UNIT 4
TEACHER EDITION

MathLinks GRADE 8

INTRODUCTION TO FUNCTIONS

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
8.EE.B	**Understand the connections between proportional relationships, lines, and linear equations.**
8.EE.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
8.F.A	**Define, evaluate, and compare functions.**
8.F.1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
8.F.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
8.F.3	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*
8.F.B	**Use functions to model relationships between quantities.**
8.F.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
8.F.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

UNIT PLANNING

* Starred resources can be accessed under Unit Resources on the Teacher Portal.

Unit Pacing* Up to 15 class hours	4.0 Opening Problem: Slides and Jumps (1 hour) 4.1 Multiple Representations (4 hours) 4.2 Function Representations (3 hours) 4.3 Rate Representations (3 hours) Review (3 hours) Assessment (1 hour)
Unit Resources* Up to 3 class hours Deleted projects	• Extra Problems • Essential Skills • Math Talks (Data, Picture, Number) • Nonroutine Problems • Technology Activities Tasks (Step by Step, Growing Squares) • Parent Support Letters
Assessment Options* See Portal Unit 4 → Other Resources → Assessment, Follow-up and Feedback for more ideas	• On the Teacher Portal ✓ Unit Quizzes ✓ Cumulative Tests ✓ Tasks ✓ The *MathLinks* Rubric (a with rubric-worthy problems) • In the Student Packet ✓ Monitor Your Progress ✓ Unit Reflection • In the Teacher Edition ✓ References to Journals ✓ Suggested problems for the *MathLinks* Rubric
Materials	• Eight counters [4.0] (2 different colors; 1 set per student) • Square tiles [4.1] (optional – handful in 2 colors per group) • General supplies (e.g., colored pencils, markers, rulers, tape, scissors, graph paper, calculators, chart paper)
Slide Decks*	S4.0 Slides and Jumps S4.1a The Pool Problem S4.1b Saving vs Spending S4.2 What is a Function?
Reproducibles*	R4-1 Slides and Jumps Board [4.0] (1 per student) R4-2 $y = 3x + 4$ Cards [Review] (1 per pair/group)
Prepare Ahead	• See Activity Routines in Program Information for directions for the *MathLinks* Rubric, Poster Problems, Why Doesn't It Belong?, and Alge-Grid [4.1, 4.3, Review]
Other Resources on the Teacher Portal	• Getting Started Videos and Resources - General Resources • Skill Boosters - Teacher Access page (fractions) • Puzzles / Games - Teacher Access page (Function Frenzy, SMASH Game 2, Alge-Grid)

PLANNING FOR DIFFERENT USERS

Student Packet (SP)

Unit 4 component options for those who support students:

For teachers	• Teacher Edition (this document) • Teacher Portal (Unit Resources, General Resources) • Program Information
For substitutes	• SP (Practice 1 – 7 may be completed independently any time after instruction; Spiral Review; Vocabulary Review) • Unfinished work from previous SP's • Extra Problems • *MathLinks* Puzzles / Games
For parents	• Resource Guide • Parent Letter (English and Spanish)

Unit 4 component options to use with all students (all available on the Teacher Portal):

- SP (Word Bank, Opening Problem, Lessons, Activity Routines, Review, Student Resources, self-monitoring, journals, reflection prompts)
- Student Packet Text file for Translation (EL)
- Extra Problems (practice or assess by lesson)
- Essential Skills (just-in-time review)
- Math Talks (whole-class discourse)
- Nonroutine Problems (enrichment)
- Tasks (multi-part problems)
- Technology Activities (variety)
- *MathLinks* Puzzles / Games (fun challenges)

Unit 4 component options for particular subgroups of students:

For English learners see pgs vii-viii for specific strategies	• SP Text File for Translation • SP features for language development (Word Bank, Vocabulary Review, consistent structure for reading and writing, grouping opportunities for speaking and listening) • SP Activity Routines for language development (rubric-worthy problems with the *MathLinks* Rubric, Poster Problems, Why Doesn't it Belong?) • Math Talks for speaking and listening
For struggling learners see pgs vii-viii for specific strategies	• SP features for math confidence (Getting Started, Review including Spiral Review, Word Bank, Vocabulary Review, consistent structure, grouping options) • SP Activity Routines for math confidence (rubric-worthy problems with the *MathLinks* Rubric, Poster Problems, Why Doesn't it Belong?, Alge-Grid) • Essential Skills for just-in-time intervention • Extra Problems (by lesson) for practice, review, or assessment • Skill Boosters (Fraction Addition and Subtraction; Fraction Multiplication and Division)
For enrichment and advanced learners	• SP enrichment (see page xi for specific options) • SP Activity Routines for enrichment (rubric-worthy problems with the *MathLinks* Rubric, Why Doesn't it Belong?) • Nonroutine Problems (including problems from the Math Olympiad) • Technology Activities for variety

MATH BACKGROUND

The Obvious Rule May Not Be the Only Rule

If only a finite number of terms of a sequence are given, and no other additional information is provided, it is always possible to find infinitely many (infinite) sequences that have the given terms.

Suppose that the first terms of a sequence are 4, 6, 8, 10, … . This suggests that each term in the sequence is obtained by adding 2 to the preceding term, that is, $a_1 = 4$ and $a_{n+1} = a_n + 2$ for $n > 1$. This may be the simplest sequence pattern with the given initial terms. But there are many other sequences that begin with the same four terms. For instance, the rule $a_1 = 4$ and $a_{n+1} = a_n + 2 + (n - 2)(n - 3)(n - 4)$ for $n > 1$ determines a sequence with the same first four terms, but whose fifth term is $a_5 = 10 + 2 + 6 = 18$, not 12.

For this reason, we are careful to say "find a rule…" rather than "find the rule…"

Different Definitions of Function

There are two routes for developing the function idea. The route we have followed is to define a <u>function</u> as an input-output rule, and to define the <u>graph of the function</u> as ordered pairs of input and output values.

Another more sophisticated route, which is common in school mathematics, is to define first a <u>relation</u> to be a set of ordered pairs, and then to define a function to be a relation with the "vertical line property." A *function*, according to this definition, is what we have defined as the *graph of the function*.

The two routes lead to essentially the same class of objects. However, we have chosen the input-output definition because it is the definition used in college mathematics, particularly calculus. Further, the input-output definition is better adapted to understanding algebraic operations on functions and the operation of composition of functions.

TEACHING TIPS

<table>
<tr><th colspan="3">Applying Standards for Mathematical Practice (SMP)</th></tr>
<tr><td colspan="3">Here is an abbreviated version of the SMPs and some ways they are applied in this unit.</td></tr>
<tr><td>SMP1</td><td>Make sense of problems and persevere in solving them.
• Understand a problem and look for entry points
• Consider simpler or analogous problems
• Monitor progress and alter solution course as needed
• Make connections between multiple representations
• Check answers with a different method</td><td>[4.0] Encourage perseverance as students find solutions for Slides and Jumps. Looking for patterns in the first steps could be productive. Finding recording techniques to keep track of moves will be useful.

[All] Slides and Jumps, The Pool Problem, Saving vs Spending, and To School and Back Home include opportunities to create multiple representations, make connections, and analyze results.</td></tr>
<tr><td>SMP2</td><td>Reason abstractly and quantitatively.
• Use numbers and quantities flexibly in computations
• Attend to the meaning of quantities
• Decontextualize a problem using symbols, manipulate them, and then interpret based on the context</td><td>[All] Students create rules, graphs, generalize patterns, and compare representations for a variety of problems. This important algebraic work helps students develop abstract reasoning.

[4.3] Given a graph, students explain how it might represent the height of water in a bathtub. Students create a graph to represent the varying speeds of a rollercoaster.</td></tr>
<tr><td>SMP3</td><td>Construct viable arguments and critique the reasoning of others.
• Use assumptions, definitions, established results, examples, and counter examples to analyze an argument and discuss its merits or flaws
• Make and test conjectures based on evidence
• Analyze situations by breaking them into cases
• Understand and analyze the approaches of others</td><td>[4.1] Students explore both geometric patterns and a saving vs. spending context. They explain how relationships within the contexts are the same and different, determine if relationships are proportional, make conjectures, and justify their reasoning.

[4.1] On the Savings vs Spending slide deck, students determine if statements made by Mateo and Talia are correct or not, and justify their reasoning.

[4.3] Students explain graphs on The Bath Graph and The Rollercoaster. Then they critique the explanations of their peers using The MathLinks Rubric.</td></tr>
</table>

Applying Standards for Mathematical Practice (SMP) Continued		
SMP4	**Model with mathematics.** • Attach meaningful mathematics to everyday problems and questions of interest • Make reasonable assumptions and approximations to simplify a situation • Identify quantities, use mathematical tools (such as multiple representations, formulas, equations) to analyze relationships • Interpret results and draw conclusions in the context of the situation	[All] Several rich problems are presented for students to explore and model using mathematical representations. These include **Slides and Jumps**, **The Pool Problem**, **Saving vs Spending**, **To School and Back Home**, **The Bath Graph**, and **The Rollercoaster**.
SMP5	**Use appropriate tools strategically.** • Select and use tools strategically (and flexibly) to visualize, explore, and compare information • Use technological tools and resources to solve problems and deepen understanding	[4.0, 4.1, Review] Manipulatives are useful for many algebra explanations. For example, counters are crucial for **Slides and Jumps**, and square tiles may be helpful for **The Pool Problem** and **Poster Problems**. [4.1, 4.3] As students begin their study of linear functions, consider Desmos technology activities designed to deepen understanding of slope and intercept.
SMP6	**Attend to precision.** • Calculate accurately and efficiently • Explain thinking using mathematical vocabulary • Use symbols appropriately • Specify units of measure	[4.2] The function definition and corresponding examples can be tricky and difficult to grasp at first. Require that students pay particularly close attention to the language that they use as they describe whether a representation (graph, table, mapping diagram, etc.) could represent a function.
SMP7	**Look for and make use of structure.** • Recognize the structure of a symbolic representation and generalize it • See complicated objects as composed of chunks of simpler object	[4.1, 4.3, Review] As students explore various real life and mathematical contexts, their understanding of the structure of proportional relationships and linear functions should grow.
SMP8	**Look for and make use of repeated reasoning.** • Identify repeated calculations and patterns • Generalize procedures based on repeated patterns or calculations • Find shortcuts based on repeated patterns or calculations	[All] Students are asked repeatedly to create tables, ordered pairs, graphs, equations, and mapping diagrams to represent mathematical situations. Based on this work, their understanding of efficiency and purpose of different representations will grow.

<table>
<tr><th colspan="3">Strategies to Support Different Learners</th></tr>
<tr><td colspan="3">Classrooms typically include students with different learning styles and needs. Here are some specific ways that MathLinks supports special populations. Strategies essential to the academic success of English learners are noted with a star (*). See Universal Design for Learning in Program Information for more details.</td></tr>
<tr><th></th><th>General Examples</th><th>MathLinks Examples</th></tr>
<tr><td>Know your Learner</td><td>✓ Understand student attributes that support or interfere with learning
✓ Determine preferred learning and interaction styles
✓ Assess student knowledge of prerequisite mathematics content
✓ Check for understanding continuously
✓ Provide differentiation opportunities for intervention to reach more learners
✓ Encourage students to write about their attitudes and feelings towards math
✓ Use contexts that link to students' cultures*</td><td>Built into the MathLinks Design:
SP: Getting Started, Spiral Review, Monitor Your Progress, Unit Reflection
TE: References to Journals
PR: Extra Problems, Essential Skills, Projects
OR: Skill Boosters, Assessment Options

[All] Provide parents with assignments ahead of time so they can help students budget their homework schedules. In addition, parent letters with examples and explanations are available for each student unit.

[All] Encourage students to use their learning strengths to maximize their versatility as skillful math students. Students should be able to interpret and present information in tables, as ordered pairs, in graphs, as mapping diagrams, with equations, and with verbal/written descriptions. Focusing on areas of strengths can help boost confidence necessary to improve areas of weakness.</td></tr>
<tr><td>Increase Academic Language through Mathematics</td><td>✓ Provide opportunities for students to read, write, speak, and listen
✓ Explain the academic vocabulary needed to access mathematical ideas, providing both examples and non-examples
✓ Use strategically organized groups that attend to language needs*
✓ Use rich mathematical contexts and sophisticated language to help ELs progress in their linguistic development*
✓ Use cognates and root words (when appropriate) to link new math terms to students' background knowledge*</td><td>Built into the MathLinks Design:
SP: Word Bank, Vocabulary Review, Student Resources
TE: Grouping suggestions, References to Journals, Suggested problems for The MathLinks Rubric
PR: Math Talks
OR: Critique student work on Slide Decks

[4.1, 4.2, 4.3] Address linguistic ambiguities in math by comparing everyday meanings of words to their mathematical uses. For example, contrast the word "step" as used in pattern problems with its usage in everyday language related to walking. The word "steep" describes slope in some sense, but has no precise mathematical meaning.*

[4.0, 4.1, Review] Encourage students not yet speaking English to indicate their understanding by color-coding or pointing to evidence of their thinking on the graph, table, or equation. For example, when exploring a growing pattern (such as in the Poster Problems), color the number of blocks by which the figure is growing.*</td></tr>
</table>

Components cited: Student Packet (SP), Teacher Edition (TE), Unit Resources (PR), Other Resource (OR)

<table>
<tr><th colspan="3">Strategies to Support Different Learners (Continued)</th></tr>
<tr><th></th><th>General Examples</th><th>MathLinks Examples</th></tr>
<tr><td>Increase Comprehensible Input</td><td>✓ Link concepts to past learning

✓ Make concepts meaningful through hands-on activities, visuals, demonstrations, and color-coding

✓ Use a think-aloud strategy to model appropriate thinking processes and academic language use

✓ Use graphic organizers to help students record information and data, see patterns, and generalize them

✓ Use multiple representations (pictures, numbers, symbols, words, contexts) of math ideas to create meaning and make connections

✓ Strategically sequence and scaffold to make mathematics accessible

✓ Simplify written instructions, rephrase explanations, and use verbal and visual clues*</td><td>Built into the MathLinks Design:
SP: Structured workspace
TE: Slide Deck Alternatives, Reproducibles, Materials
OR: Slide Decks

[4.1] Building geometric patterns with manipulatives and then sketching them provides for a transition from concrete to semi-concrete. Recording numbers in a table, graphing the data, and writing algebraic rules, both in words and with symbols, are more abstract representations.

[4.2] Have students focus on visual representations to see if a relationship is a function. Specifically, mapping diagrams can be helpful because arrows help students more easily see if a domain value “maps to” more than one range value. Mapping diagrams can be created for most examples in this lesson to aid student understanding.

[4.1, 4.3] Use the contexts in Saving vs Spending and To School and Back Home to help students make sense of positive and negative rates of change.</td></tr>
<tr><td>Promote Student Interaction</td><td>✓ Use flexible group configurations that support content objectives

✓ Use strategies and activities that promote teacher/student and student/student interactions (e.g., think-pair-share, Poster Problems)

✓ Encourage elaborate responses through questioning

✓ Allow processing time and appropriate wait time, recognizing the importance of the different requirements for speaking, reading, and writing in a new language*

✓ Allow alternative methods to express mathematical ideas (e.g., visuals, students’ first language)*</td><td>Built into the MathLinks Design:
SP: Lesson and Review activities
TE: References for Journals, Suggested problems for The MathLinks Rubric
PR: Math Talks, various games and puzzles
OR: Slide Decks, Activity Routines

[4.0] Allow appropriate time and encourage productive struggle as students look for patterns in Slides and Jumps.

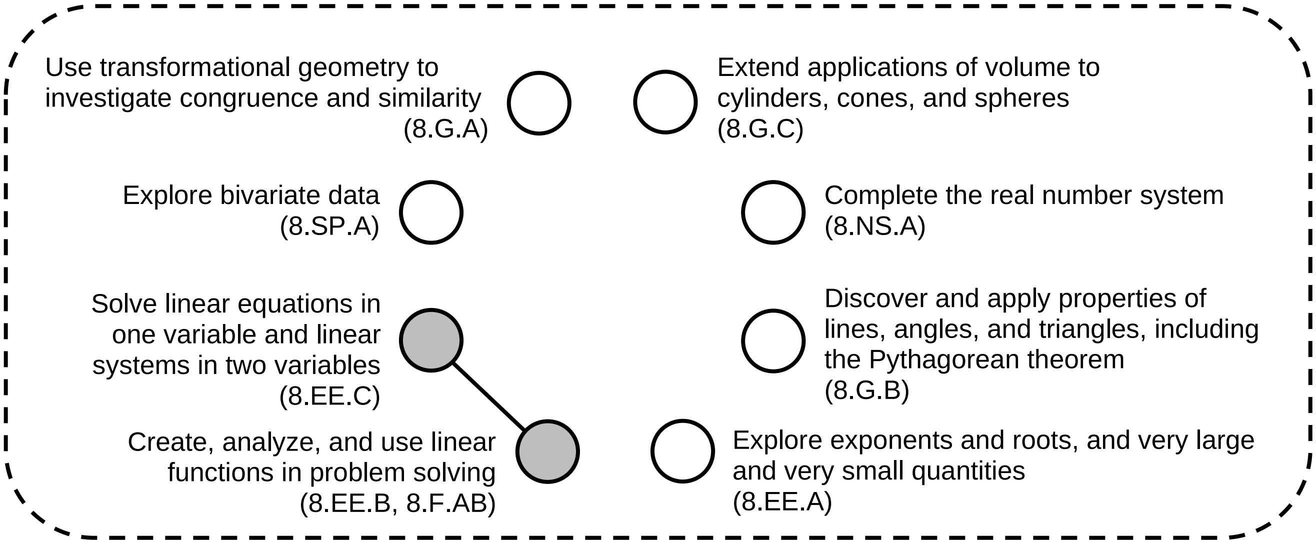
[4.1, 4.3, Review] Encourage students to routinely compliment each other’s effort and politely critique each other’s work. Explicit opportunities are included in the slide deck for Saving vs Spending. The Bath Graph, The Rollercoaster, and the Review activities offer opportunities to critique student work as well.</td></tr>
</table>

Components cited: Student Packet (SP), Teacher Edition (TE), Unit Resources (PR), Other Resource (OR)

Big Ideas and Connections

The Center for Mathematics and Teaching is dedicated to igniting and nurturing passion for mathematics in middle school students. We see the classroom as a place of joy and wonder, collaboration and purpose, perseverance and empowerment. We want all students to succeed in mathematics, as they explore its beauty in patterns, concepts, connections, and applications.

MathLinks: Grade 8 is organized around seven big ideas. This graphic provides a snapshot of the ideas in Unit 4 and their connections to each other.



These ideas build on past work and prepare students for the future. Some of these include:

Prior Work	What's Ahead
• Generate number and shape patterns from a rule (4.OA.C) • Generate two patterns and simultaneously graph ordered pairs, comparing corresponding coordinates (5.OA.B, 5.G.A) • Explore ratios and rates using multiple representations (6.RP.A) • Recognize when a representation illustrates a proportional relationship (7.RP.A)	• Formally define slope of a line (8.F.A) • Develop the slope-intercept form of a line (8.F.A) • Introduction to transformational geometry as an example of a function (8.F.A, 8.G.A) • Deepen understanding of functions (HS) • Study a variety of classic mathematical relationships, including linear, quadratic, exponential, and trigonometric functions (HS)

Students May Wonder…

Slides and Jumps is a classic problem/game. Students may enjoy looking for online videos with explanations of this game and its variations.

Encourage students to research: ***Why are functions useful? What are some examples of functions in real life?***

Developing Language Skills through *MathLinks*

Language (reading, writing, speaking, and listening) helps students communicate math ideas and understand concepts. Here are some language examples in Unit 4.

Language Objectives

Student will:

(Lesson 1) Use color coding to see the relationship between the pattern, table, graph, and equation.

(Lesson 2) Focus on vocabulary to determine if a function is increasing, decreasing, linear or nonlinear.

(Lesson 3) Use multiple representations to represent rates and compare functions.

(Review) Collaborate in discussion on all activities with partners. Student groupings should reflect a safe environment for expressing ideas orally and in writing.

Group Discussions to Promote Reading, Listening, and Speaking (2+)

Critique reasoning situations appear on Slide Deck S4.1b. Use a partner strategy such as “turn and talk” to encourage students to respectfully critique the reasoning of other students.

On Interpreting Tables, Equations, and Graphs (pg 3), provide time for pairs or small groups to brainstorm similarities and differences prior to writing a response.

The Bath Graph (pg 23) can present a challenge in describing what is happening in the graph. Use think-pair-share so that students can first work in pairs and then also share with another individual or pair.

On $y = 3x + 4$ (pg 25), provide some time for pairs/trios to compare and organize the cards together. Discuss and determine if cards model an equation. Were there any difficulties in organizing the cards and completing the graph?

Journal Ideas to Promote Writing

(Explaining Concepts) Journal ideas appear on pages 3, 5, and 7.

(New Language) Which representation for function makes the most sense to you? Explain how you can tell which representation represents a function.

(Language in the Real World) Choose a situation familiar to you in the real world (for example, saving for an item you would like to purchase). Write a problem for this context that leads to an equation for your situation and solve it.

Enrichment and Challenges for Advanced Learners

MathLinks: Grade 8 materials provide multiple opportunities for advanced students to investigate grade-level mathematics at a higher level of complexity without doing more work than their peers.

Within this Student Packet, here are some pages that have accessible entry points and opportunities for extensions (low floors / high ceilings).

- Slides and Jumps (pg 1)
- Analyzing Saving vs Spending (pg 9)
- To School and Back Home (pgs 19 – 20)
- The Bath Graph (pg 23)
- The Rollercoaster (pg 24)

Encourage students to write a story similar to **The Bath Graph** or **The Rollercoaster** and describe it using mathematical representations and language.

Consider speeding up instruction and skipping some Practice and Spiral Review. After Lesson 1 use more challenging Function Frenzy (Portal access --> Puzzles / Games). Use more challenging Alge-Grids (Portal access → Puzzles / Games). See also Planning for Different Users (TE, pg iii) and Enrichment for Advanced Learners and Those with Undiscovered Hidden Talents (Program Information → Universal Design for Learning) for more ideas.

Using Multiple Representations to Develop Algebra Skills and Concepts

These lessons help prepare students to develop algebra skills and concepts through the use of multiple representations—namely numbers, pictures, symbols, and words—to solve problems. Students organize data, recognize patterns, use inductive reasoning to find an equation for a particular pattern, and graph the relationship. Driscoll refers to this as “building rules to represent functions.”

Students will recognize the utility of algebra as they use variables, expressions, and equations to interpret information and translate among different representations. Flexibility with multiple representations gives students tools to complete a procedure and reverse the procedure. Driscoll refers to this as “doing-undoing.” In these lessons, given an input-output rule, an example of “doing” is to find an output given an input value, and an example of “undoing” is to find an input given an output value.

Mark Driscoll (1999). Fostering Algebraic Thinking: A Guide for Teachers Grades 6-10. Published by Heinemann.

Avoiding Learned Helplessness

When extending and generalizing patterns, students may want immediate reassurance from an expert that their patterns and rules are correct. We encourage instructors to refrain from providing answers too quickly. For students to become independent, confident thinkers, they must be given adequate time to work through a problem, and sufficient opportunities to resolve misconceptions and unclear concepts through discussion with peers. Teachers unwittingly contribute to “learned helplessness” when they provide too much feedback without allowing, and demanding, adequate effort from students to work through problems themselves.

The Algebra Progression in *MathLinks*: Grade 8

Algebra topics primarily appear in the CCSS-M Expressions and Equations and Functions domains. They are also in the Statistics and Probability domain. These areas are the focus of six units in *MathLinks*: Grade 8.

Unit 1 (**Plane and Solid Figures**) and Unit 2 (**Real Numbers and the Pythagorean Theorem**) apply 7th grade algebra to new 8th grade topics.

- In Unit 3, **The Algebra of Exponents and Roots**, students observe patterns in numerical expressions with exponents and generalize them to obtain the product, power, and quotient rules for exponents. They also use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number.

- In Unit 4 (this unit), **Introduction to Functions**, students continue the work started in 6th and 7th grades with multiple representations by connecting real-life or visual contexts to tables, graphs, and algebraic equations. Functions are formally defined and students represent them in tables, graphs, and mapping diagrams. They interpret different characteristics of functions (e.g., increasing or decreasing, linear or nonlinear).

- In Unit 5, **Linear Functions**, slope is formally defined as students find slopes of lines by counting on a grid, and by using the slope formula, which leads into using the slope-intercept form of a line, and applying this knowledge to solving various problems.

- In Unit 6, **Bivariate Data**, students graph bivariate numerical data as scatter plots, describe patterns of association (if they exist), estimate lines of best fit to points showing linear associations, write equations of these estimated lines from which predictions are made, and draw conclusions from the data.

- In Unit 7, **Linear Equations and Systems 1**, students solve linear systems of equations by graphing, noting that these systems have exactly one solution, no solutions, or infinitely many solutions. Estimating solutions creates the need for a more precise solution method. The process of substitution is introduced as a way to take a system of two equations in slope-intercept form and creating one equation in one variable. This creates a need to learn to solve equations with variables on both sides.

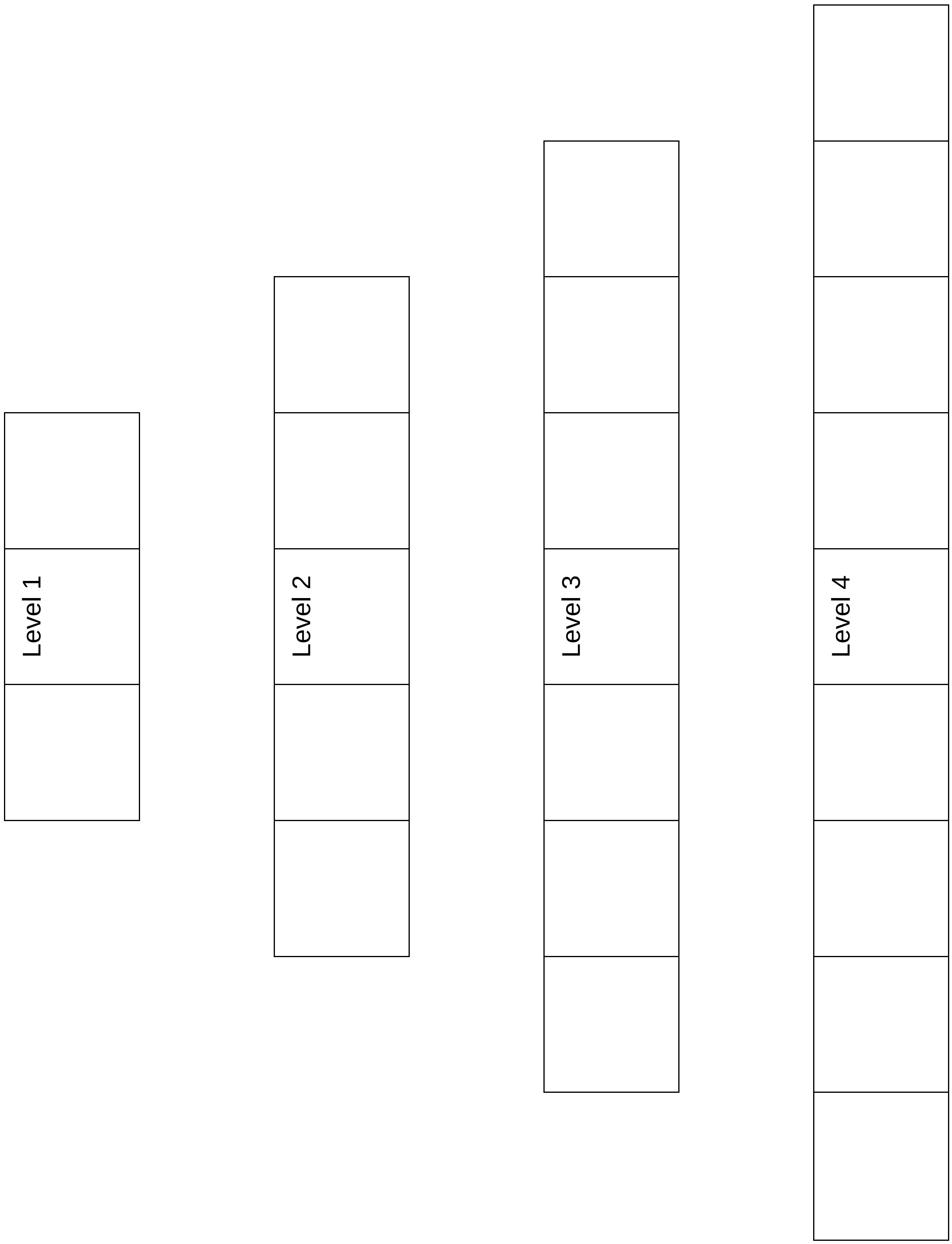
  Students then revisit the “cups and counters” model (introduced in Grade 7) for solving equations. The model aids the transition to formal procedures. Students see a parallel to the work done with systems, as linear equations in one variable may have one, infinitely many, or no solutions.

- In Unit 8, **Linear Equations and Systems 2**, students revisit the use of procedures to solve equations in one variable, though made harder by the introduction of non-integer values. With newly acquired equation-solving skills, students use the substitution method to solve systems algebraically. The elimination method is introduced as an alternative algebraic method, but is not rigorously pursued, being left to high school mathematics. The unit culminates with applications that utilize skills learned in Units 7 and 8.

The concept of a function is revisited in Unit 9 (**Congruence**) and Unit 10 (**Similarity**) as a mapping diagram, and transformations are defined as functions that map points in the plane to points in the plane. Algebraic rules are used to describe translations on coordinate planes.

REPRODUCIBLES

R4-1 SLIDES AND JUMPS BOARD



R4-2 $y = 3x + 4$ CARDS

A

Jakob charges $4 per hour for babysitting. Parents are charged $3 if they arrive home later than scheduled. Mrs. Lam always arrives late. How much does she pay Jakob for *x* hours of babysitting?

B

There are 4 guppies in the fish tank to start. Each month thereafter the number of guppies is 3 times the number in the month before. How many guppies are there after *x* months?

C

Lexie is making simple cloth facemasks. She sells them for $3 each to a local store, and they generously tip her an extra $4 each. How much money does she make from this store after selling them *x* facemasks?

D

Sara works delivering groceries to help with the family bills. She sees a sweatshirt that she really wants. Sara has an extra $4 set aside, and saves $3 per shift. How much money does she have after *x* shifts?

E

With an initial *y*-value of 4, each increase in the *x*-value by 1 results in an increase in the *y*-value of 3.

F

With an initial *y*-value of 3, each increase in the *x*-value by 1 results in an increase in the *y*-value of 4.

G

With a *y*-intercept equal to 4, the graph increases horizontally by 3 for every vertical increase of 1.

H

With a *y*-intercept equal to 4, the graph increases vertically by 3 for every horizontal increase of 1.

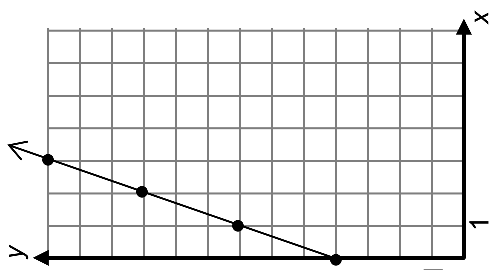
I

y	4	7	10	13	16	19
x	0	1	2	3	4	5

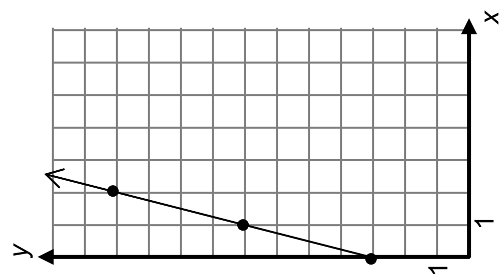
J

y	3	4	7	10	13	17
x	0	1	2	3	4	5

K



L



Who's in charge in a pencil case? The ruler.

Name ______________________________ Period __________ Date ______________

UNIT 4
ANSWER KEY

MathLinks GRADE 8

INTRODUCTION TO FUNCTIONS

Materials | Grouping | Reproducibles | Slide Deck | Journal Idea

Parent (or Guardian) signature ____

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

When a vocabulary word first comes up in context, take the time to support students in writing something that is meaningful to them, whether it's an explanation of the vocabulary in their own words, an example, and/or a picture.

function	graph of a function
input-output rule	proportional proportional relationship
unit rate	*y*-intercept

OPENING PROBLEM: SLIDES AND JUMPS

[SMP 1, 2, 4, 5, 8]

Follow your teacher's directions for (1) – (2).

(1) *Record slides and jumps for Level 1 through Level 5.*

Level #	Slides and Jumps
1	S J S
2	S J S JJ S J S
3	S J S JJ S JJJ S JJ S J S
4	S J S JJ S JJJ S JJJJ S JJJ S JJ S J S
5	S J S JJ S JJJ S JJJJ S JJJJJ S JJJJ S JJJ S JJ S J S

(2) *Record moves for Level 1 through Level 5, and generalize for Level x.*

Level #	# of Slides	# of Jumps	Total # of Moves
1	2	1	3
2	4	4	8
3	6	9	15
4	8	16	24
5	10	25	35
x	$2x$	x^2	$2x + x^2$

3. Record the missing values in the table below. Show your work on this page as needed.

Level #	# of Slides	# of Jumps	Total # of Moves
10	20	100	120
20	40	400	440
25	50	625	675
50	100	2,500	2,600
100	200	10,000	10,200
1,000	2,000	1,000,000	1,002,000
n	$2n$	n^2	$n^2 + 2n$

4 counters of one color,
4 counters of another color

S4.0 2+ R4-1

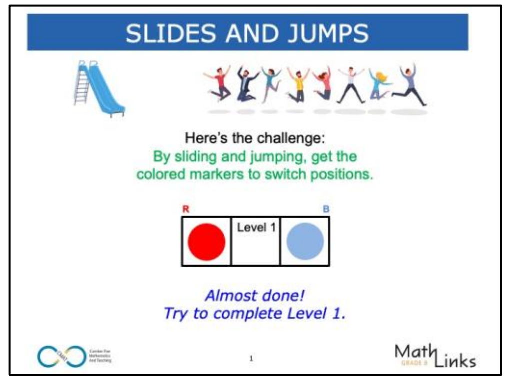
LESSON NOTES S4.0: SLIDES AND JUMPS

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

This game requires perseverance and patience for many teachers and students, though some will catch on quickly. Introduce the game on slides 1 – 3, and give students individual and collaborative time to work on game strategy. Revisit the problem over a few class periods or as homework. Then return to slides 4 – 5, where students record in a table, look for patterns, analyze moves, and generalize with algebra.

- Slide 1: Give each student R4-1, four counters of one color, and four counters of another. It is helpful to make note of the starting location of each color (see the red R and blue B on the slide) because it's easy to confuse which color is intended to go in which direction as levels increase.

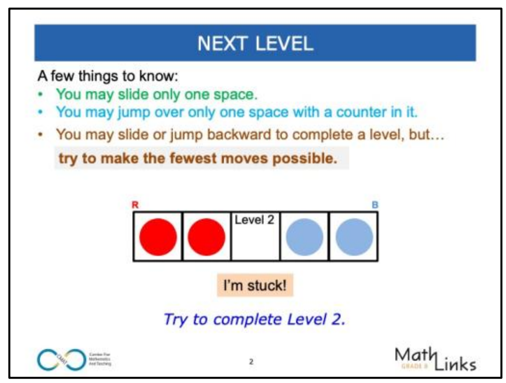
  Demonstrate the start of Level 1 and ask students to finish on their own. This will be accomplished quickly.



- Slide 2: Level 2 is trickier. Demonstrate "getting stuck" while discussing the rules and the goal of making the fewest moves per level. Allow adequate time to play with this level.

  What happens when colors "bunch up?" What can we do to keep this from happening? We get stuck, so we should take care to keep colors separate at the start.

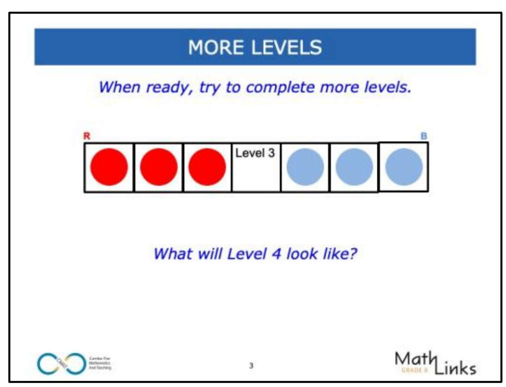
  How many moves does it take to complete Level 2? 8 Encourage students to demonstrate, both for "proof," and to assist those who want the help.



- Slide 3: Level 3 is shown here. Check for understanding by asking students what Level 4 will look like.

  Consider efficient and effective uses of time. All students will not move at the same pace, and it may not be realistic to expect all students to successfully complete each level. Students who move quickly may want to attempt Level 5.

  Pause here for more discussion, student demonstration, and asking of questions before moving on.



LESSON NOTES S4.0: SLIDES AND JUMPS

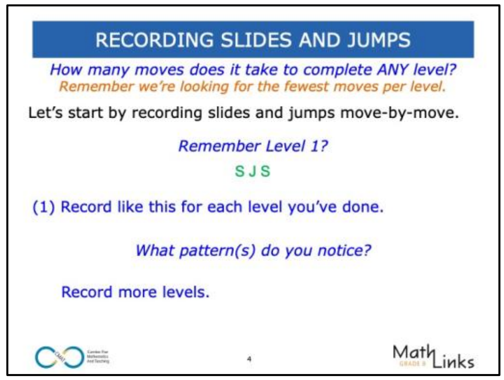
Continued

- Slide 4: ***How many moves does it take to complete ANY level?*** This question is intended to make students think in terms of generalizing and they will answer it later.

  When students were playing the game, they may not have been thinking much about the number of moves, or whether those moves were slides or jumps. Discuss a recording method prior to posing (1). If writing "S's" and "J's" doesn't work for some students, suggest they choose something that better suits them.

  After recording at least three levels, begin to discuss patterns. This will allow anyone to record to Level 5 or beyond without even completing those levels.

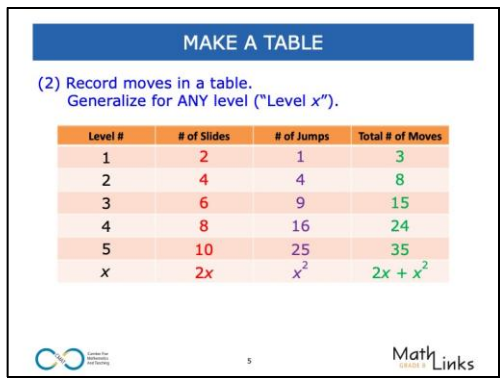
  What pattern(s) do you notice? Answers may vary. One possible answer: Level 1 is simply S J S. Then each subsequent level begins and ends with S J S. In between, Level 2 has 2 J's. Level 3 has 2J's, 3J's, 2J's, each separated by a single S. Level 4 has 2J's, 3J's, 4 J's, 3J's, 2J's, each separated by a single S. And so on.



- Slide 5: For (2), students record the numbers of slides, jumps, and total moves in a table through Level 5, and generalize for Level x. Allow for student work-time to write each variable expression.

  Share and discuss. The completed table is on this slide.

  Problem 3 demonstrates the power of algebra. Once generalized, the work with larger values is much easier to handle.



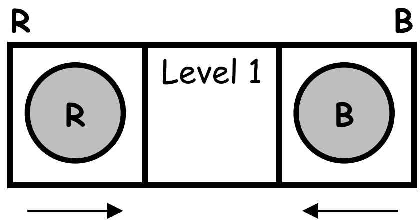
SLIDE DECK ALTERNATIVE S4.0: SLIDES AND JUMPS

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slides 1 – 2

Here's the challenge:

By sliding and jumping, get the colored markers to switch positions

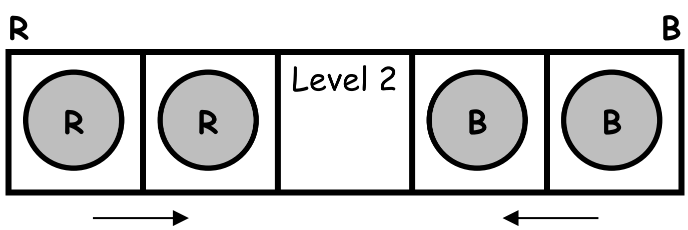


Try to complete Level 1

A few things to know:

- You may slide only one space
- You may jump over only one space with a counter in it
- You may slide or jump backward to complete a level, but… try to make the fewest moves possible

Try to complete Level 2

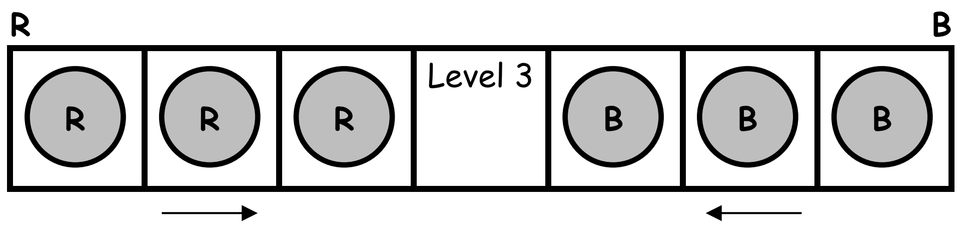


SLIDE DECK ALTERNATIVE S4.0: SLIDES AND JUMPS

Continued

Slide 3

When ready, try to complete more levels.



What will Level 4 look like?

Slides 4 – 5

How many moves does it take to complete ANY level?
Remember, we're looking for the fewest moves per level.

Let's start by recording slides and jumps move-by-move.

What was it for Level 1?

(1) Record moves for each level you've completed.

What pattern(s) do you notice?

Given the pattern(s) you noticed, record up through Level 5, even though you may not have physically completed all the levels.

(2) Record moves in a table. Generalize for ANY level (Level x).

Level #	# of Slides	# of Jumps	Total # of moves
1			
2			
3			
4			
5			
x			

MULTIPLE REPRESENTATIONS

We will use words, pictures, tables of numbers, and graphs to represent, describe, and analyze situations involving area and money.

[8.EE.5, 8.F.2, 8.F.3, 8.F.4; SMP1, 2, 3, 4, 7, 8]

GETTING STARTED

Fill in missing numbers and blanks based on the suggested numerical patterns. In the tables below, the *x*-value is considered the input value (independent variable) and the *y*-value is the output value (dependent variable).

1. Table I

x	0	1	2	3	4	5	6
y	0	4	8	12	16	20	24

a. Rate of change: for every increase of x by 1, y increases by __4__.

b. Input-output rule (words): multiply the *x*-value by __4__ to get the corresponding *y*-value.

c. Input-output rule (equation): y = __4x__. When $x = 0$, y = __0__.

2. Table II

x	0	1	2	3	4	5	6
y	1	5	9	13	17	21	25

a. Rate of change: for every increase of x by 1, y increases by __4__.

b. Input-output rule (words): multiply the *x*-value by __4__, then __add 1__ to get the corresponding *y*-value.

c. Input-output rule (equation): y = __4x + 1__. When $x = 0$, y = __1__.

3. Table III

x	0	1	2	3	4	5	6
y	-1	3	7	11	15	19	23

a. Rate of change: for every increase of x by 1, y increases by __4__.

b. Input-output rule (words): multiply the *x*-value by __4__, then __subtract 1__ to get the corresponding *y*-value.

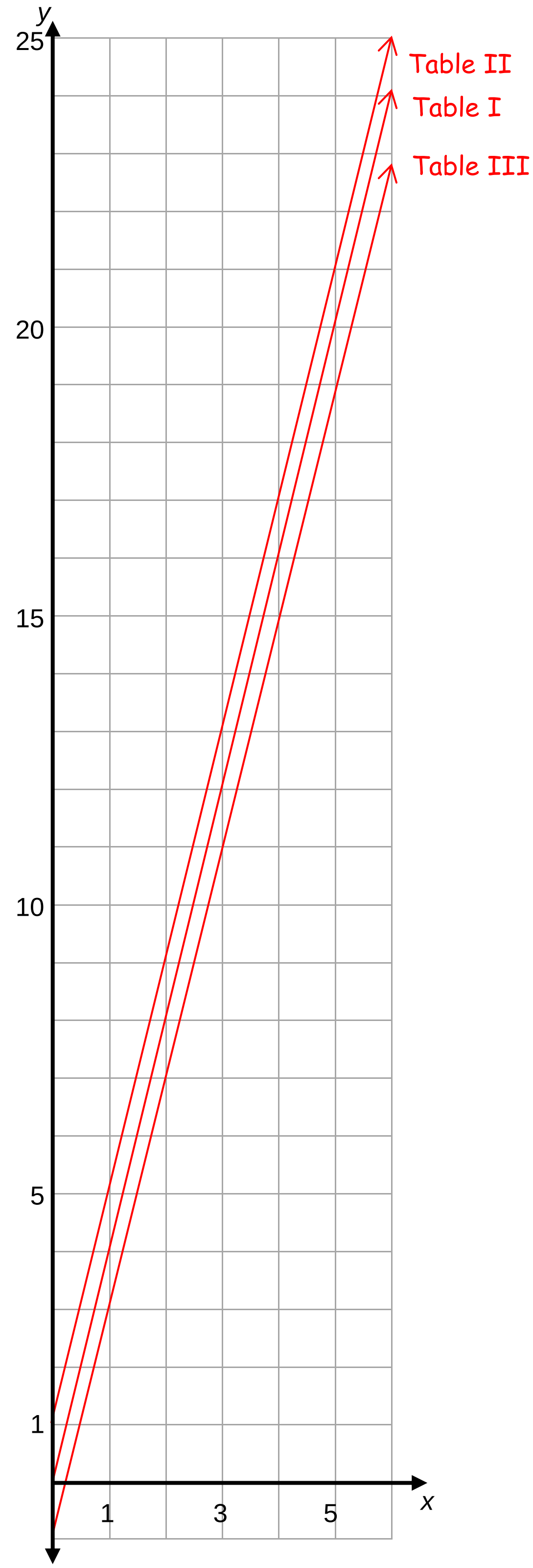
c. Input-output rule (equation): y = __4x - 1__. When $x = 0$, y = __-1__.

4. Record the meaning of input-output rule in **My Word Bank**.

INTERPRETING TABLES, EQUATIONS AND GRAPHS

The *MathLinks* Rubric: See Activity Routines on the Teacher Portal for directions. [SMP1, 2, 3, 7, 8]

Answers may vary. Some possible answers:



1. Graph each of the tables of numbers from **Getting Started** and connect the points with a line.

   Clearly label all three lines.

2. Describe each table of numbers. What is…

   - the same:
     As each x-value increases by 1 (from 0 to 6), each y-value increases by 4.

   - different:
     Since the initial y-values are different, the subsequent y-values are different.

3. Describe each equation. What is…

   - the same:
     All have an x-term with 4 as a coefficient.

   - different:
     All have a different constant term (0, 1, and -1, respectively).

4. Describe each graph. What is…

   - the same:
     All are straight lines. They are parallel, because they each increase vertically by 4 for each horizontal increase of 1.

   - different:
     Due to different initial values, they all intersect the y-axis at different places.

5. Explain which table of values/equation/graph describes a proportional relationship.
   Table 1 represents a proportional relationship because the y-values are multiples of the x-values, the equation is in the form $y = kx$, and the graph is a line through the origin.

6. Record the meanings of <u>proportional</u> and <u>proportional relationship</u> in **My Word Bank.**

THE POOL PROBLEM

[SMP1, 2, 3, 4, 5]

Follow your teacher's directions.

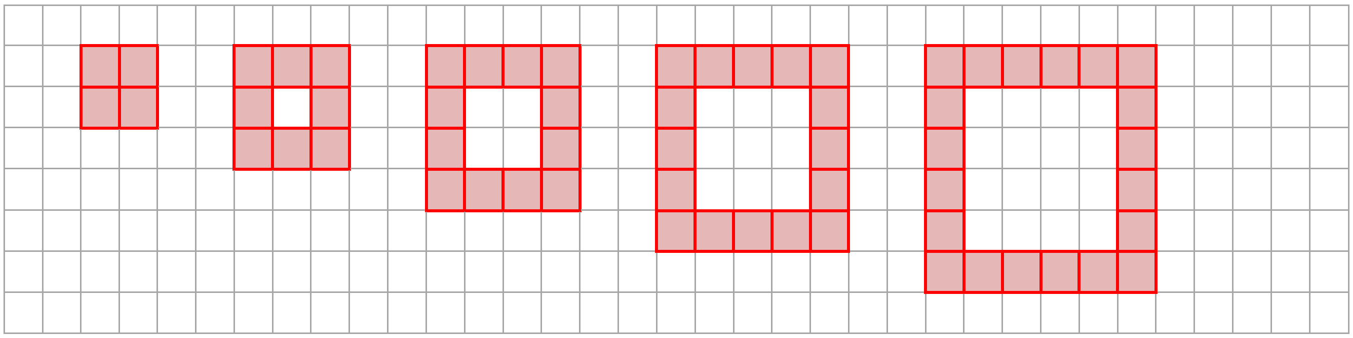
(1) ***Describe in words the way the pattern is growing.*** Answers may vary. Some possible answers:

- Border: the "outer" small squares create a 3×3 square, a 4×4 square, and a 5×5 square
- Pool: Area is a 1×1 square, a 2×2 square, and a 3×3 square

(2) – (4) Draw pools 1 – 4; then go back to "Pool 0;" make tables; make graphs.

(2) Picture

Pool 0 Pool 1 Pool 2 Pool 3 Pool 4



(3) Table

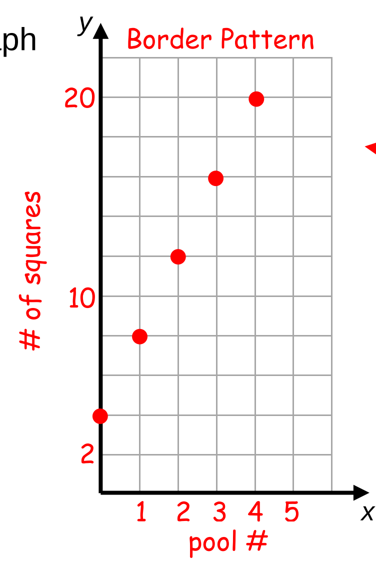
Border Pattern	
pool # (*x*)	# of squares (*y*)
0	4
1	8
2	12
3	16
4	20
5	24

Leave blank till later

(3) Table

Water Pattern	
pool # (*x*)	# of squares (*y*)
0	0
1	1
2	4
3	9
4	16
5	25

(4) Graph



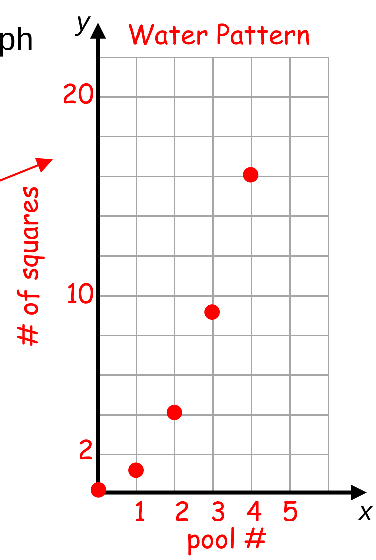
scaling may vary

linear

nonlinear

trend lines/curves may be drawn to stress these (non-proportional) relationships

(4) Graph



Square tiles (optional)

S4.1a

2+

LESSON NOTES S4.1a: THE POOL PROBLEM

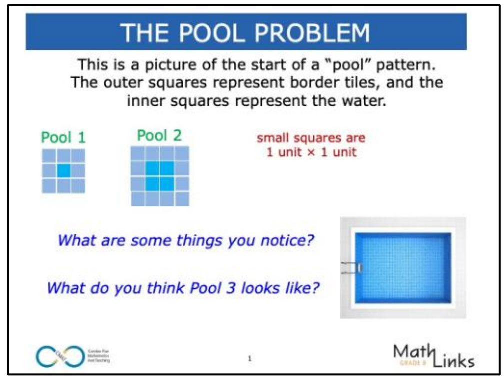
On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

We focus on growing visual patterns as we record numerical data in tables and graphs, use an area context that shows a relation between two variables, and explore rates of change. This work will continue in coming lessons to formalize the study of linear functions. Only one of the relationships in this part of the lesson is linear for contrast, and neither are proportional. If building pools, use two colors of square tiles. When drawing them, shade the border squares and leave the water squares unshaded.

- Slide 1: Students observe the first two pools in a pattern, and are directed to focus on the outer border squares and the inner water squares (small squares are 1×1).

  What do you notice? Possibilities include that Pool 1 is 3×3 and Pool 2 is 4×4; both are increasing (border squares from 8 to 12, and water squares from 1 to 4); water squares form perfect square numbers; the pool number corresponds to the side length of the water square.

  What do you think Pool 3 looks like? Dignify all responses and ask for justification.



- Slide 2: Reveal Pool 3. Tell students that this is the pattern that will be studied now. If other patterns of interest emerge from the discussion, they may be explored later.

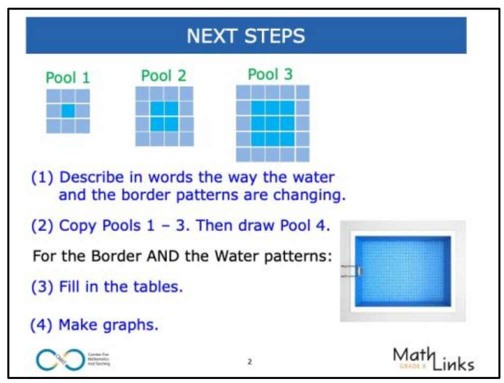
  For (1), students describe the changes they see in writing.

  For (2), students copy Pools 1 – 3 onto their grid paper, and then draw Pool 4.

  For (3) and (4), students fill in tables and draw graphs. Be sure to discuss appropriate table headings, use of variables, and graph labels and scale.

  Why does it make sense to NOT connect points with a line or curve? The context only lends itself to whole numbers of pools. But trend lines may be drawn to show the linear and nonlinear patterns.

  Follow-up problems are on the next student page.



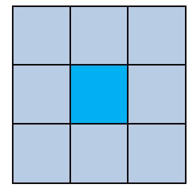
SLIDE DECK ALTERNATIVE S4.1a: THE POOL PROBLEM

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction.
The Slide Deck Alternative offers a modified option.

Slides 1 – 2

This is a picture of the start of a "pool" pattern.

Pool 1

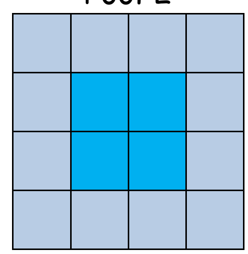


(Pool 1, and then Pool 2. The small squares are each 1×1.)

The outer squares represent border tiles.

The inner squares represent the water.

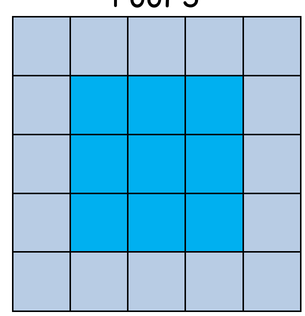
Pool 2



What are some things you notice?

What do you think Pool 3 looks like?

Pool 3



(1) Describe in words the way the water and the border patterns are changing.

(2) Copy Pools 1 – 3. Then draw Pool 4.

For the Border AND the Water Pattern:

(3) Fill in the tables.

(4) Draw graphs.

ANALYZING THE POOL PROBLEM

[SMP1, 2, 4, 7, 8]

Refer to **The Pool Problem**.

Answers may vary. Some possible answers:

1. Draw and label “Pool 0” in the space to the left of Pool 1. How is it different from the other pools? Following the patterns backward, it must have4 border squares and 0 water squares. There is no water.

2. In the top row of the tables, write the entries for Pool 0. Graph the points for Pool 0.

3. When does the Border Pattern have more squares than the Water Pattern?
   Pools 0 – 4.

   When does the Water Pattern have more squares?
   From pool 5 on.

4. Write equations to represent the number of squares for each pool number.

   a. Border: $y = 4x + 4$ b. Water: $y = x^2$

5. Write the number of squares for each pattern for Pool 20.

   a. Border: 84 b. Water: 400

6. Which pool has 48 border squares and 121 water squares? Pool 11

7. Explain what (0, 4) and (0, 0) represent in the context of **The Pool Problem**.
   Pool 0 has 4 border squares and 0 water squares.

   Where are these points found on the graphs? On the y-axis.

8. Does the Border Pattern grow at a constant rate? Explain.
   Yes. There is an increase of 4 squares for each increase of 1 pool # in the table and on the graph (a straight line), which is a constant rate of increase.

   Does it represent a proportional relationship? Explain.
   No. The straight line does not go through the origin (the equation is not in the form $y = ax$). In the table, outputs do not represent constant multiples of inputs.

9. Does the Water Pattern grow at a constant rate? Explain.
   No. The increases are by 1, then 3, then 5, then 7, … in the table. The graph is a curve that increases at a slower/flatter rate at the start, and then becomes quicker/steeper.

   Does it represent a proportional reationship? Explain.
   No. The graph goes through the origin, but the table entries are not constant multiples of one another. The equation involves squaring (not in the form $y = ax$).

2+

Journal Problem 8, 9

PRACTICE 1

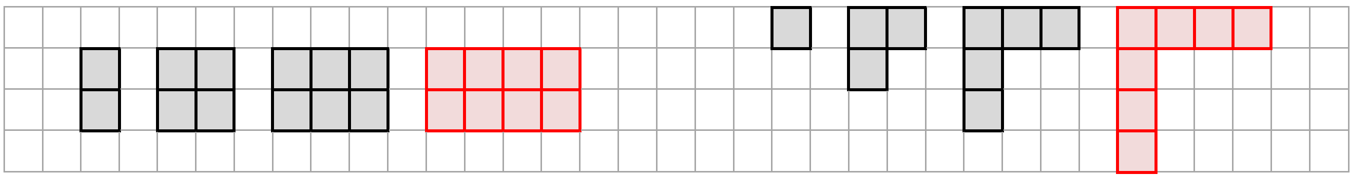
For enrichment and challenge, use "Function Frenzy" Puzzles (Portal access → Puzzles and Games) [SMP1, 2, 3]

1. Below are two different square patterns. Draw the 4th step for each pattern. Fill in the tables. Draw graphs with titles and labels.

Watch for (1): Does the information match for each respective pattern, table, and graph?

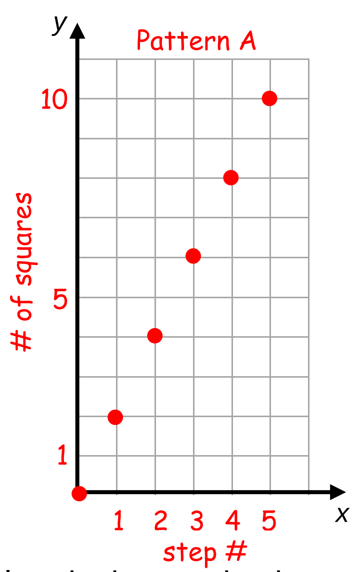
Pattern A
Step numbers:
1 2 3 4

Pattern B
Step numbers:
1 2 3 4

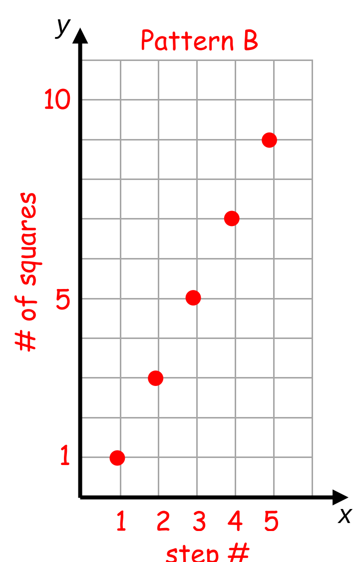


Pattern A	
step # (x)	# of squares (y)
0	0
1	2
2	4
3	6
4	8
5	10

Pattern B	
step # (x)	# of squares (y)
0	-1
1	1
2	3
3	5
4	7
5	9



trend lines may be drawn to stress linear relationships



2. If not already done, write the entries for step 0 in the top row of the tables.

3. Given the pictures and numbers, does "step 0" make sense for either pattern?
   For A, it could makes sense that step 0 has 0 squares. For B, it does not make sense that step 0 has a negative number of squares.

4. Graph a point for step 0 only if it makes sense for the pattern.

PRACTICE 1

Continued

Answers may vary. Some possible answers:

5. Write equations to represent the number of squares for each pattern.

   a. Pattern A: $y = 2x$

   b. Pattern B: $y = 2x - 1$

6. For Pattern A, find…

   a. The number of squares in step 80: 160

   b. The step number for 200 squares: 100

7. For Pattern B, find…

   a. The number of squares in step 70: 139

   b. The step number for 101 squares: 51

8. Consider the tables, graphs, and rules used to represent both patterns.

   a. List some things that are the same for both.
   For each step increase of 1, there is an increase of 2 squares. The graphs are straight lines. The coefficients of x in the equations are 2.

   b. List some things that are different for both.
   The corresponing values in the table for B is always 1 less than for A. The line for A goes through the origin; this is not true for B. The equations for A and B are in different forms ($y = ax$ vs $y = ax + b$).

9. Why does Pattern A represent a proportional relationship, while Pattern B does NOT?
   A represents a proportional relationship because all the entries in the table are constant multiples of one another, the graph is a line through the origin, and its equation is in the form $y = ax$. None of this is true for B.

10. For both patterns:

    a. In the tables, as the x-value increases by 1, the y-value increases by __2__.

    b. On the graphs, the y-coordinate moves up by __2__ as the x-coordinate moves right by 1.

    c. For the equations, the coefficient of x is __2__.

1+

Journal Problem 8

SAVING VS SPENDING

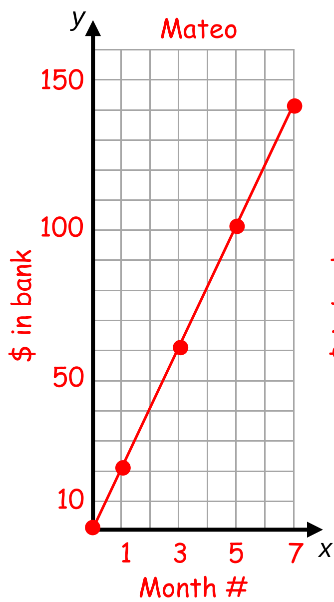
[SMP1, 2, 3, 4, 8]

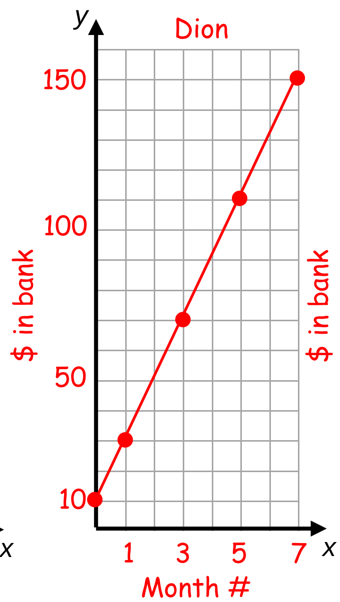
Follow your teacher's directions for (1) – (3).

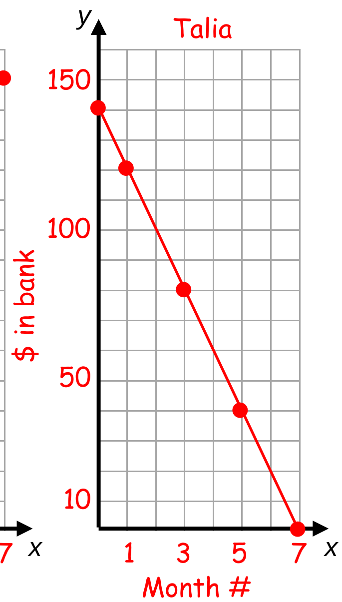
(1) ***Complete the table (all increases and decreases are at a constant rate).***

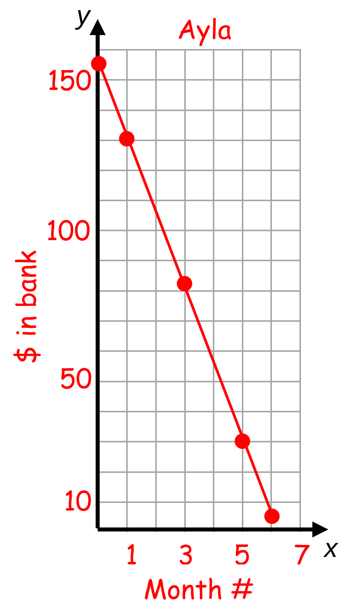
Month # (x)	**Mateo** $ in bank (y)	**Dion** $ in bank (y)	**Talia** $ in bank (y)	**Ayla** $ in bank (y)
0	0	10	140	155
1	20	30	120	130
2	40	50	100	105
3	60	70	80	80
4	80	90	60	55
5	100	110	40	30
6	120	130	20	5
7	140	150	0	-20

(2) ***Draw graphs of each student's data. Label appropriately.*** Some points are graphed. Trend lines are shown.









(3) ***Write equations to represent each student's data.***

Mateo: $y = 20x$ Dion: $y = 20x + 10$ Talia: $y = -20x + 140$ Ayla: $y = -25x + 155$

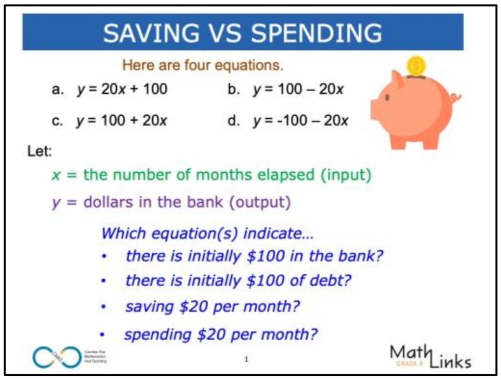
4. Record the meaning of <u>*y*-intercept</u> in **My Word Bank**.

LESSON NOTES S4.1b: SAVING VS SPENDING

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

Students continue to use multiple representations to make connections that are important in the study of linear functions. The context used here allows for discussions about functions that increase as well as those that decrease, and an introduction to the y-intercept (as an initial value), and slope (informally, as a rate of change). Slope is formally defined in the next unit.

- Slide 1: Students observe four equations for discussion. After the variables are defined (elapsed time in months and money in the bank in dollars), ask students to analyze and explain the meaning of the equations in terms of the context provided.

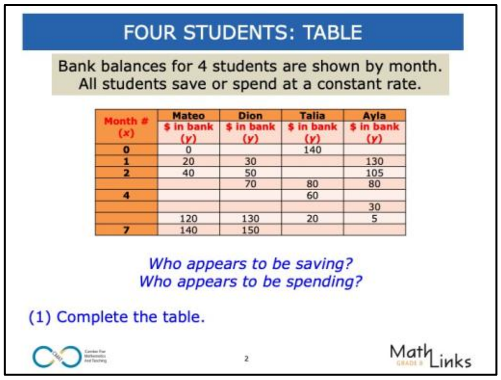
  Which of these equations could represent starting with a $100 initial value? Equations a, b, and c.

  How could the -100 be interpreted for equation d? An initial value of -$100 could represent $100 of debt.

  Of these, which could represent saving $20 per month? Since saving is an increase in money, equations a and c, which are equivalent.

  Which of these could represent spending $20 per month? Since spending is a decrease in money, equations b and d.

- Slide 2: Introduce monthly balance data for four students. This data does NOT correspond to the equations on the previous slide, but the discussion about those equations are very relevant to this data. Ask students to analyze the table that has missing data.

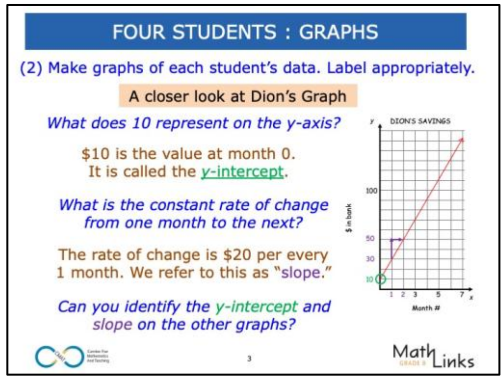
  Who appears to be saving? Mateo and Dion. ***Who appears to be spending?*** Talia and Ayla.

  For (1), students complete the table.

LESSON NOTES S4.1b: SAVING VS SPENDING

Continued

- Slide 3: Pose (2). Allow time for drawing graphs and discussion. Use the context to INFORMALLY introduce the concepts of *y*-intercept (the initial amount in the bank - where the graph intersects the *y*-axis), and slope (the change in dollar amount from one month to the next).

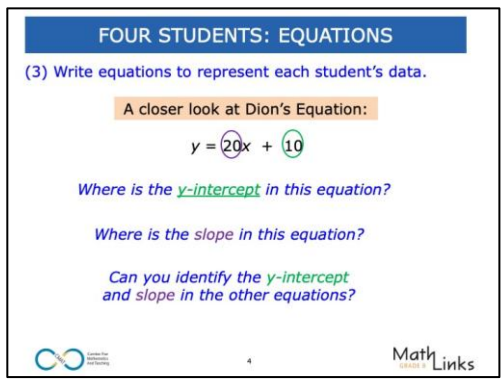


  On the graphs, where are the initial values located? On the *y*-axis; the *y*-value where $x = 0$. This number is called the <u>*y*-intercept</u>. For Dion, the initial amount of money in the bank is $10, which is indicated by the point (0, 10). We say the *y*-intercept is 10.

  On the graph, how can you determine the rate of change from one month to the next? Look at the vertical change per unit of horizontal change. We will refer to this as "slope," and define it more formally in the next unit. For Mateo and Dion, this is an increase of $20 each month.

  Ask students to identify the initial values (*y*-intercept) and rates of change (slope) on the other three graphs.

- Slide 4: Pose (3). Allow time for writing equations and discussion.

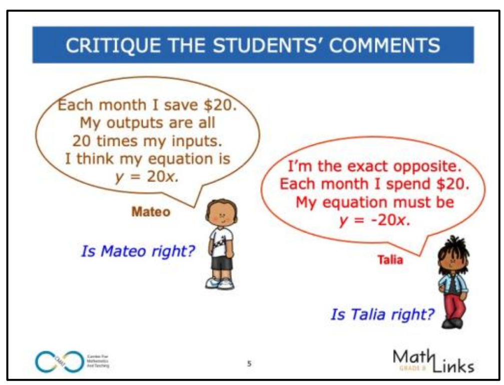


  How can you determine the initial value (or y-intercept) from the equation? It's the constant term that's "added on" to the *x*-term.

  How can you determine the rate of change (or slope) from the equation? It's the coefficient of *x*.

  Ask students to identify the initial values (*y*-intercept) and rates of change (slope) of the other three equations.

- Slide 5: Critique student statements.



  Is Mateo right? Students may substitute values to verify this equation is correct. ***Is Talia right?*** Students may substitute values to verify this equation is NOT correct. The initial value of $y = 140$ must factor in.

SLIDE DECK ALTERNATIVE S4.1b: SAVING VS SPENDING

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slide 1

Here are four equations.

a. $y = 20x + 100$ b. $y = 100 - 20x$

c. $y = 100 + 20x$ d. $y = -100 - 20x$

Let x = the number of months elapsed (input; independent variable)
Let y = dollars in the bank (output; dependent variable)
money is being saved or spent.

Which equation(s) indicate …

- *there is initially $100 in the bank?*
- *there is initially a $100 debt?*
- *a savings of $20 per month?*
- *a spending of $20 per month?*

Slide 2

STUDENT SAVING AND SPENDING

Month # (x)	Mateo $ in bank (y)	Dion $ in bank (y)	Talia $ in bank (y)	Ayla $ in bank (y)
0	0		140	
1	20	30		130
2	40	50		105
		70	80	80
4			60	
				30
	120	130	20	5
7	140	150		

Who appears to be saving?
Who appears to be spending?

(1) Complete the table.

SLIDE DECK ALTERNATIVE S4.1b: SAVING VS SPENDING
Continued

Slide 3 – 4

(2) Draw graphs of each student's data. Label each graph appropriately.

For Dion's graph, ***what does 10 on the y-axis represent?***

$10 is the value at month 0. This is the <u>y-intercept</u>.

What is the constant rate of change from one month to the next?

Rate of change is $20 per month. We refer to this as the "slope."

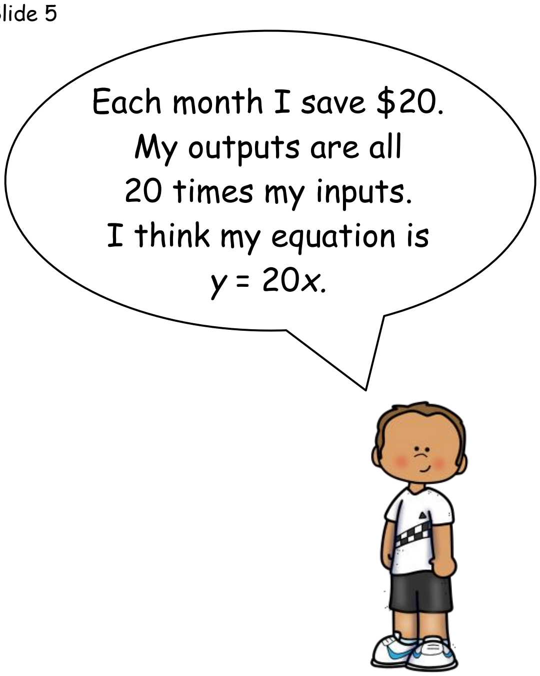
Can you identify the y-intercept and slope on the other graphs?

(3) Write equations to represent each student's data.

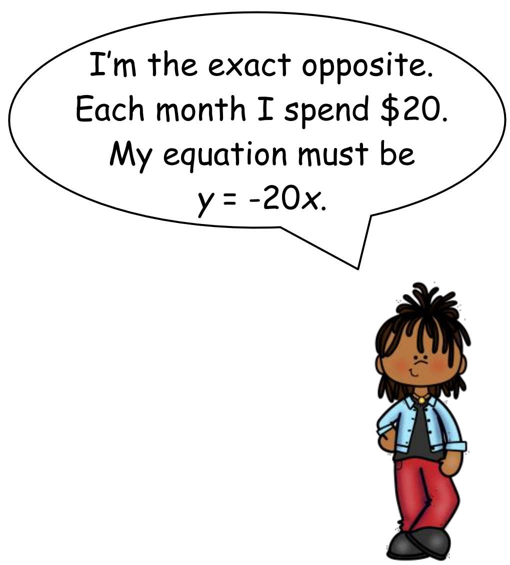
For Dion's equation, where is the y-intercept? The slope?

Can you identify the y-intercept and slope in these equations?

Slide 5



Is Mateo right?



Is Talia right?

ANALYZING SAVING VS SPENDING

The *MathLinks* Rubric: See Activity Routines on the Teacher Portal for directions. [SMP1, 2, 3, 4, 7]

Refer to **Saving vs Spending** on the previous page. As a convention, we read graphs from left to right.

1. For which students do table values and graphs appear to be increasing? Mateo and Dion

   Decreasing? Talia and Ayla

2. Compare tables and graphs for pairs of students.

Compare:	How are they the same?	How are they different?
Mateo and Dion	Both are increasing by \$20 per month. Both have straight, increasing lines graphed. The lines appear to be parallel.	For Mateo, the initial amount is \$0. For Dion, initial amount is \$10.
Dion and Talia	Both are changing by \$20 per month; both have straight lines graphed.	For Dion there is an increase in values (money saved) and the graph is increasing. For Talia there's a decrease in values (money spent) and the graph is decreasing.
Talia and Ayla	Both are spending (decreasing) an amount of money each month. Both are straight lines.	Ayla starts with more money. Ayla is spending faster than Talia.

3. How do initial amounts (in these cases, this is Month 0) appear to be shown in the equations?
   These seem to be the values added to the x-term. For Mateo, this is \$0; for Dion, this is \$10; etc.

4. How do rates of change (slope) appear in the equations?
   They seem to be the coefficients of x. For Talia, this is -20; for Ayla, this is -25; etc.

5. Do any of these situations represent a proportional relationship? Explain.
   Mateo has the only table in which all entries are constant multiples of one another, the only graph that is a straight line through the origin, and the only equation in the form $y = ax$.

PRACTICE 2

[SMP1, 2, 3]

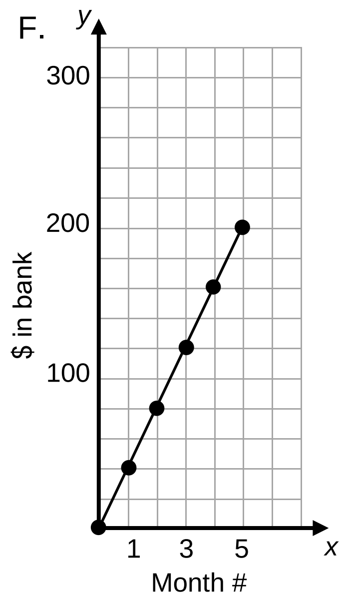
Use the representations A – G to fill in the table below.

A. With an initial value of $260, Jocelyn's bank account balance goes down at a constant rate of $30 per month.	B. With an initial value of $50 Jayson's bank account balance goes up at a constant rate of $20 per month.
C. $y = 25x + 100$ (let x be month #, and y be $ in bank)	D. $y = -30x + 280$ (let x be month #, and y be $ in bank)

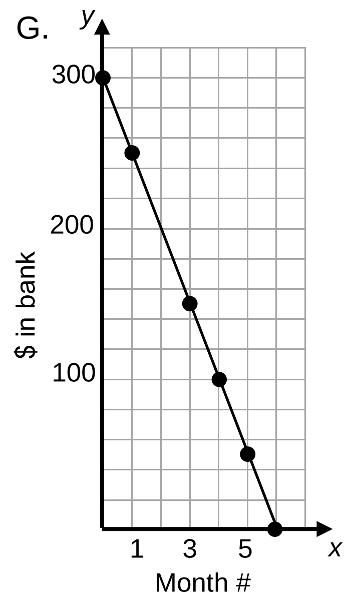
E.

Month # (x)	$ in bank (y)
0	260
1	220
2	180
3	140
4	100
5	60
6	20

F.



G.



	An increase or a decrease?	Initial value ($ at month 0)	Rate of change (slope)	A proportional relationship?
A.	decrease	$260	down $30/mo (-30)	no
B.	increase	$50	up $20/mo (20)	no
C.	increase	$100	up $25/mo (25)	no
D.	decrease	$280	down $30/mo (-30)	no
E.	decrease	$260	down $40/mo (-40)	no
F.	increase	$0	up $40/mo (40)	yes
G.	decrease	$300	down $50/mo (-50)	no

FUNCTION REPRESENTATIONS

We will explore the concept of a function. We will define function and graph of a function. We will describe examples of functions and examples of non-functions.

[8.F.1, 8.F.3, 8.F.5; SMP1, 2, 4, 5, 6]

GETTING STARTED

1. Refer to the table to the right about professional football in the 1970s.

Year	Football Champion
1970	Kansas City
1971	Baltimore
1972	Dallas
1973	Miami
1974	Miami
1975	Pittsburgh
1976	Pittsburgh
1977	Oakland
1978	Dallas
1979	Pittsburgh

   a. Which cities won exactly once?
      Kansas City, Baltimore, and Oakland

   b. Which cities won exactly twice?
      Dallas and Miami

   c. Which city won exactly three times?
      Pittsburgh

   d. If you are given a specific year (input), can you always tell which team won (output)? yes
      Give an example.
      Given the year 1970, we know that Kansas City won the championship.

   e. If you are given a specific team (input), can you always tell which one year they won (output)? no Give an example.
      Given Dallas, we do not know if the year is 1972 or 1978 because they won in both years.

2. For the equation $y = x + 1$, fill in the table, write ordered pairs to correspond with table entries, and draw a graph.

Table:

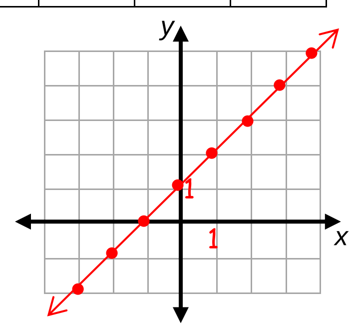
input (x)	3	2	1	0	-1	-2	-3
output (y)	4	3	2	1	0	-1	-2

Ordered Pairs:

(3, 4) (2, 3), (1, 2), (0, 1),

(-1, 0), (-2, -1), (-3, -2)

Graph:

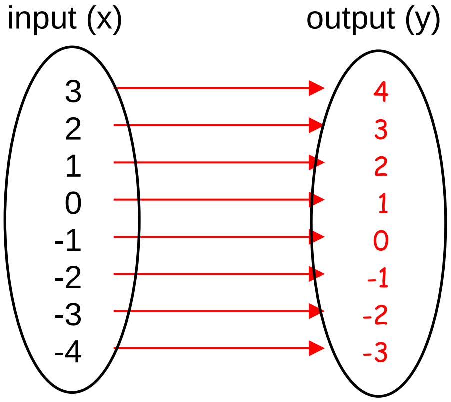


WHAT IS A FUNCTION?

[SMP1, 2, 6]

Follow your teacher's directions for (1) – (6).

(1) ***Represent the equation $y = x + 1$ with a mapping diagram.***



(2) ***Look at the representations in Getting Started and in this slide deck. Explain why $y = x + 1$ is a function.***

- With every table input, we can only get one output.
- Any ordered pair's x-value has exactly one y-value that corresponds to it.
- Every graphed x-value has exactly one y-value that corresponds to it.
- Each input in the diagram maps to exactly one output.

(3) ***Represent the equation $y = x^2 + 1$ with a table, ordered pairs, graph, and mapping diagram.***

Equation: $y = x^2 + 1$

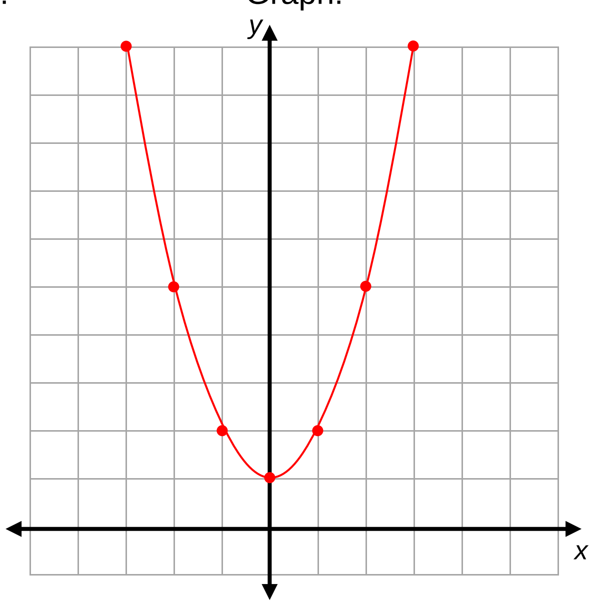
Table:

input (x)	3	2	1	0	-1	-2	-3
output (y)	10	5	2	1	2	5	10

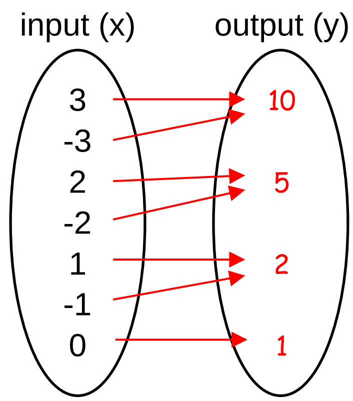
Ordered Pairs:

(3, 10), (2, 5),
(1, 2), (0, 1),
(-1, 2), (-2, 5),
(-3, 10)

Graph:



Mapping Diagram:



(4) ***Is $y = x^2 + 1$ a function? Explain.***

Yes. Every x-value is matched with exactly one y-value.

WHAT IS A FUNCTION?

Continued

(5) ***The graph for $x = y^2 + 1$ is shown. Use it to fill in the table, write ordered pairs, and create a mapping diagram.***

Equation: $x = y^2 + 1$

Order may vary for table entries.

Table:

input (x)	10	5	2	1	2	5	10
output (y)	3	2	1	0	-1	-2	-3

Ordered Pairs:

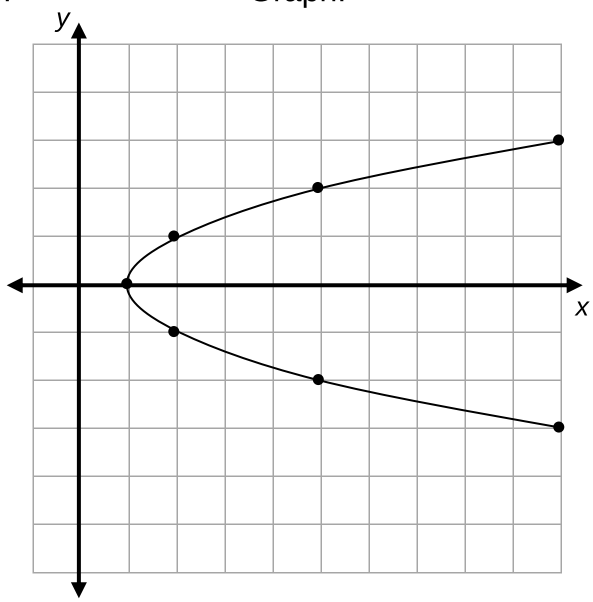
(1, 0),

(2, 1), (2, -1),

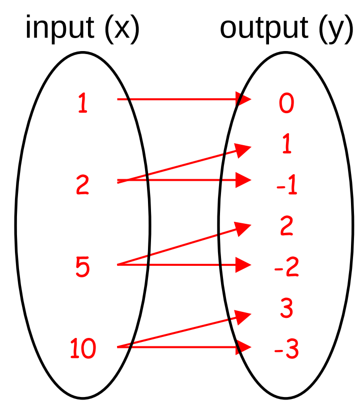
(5, 2), (5, -2),

(10, 3), (10, -3)

Graph:



Mapping Diagram:



(6) ***Is $x = y^2 + 1$ a function? Explain.***

No. Most of the x-values correspond to two different y-values. For example (2, 1) and (2, -1) are both on the graph.

7. Go back to the football champion problem in **Getting Started**. Does the situation represented by the ordered pairs (year, champion) represent a function? __Yes__

How about (champion, year)? __No__

Explain your reasoning.

If you give a year as input, then there is only one team that won. But if you give a team as input, there could be more than one year that they won. In other words, if you name a team, the year they won is not unique.

8. Record the meanings of <u>function</u> and <u>graph of a function</u> in **My Word Bank**.

LESSON NOTES S4.2: WHAT IS A FUNCTION?

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

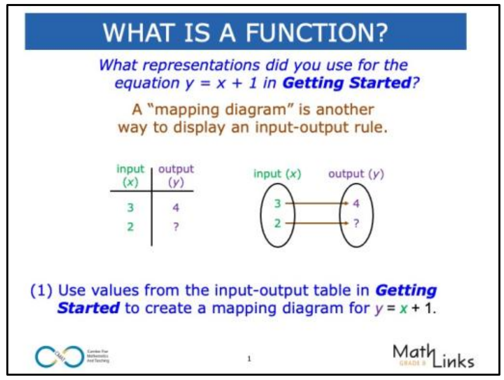
Students investigate three examples, each including multiple representations, to determine whether or not each situation represents a function.

- Slide 1: Discuss $y = x + 1$ from **Getting Started**.

  What others ways did we represent this equation? A table, a list of ordered pairs, and a graph.

  Introduce a mapping diagram and explain how it connects to other representations.

  Pose (1) and discuss as needed.

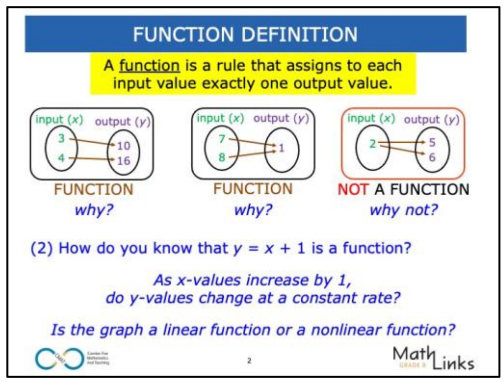


- Slide 2: Define <u>function</u> and use the mapping diagrams to illustrate its meaning.

  Why do the first two mapping diagrams represent functions? Each input is mapped to exactly one output.
  Why does the third one not represent a function? An input is mapped to more than one output.

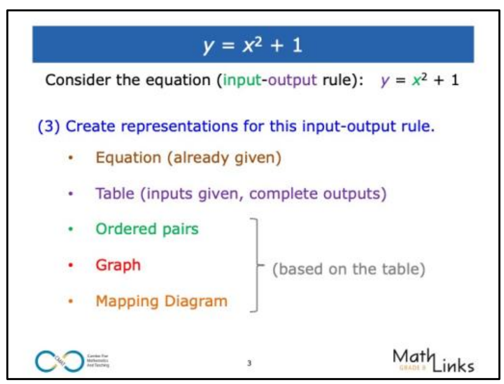
  Pose (2). Encourage students to explain orally and in writing why $y = x + 1$ is a function using specific examples from the five representations they have learned (equation, table, ordered pairs, graph, mapping diagram).

  As the x-values change by 1, how do the corresponding y-values change? Also by 1. ***Is the graph a linear function?*** Yes.



- Slide 3: For (3), students create representations for the equation $y = x^2 + 1$. Discuss as needed.

  How are a <u>function</u> and <u>graph of a function</u> different? Encourage students to look at the definitions. Function is a big idea in mathematics because a rule where each input maps to exactly one output is very useful. Graph of a function is an important way to display a function. Visually, we can tell if a graph is a function using a "vertical line test". If some vertical line intersects the set in more than one point, then the set is NOT the graph of a function. This can be tied back to the definition of function.

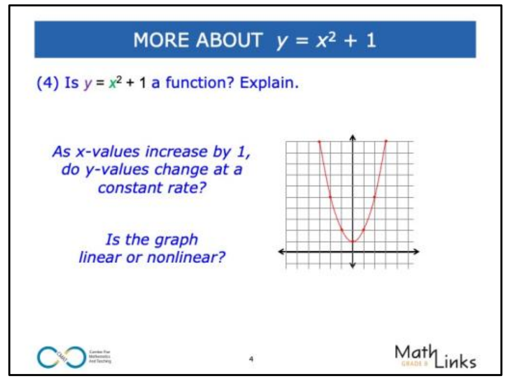


LESSON NOTES S4.2: WHAT IS A FUNCTION?

Continued

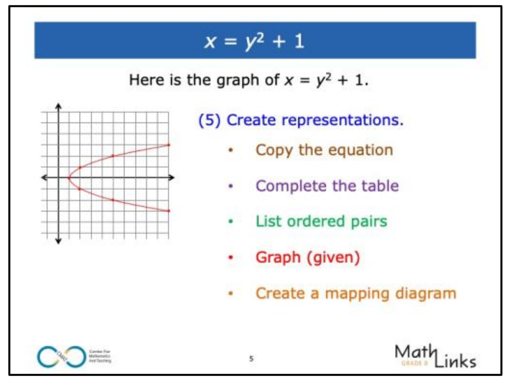
- Slide 4: Pose (4). Encourage students to explain orally and in writing why $y = x^2 + 1$ is a function using specific examples from the five representations.

  As x-values increase by 1, do y-values increase at a constant rate? No. On the left of the *y*-axis, the graph decreases (at a non-constant rate), and on the right of the *y*-axis it increases (at a non-constant rate). ***Is the graph linear?*** No, it "curves" (it's a parabola).



- Slide 5: For (5), students create representations for the equation $x = y^2 + 1$. Discuss as needed.

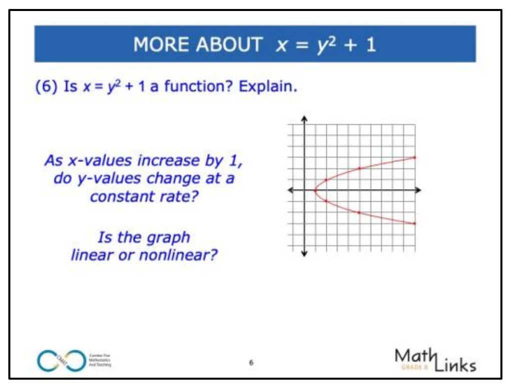
  When the input (the x-value) is 2, what is the output (y-values)? Students should realize there are two outputs, namely 1 and -1. Tables, ordered pairs, and mapping diagrams should reflect this.



- Slide 6: Pose (6). Encourage students to explain orally and in writing why $x = y^2 + 1$ is NOT a function using specific examples from the five representations.

  As x-values increase by 1, do y-values increase at a constant rate? No. Below the *x*-axis the graph decreases (at a non-constant rate), and above the *x*-axis it increases (at a non-constant rate). ***Is the graph linear?*** No.

  Look back at our three examples. What conditions do you think are necessary for rate of change to be a constant rate? Students may conjecture that for rate of change to be constant, the graphical display will be a line. This also connects to the nature of the variables in the equations we observed (whether an exponent other than 1 exists).



SLIDE DECK ALTERNATIVE S4.2 WHAT IS A FUNCTION?

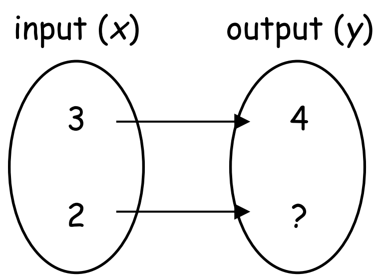
Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slides 1 – 2

Consider the equation (input-output rule): $y = x + 1$.

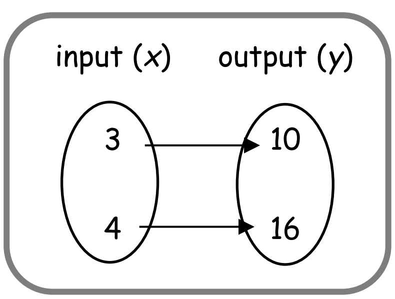
What other representations did you use for the equation $y = x + 1$ in ***Getting Started****?*

A “mapping diagram” is another way to display an input-output rule.

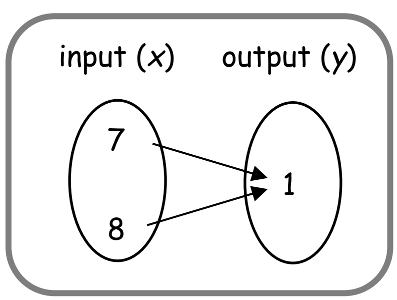


(1) Use values in your table to create a mapping diagram for $y = x + 1$.

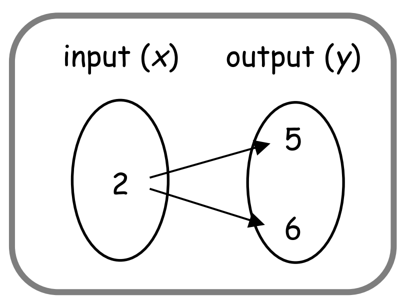
A <u>function</u> is a rule that assigns to each input value exactly one output value.



FUNCTION
why?



FUNCTION
why?



NOT A FUNCTION
why?

(2) How do you know that $y = x + 1$ represents a function?

As x-values increase by 1, do y-values change at a constant rate?

Is the graph a linear function or a nonlinear function?

SLIDE DECK ALTERNATIVE S4.2 WHAT IS A FUNCTION?

Continued

Slide 3 – 4

(3) Consider the equation $y = x^2 + 1$. Record five representations (equation, table, ordered pairs, graph, mapping diagram) for this input-output rule.

(4) Is $y = x^2 + 1$ a function? Explain.

As x-values increase by 1,
do y-values change at a constant rate?

Is the graph linear or a nonlinear?

Slides 5 – 6

(5) Consider the equation $x = y^2 + 1$. Record five representations (equation, table, ordered pairs, graph, mapping diagram) for this input-output rule.

(6) Is $x = y^2 + 1$ a function? Explain.

As x-values increase by 1,
do y-values change at a constant rate?

Is the graph linear or nonlinear?

PETS AND APARTMENTS

[SMP1, 2, 4, 5]

Mary has three pets, Kerry has one pet, and both Larry and Barry have no pets.

Let friend names be the input values.
Let the number of pets they each own be the output values.

1. Represent this situation with an input-output table, ordered pairs, and a mapping diagram.

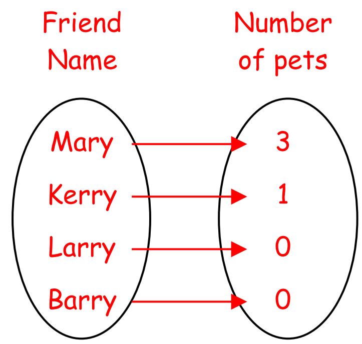
Input Output

Name	Number of pets
Mary	3
Kerry	1
Larry	0
Barry	0

Ordered pairs

(Mary, 3), (Kerry. 1), (Larry, 0), (Barry, 0)

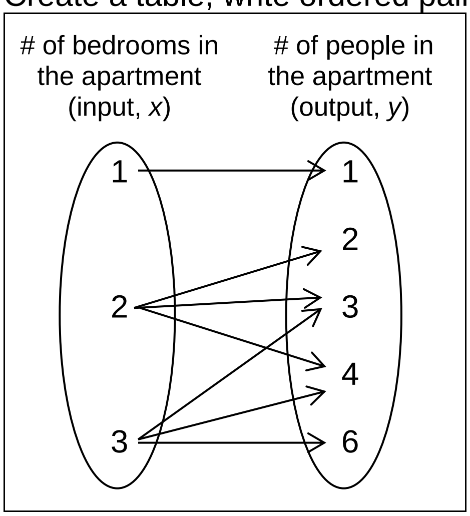
Mapping diagram



2. Explain why this situation represents a function.
Each friend can have only one specific number of pets. For example, if Mary has 3 pets, she cannot have a different number of pets also.

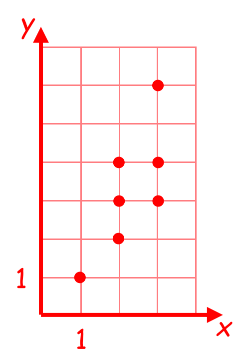
The mapping diagram below shows the number of bedrooms in an apartment building and the number of people who live in the apartment.

3. Create a table, write ordered pairs, and draw a graph for this situation.



Input (x) # of bedrooms in bldg.	Output (y) # of people per apt.
1	1
2	2
2	3
2	4
3	3
3	4
3	6

(1, 1), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (3, 6)



4. Does this situation represent a function? Explain.
No. For example, in 2-bedroom apartments, there are 2, 3, or 4 inhabitants.

PRACTICE 3

[SMP1, 2, 5]

Watch for (all): Are students applying the definition of function correctly for each representation?

1. Which of the following input-output tables could represent functions when x is used for the input value and y for the output value?

a. yes

x	y
0	4
3	6
6	4
9	6
12	4

b. no

x	y
0	10
0	9
2	8
2	7
4	6

c. no

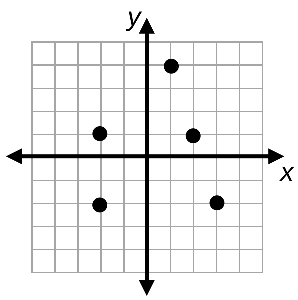
x	9	9	9	9	9
y	1	2	3	4	5

d. yes

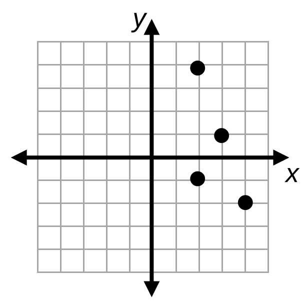
x	1	2	3	4	5
y	9	9	9	9	9

2. Which of the following graphs of points could represent functions?

a. yes



b. no



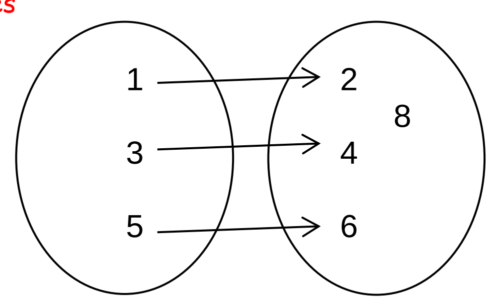
3. Which of the following sets of ordered pairs could represent functions?

a. (1, 5), (2, 6), (3, 5), (4, 6) yes

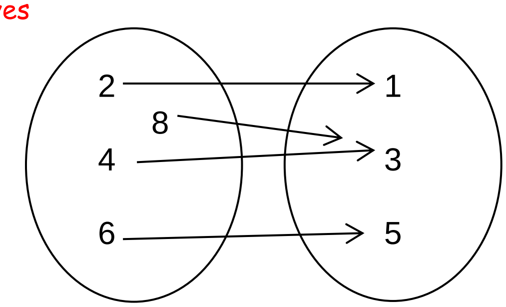
b. (10, -20), (-20, 10), (-10, -5), (10, 5) no

4. Which of the following mapping diagrams could represent functions?

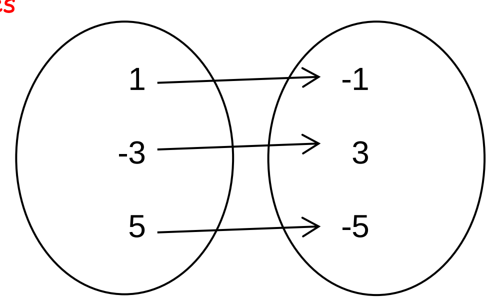
a. yes



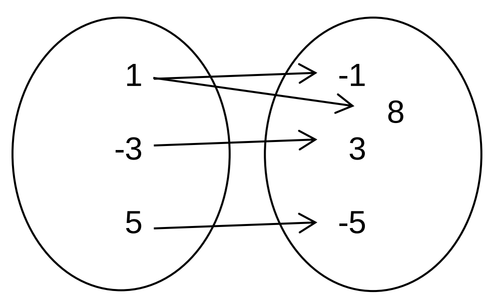
b. yes



c. yes



d. no

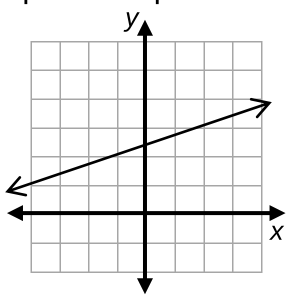


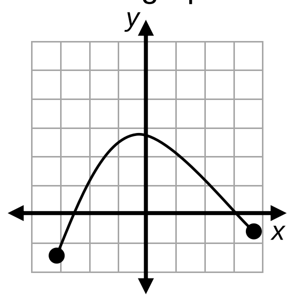
PRACTICE 4

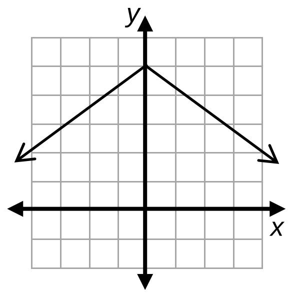
[SMP1, 2, 5]

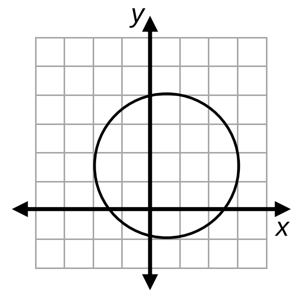
We say a function is **increasing** if a graph of the output values increases from left to right, and **decreasing** if a graph of the output values decreases from left to right. **Piece-wise** linear functions are considered to be linear.

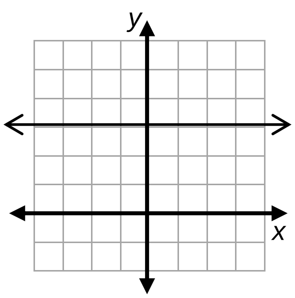
Complete the problems below about graphs A – H.

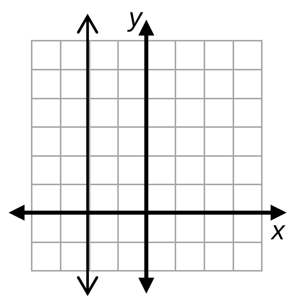
A.


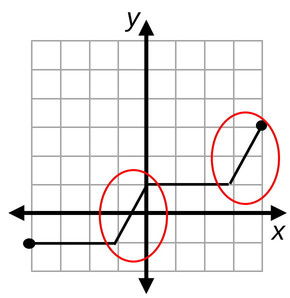
B.


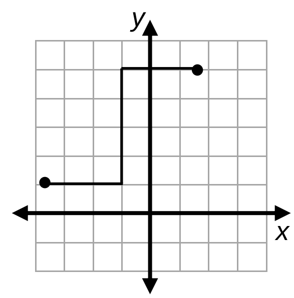
C.


D.


E.


F.


G.


H.


Watch for (1): Are students making sense of the vocabulary in relation to the graphs?

1. List all graphs that are:
   a. Linear, but NOT piece-wise linear. A, E, F
   b. Linear, but NOT a function. F, H
   c. A function, but nonlinear. B
   d. Nonlinear and not a function. D
   e. An increasing function that is not decreasing anywhere. A, G

2. Describe where the graph of C is increasing and where it is decreasing.
   C is increasing to the left of $x = 0$ (to the left of the y-axis), and it is decreasing to the right of $x = 0$ (to the right of the y-axis).

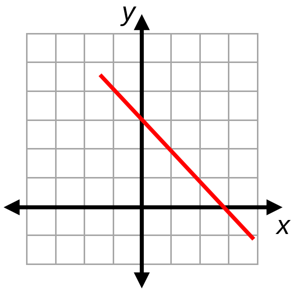
3. The graph of G has four line segments. How many of them are increasing? 2
   Circle those segments on the graph.

PRACTICE 5

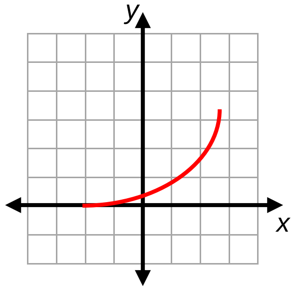
[SMP1, 2, 5]

Draw the four graphs as described. Graphs will vary. Some possible graphs:

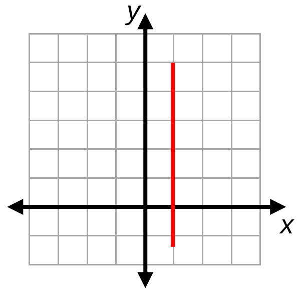
1. A decreasing linear function



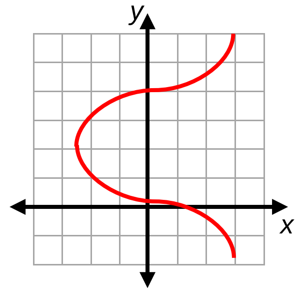
2. An increasing nonlinear function



3. A linear non-function



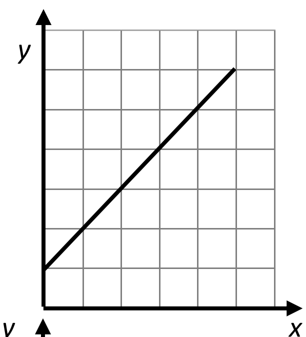
4. A nonlinear non-function



Estimate appropriate ordered pairs for each graph. Circle one **bold** choice for each bulleted statement. Ordered pairs may vary. Some possible answers:

5.

x	y
0	1
1	2
2	3
3	4
4	5
5	6

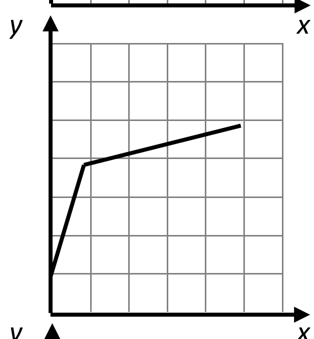


This graph:

- **could** **could not** represent a function
- is **increasing** **decreasing**
- is **linear** **nonlinear**

6.

x	y
0	1
1	4
2	4.2
3	4.5
4	4.8
5	5

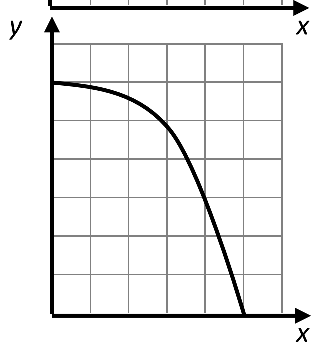


This graph:

- **could** **could not** represent a function
- is **increasing** **decreasing**
- is **linear** **nonlinear**

7.

x	y
0	6
1	5.9
2	5.6
3	5
4	3
5	0



This graph:

- **could** **could not** represent a function
- is **increasing** **decreasing**
- is **linear** **nonlinear**

8. Describe the change you observe in the table and graph in problem 7.
   As x-values increase by 1, y-values decrease slowly at first (by smaller amounts), and then more quickly (by larger amounts).

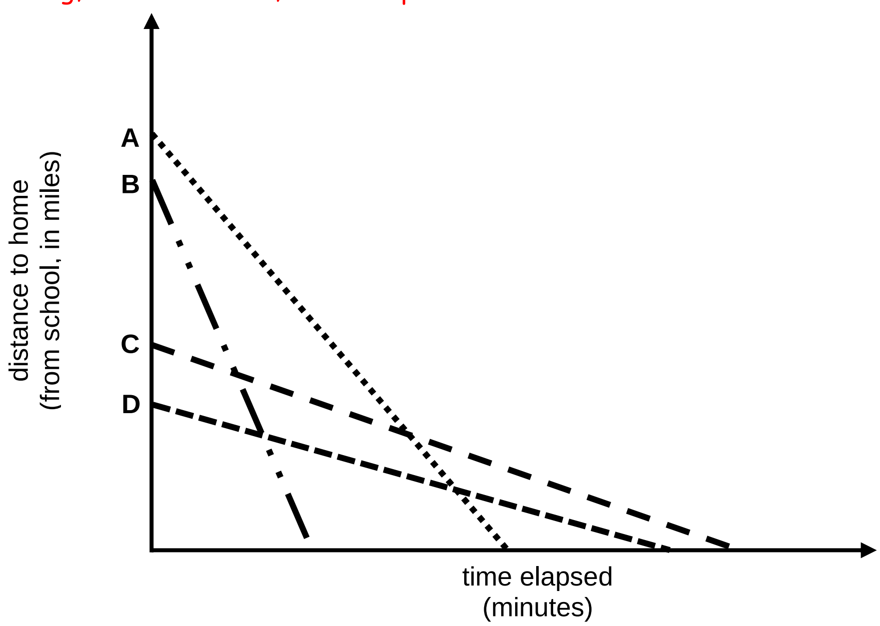
RATE REPRESENTATIONS

We will use words, tables of numbers, equations, and graphs to represent rates. We will compare representations of functions.

[8.EE.5, 8.F.1, 8.F.2, 8.F.3, 8.F.4, 8.F.5; SMP1, 2, 3, 4, 5, 7, 8]

GETTING STARTED

Andre, Bethany, Claudia, and Derek all walked home from John Baxter Taylor Middle School today to each of their respective homes. The graph below shows the distance from school to home, and the time elapsed during their walks. Use the letters A, B, C, and D to identify each person. Time permitting, discuss Baxter, an accomplished athlete.



1. Why do all walkers appear to have constant rates of speed?
   All of their graphs are straight lines.

2. Who started farthest from home? __A__ Closest to home? __D__

3. Who got home first? __B__ Last? __C__

4. Who walked at the fastest rate? __B__ Slowest? __D__

5. Which pair appears to walk at the closer rate of speed, A-B, or C-D? __C-D__

6. Could all of these graphs represent functions? Yes

7. Which of these graphs are increasing and which are decreasing? All are decreasing.

TO SCHOOL AND BACK HOME

Discuss the context and meaning of start values. Be sure students understand that x represents minutes and y represents miles from home in both cases. [SMP1, 2, 3, 4, 7, 8]

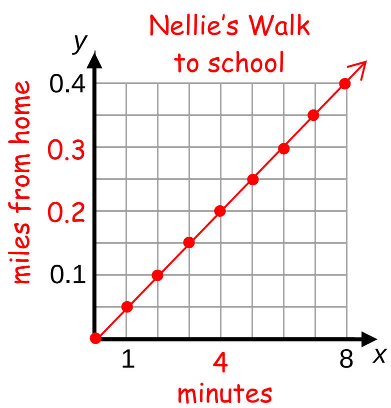
Nellie walks to Marjorie Lee Brown Middle School each morning at a constant rate of 0.05 miles per minute, and jogs home in the afternoon at a constant rate of 0.08 miles per minute.

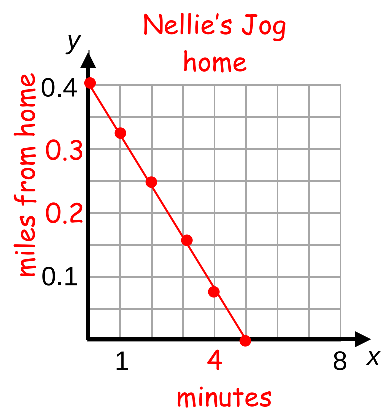
Time permitting, discuss Brown, an accomplished mathematician.

School and home are $\frac{4}{10}$ of a mile apart.

1. Fill in both columns in the table below and draw graphs based upon the given data.
   Look for data points that don't make sense (e.g., "to home" data after 5 minutes elapsed is negative).

Minutes Elapsed (*x*)	0	1	2	3	4	5	6	7	8
To School: Miles from home (*y*)	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4
To Home: Miles from home (*y*)	0.4	0.32	0.24	0.16	0.08	0	N/A	N/A	N/A





2. Why does it make sense to draw lines for the graphs with this context?
   The rates are constant. Time is "continuous." Minute values between whole numbers make sense.

3. Which graph is increasing? Walking to school Decreasing? Jogging home

4. Does either one of these situations represent a proportional relationship? Explain.
   Walking to school; the graph is a straight line through the origin; entries in the table are constant multiples of one another.

5. For walking to school, what is the unit rate? 0.05 miles for every 1 minute.

6. Write an equation for each situation.

   Walking to school: $y = 0.05x$ Jogging home: $y = 0.4 - 0.08x$

TO SCHOOL AND BACK HOME

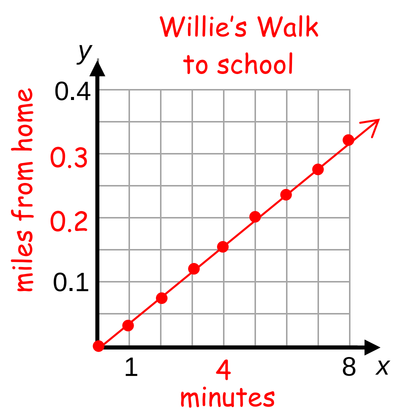
Continued

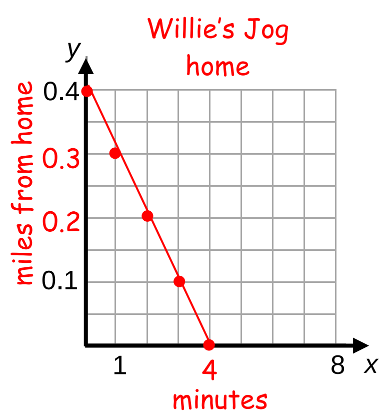
Nellie's brother, Willie, lives in the same house, goes to the same school, and he also walks there and jogs back.

- His "jogging home" data is in the table below
- The equation that represents his walk to school is $y = 0.04x$

7. Fill in the rest of the table below and draw graphs based on the given data.

Minutes Elapsed (x)	0	1	2	3	4	5	6	7	8
To School: Miles from home (y)	0	0.04	0.08	0.12	0.16	0.2	0.24	0.28	0.32
To Home: Miles from home (y)	0.4	0.3	0.2	0.1	0	N/A	N/A	N/A	N/A





8. For walking to school, what is the unit rate?
   0.04 miles for every 1 minute.

   What does an initial value of $x = 0$ mean?
   At $x = 0$ minutes, distance from home is $y = 0$ (at home).

9. Write an equation for jogging home. $y = 0.4 - 0.1x$

10. Who walked to school at a faster rate? Explain.
    Nellie; 0.05 minutes per mile is greater than 0.04 minutes per mile. Nellie's graph is "steeper" (more miles covered per minute).

11. Who jogged home at a faster rate? Explain.
    Willie; 0.1 minutes per mile is greater than 0.08 minutes per mile. His graph is "steeper."

PRACTICE 6

[SMP1, 2, 4, 8]

Below is information for four different cyclists, Tamika, Vinnie, Wanda, and Zach, all of whom ride at constant rates of speed. Use the letters T, V, W and Z to identify each person.

Let x be time elapsed in hours (hr) and y be distance traveled in miles (mi).

One representation is given for each. Complete the remaining representations in any order

1. Word descriptions.

T	Tamika rides at a constant rate of 18 miles per hour.
V	Vinnie rides at a constant rate of 15 miles per hour.
W	Wanda rides at a constant rate of 12 miles per hour.
Z	Zach rides at a constant rate of 10 miles per hour.

2. Entries in the table and equations.

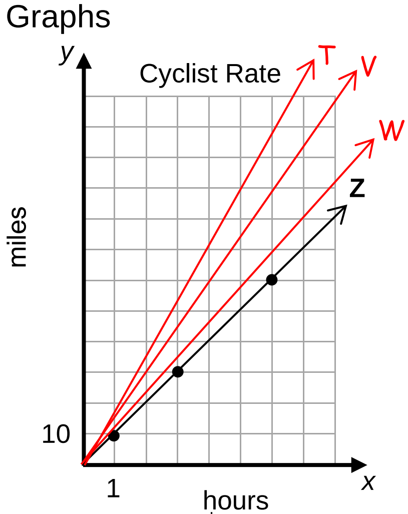
T	x	0	1	2	3	4	5	$y = 18x$
	y	0	18	36	54	72	90	

V	x	0	1	2	3	4	5	$y = 15x$
	y	0	15	30	45	60	75	

W	x	0	1	2	3	4	5	$y = 12x$
	y	0	12	24	36	48	60	

Z	x	0	1	2	3	4	5	$y = 10x$
	y	0	10	20	30	40	50	

3. Graphs



4. What are the initial values (y-intercepts) for each rider?
$y = 0$

5. What are the speeds in miles per hour (rates of change) for each rider? Where do you see these rates in the equations?
T: 18, V: 15, W: 12, Z: 10; these values are all the coefficients of x in the respective equations.

6. How are the graphs of the fastest and slowest riders different?
The fastest, T, has the steepest graph (line), and the slowest, Z, has the flattest.

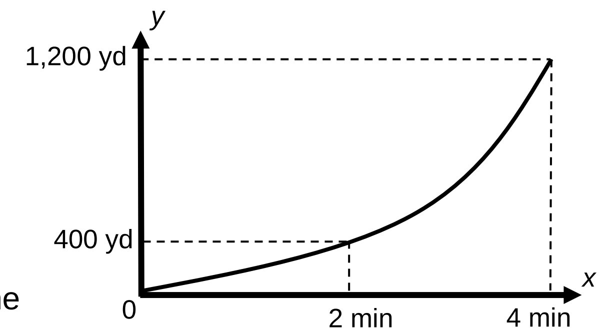
PRACTICE 7

[SMP1, 2, 4]

Chris went jogging in Malala Yousafzai Park on Monday.

Time permitting, discuss Yousafzai, an education activist.

Monday



1. Could Chris' graph represent a function? yes

2. Does it appear to be linear? no

3. Is it increasing or decreasing? increasing

4. Use the Monday graph to the right to complete the table below.

Time Period	Number of Minutes	Yards Traveled	Average Rate of Speed
From 0 to 2 minutes	2	400	200 yd/min
From 2 to 4 minutes	2	800	400 yd/min
From 0 to 4 minutes	4	1,200	300 yd/min

5. In what part of the jog did Chris run faster, the initial two minutes or the last two minutes? Explain by referencing numbers and the shape of the graph.
   For the last two minutes Chris jogged twice as fast (400 yd/min compared to 200 yd/min) as the first two minutes. That part of the graph is steeper (greater distance traveled over the same amount of time).

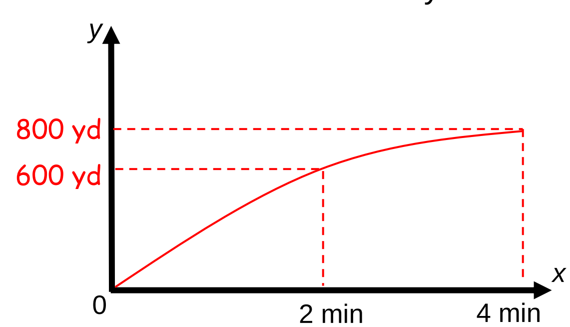
Chris went jogging in the park again on Wednesday.

6. Complete the table below, and sketch and label a graph.

Time Period	Number of Minutes	Yards Traveled	Average Rate of Speed
From 0 to 2 minutes	2	600	300 yd/min
From 2 to 4 minutes	2	200	100 yd/min
From 0 to 4 minutes	4	800	200 yd/min

7. In what part of the jog did Chris run faster, the initial two minutes or the last two minutes? Explain by referencing numbers and the shape of the graph to the right.
   In the first two minutes Chris jogged three times as fast (300 yd/min compared to 100 yd/min). That part of the graph is steeper (greater distance traveled over the same amount of time).

Wednesday

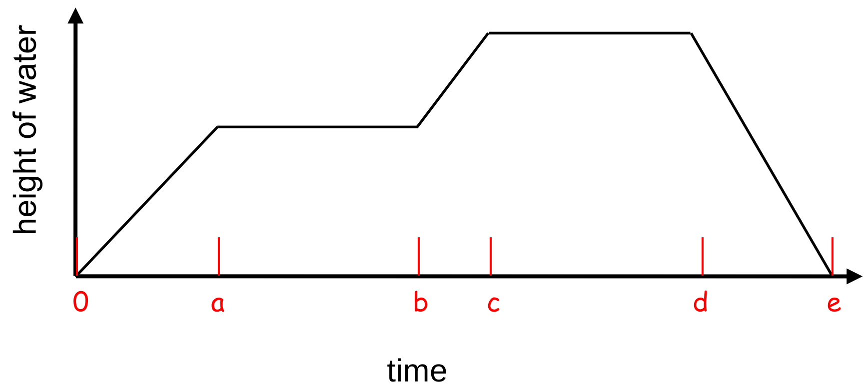


THE BATH GRAPH

The *MathLinks* Rubric: See Activity Routines on the Teacher Portal for directions. [SMP1, 2, 3, 4, 5]

Explanations will vary. Numbers are not required. One possible answer:

Write several sentences to explain what story this graph could be telling. Explain in the context of the story why this graph must represent a function.



This graph shows water filling a bathtub.

At the start (from t = 0 to a) the bath is filling up with water at a constant rate.

Then the water stops for a while and height stays the same (from t = a to b).

Then the height rises at a constant rate some more (from t = b to c). This could be due to putting more water in, or from a person slowly getting in the tub.

Then the water stays the same for a while longer (from t = c to d).

Finally, the water all drains out at a constant rate (from t = d to e).

This graph must represent a function because at any given time, there can only be one water height that corresponds to it.

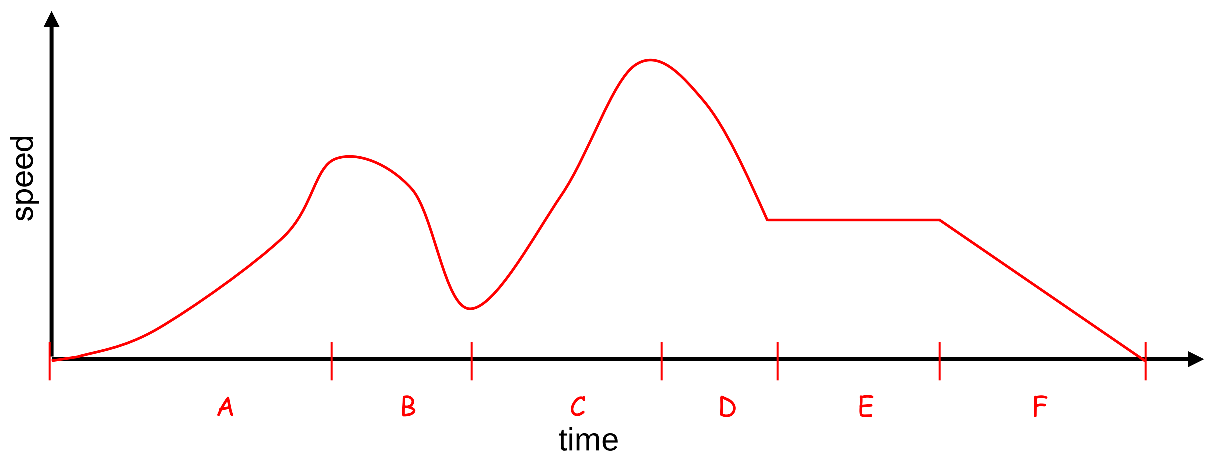
THE ROLLERCOASTER

The *MathLinks* Rubric: See Activity Routines on the Teacher Portal for directions. [SMP1, 2, 3, 4, 5]

Graphs and explanations will vary. Numbers are not required. One possible answer:

1. Draw a reasonable graph for a typical rollercoaster ride, based on the following information. Label each section by letter (each segment or curved portion of your graph) based upon the descriptions below, A – F. Note that the vertical axis represents speed, NOT HEIGHT.

   A. The rollercoaster starts slowly and gradually builds speed.
   B. It comes to a hill and climbs up slowly.
   C. It races downhill.
   D. It does a full loop.
   E. It continues at a constant speed.
   F. It gradually comes to a stop.



2. Write a few sentences to summarize your work and explain how you know you have drawn a good depiction of the rollercoaster ride described. Include in your explanation if this graph could represent a function and why.

   - This curve represents a function because for each segment of time, there is one speed
   - **Section A**: speed increases slowly – flatter curve at first but steeper as speed increases more rapidly
   - **Section B**: speed decreases to almost nothing because of the hill climb
   - **Section C**: speed increases rapidly because of the hill (notice that height would decrease, but speed is increasing)
   - **Section D**: speed decreases because of the loop, but not too much because some speed is required to do a loop
   - **Section E**: constant speed (neither increasing nor decreasing)
   - **Section F**: speed gradually decreases to zero (stopping);
     it could be represented by a curve like the one drawn to the right as well

REVIEW

y = 3*x* + 4

[SMP1, 2, 7]

1. Your teacher will give you some cards.

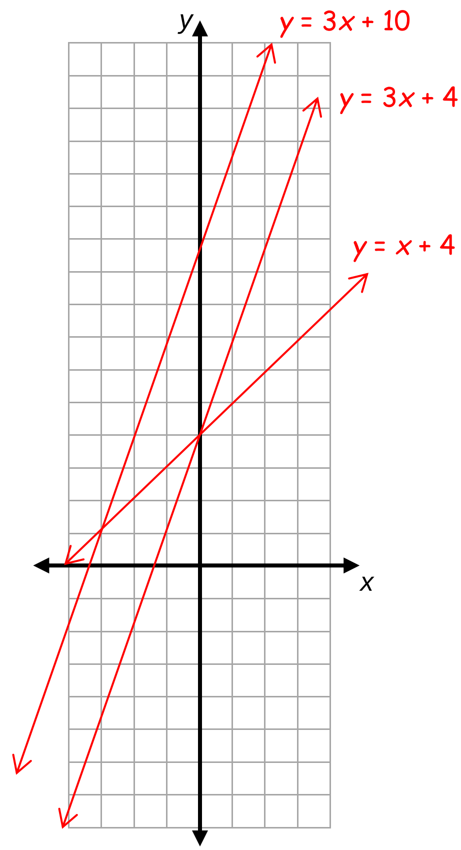
   Sort them into two piles and list the appropriate cards below.

   DOES model $y = 3x + 4$: B, D, E, H, I, K

   DOES NOT model $y = 3x + 4$: A, C, F, G, J, L

2. On the grid to the right, graph and label:

   - $y = 3x + 4$
   - a different equation that has the same *y*-intercept as $y = 3x + 4$
     Graphs may vary. One possible answer is $y = x + 4$
   - a different equation that is parallel to $y = 3x + 4$
     Graphs may vary. One possible answer is $y = 3x + 10$



3. Use different representations (numbers, words, and pictures) to describe at least four features of the equation $y = 3x + 4$.
   Answers will vary. Some possible answers:

   - It represents a function.
   - It does not represent a proportional relationship.
   - The graph is increasing.
   - Input-output values include the following—and infinitely more—written as ordered pairs (0, 4), (1, 7), (2, 10), (3, 13), (4, 16), (-1, 1), (-2, -2), (-3, -5), (-4, -8).
   - For each increase *x* of 1, *y* increases by 3.
   - For each decrease of 1, *y* decreases by 3.
   - Its graph is a straight line (linear) with *y*-intercept 4.

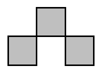
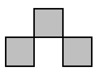
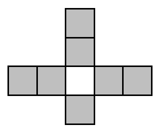
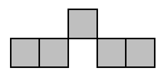
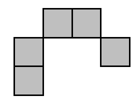
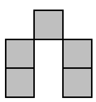
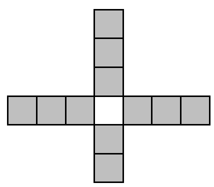
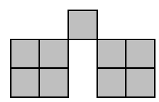
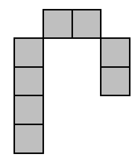
POSTER PROBLEMS: INTRODUCTION TO FUNCTIONS

See Activity Routines on the Teacher Portal for directions. [SMP1, 2, 5, 8]

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _______.
- Each group will have a different colored marker. Our group marker is _________.

Part 2: Do the problems on the posters by following your teacher's directions.

	Poster 1 (or 5)	**Poster 2 (or 6)**	**Poster 3 (or 7)**	**Poster 4 (or 8)**
Step 1				
Step 2				
Step 3				

A. Copy steps 1 – 3 onto the poster and draw step 4. Explain your step 4 in words.

B. Make a table, label it appropriately, and record values for steps 0 through 5. Make note of the initial value and the rate of increase.

C. Make a graph and label it appropriately.

D. Write an input-output rule that relates the total number of tiles to the step number, and then find the number of tiles for step 100.

Note: Before going on, write the equations for Posters 2 (or 6) and 3 (or 7) on the board, as well as the *y*-intercept for all posters.

Part 3: Return to your seats. Work with your group, and show all your work.

1. The ordered pair (1, 1) is on both Posters 1 and 3. What does it represent?
   Both patterns have one tile at step 1.

2. Find the step number that has exactly 155 tiles in poster 2. step 39

3. Find the step number that has exactly 185 tiles in poster 4. step 62

4. What does the *y*-intercept mean in each poster?
   It has no meaning in this context since you can't have less than zero tiles in a step.

Chart paper or board space, markers

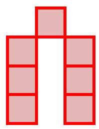
4

POSTER PROBLEMS: INTRODUCTION TO FUNCTIONS

Answer Key

Answer Key. Likely patterns illustrated below, though they may vary if justified.

Poster 1 (or 5)

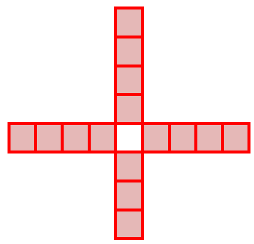


Step # (x)	# of tiles (y)
0	-1
1	1
2	3
3	5
4	7
5	9

$y = 2x - 1$

Step 100: 199 tiles

Poster 2 (or 6)

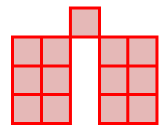


Step # (x)	# of tiles (y)
0	-1
1	3
2	7
3	11
4	15
5	19

$y = 4x - 1$

Step 100: 399 tiles

Poster 3 (or 7)

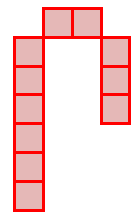


Step # (x)	# of tiles (y)
0	-3
1	1
2	5
3	9
4	13
5	17

$y = 4x - 3$

Step 100: 397 tiles

Poster 4 (or 8)

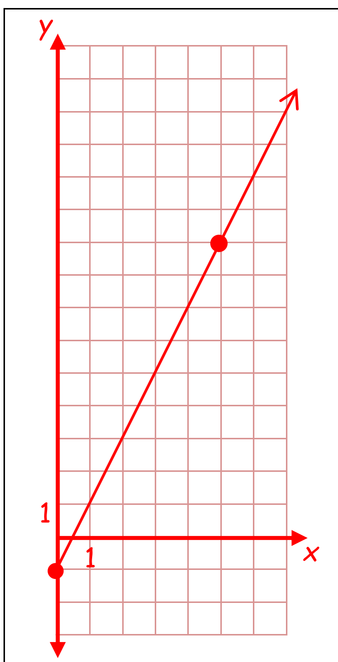


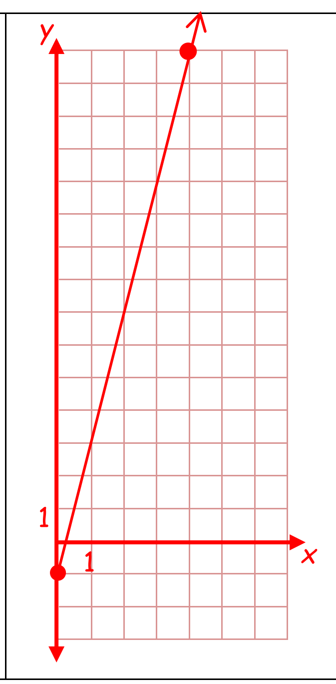
Step # (x)	# of tiles (y)
0	-1
1	2
2	5
3	8
4	11
5	14

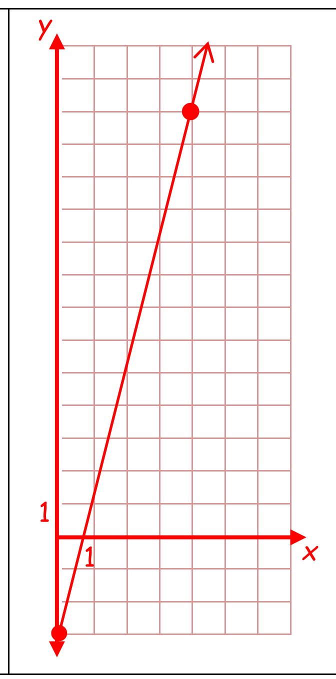
$y = 3x - 1$

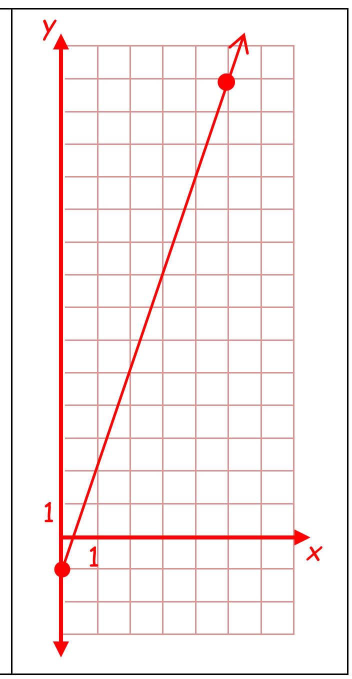
Step 100: 299 tiles

Points may be connected on each graph to show a trend line.









WHY DOESN'T IT BELONG?: INTRODUCTION TO FUNCTIONS

See Activity Routines on the Teacher Portal for directions. [SMP1, 2]

For more challenging Alge-Grid puzzles for advanced learners, see Puzzles and Games in the Teacher Portal.

A. **Table:**

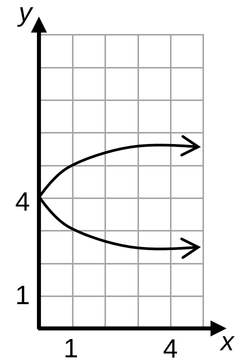
Input (x)	Output (y)
0	0
1	1
2	4
3	9
4	16

B. **Equation:** $y = -2x + 1$

C. **Context:**

Sal skateboards to and from work every day at an average rate of 6 miles per hour. He uses this information to keep track of how far he travels after any number of hours.

D. **Graph**



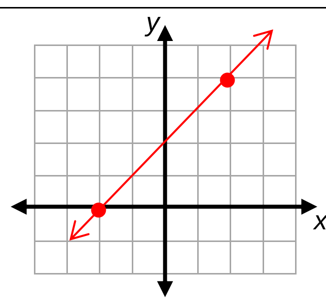
Avoid the obvious differences, such as "It's a graph."

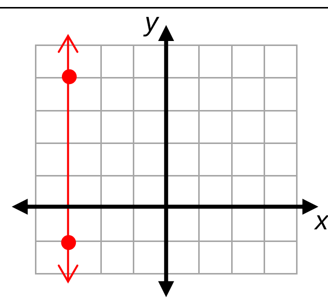
Explanations will vary. Some possible explanations:

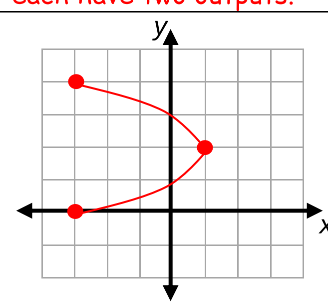
1. Choose one representation A – D above and explain why it does not belong with the others.	2. Now choose a different representation and explain why it does not belong.
A does not belong because it's an increasing nonlinear function B does not belong because it's a decreasing linear function	C does not belong because it's an increasing linear function D does not belong because it's a nonlinear non-function

Graph each of the described situations below, answer the questions, and explain.

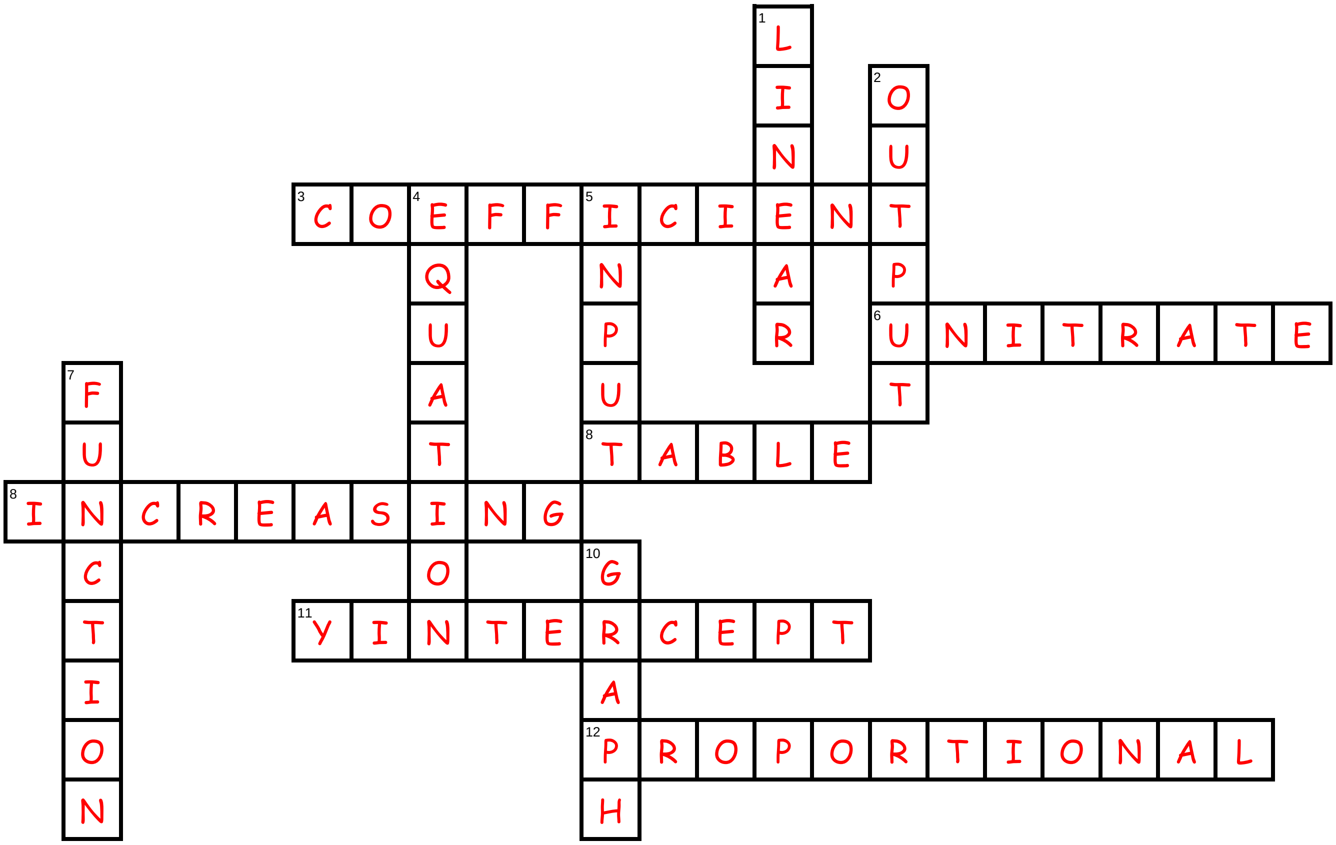
3. I am a linear function. Two of my points are located at (-2, 0) and (2, 4).	4. I am a line. Two of my points are (-3, 4) and (-3, -1).	5. Graph and connect my three points in this order: (-3, 4), (1, 2), and (-3, 0).
My y-intercept is: 2 Am I increasing or decreasing? Explain. Increasing; as the x-values increase from left to right, the y-values do as well.	Am I a function? Explain. No; for the input given, -3, there are more than one output.	Am I a function? Explain. No; curves are depicted below, but could be straight segments/rays too; regardless, multiple inputs each have two outputs.







VOCABULARY REVIEW



Across

3 In the equation *y* = 4*x* + 2, the rate of change is the ___ of *x.*

6 in a proportional relationship, the rate of change per unit (two words)

8 Entries in this organized chart represent ordered pairs of numbers.

9 The graph of a function that has a positive rate of change is ___.

11 number where the graph crosses the *y*-axis (hyphenated word)

12 relationship where each output is a constant multiple of the input

Down

1 function represented by a straight line

2 dependent variable, *y*-value

4 symbolic representation of a mathematical rule

5 independent variable, *x*-value

7 rule where every input has a unique output

10 visual representation of a mathematical rule (usually on a grid)

SPIRAL REVIEW

See Activity Routines on the Teacher Portal for directions.

1. **Alge-Grid: What's the *a*?** Each clue gives the value of a corresponding cell. Use clues to find *a*, which has the same value in all cells. Once evaluated, the cells will contain the whole numbers 1 – 9, exactly once each.

   $a = 2$

The Alge-Grid

$\frac{3}{4}a + 0.5$ 2	a^2 4	$(a + 1)^2$ 9
$3a$ 6	a^0 1	$a^2 - 1$ 3
$a^2 + 1$ 5	$a^2 + 2a - 1$ 7	a^3 8

The Clues

Perfect number	Factor of all numbers	
		(Prime)^3

2. Solve each equation below.

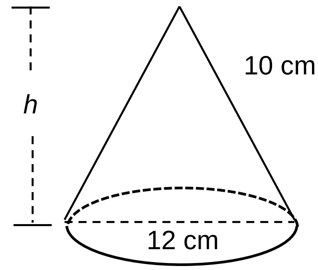
a. $6(g - 7) = 18$ $g = 10$	b. $6 = \frac{x}{4} - 2$ $x = 32$
c. $\frac{m + 6}{3} = -5$ $m = -21$	d. $3(p - 5) + 2p = -55$ $p = -8$

SPIRAL REVIEW

Continued

Figures not drawn to scale. Answers on this page may vary due to rounding.

3. Find the height and the volume of the cone to the right if the diameter is 12 cm and the slanted edge is 10 cm. Round to the nearest tenth. Use π = 3.14.



Using the Pythagorean theorem: $6^2 + h^2 = 10^2$

$\frac{1}{3}\pi(36)(8)$

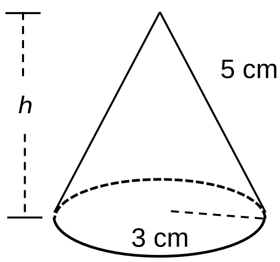
$\frac{288\pi}{3} = 96\pi$

h = 8 cm

v = 301.6 cm^3

4. Find the height and the volume of each cone below. Round to the nearest tenth. Use π = 3.14.

a.



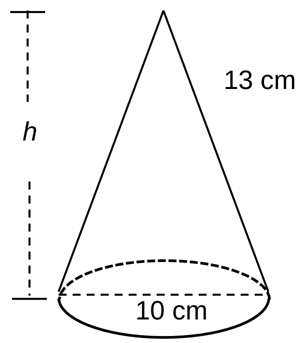
$\frac{1}{3}\pi(9)(4)$

12π

h = 4 cm

v = 37.7 cm^3

b.



$\frac{1}{3}\pi(25)(12)$

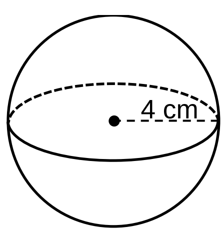
100π

h = 12 cm

v = 314 cm^3

5. Find the volume of each sphere below. Use π = 3.14. Round to the nearest hundredth.

a.

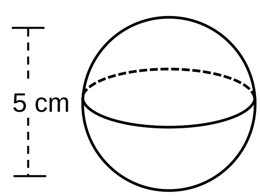


$\frac{4}{3}\pi(4)^3$

$\frac{256\pi}{3}$

v = 268.08 cm^3

b.



$\frac{4}{3}\pi(2.5)^3$

$\frac{625\pi}{3}$

v = 65.45 cm^3

SPIRAL REVIEW

Continued

6. Solve and show your work.

A store is having a sale on bikes. The original cost of a bike that Epic wants to buy is $190.

a. If the bike is marked down to $110, what is the percent discount?

190 – 110 → decrease of 80

$\frac{80}{190} = 0.42 = 42\%$ → A 42% discount.

b. Sales tax in this location is 9.6%. What is the sales tax amount for the discounted bike?

(0.096)(110) = 10.56

Sales tax is $10.56.

c. Epic has $125 to spend on the bike. Will this be enough money? Explain.

110 + 10.56 = 120.56. Yes, Epic has enough money for the bike.

7. Compute

a. $4^{-2} \bullet 4^{3}$ 4	b. $5^{7} \bullet 5^{-7}$ 1	c. $3^{-2} \bullet 3^{-3} \bullet 3^{8}$ 27
d. $(2^{3})^{4} \bullet (4^{2})^{-3}$ 1	e. $\frac{10^{5} \bullet 10^{8}}{10^{10}}$ 1,000	f. $\sqrt[3]{27}$ 3
g. $\sqrt[3]{-27}$ -3	h. $(\sqrt{25})(\sqrt{4})$ 10	i. $\left(\frac{\sqrt{36}}{\sqrt{16}}\right)$ 1.5

REFLECTION

Answers will vary. Some possible answers:

1. **Big Ideas**. Shade all circles that describe big ideas in this unit. Draw lines to show connections that you noticed. See Teaching Tips for connections.

Use transformational geometry to investigate congruence and similarity (8.G.A)

Extend applications of volume to cylinders, cones, and spheres (8.G.C)

Explore bivariate data (8.SP.A)

Complete the real number system (8.NS.A)

Solve linear equations in one variable and linear systems in two variables (8.EE.C)

Discover and apply properties of lines, angles, and triangles, including the Pythagorean theorem (8.G.B)

Create, analyze, and use linear functions in problem solving (8.EE.B, 8.F.AB)

Explore exponents and roots, and very large and very small quantities (8.EE.A)

Give an example from this unit of one of the connections above.
Solving equations to find missing values in various pattern problems.

2. **Unit Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before.

3. **Mathematical Practice.** Explain how you used multiple representations to model and analyze relationships [SMP4]. Then circle one more SMP on the back of this packet that you think was addressed in this unit and be prepared to share an example.
Throughout this unit, problems involving visual patterns and contexts were explored using words, numbers, graphs, and equations (**Slides and Jumps**, **The Pool Problem**, **Saving vs Spending**, **To School and Back Home**).

4. **Making Connections.** Give examples of how a function helps us to explore changing quantities and predict what might happen.
Contexts that can be represented by input-output rules (functions) allow for modelling a situation and predicting an output given an input (or vice versa). Examples from this unit include **Slides and Jumps**, **The Pool Problem**, and **Saving vs Spending**.

STUDENT RESOURCES

<table>
<tr><th>Word or Phrase</th><th>Definition</th></tr>
<tr><td>coefficient</td><td>A coefficient is a number or constant factor in a term of an algebraic expression.

In the expression $3x + 5$, 3 is the coefficient of the term $3x$, and 5 is the constant term.</td></tr>
<tr><td>dependent variable</td><td>A dependent variable is a variable whose value is determined by the values of the independent variables. See independent variable.</td></tr>
<tr><td>function</td><td>A function is a rule that assigns to each input value exactly one output value.

For $y = 3x + 6$, any input value, say $x = 10$, has a unique output value, in this case $y = 36$.

For $y = x^2 + 1$, $x = 2$ has the unique output value $y = 2^2 + 1 = 5$.</td></tr>
<tr><td>graph of a function</td><td>The graph of a function is the set of all ordered pairs (x, y) where y is the output for the input value x. If x and y are real numbers, then we can represent the graph of a function as points in the coordinate plane.</td></tr>
<tr><td>independent variable</td><td>An independent variable is a variable whose value may be specified. Once specified, the values of the independent variables determine the values of the dependent variables.

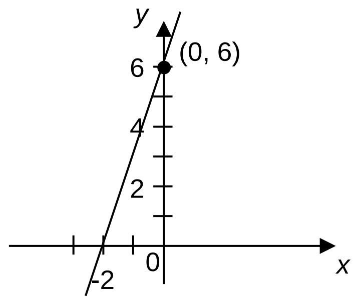
For the equation $y = 3x$, y is the dependent variable and x is the independent variable. We may assign a value to x. The value assigned to x determines the value of y.</td></tr>
<tr><td>input-output rule</td><td>An input-output rule for a sequence of values is a rule that establishes explicitly an output value for each given input value.

<table>
<tr><td>input value (x)</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>x</td></tr>
<tr><td>output value (y)</td><td>1.5</td><td>3</td><td>4.5</td><td>6</td><td>7.5</td><td>$1.5x$</td></tr>
</table>

In the table above, the input-output rule could be $y = 1.5x$. To get the output value, multiply the input value by 1.5. If $x = 100$, then $y = 1.5(100) = 150$.</td></tr>
<tr><td>proportional</td><td>Two variables are proportional if the values of one are the same constant multiple of the corresponding values of the other. The variables are said to be in a proportional relationship, and the constant is referred to as the constant of proportionality.

If Wrigley eats 3 cups of kibble each day, then the number of cups of kibble is proportional to the number of days. If x is the number of days, and y is the number of cups of kibble, then $y = 3x$. The constant of proportionality is 3.</td></tr>
<tr><td>unit rate</td><td>The unit rate associated with a ratio $a : b$ of two quantities a and b, $b \neq 0$, is the number $\frac{a}{b}$, to which units may be attached. This is sometimes referred to as the value of the ratio.

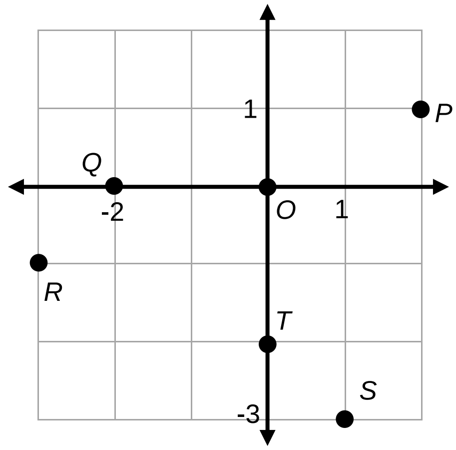
The ratio of 40 miles for every 5 hours has a unit rate of 8 miles per hour.</td></tr>
</table>

Word or Phrase	Definition
y-intercept	The y-intercept of a line is the y-coordinate of the point at which the line crosses the y-axis. It is the value of y that corresponds to $x = 0$. The y-intercept of the line $y = 3x + 6$ is 6. If $x = 0$, then $y = 6$. 

The Coordinate Plane

A coordinate plane is determined by a horizontal number line (the x-axis) and a vertical number line (the y-axis) intersecting at the zero on each line. The point of intersection (0, 0) of the two lines is called the origin. Points are located using ordered pairs (x, y).

- The first number (x-coordinate) indicates how far the point is to the right or left of the y-axis.
- The second number (y-coordinate) indicates how far the point is above or below the x-axis.



Point, coordinates, and interpretation

O (0, 0) → This is the intersection of the axes (origin).

P (2, 1) → start at the origin, move 2 units right, then 1 unit up

R (-3, -1) → start at the origin, move 3 units left, then 1 unit down

S (1, -3) → start at the origin, 1 unit right, then 3 units down

Q (-2, 0) → start at the origin, move 2 units left, then 0 units up or down

T (0, -2) → start at the origin, 0 units right or left, then 2 units down

Functions

Some ways to represent rules in mathematics are input-output tables, mapping diagrams, ordered pairs, equations, and graphs.

Examples that are Functions

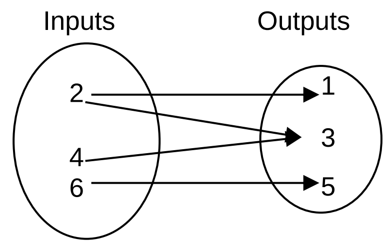
Input-Output Table

x input	*y* output
1	1
3	3
5	5
7	7
9	9

This table lists input values with unique output values.

Examples that are NOT Functions

Mapping Diagram



This mapping diagram is not a function. It is not permissible for the same input value (in this case 2) to be assigned two different output values. However, all other input-output mappings above are fine.

Examples that are Functions

Ordered Pairs

(0, 2), (1, -2), (2, 2), (3, -2)

In this set of ordered pairs, each input value is assigned to a unique output value. Note that different input values may be assigned the same output value. In this example, both 1 and 3 are assigned the output value -2.

Examples that are NOT Functions

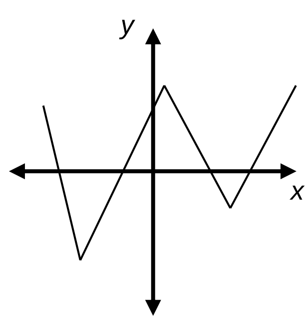
Equation (with Ordered Pairs)

Consider the set of pairs (x, y) that satisfy $x = y^2$, such as (0, 0), (25, 5), and (25, -5). Since the input value, x = 25, corresponds to two different output values (y = 5 and y = -5), the y-values are not a function of the x values.

Examples that are Functions

Graph

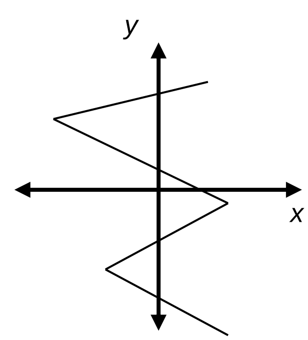
This graph represents a function because every vertical line through it intersects at most one point of the graph. In other words, each possible x-value corresponds to a unique y-value.



Examples that are NOT Functions

Graph

This graph does not represent a function because some vertical lines (for example, the y-axis) intersect the graph in more than one point. In other words, some x-values correspond to more than one y-value.



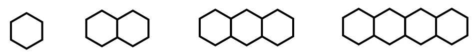
Using Multiple Representations to Describe Linear Functions

Here are four representations commonly used to approach a math problem:

- Numbers (numerical approach, as by making a table)
- Pictures (visual approach, as with a picture or graph)
- Symbols (approaching the problem using algebraic symbols)
- Words (verbalizing a solution, orally or in writing)

Each approach may lead to a valid solution. Collectively they should lead to a complete and comprehensive solution, one that is readily accessible to more people and that provides more insight.

Example 1: Describe this pattern of hexagons using numbers, pictures, words, and symbols.



Numbers

Step #	**number of segments**	**Breaking apart numbers sometimes helps you see an input-output rule.**	
1	6	6	= 6 + (0)5
2	11	6 + 5	= 6 + (1)5
3	16	6 + 5 + 5	= 6 + (2)5
4	21	6 + 5 + 5 + 5	= 6 + (3)5
5	26	6 + 5 + 5 + 5 + 5	= 6 + (4)5
n	5*n* + 1	5*n* + 1	= 6 + (*n* – 1)5

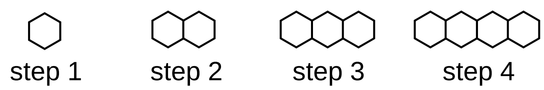
Words

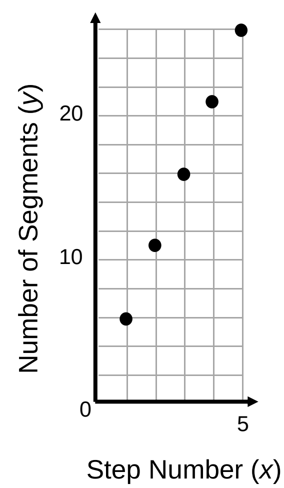
One way to describe the hexagonal pattern is to start with 6 segments and add 5 more segments at each subsequent step. Notice that the number of 5’s added at each step is equal to 1 less than the step number.

Symbols

A rule for finding the number of segments at step *n* is 6 + (*n* – 1)5, which can be simplified to 5*n* + 1.

Pictures





Note: we consider a graph to be a picture.

Using Multiple Representations to Describe Linear Functions (Continued)

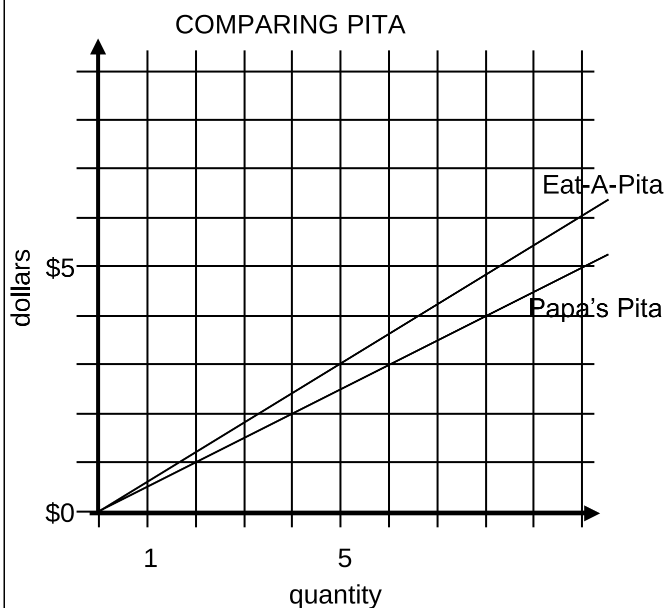
Example 2: At Papa's Pitas, 2 pitas cost $1.00. At Eat-A-Pita, 5 pitas cost $3.00. Assuming a proportional relationship between the number of pitas and their cost, use multiple representations to explore which store offers the better buy for pitas.

Numbers (make a table)

PAPA'S PITAS	
# of pitas (x)	cost (y)
2	$1.00
4	$2.00
6	$3.00
8	$4.00
10	$5.00

EAT-A-PITA	
# of pitas (x)	cost (y)
5	$3.00
10	$6.00
15	$9.00
20	$12.00
25	$15.00

Pictures (make a graph)



Words (write sentences)

Based on the table, Papa's Pitas is the better buy.

At Papa's Pitas, you get 6 pitas for $3.00.
This means the unit price (cost for one pita) is $0.50.

At Eat-A-Pita you only get 5 pitas for $3.00. This means the unit price (cost for one pita) is $0.60.

Symbols (write equations to relate the number of pitas to cost)

PAPA'S PITAS $y = 0.5x$

EAT-A-PITA $y = 0.6x$

Notice that $0.50 is the cost of one pita at Papa's Pita. This corresponds to the point (1, 0.5) on the graph.

Notice that $0.60 is the cost of one pita at Eat-A-Pita. This corresponds to the point (1, 0.6) on the graph.

The equations above are both in the form $y = mx$. This equation form represents a proportional relationship because y is a constant multiple of x. Graphs of equations in this form are always lines going through the origin. They will be explored more in the next unit and contrasted with equations in the form $y = mx + b$.

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
8.EE.B	**Understand the connections between proportional relationships, lines, and linear equations.**
8.EE.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
8.F.A	**Define, evaluate, and compare functions.**
8.F.1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
8.F.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
8.F.3	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*
8.F.B	**Use functions to model relationships between quantities.**
8.F.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
8.F.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

STANDARDS FOR MATHEMATICAL PRACTICE	
SMP1	Make sense of problems and persevere in solving them.
SMP2	Reason abstractly and quantitatively.
SMP3	Construct viable arguments and critique the reasoning of others.
SMP4	Model with mathematics.
SMP5	Use appropriate tools strategically.
SMP6	Attend to precision.
SMP7	Look for and make use of structure.
SMP8	Look for and express regularity in repeated reasoning.

Why did the students get upset when their teacher called them average?
Because it was a mean thing to say.

9 781614 450603