**MathLinks**

# EXPRESSIONS AND EQUATIONS 1 STUDENT PACKET

## VARIABLES AND BALANCE

Parent (or Guardian) Signature _____

*MathLinks*: Essentials ©CMAT (Expressions and Equations 1: Student Packet)

## MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. (See section 1.5.) Key mathematical vocabulary is underlined throughout the packet.

| distributive property | equation |
|---|---|
| evaluate | expression |
| simplify | solve an equation |
| substitution | variable |

**OPENING PROBLEM: CONTAINERS**

On this page, the same containers have the same value or "weight." What is the value of each container? Explain your reasoning.

 +  +  = 24

 +  = 25

 −  = 

 +  +  = ??

## BALANCE

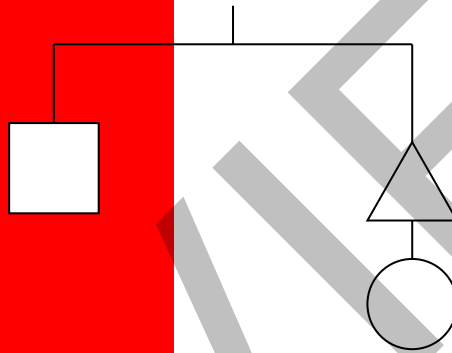We will explore concepts related to variables, expressions, and equations through puzzles and problems.

### GETTING STARTED

On this puzzle, the same shapes have the same value (or weight). The horizontal bar shows balance.



1. Suppose the total weight is 8 and $\triangle$ = 1. Find the values of the other shapes.

   a.  $\bigcirc$ = _____                                      $\square$ = _____

   b. Write true equations, based on these facts.

      _____ + 1 = _____                  _____ + _____ + _____ = 8

2. Suppose $\square$ = 20. Find possible values of the other shapes.

   a.  $\bigcirc$ = _____                              $\triangle$ = _____

   b. Write true equations, based on part a above.

      _____ + _____ = _____                  _____ + _____ + _____ = _____

3. Compare your solutions to problems 1 and 2 above with at least one partner in class. Do your solutions agree? If they are different, are they still correct?

# MOBILES

Follow your teacher's directions. Each mobile is independent of the other.

(1)

(2)

(3)

(4)

(5)

## MAKING EQUATIONS TRUE

1.  Find the word <u>equation</u> in section 1.5 and explain what it means in My Word Bank.

Fill in the blank shapes below with numbers to make each equation true. Squares and circles can represent different values in different equations, but the same shape must represent the same value in the same equation.

2.    $20 = \square + \bigcirc$

3.    $20 = \square - \bigcirc$

4.    $20 = \square \bullet \bigcirc$

5.    $20 = \square \div \bigcirc$

6.    $20 = \square + \square + 4$

7.    $20 = \square \bullet 4$

8.    $20 = 2 \bullet \square - \square$

9.    $20 = \dfrac{60}{\bigcirc}$

10.   $8 \bullet 6 = 39 + \bigcirc$

11.   $7 \bullet 6 = \square - \bigcirc$

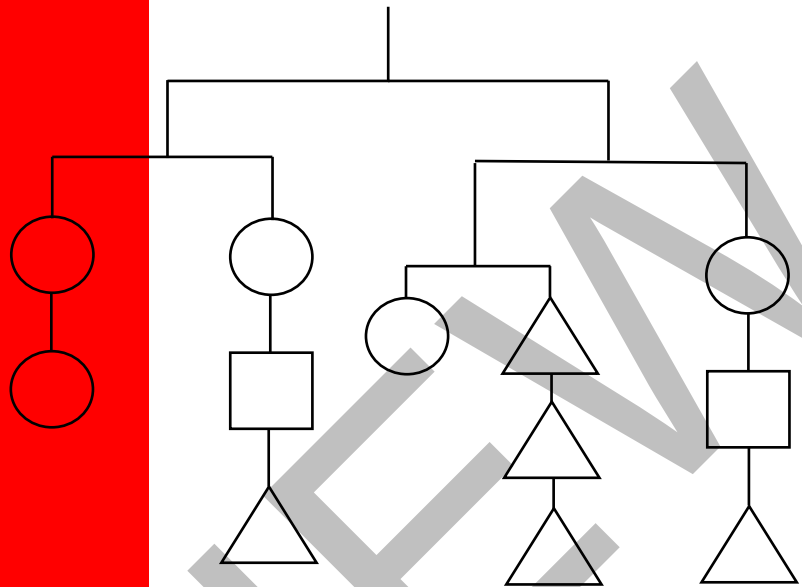12. Which problems must have exactly one (a unique) correct answer? Explain.

13. Choose two problems above where the values of the squares and circles are not unique. Copy them below, rewriting them with different solutions.

## PRACTICE 1

The total weight of this mobile is 48.

1.  Find the weight that each shape represents.

    Explain how you found the weight of the circle.

2.  Write three statements or equations based on the weights on the mobile.

Fill in the blank shapes with numbers to make each equation true. Squares and circles can represent different values in different equations, but the same shape must represent the same value in the same equation.
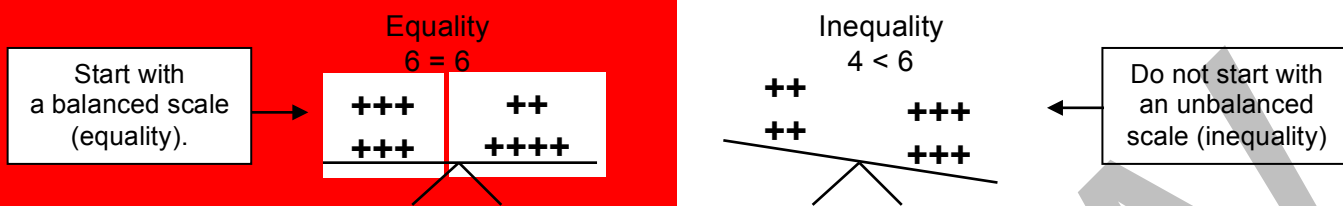
3.    24 = ☐ + ☐ + ☐

4.    ◯ • ☐ = 40

5.    ☐ ÷ 2 + ☐ = 9

6.  Which problem above does not have a unique solution? _____ Why?

# BALANCE SCALES

| | Equality 6 = 6 | Inequality 4 < 6 | |
|---|---|---|---|

Start with a balanced scale (equality). → 
+++
+++  |  ++
++++

++
++      +++
           +++

← Do not start with an unbalanced scale (inequality)

| An <u>equal sign</u> signifies that two expressions have the same value. |
|---|

We can picture equalities or inequalities with balance scales.
Imagine that each **+** represents one unit of weight.

For each problem, start with the **balanced** scale above (6 = 6). Draw a sketch to illustrate the action. Write the resulting equality or inequality.
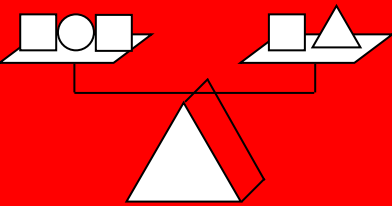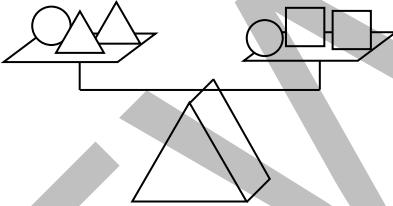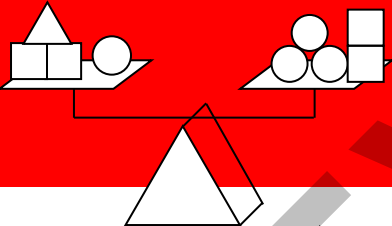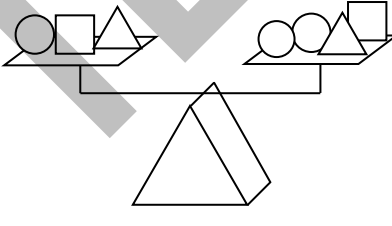
| 1. Four units are removed from the right side of the original scale.<br><br>Equality or inequality: | 2. Three units are added to the right side and 1 unit is added to the left side of the original scale.<br><br>Equality or inequality: |
|---|---|
| 3. Two units are removed from both sides of the scale.<br><br>Equality or inequality: | 4. One unit is added to both sides of the scale.<br><br>Equality or inequality: |
| 5. The number of units on both sides of the scale is doubled.<br><br>Equality or inequality: | 6. Only one-third the numbers of units on each side of the scale remain.<br><br>Equality or inequality: |

7. Under what conditions does a scale in balance remain in balance?

8. Under what conditions does a scale in balance become unbalanced?

# BALANCE SCALE PUZZLES

In each problem below, all shapes have some weight, and the same shapes have the same weight. All problems are independent of one another. Use what you know about balance to determine the answer to each question.

| 1. What weighs the same as △ ? | 2. What weighs the same as □ ? |
|---|---|
| 3. What weighs the same as △ ? | 4. Why is this picture impossible? |

5.  Let each circle on the scale to the right have a weight equal to 4. Give all possible whole number weights for the square and triangle in an organized table.

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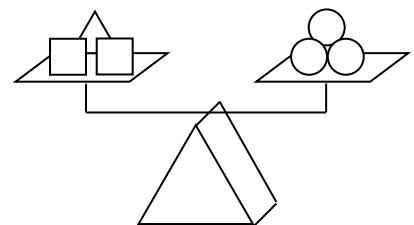
## MAMA'S PIZZA

We will use variables to write algebraic expressions and evaluate the expressions. We will find values that make equations true.

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## GETTING STARTED

| MAMA'S PIZZA MENU | |
| --- | --- |
| **Pizza** | **Drinks** |
| Cheese slice                        $1.00<br>Pepperoni slice                    $1.25 | Small drink                          $0.95<br>Medium drink                      $1.20<br>Large drink                         $1.60 |

1.  What would you order at Mama's Pizza?


2.  What is the cost of your order?


3.  What is the cost of 6 medium drinks? Find your answer in two or three different ways.

| Method 1: | Method 2: | Method 3: |
| --- | --- | --- |
|  |  |  |

# VARIABLES AND EXPRESSIONS

Follow your teacher's directions to complete this page.

| MAMA'S PIZZA MENU | |
|---|---|
| **Pizza** | **Drinks** |
| Cheese slice $1.00<br>Pepperoni slice $1.25 | Small drink $0.95<br>Medium drink $1.20<br>Large drink $1.60 |

Let _____ = the cost of a _____

Let _____ = the cost of a _____

Let _____ = the cost of a _____

Let _____ = the cost of a _____

Let _____ = the cost of a _____

# PRACTICE 2

| MAMA'S PIZZA MENU | | | |
| --- | --- | --- | --- |
| (The variable represents the cost of the item.) | | | |
| **Pizza** | | **Drinks** | |
| Cheese slice (*c*) | $1.00 | Small drink (*s*) | $0.95 |
| Pepperoni slice (*p*) | $1.25 | Medium drink (*m*) | $1.20 |
| | | Large drink (*L*) | $1.60 |

A group of friends decide to go to the Mama's Pizza for lunch.
- Miguel orders a slice of cheese pizza, a slice of pepperoni, and a medium drink.
- Barry orders two slices of pepperoni pizza and a large drink.
- Susie orders a slice of pepperoni pizza and a medium drink.
- Ronni orders two slices of cheese pizza and a large drink.

In the table below, record the variable expressions representing the costs of each order separately, and then the total order.

| | Expression for the cost of the order | Evaluate to find the cost |
| --- | --- | --- |
| 1.  Miguel | | |
| 2.  Barry | | |
| 3.  Susie | | |
| 4.  Ronni | | |
| **5.  Total** (in simplest form) | | |

6. Explain why 3*p* + 2*p* is equivalent to 5*p*, regardless of the cost of a slice of pepperoni.

The pizza shop owner decides to take $0.10 off the cost of each slice of pizza.

7. Write a numerical expression for the total cost of the order in problem 5, including this discount.

8. Find the cost of the order, including this discount.

# VARIABLES AND EQUATIONS

Follow your teacher's directions to complete this page.

| MAMA'S PIZZA MENU – NEW AND IMPROVED! | | | |
|---|---|---|---|
| (The variable represents the cost of the item.) | | | |
| **Pizza** | | **Drinks** | |
| Cheese slice ($c$) | $1.00 | Small drink ($s$) | $0.95 |
| Pepperoni slice ($p$) | $1.25 | Medium drink ($m$) | $1.20 |
| Daily Special ($d$) | $1.75 | Large drink ($L$) | $1.60 |
| | | Extra-large drink ($x$) | $1.90 |

## PRACTICE 3

| BOOM BURGERS MENU |
| --- |
| (The variable represents the cost of the item.) |

| Burgers | | Drinks | |
| --- | --- | --- | --- |
| Hamburger (*h*) | $4.00 | Small drink (*s*) | $1.00 |
| Cheeseburger (*c*) | $4.25 | Medium drink (*m*) | $1.25 |
| Veggie burger (*v*) | $4.75 | Large drink (*L*) | $1.50 |
| | | Extra-large drink (*x*) | $1.75 |

For each equation below, find a menu item above with a cost that makes the equation true. Within the same problem below, the ☐ refers to the same item. In different problems, the ☐ need not represent the same menu item.

1.  $2h + \boxed{\phantom{x}} = 2v$

Menu item: _____

Cost of menu item: _____

2.  $2c = \boxed{\phantom{x}} + 2m + 2s$

Menu item: _____

Cost of menu item: _____

3.  $h + s + m + L + x = 2 \bullet \boxed{\phantom{x}}$

Menu item: _____

Cost of menu item: _____

4.  $2h = 2 \bullet (\boxed{\phantom{x}} + m + s)$

Menu item: _____

Cost of menu item: _____

5.  $3(s + m + h) = 4 \bullet \boxed{\phantom{x}} + x$

Menu item: _____

Cost of menu item: _____

6.  $2L + \boxed{\phantom{x}} = 4 \bullet \boxed{\phantom{x}}$

Menu item: _____

Cost of menu item: _____

## SOLVING EQUATIONS: SUBSTITUTION

We will solve and check equations using substitution strategies.

### GETTING STARTED

Complete each sentence using the word bank below. Use section 1.5 to help you, and record definitions in My Word Bank.

| equation | evaluate | expression |
|----------|----------|------------|
| simplify | solution | solve |

1.   $3p + p + 5$ is an example of a(n) _____.

   If I _____ it, the result is  $4p + 5$.

   If I _____ it when $p = 2$, the result is 13.

2.   $3p = 21$ is an example of a(n) _____.

   If I _____ it, then $p = 7$.

   Therefore, $p = 7$  is a(n) _____.

3.  Describe in your own words the difference between an expression and an equation. Include examples.

4.  Describe in your own words the difference between simplify, evaluate, and solve.

# EQUATION SOLVING STRATEGY: SUBSTITUTION

Follow your teacher's directions to complete this page.

| (1) | (2) | (3) |
|---|---|---|
| (4) | (5) | (6) |

Solve each equation using mental math and substitution. Check by substitution.

| 7.    $56 = 8x$ | 8.    $37 = y + 17$ | 9.    $p - 20 = 14$ |
|---|---|---|
| 10.    $12 - x = 3$ | 11.    $\dfrac{w}{6} = 6$ | 12.    $\dfrac{6}{t} = 6$ |

## PRACTICE 4

Solve each equation using mental math and substitution. Check by substitution.

| 1. $16 = n + 2$ | 2. $5 - x = 1$ |
|---|---|
| 3. $7m = 42$ | 4. $\dfrac{y}{8} = 3$ |
| 5. $a - 13 = 62$ | 6. $d + 99 = 157$ |
| 7. $148 = 4p$ | 8. $\dfrac{k}{3} = 29$ |
| 9. $184 = v - 42$ | 10. $500 = w + 250$ |

11. The weight of a bag of apples, $a$, is unknown. There are 15 bags of apples that are all of this weight.

| a. Write an expression for the total weight of the bags of apples you have. | b. Write an equation to show that the total weight of the apple bags is 180 pounds. Then solve the equation. |
|---|---|
|  |  |

## MORE EQUATION SOLVING WITH SUBSTITUTION

For 1-4, follow your teacher's directions.
For 5-9, solve each equation using "cover up" and substitution. Check by substitution.

| | | |
|---|---|---|
| (1) | (2) | (3) |
| (4) | 5.  $50 = 35 + 5n$ <br><br><br> Check: | 6.  $6(x - 5) = 54$ <br><br><br> Check: |
| 7.  $98 = 9y - 1$ <br><br><br> Check: | 8.  $\dfrac{w - 12}{6} = 6$ <br><br><br> Check: | 9.  $\dfrac{1}{3}(2 + 2x) = 4$ <br><br><br> Check: |

# PRACTICE 5

Solve each equation. Check your solution using substitution.

| 1. $4 + 12b = 100$ | 2. $78 = 5y + 23$ | 3. $6n - 15 = 75$ |
|---|---|---|
| 4. $0 = 6(5 - x)$ | 5. $\dfrac{m}{4} + 6 = 7$ | 6. $4(p - 8) = 100$ |
| 7. $6 = \dfrac{42}{n}$ | 8. $\dfrac{2}{3}(x + 7) = 8$ | 9. $0.75x = 2.25$ |

10. The weight of a bag of apples, *a*, is unknown. The weight of a bag of oranges is 5 pounds.

| a. Write an expression for the weight of a grocery bag filled with a bag of oranges and a bag of apples. | b. Write an expression for the weight of 3 grocery bags, each filled with a bag of oranges and a bag of apples. |
|---|---|
| c. Write an equation to show that the total weight of the 3 grocery bags is 36 pounds. Then solve the equation. | d. What does the solution to the equation represent? |

## REVIEW

**BIG SQUARE PUZZLE**

# ONE-STEP TARGET EQUATIONS

For problems 1-3, use the equation structure to the right:
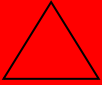
$$\square + x = \square\square$$
$$x = \square$$

1. Use exactly four of the digits 1 through 9 one time each. Write a true equation and find its solution.

2. Use exactly four of the digits 1 through 9 one time each (again). Write an equation with the greatest solution that you can find.

3. How many different equations can you write with the greatest solution? How do you know you have them all?

For problems 4-6, use the equation structure to the right:

$$\square + x = \square\square$$
$$x = \square\square$$
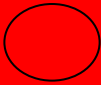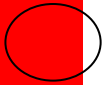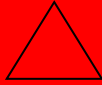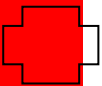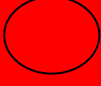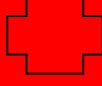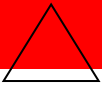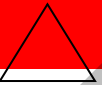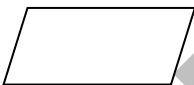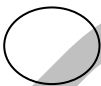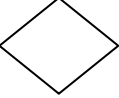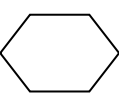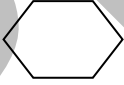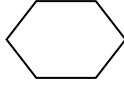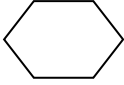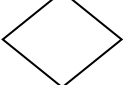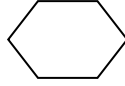
4. Use exactly four of the digits 1 through 9 one time each. Write a true equation and solution.

5. Use exactly four of the digits 1 through 9 one time each (again). Write an equation with the greatest solution that you can find.

6. How many different equations can you write with the greatest solution? How do you know you have them all?

## EQUATION CHALLENGE

The ten shapes in these nine equations represent the digits 0 through 9. Each shape represents the same digit in all equations, and different shapes represent different digits. Two shapes together (no equal sign or operation symbol between them) represent a 2-digit number. Determine the value of each shape.

1. $\square$ + $\bigcirc$ = $\bigcirc$

2. $\triangle$ • $\triangle$ = (trapezoid)

3. $\heartsuit$ ÷ $\star$ = (cross)

4. $\bigcirc$ − (cross) = $\triangle$

5. $\triangle$ + $\triangle$ = (parallelogram)

6. (parallelogram) • (diamond) = (parallelogram)

7. $\bigcirc$ • $\triangle$ = $\star$ (diamond)

8. (square) ÷ $\star$ = (hexagon)

9. (hexagon) + (hexagon) + (hexagon) = (diamond)(hexagon)

10.     Choose three shapes. Explain how you found their values.

# VOCABULARY REVIEW
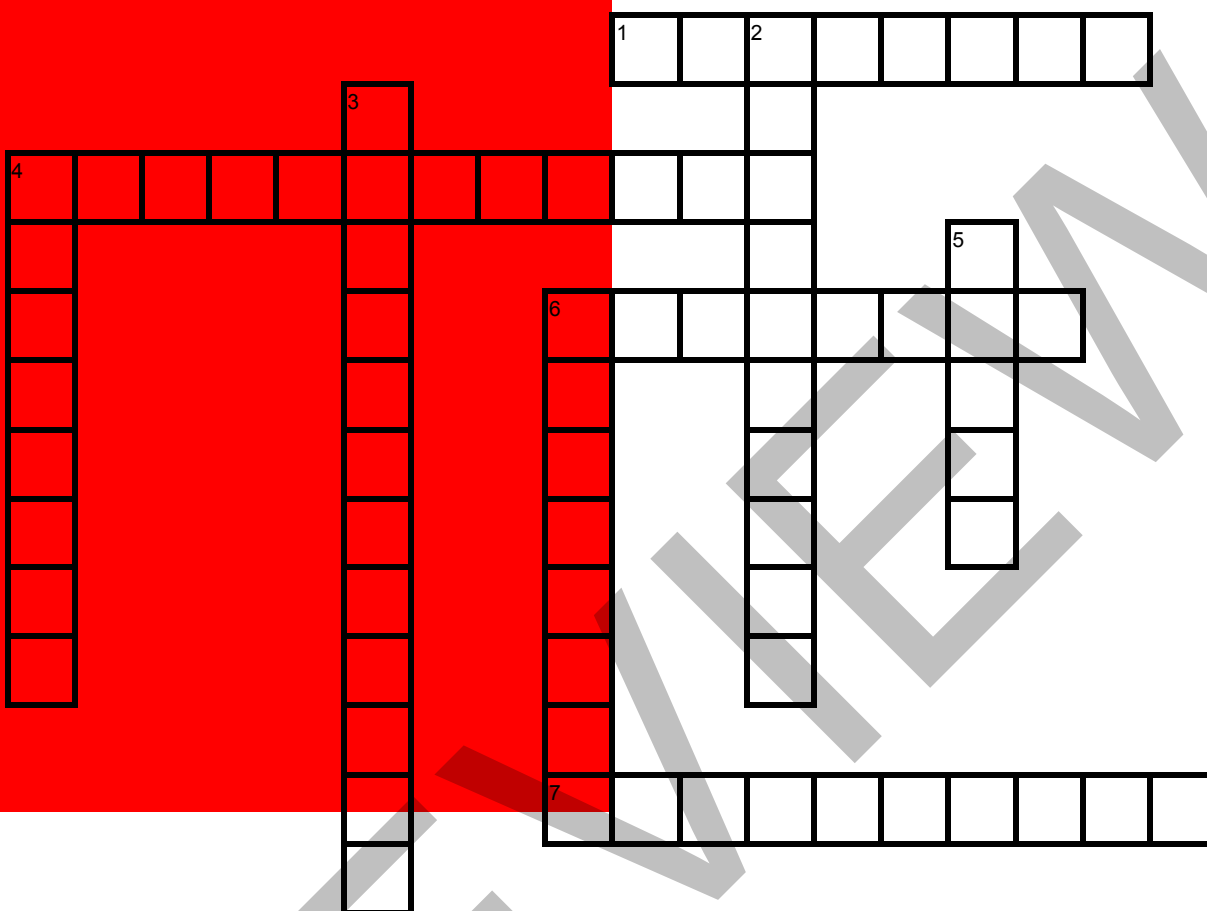
## Across

1  Write an expression in a simpler form.

4  _____ refers to replacing a quantity with one that is equal to it.

6  A statement asserting that two expressions are equal.

7  A combination of numbers, variables, and operation symbols.

## Down

2  A strategy for solving an equation that involves substitution (2 words).

3  A property that rewrites a product as a sum.

4  A value that makes an equation true.

5  Find a value that makes an equation true.

6  Find the value of an expression.

# DEFINITIONS, EXPLANATIONS, AND EXAMPLES

| Word or Phrase | Definition |
|---|---|
| distributive property | The distributive property states that $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for any three numbers $a$, $b$, and $c$.<br><br>$3(4 + 5) = 3(4) + 3(5)$ and $(4 + 5)8 = 4(8) + 5(8)$ |
| equation | An equation is a mathematical statement that asserts the equality of two expressions.<br><br>$18 = 8 + 10$ is an equation that involves only numbers.<br><br>$18 = x + 10$ is an equation that involves numbers and a variable.<br><br>$y = x + 10$ is an equation that involves a number and two variables. |
| evaluate | Evaluate refers to finding a numerical value. To evaluate an expression, replace each variable in the expression with a value and then calculate the value of the expression.<br><br>To evaluate the expression $3 + 4(5)$, we calculate<br>$3 + 4(5) = 3 + 20 = 23$.<br><br>To evaluate the expression $2x + 5$ when $x = 10$, we calculate<br>$2x + 5 = 2(10) + 5 = 20 + 5 = 25$. |
| expression | A mathematical expression is a combination of numbers, variables, and operation symbols. When values are assigned to the variables, an expression represents a number.<br><br>Some mathematical expressions are $7x$, $a + b$, $4v - w$, $\dfrac{8 + x}{10}$, and $19$. |
| simplify | Simplify refers to converting a numerical or variable expression to a simpler form. A variable expression might be simplified by combining like terms. A fraction might be simplified by dividing numerator and denominator by a common divisor.<br><br>$2x + 6 + 5x + 3 = 7x + 9$<br><br>$\dfrac{8}{12} = \dfrac{2}{3}$ |
| solution to an equation | A solution to an equation involving variables consists of values for the variables which, when substituted, make the equation true.<br><br>The value $x = 8$ is a solution to the equation $10 + x = 18$.<br>If we substitute $8$ for $x$ in the equation, the equation becomes true:<br>$10 + 8 = 18$. |
| solve an equation | To solve an equation refers to finding all values for the variables in the equation that, when substituted, make the equation true. Values that make an equation true are called solutions to the equation.<br><br>To solve the equation $2x = 6$, one might think "two times what number is equal to 6?" Since $2(3) = 6$, the only value for $x$ that satisfies this condition is $3$.<br>Therefore 3 is the solution. |

| Word or Phrase | Definition |
|---|---|
| substitution | Substitution refers to replacing a value or quantity with an equivalent value or quantity.<br><br>If $x + y = 10$, and $y = 8$,<br>then we may substitute this value for $y$ in the equation to get $x + 8 = 10$. |
| variable | A variable is a quantity whose value has not been specified. Variables are used in many different ways. They may refer to functions, to quantities that vary in a relationship, or to unknown quantities in equations and inequalities.<br><br>In the equation $d = rt$, the quantities $d$, $r$, and $t$ are variables.<br><br>In the equation $2x + 4 = 10$, the variable $x$ may be referred to as the unknown. |

| Variables in Algebra | |
|---|---|

Loosely speaking, variables are quantities that can vary. Variables are represented by letters or symbols. Variables have many different uses in mathematics. The use of variables, together with the rules of arithmetic, makes algebra a powerful tool.

Three important ways that variables appear in algebra are the following.

| Usage | Examples |
|---|---|
| Variables can represent an unknown quantity in an equation or inequality. In this case, the equation is valid only for specific value(s) of the variable. | $x + 4 = 9$<br>$5n = 20$<br>$y < 6$ |
| Variables can represent quantities that vary in a relationship. In this case, there is always more than one variable in the equation. | Formula: $P = 2\ell + 2w$, $A = s^2$<br><br>Function (input-output rule): $y = 5x$, $y = x + 3$ |
| Variables can represent quantities in statements that generalize rules of arithmetic.<br><br>In this case, there may be one or more variables. | Commutative property of addition: $x + y = y + x$<br><br>Distributive property: $x(y + z) = xy + xz$ |

## Using Shapes to Represent Variables

If the same shape (variable) is used more than once in an equation (or system of equations), it must represent the same value each place where it appears. Two different shapes (variables) in an equation (or system of equations) may represent the same values or different values.

| This is allowed | This is allowed | This is NOT allowed |
|---|---|---|
| □ + □ = ○ | □ + ○ = ◇ | ○ + ○ = △ |
| 7 + 7 = 14 | 6 + 6 = 12 | 6 + 4 = 10 |

## Writing Expressions

The notation used for algebra is sometimes different from the notation used for arithmetic.
For example:

- 54  means the sum of five tens and four ones, that is,  5(10) + 4.

- $5\frac{1}{2}$  means the sum of five and one-half. that is,  $5 + \frac{1}{2}$ .

- $5x$  means the product of  5  and  $x$, which can also be written  $5(x)$  or  $5 \bullet x$. We typically do not write  $5 \times x$  because the multiplication symbol '$\times$' is easily confused with the variable  $x$.

## Evaluate or Simplify?

We use the word "evaluate" when we want to calculate the value of an expression.

To evaluate 16 – 4(2), follow the rules for order of operations and compute:
$$16 – 4(2) = 16 – 8 = 8.$$

To evaluate  $6 + 3x$  when  $x = 2$, substitute  2  for  $x$  and calculate:
$$6 + 3(2) = 6 + 6 = 12.$$

We use the word "simplify" when rewriting a number or an expression in a form more easily readable or understandable.

To simplify $2x + 3 + 5x,$ combine like terms: $2x + 3 + 5x = 7x + 3.$

Sometimes it may not be clear what is the simplest form of an expression. For instance, by the distributive property, $4(x + 2) = 4x + 8$. For some applications, $4(x + 2)$ may be considered simpler than $4x + 8$, but for other applications, $4x + 8$ may be considered simpler than $4(x + 2)$.

| How to Determine if an Equation is True |
|---|

**PIZZA SHOP MENU**
(The variable represents the cost of an item.)

| Pizza | | Drinks | |
|---|---|---|---|
| Cheese slice ($c$) | $1.00 | Small drink ($s$) | $0.95 |
| Pepperoni slice ($p$) | $1.25 | Large drink ($L$) | $1.75 |

.

What value from the menu above makes this equation true?

$$p + \boxed{\phantom{x}} = 3c$$

Substitute:   $1.25 + \boxed{\phantom{x}} = 3(1.00)$

$1.25 + 1.00 = 3.00$ ?  NO

$1.25 + 1.25 = 3.00$ ?  NO

$1.25 + 0.95 = 3.00$ ?  NO

$1.25 + 1.75 = 3.00$ ?  YES

The equation is true when $\boxed{\phantom{x}}$ represents the cost of a large drink ($L = 1.75$).

## Solving Equations Using a Substitution Strategy

To solve an equation using substitution, apply your knowledge of arithmetic facts to find values that make the equation true.

| Solve $3x = 15$ | Solve $12 = 20 - k$. |
|---|---|
| • Think: ***What number times 3 is 15?*** | • Think: ***20 minus what number equals 12?*** |
| • Since $3(5) = 15$, $x = 5$. | • Since $20 - 8 = 12$, $k = 8$. |
| Check: $3(5) = 15$ <br> $15 = 15$ | Check: $12 = 20 - 8$ <br> $12 = 12$ |

You can also "cover up" the expression containing the unknown and use your knowledge of arithmetic facts to find a value under your finger that makes the equation true. Then use that information to find the value of the unknown.

| Solve $\dfrac{n + 20}{3} = 8$ | Solve $5(m - 2) = 20$. |
|---|---|
| • Cover up $n + 20 \rightarrow \dfrac{\phantom{n+20}}{3} = 8$ | • Cover up $m - 2 \rightarrow 5(\phantom{m-2}) = 20$ |
| • Think ***What number divided by 3 equals 8?*** | • Think: ***5 times what number equals 20?*** |
| • Since $\dfrac{24}{3} = 8$, you are covering up 24. | • Since $5(4) = 20$, you are covering up 4. |
| • Think: ***What number plus 20 equals 24?*** | • Think: ***What number minus 2 equals 4?*** |
| • Since $4 + 20 = 24$, $n = 4$. | • Since $6 - 2 = 4$, $m = 6$. |
| Check: $\dfrac{4 + 20}{3} = 8$ <br> $8 = 8$ | Check: $5(6 - 2) = 20$ <br> $5(4) = 20$ <br> $20 = 20$ |

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