ y - 4x = 2 \end{cases}$	6. $\begin{cases} x = -y + 5 \\ y = 3x + 1 \end{cases}$

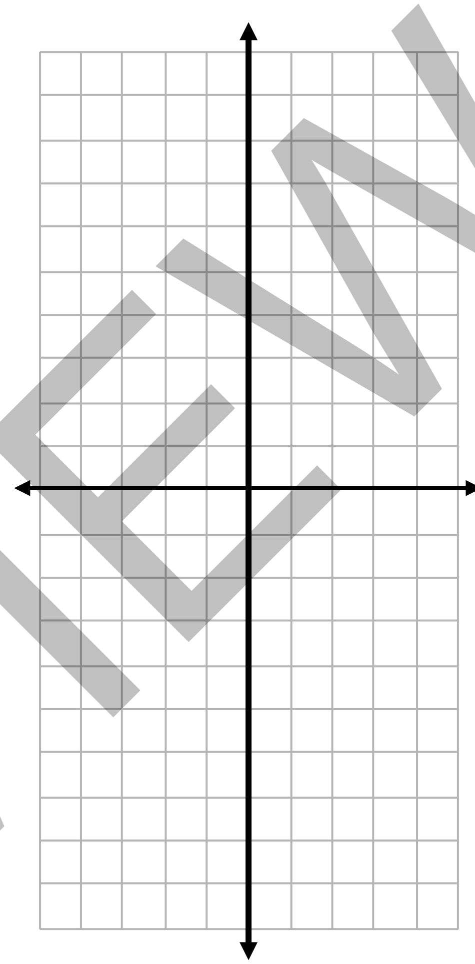
One of these systems has no solutions and one has infinite solutions. Use any strategy to decide which is which, and explain how you know.

7. $\begin{cases} y = x \\ y = x + 2 \end{cases}$	8. $\begin{cases} y = 2x + 1 \\ 2y = 4x + 2 \end{cases}$

WHY USE ALGEBRA?

Consider this system of equations:

$$\begin{cases} y = -4x + 2 \\ y = 8x - 1 \end{cases}$$



1. Graph the system.

2. Ike looked at the graphs and said,
   "I can't tell for sure what the solution is."
   Explain why you think Ike said this.

3. Estimate what the solution to this system might be.

4. Solve by using substitution.

5. Explain why you think substitution might be better than graphing for solving this system of equations.

SOLVING SYSTEMS BY SUBSTITUTION

1. Follow your teacher's instructions to solve a system of linear equations using substitution.

2. Solve the following system using substitution.
   (Hint: Isolate one variable in one of the equations.)

$$\begin{cases} x - y = -1 \\ 3x - 4y = -2 \end{cases}$$

PRACTICE 5

Solve each system using substitution.

1. $\begin{cases} x = 3y + 1 \\ x = y + 5 \end{cases}$

2. $\begin{cases} y - 2x = 6 \\ y = 2x + 2 \end{cases}$

3. $\begin{cases} x - y = 1 \\ 3y + x = -7 \end{cases}$

4. $\begin{cases} 4x = 6 + y \\ 2x - y = 4 \end{cases}$

5. The sum of two numbers is 136. One number is 5 less than twice the other. Fill in the blanks to set up a system of equations that can be used to solve this problem. Then find the numbers using substitution.

   Let ______ = one number and ______ = the other number

   $\begin{cases} x + y = \_\_\_\_\_\_ \\ x = \_\_\_ y - \_\_\_ \end{cases}$

PRACTICE 6

Solve each system using substitution.

1. $\begin{cases} 2y + x = -5 \\ -y + 1 = x \end{cases}$	2. $\begin{cases} 3x = y + 1 \\ 2y = 6x - 2 \end{cases}$
3. $\begin{cases} x - y = -10 \\ -3x + 50 = y \end{cases}$	4. $\begin{cases} -x + 2 = y \\ \frac{1}{2}x + \frac{1}{2}y = 4 \end{cases}$

5. The sum of two numbers is 43. One number is one more than twice the other. Find the numbers using the substitution method.

SOLVING LINEAR SYSTEMS BY ELIMINATION

We will solve systems of linear equations by elimination.

GETTING STARTED

(1) You and a friend both have the same number of pens in your backpack. If someone gives each of you 4 more pens, will you both still have the same amount?

(2) You and a friend both earned the same amount of money doing some work for a neighbor last week. If you both earn double the amount next week, will you earn the same amount next week?

Let m be some number. Write true (T) or false (F) next to each statement.

3. $m + (2 + 5) = m + 7$
4. $m + (-4) = m – 4$
5. $m – 8 = 8 – m$
6. $\frac{m}{3} = \frac{3}{m}$
7. $2(m + 5) = 2m + 5$
8. $2(m + 5) = 2m + 10$
9. $m – (2 – 6) = m – 4$
10. $(2 + 5)(m) = (3 + 4)(m)$

For each problem, create a new equation by adding or subtracting the expressions on the left side and the right side as indicated. (The equations are written in vertical alignment to make adding or subtracting easier.)

11.
$$\begin{array}{rcrcr} (3 & + & 6) & = & (9) \\ \underline{+(2} & \underline{+} & \underline{5)} & \underline{=} & \underline{+(7)} \end{array}$$

Why can you add 2 + 5 to the left side of the first equation and 7 to the right side?

12.
$$\begin{array}{rcrcr} (3 & + & 6) & = & (9) \\ \underline{-(2} & \underline{+} & \underline{5)} & \underline{=} & \underline{-(7)} \end{array}$$

Why can you subtract 2 + 5 from the left side of the first equation and 7 from the right side?

SOLVING SYSTEMS BY ELIMINATION

Given a system of two linear equations, properties of equality can be used to create new systems that have the same solution as the original ones. Using this idea strategically to eliminate one of the variables is called solving a system by elimination.

Follow your teacher's directions to solve this system by elimination in two different ways.

1. Consider this system of equations:

$$\begin{cases} 2x + y = 10 \\ -y = -x + 2 \end{cases}$$

SOLVING SYSTEMS BY ELIMINATION
(Continued)

Solve the following system of equations.

2. Given system: $\begin{cases} y + x = 5 \\ y = 6 - 2x \end{cases}$

   a. Rewrite both equations in the form $ax + by = c$.

   b. Solve by eliminating the y's.

   c. Solve again. This time eliminate the x's.

   d. Write the solution as equations and as an ordered pair.

3. Given system: $\begin{cases} 4y - 3x = 1 \\ 3x = 2 - 2y \end{cases}$

   a. Rewrite both equations in the form $ax + by = c$.

   b. Solve by eliminating the x's.

   c. Solve again. This time eliminate the y's.

   d. Write the solution as equations and as an ordered pair.

PRACTICE 7

Solve each system using elimination.

1. $\begin{cases} 3x + y = 12 \\ y - 2 = -5x \end{cases}$

2. $\begin{cases} 3x = y + 3 \\ 2y = 5x - 1 \end{cases}$

3. $\begin{cases} y - x = -20 \\ y = 100 - 2x \end{cases}$

4. $\begin{cases} 2y = 5x - 1 \\ 2 - 10x = -4y \end{cases}$

PRACTICE 8

Use elimination or substitution to solve these problems.

1. $\begin{cases} x = y \\ x + y = 1 \end{cases}$	2. $\begin{cases} -2x + 2 = y \\ \frac{1}{2}x + \frac{1}{2}y = 1 \end{cases}$
3. There are pigs and chickens on the farm. There are 65 heads and 226 legs. How many chickens are there?	4. A collection of dimes and quarters totals $14.40. There are twice as many quarters as dimes. How many quarters are there?

REVIEW

TAFFY'S COIN JAR REVISITED

1. Name three methods you learned for solving a system of equations.

2. Recall that Taffy's coin jar contained 33 dimes and quarters totaling $5.10. Determine the number of dimes and quarters in the jar. Solve this problem twice. Use two different methods you have learned.

   Method 1: Method 2:

3. In a coin collection, there are twice as many pennies as quarters. The value of those coins is $5.40. Find the number of pennies and quarters using substitution.

POSTER PROBLEMS: SYSTEMS OF EQUATIONS

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is ________.
- Each group will have a different colored marker. Our group marker is __________.

Part 2: Do the problems on the posters by following your teacher's directions.

Poster 1 (or 5)	Poster 2 (or 6)	Poster 3 (or 7)	Poster 4 (or 8)
$\begin{cases} y-1=5x \\ y-2x=4 \end{cases}$	$\begin{cases} y-x=6 \\ -4x+y=0 \end{cases}$	$\begin{cases} x+y=1 \\ 2x+2y=4 \end{cases}$	$\begin{cases} x+y=7 \\ 0=x-y \end{cases}$

A. Copy the problem. Solve the system by graphing. Scale appropriately.

B. Solve the system by substitution.

C. Solve the system by elimination.

D. Which method(s) worked best for this problem? Why?

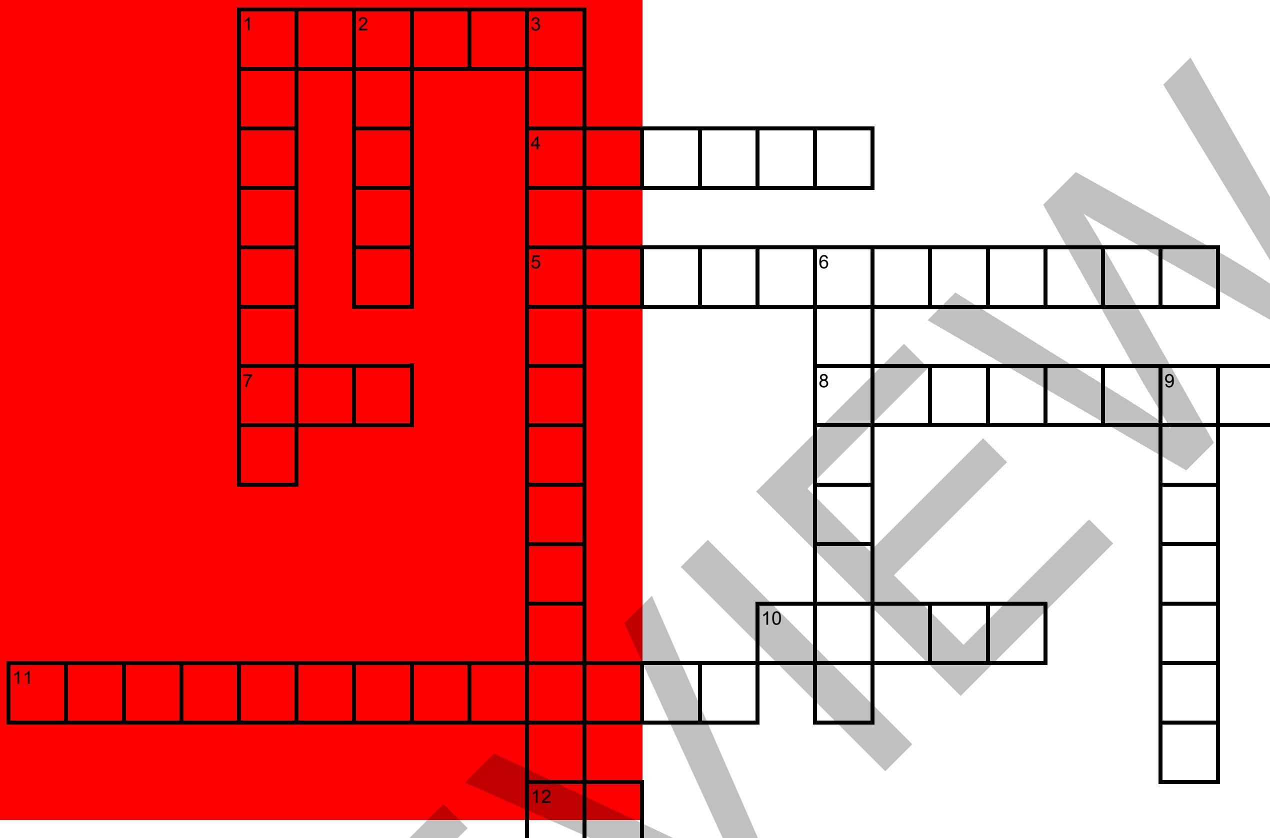
Part 3: Return to your seats. Work with your group.

Use your "start problem." Write a story or situation to match the system. Be prepared to share your story and the solution.

BIG SQUARE PUZZLE: SYSTEMS OF EQUATIONS

Your teacher will give you a “big square puzzle” to complete. Assemble it so that each system matches its solution on the edges. Tape it here.

VOCABULARY REVIEW



Across

1 A set of two or more equations in the same variables.

4 A function written in the form *y* = *mx* + *b*.

5 Where two lines meet is called the point of _____.

7 If two lines intersect, they represent a system of equations with _____ solution(s).

8 The _____ property of equality: if *a* = *b*, then 3 + *a* = 3 + *b*.

10 A visual representation of a function.

11 Algebraic strategy for solving a system of equations (not elimination).

12 If two lines are parallel, they represent a system of equations with _____ solution(s).

Down

1 The values of the variables that make all equations in a system true.

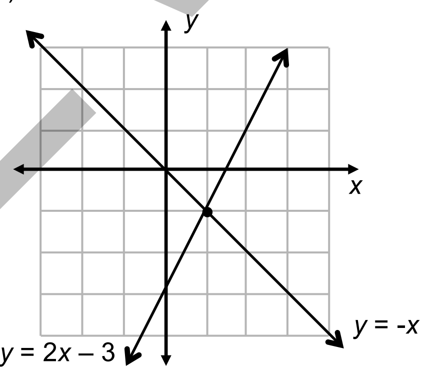
2 A measure of steepness of a line.

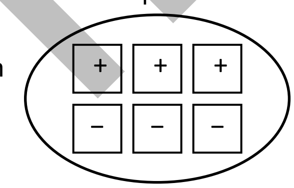
3 The _____ property of equality: if *c* = *d*, then 4*c* = 4*d*.

6 The _____ form of an equation: *ax* + *by* = *c*.

9 A pair of numbers with a specified order is called an _____ pair.

DEFINITIONS, EXPLANATIONS, AND EXAMPLES

Word or Phrase	Definition
addition property of equality	The addition property of equality states that if $a = b$ and $c = d$, then $a + c = b + d$. In other words, equals added to equals are equal. If 3 = 2 + 1, and 8 = 4 + 4, then (3) + (8) = (2 + 1) + (4 + 4) Check: 11 = 11 ✓
multiplication property of equality	The multiplication property of equality states that if $a = b$ and $c = d$, then $ac = bd$. In other words, equals multiplied by equals are equal. If 4 + 2 = 5 + 1, and 10 = 2 • 5, then (10) • (4 + 2) = (2 • 5) • (5 + 1) Check: 60 = 60 ✓
point of intersection	A point of intersection of two lines is a point where the lines meet. The two straight lines in the plane with equations $y = -x$ and $y = 2x - 3$ have point of intersection (1, -1). 
slope-intercept form	The slope-intercept form of the equation of a line is the equation $y = mx + b$, where m is the slope of the line, and b is the y-intercept of the line. The equation $y = 2x + 3$ determines a line with slope 2 and y-intercept 3.

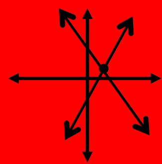
Word or Phrase	Definition
standard form	The <u>standard form</u> of a linear equation in two variables is $ax + by = c$. The equation $4x + 3y = 9$ is in standard form.
substitution	<u>Substitution</u> refers to replacing a value or quantity with an equivalent value or quantity. If $x + y = 10$, and we know that $y = 8$, then we may substitute this value for y in the equation to get $x + 8 = 10$.
system of linear equations	A <u>system of linear equations</u> is a set of two or more linear equations in the same variables. An example of a system of linear equations in x and y: $\begin{cases} x + y = 1 \\ x + 2y = 4 \end{cases}$
zero pair	In the signed counters model, a positive and a negative counter together form a <u>zero pair</u>. Let "+" represent a positive counter, and let "−" represent a negative counter. Then  is an example of a collection of (three) zero pairs.

Systems of Linear Equations

A system of equations is a set of two or more equations in the same variables. A solution to the system of equations consists of values for the variables which, when substituted, make all equations simultaneously true.

A system of linear equations has exactly one solution, infinitely many solutions, or no solution.

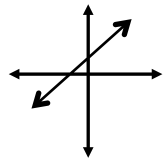
$$\begin{cases} y = -x + 3 \\ y = 2x - 3 \end{cases}$$



One solution at (2, 1)

These two lines intersect in exactly one point.This is the only pair of x- and y-values that statisfies these two equations simultaneously.

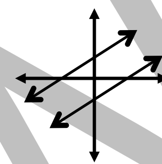
$$\begin{cases} y = x + 1 \\ 2y = 2x + 2 \end{cases}$$



Infinitely many solutions

Since the equations are equivalent, the two lines coincide. Every point on the line represents a solution.

$$\begin{cases} y = x - 2 \\ y = x + 2 \end{cases}$$



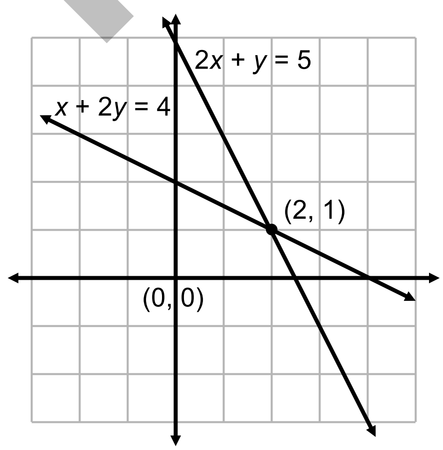
No solution

Since these two lines are parallel, they do not intersect. Thus these two equations have no solution in common.

Solving a System of Linear Equations by Graphing

To solve a system of linear equations by graphing, graph both lines on the same set of axes and observe the point(s) of intersection, if any.

Solve by graphing: $\begin{cases} 2x + y = 5 \\ x + 2y = 4 \end{cases}$



1. Change each to slope-intercept form, $y = mx + b$.

$$2x + y = 5 \longrightarrow y = -2x + 5$$

$$x + 2y = 4 \longrightarrow y = -\frac{1}{2}x + 2$$

2. Graph each linear equation.

3. Observe the intersection of the lines, (2, 1). This represents the solution to the system. In other words, these are the x- and y- values that satisfy both equations. Remember that not every system of equations has exactly one solution.

4. Check by substituting solutions in the original equations to be sure they are correct.

$$2x + y = 5 \longrightarrow 2(2) + 1 = 5 \quad \text{(true)}$$

$$x + 2y = 4 \longrightarrow 2 + 2(1) = 4 \quad \text{(true)}$$

Solving a System of Linear Equations by Substitution

Example: Solve this system of linear equations by substitution.

$$\begin{cases} 2x + y = 5 \\ x + 2y = 4 \end{cases}$$

1. In at least one of the equations, solve for one variable in terms of the other. For example,

$$2x + y = 5 \longrightarrow y = -2x + 5$$

2. Use the substitution property to replace the expression equivalent to y in the second equation and solve for x.

y from step 1

$$x + 2y = 4 \longrightarrow x + 2(-2x + 5) = 4$$
$$x - 4x + 10 = 4$$
$$-3x = -6$$
$$x = 2$$

3. Substitute x into one of the original equations to find y.

x from step 2

$$2x + y = 5 \longrightarrow 2(2) + y = 5$$
$$y = 1$$

Solution:

$x = 2$, $y = 1$

(2, 1)

4. Substitute x and y into the other original equation to check.

x from step 2 | y from step 3

$$x + 2y = 4$$
$$2 + 2(1) = 4$$
$$4 = 4 \quad \text{(true)}$$

Solving a System of Linear Equations by Elimination

Elimination, which applies the multiplication property of equality and the addition property of equality, is another method for solving a system of linear equations. Here is an example.

Example: Solve this system of linear equations by elimination.

$$\begin{cases} 2x + y = 5 & [1] \\ x + 2y = 4 & [2] \end{cases}$$

> We will number each equation with brackets to help keep track of them.

1. Use the multiplication property of equality. Multiply both sides of one (or both) equations by some number that will make one of the variable expressions in each equation opposites of each other. In this case, we might multiply both sides of the first equation by -2.

$$[1]\quad -2(2x + y) = -2(5) \rightarrow -4x - 2y = -10 \quad [3]$$

2. Use the addition property of equality. Add expressions on each side of the equation together to eliminate one variable. Solve the new equation.

$$\begin{aligned} -4x - 2y &= -10 \quad [3] \\ \underline{x + 2y} &= \underline{4 \quad\quad [2]} \\ -3x &= -6 \\ x &= 2 \end{aligned}$$

3. Substitute into one of the original equations to find y.

$$[1]\quad 2x + y = 5 \rightarrow 2(2) + y = 5$$

$$y = 1$$

Solution:
$x = 2$, $y = 1$
(2, 1)

4. Substitute into the other original equation to check.

$$[2]\quad x + 2y = 4 \longrightarrow 2 + 2(1) = 4 \quad \text{(true)}$$

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