

8-4 NONROUTINE PROBLEMS OPEN MIDDLE PROBLEMS

Answers may vary. Examples are shown. Using graph paper may be helpful.
Use any of the integers -9 to 9 at most once in each table below.

1. Create an input-output table for:

a. any linear function

x	0	1	2	3
y	-4	-3	-2	-1

b. any non-linear function
("piecewise" linear is okay)

x	-3	-2	0	1
y	-9	-8	-1	4

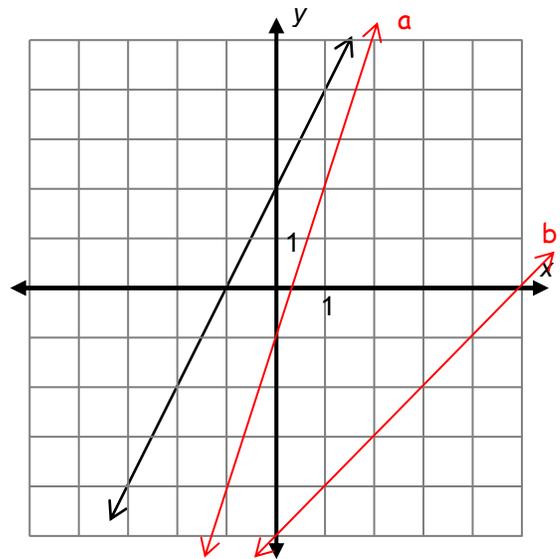
2. For the given graphed line and directions, create input-output values in the tables. Graph some ordered pairs to demonstrate that they are along the type of lines required.

a. a line that is "steeper" than the pictured line.

x	-2	-1	1	3
y	-7	-4	2	8

b. a line that is "flatter" than the pictured line.

x	-1	0	1	2
y	-6	-5	-4	-3



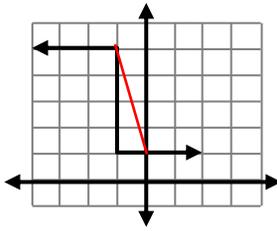
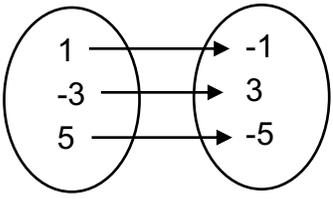
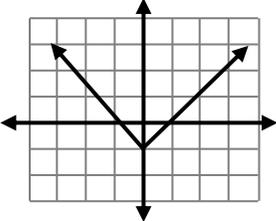
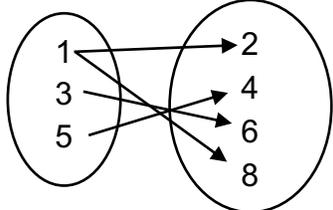
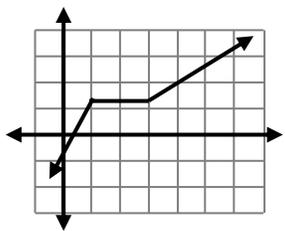
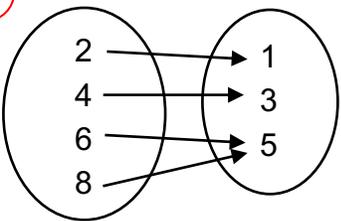
3. Why is it impossible to create an input-output table that does not represent a function?

It is not possible with integer entries because they cannot be used more than once for x .
However, one may argue that the integers might be used to create equivalent fractions for x -values to make a non-function possible (for example, if one accepts $\frac{-1}{2}$ and $\frac{-3}{6}$ as inputs with different outputs).

8-4 NONROUTINE PROBLEMS IS IT A FUNCTION?

Below are different representations of sets of ordered pairs. Circle the ones that appear to represent functions. If it is **not** a function, change it to represent one.

Changes will vary. Examples are given.

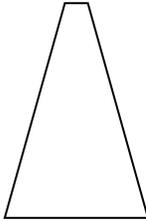
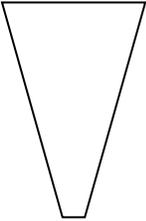
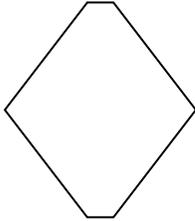
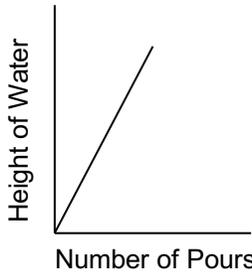
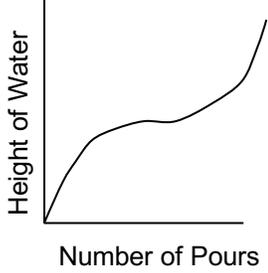
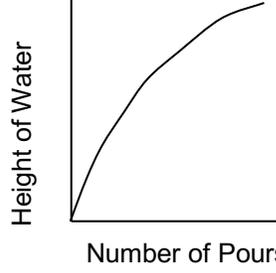
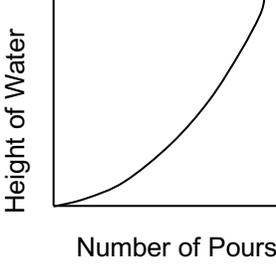
<p>1.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th style="padding: 5px;">x</th> <th style="padding: 5px;">y</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">1</td> <td style="padding: 5px;">-2</td> </tr> <tr> <td style="padding: 5px;">2</td> <td style="padding: 5px;">-2</td> </tr> <tr> <td style="padding: 5px;">3</td> <td style="padding: 5px;">-2</td> </tr> <tr> <td style="padding: 5px;">4</td> <td style="padding: 5px;">-2</td> </tr> </tbody> </table>	x	y	1	-2	2	-2	3	-2	4	-2	<p>2.</p>  <p style="color: red; text-align: center;">Replace the 4 unit vertical and 1 unit horizontal segments with this "slanted" one</p>	<p>3.</p> 
x	y											
1	-2											
2	-2											
3	-2											
4	-2											
<p>4.</p> 	<p>5.</p>  <p style="color: red; text-align: center;">remove the arrow from 1 to 8</p>	<p>6.</p> <p style="text-align: center;">(6,3), (3,6), (-3,-6), (-6,-3)</p>										
<p>7.</p> <p>$\{(4,3), (4,4), (4,5), (4,6)\}$ reverse the coordinates for x and y to get: $(3,4), (4,4), (5,4), (6,4)$</p>	<p>8.</p> 	<p>9.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th style="padding: 5px;">x</th> <th style="padding: 5px;">y</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">5</td> <td style="padding: 5px;">4</td> </tr> <tr> <td style="padding: 5px;">5</td> <td style="padding: 5px;">4</td> </tr> <tr> <td style="padding: 5px;">6</td> <td style="padding: 5px;">7</td> </tr> <tr> <td style="padding: 5px;">7</td> <td style="padding: 5px;">8</td> </tr> </tbody> </table>	x	y	5	4	5	4	6	7	7	8
x	y											
5	4											
5	4											
6	7											
7	8											
<p>10.</p> 	<p>11.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th style="padding: 5px;">x</th> <th style="padding: 5px;">y</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">-2</td> <td style="padding: 5px;">1</td> </tr> <tr> <td style="padding: 5px;">2</td> <td style="padding: 5px;">4</td> </tr> <tr> <td style="padding: 5px;">-2</td> <td style="padding: 5px;">3</td> </tr> <tr> <td style="padding: 5px;">0</td> <td style="padding: 5px;">-6</td> </tr> </tbody> </table> <p style="color: red; text-align: center;">change $(-2,1)$ to $(-1,1)$</p>	x	y	-2	1	2	4	-2	3	0	-6	<p>12.</p> <p style="text-align: center;">$(4,3), (2,5), (1,8), (0,3)$</p>
x	y											
-2	1											
2	4											
-2	3											
0	-6											

8-4 NONROUTINE PROBLEMS POURING WATER

To simulate pouring water, use a pencil and shade slowly back and forth from bottom to top of a container. Imagine pouring water into each of these containers at a constant rate.

- Match each container with an appropriate graph below.
- Write one or two sentences to justify each choice.

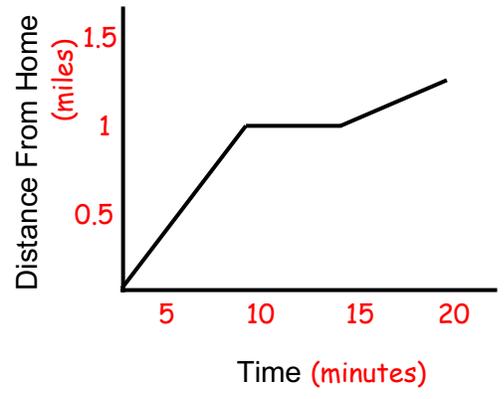
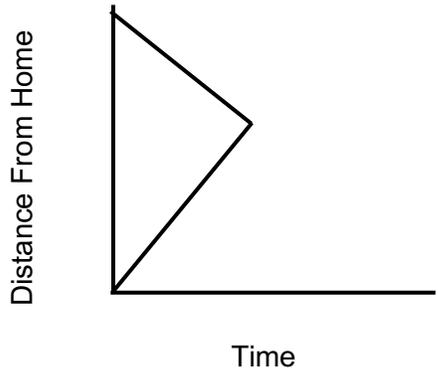
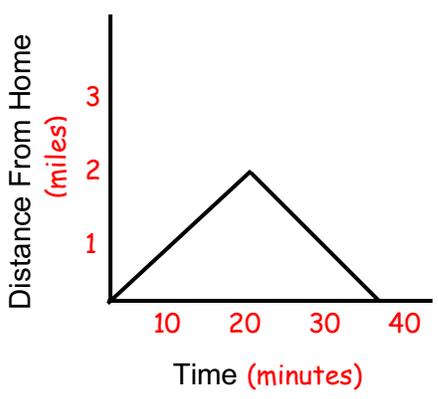
Example justifications are shown.

<p>1. Container 1</p> <div style="text-align: center; margin: 10px 0;">  </div> <p>Graph: <u> A </u></p> <p>Explain: Because of the rectangular shape, the container will fill at a constant rate (a straight-line graph).</p>	<p>2. Container 2</p> <div style="text-align: center; margin: 10px 0;">  </div> <p>Graph: <u> D </u></p> <p>Explain: Because it is wider on the bottom, the container will fill more slowly at first and then speed up (the curve on the graph is flatter at first and then steeper).</p>		
<p>3. Container 3</p> <div style="text-align: center; margin: 10px 0;">  </div> <p>Graph: <u> C </u></p> <p>Explain: Because it is narrower on the bottom, the container will fill more rapidly at first and then slow down (the curve on the graph is steeper at first and then flatter).</p>	<p>4. Container 4</p> <div style="text-align: center; margin: 10px 0;">  </div> <p>Graph: <u> B </u></p> <p>Explain: Because it is smaller at the top and bottom bases, the container will start to fill rapidly, slow down, and then fill more rapidly (the curve on the graph is steeper at first, then flatter, then steeper again).</p>		
<p>A.</p> <div style="text-align: center;">  </div>	<p>B.</p> <div style="text-align: center;">  </div>	<p>C.</p> <div style="text-align: center;">  </div>	<p>D.</p> <div style="text-align: center;">  </div>

8-4 NONROUTINE PROBLEMS RUNNING STORIES

Create stories about walking or running that each graph could represent. If the situation is not possible, explain. Include appropriate scales if using numbers to describe the story.

Answers will vary. Example numbers and stories are given.

Graph	Story
<p>A.</p> 	<p><i>Times and distances approximate; rates of speed are constant:</i></p> <p><i>A runner ran 1 mile from home in 10 minutes, stopped to rest for 5 minutes, then walked about a quarter mile to cool down over 5 minutes.</i></p>
<p>B.</p> 	<p><i>This one is not possible (in the real world) because the runner cannot be in two places at the same time. At time equal to zero, for example, the runner is both at home and some distance away from home.</i></p> <p><i>Another way to think about it: as distance from home is increasing, time increases and then decreases. How does time decrease?</i></p>
<p>C.</p> 	<p><i>A runner ran for about 20 minutes at a constant rate to a location that was about 2 miles away from home, and then ran back home at about the same constant rate. The runner finished in about 37 minutes and ran at a rate of about 9.25 miles per hour.</i></p>

8-4 NONROUTINE PROBLEMS MIXED PROBLEMS

1. The table below represents a function where x is the input value and y is the output value.

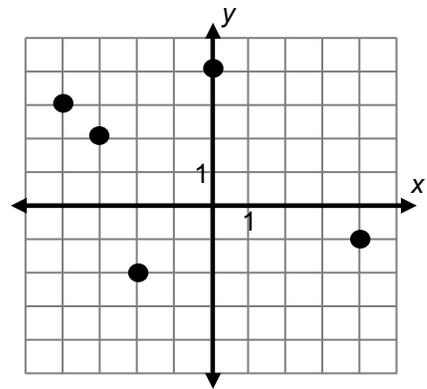
x	-3	-2	0	2
y	7	5	1	-3

Select ALL ordered pairs that could be included in the table so that it remains a function.

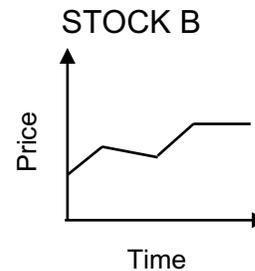
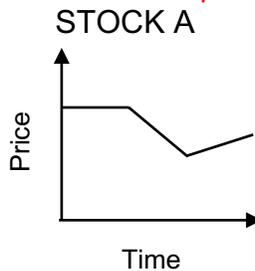
- a. (0, 0) **b.** (5, -2) **c.** (1, 1)
- d. (2, 3) **e.** (7, 0) f. (-3, 4)

2. The graph to the right represents y as a function of x . Which additional point can be plotted so that the graph continues to represent y as a function of x ?

- a. (0,1) **b.** (1,0) c. (4,2) d. (-2, 4)



3. The graphs below show the prices of two stocks during one day. Assume that they are scaled the same. *Answers may vary. Examples are shown.*



- a. Make two comparisons from the graphs.
*The two stocks appear to be about the same price at the end of the day.
 Both stocks had some periods of growth, some loss, and some stability over the course of the day.*
- b. Explain which stock, if either, you would choose to buy as an investment.
One day of performance for stocks at similar prices is not enough information to make an educated decision on an investment. And even this one day does not seem to show one is performing better than the other. Any justified response is acceptable, though.

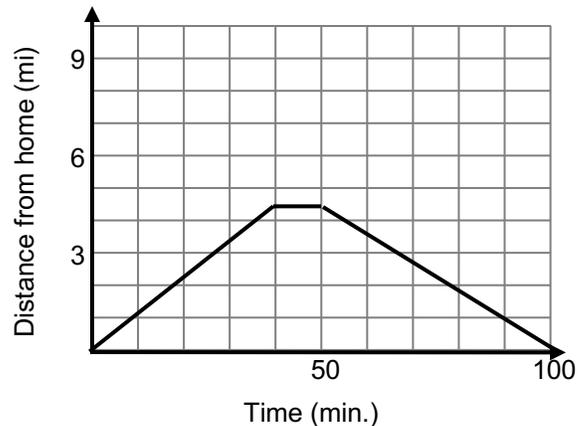
8-4 NONROUTINE PROBLEMS MIXED PROBLEMS Continued

4. Determine which equations define y as a linear function of x , and which equations define y as a non-linear function of x . Consider using a table of values or a graphing calculator.

Equation	Linear Function	Non-Linear Function
$y = 1.2x^2$		✓
$2y = x$	✓	
$x + y = 7$	✓	
$y = 4x + 3^2$	✓	
$y = 4 + 3x^2$		✓
$y = 2x^3 - 1$		✓
$y = \frac{x}{5}$	✓	

5. The graph to the right shows a runner's distance from home over time.

Based on the graph, determine whether each statement is true or false.



a. The runner's distance from home is increasing between 40 minutes and 50 minutes.

false

b. The runner's distance from home is increasing from 10 minutes to 40 minutes.

true

c. The runner's distance from home is increasing from 50 minutes to 100 minutes.

false

d. The runner ran at a constant speed the entire time.

false

8-4 NONROUTINE PROBLEMS FROM THE MATH OLYMPIAD

1. A cricket chirps 6 times every 8 seconds. At that rate, how many times does the cricket chirp in 2 minutes?

90 chirps (2 minutes = 120 seconds)

Number of Chirps	6	3	30	60	90
Time (seconds)	8	4	40	80	120

2. If 4 people can paint 2 fences in 5 hours, how many hours will it take for 8 people to paint 8 fences?

10 hours; one explanation: If 2 fences can be painted in 5 hours by 4 people, at that rate, 8 fences can be painted by those same 4 people in 4 times the number of hours, which is 20 hours. If it takes 4 people 20 hours to paint 8 fences, it will take 8 people half as long to paint those same 8 fences, which would be 10 hours.

# people	# fences	# hours	reasoning
4	2	5	given
4	8	20	4 times as many fences will take 4 times as long with the same # of people
8	8	10	Double the # of people above and it takes half the time

3. Suppose that toothpicks are used to make rows of figures. Rows of 1, 2, and 3 figures are pictured for some figures below. There is one toothpick per side. How many toothpicks are needed for a row with 100 figures?

a. squares  301

b. pentagons  401

Squares: start with 4 toothpicks to make the first square, and add 3 toothpicks to make each additional square. Since 100 figures will be made with 4 toothpicks (1st square) plus 3 times some number (the 99 additional squares), $4 + 3(99) = 301$.

Pentagons: start with 5 toothpicks to make the first pentagon, and add 4 toothpicks to make each additional pentagon. Since 100 figures will be made with 5 toothpicks (1st pentagon) plus 4 times some number (the 99 additional pentagons), $5 + 4(99) = 401$.

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