UNIT 1
TEACHER EDITION

MathLinks
GRADE 8

Annotated
for new users

LANE AND SOLID FIGURES

Unit Planning Information
(TE-UPI)
Numbered i, ii, iii, …

Annotated Answer Key,
including Student Packet
and Lesson Notes
(TE-AK)
Numbered 0, 1, 2, …

Each *MathLinks* Unit is organized this way

The start of TE-UPI

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
8.G.A	**Understand congruence and similarity using physical models, transparencies, or geometry software.**
8.G.5	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, ~~and the angle-angle criterion for similarity of triangles~~. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
8.G.C	**Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**
8.G.9	Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Common Core State Standards for the Unit are listed here and also at the end of the Student Packet.

Individual sets of standards are listed at the beginning of each lesson.

Standards for the entire grade level are in Program Information.

UNIT PLANNING

* Starred resources can be accessed under Unit Resources on the Teacher Portal.

Unit Pacing* Up to 14 class hours Pacing detail (estimates)	1.0 Opening Problem: Paper Solids (1 hour) 1.1 Volume of Cylinders (3 hours) 1.2 Volume of Cones and Spheres (2 hours) 1.3 Lines, Angles, and Triangles (4 hours) Review (3 hours) Assessment (1 hour)
Unit Resources* Up to 3 class hours On the Teacher Portal	• Extra Problems • Essential Skills • Math Talks (Data, Picture) • Nonroutine Problems • Technology Activities • Tasks (All About Angles, The Math of Collecting Rain Water) • Projects (Packing Problems, Shapes in our World) • Parent Support Letters
Assessment Options* See Portal Unit 1 → Other Resources → Assessment, Follow-up Details in Program Information	• On the Teacher Portal ✓ Unit Quizzes ✓ Cumulative Tests ✓ Tasks ✓ Projects The *MathLinks* Rubric (with rubric-worthy problems) • In the Student Packet ✓ Monitor Your Progress ✓ Unit Reflection • In the Teacher Edition ✓ References to Journals ✓ Suggested problems for the *MathLinks* Rubric
Materials See Program Information for supplies for the year	• Scratch paper [1.0] (3 per student or group) • A few dimes and quarters [1.1] (optional) • Geometric volume set, water, paper towels [1.2] (optional) • Protractor [1.3] (1 per student) • Toilet paper rolls [1.3] (1 per student, pair, or group) • General supplies (e.g., colored pencils, markers, rulers, tape, scissors, graph paper, calculators, chart paper)
Slide Decks* On the Teacher Portal	S1.0 Paper Solids S1.1 Volume of a Cylinder S1.2 Volume of a Cone and a Sphere S1.3a Two Investigations Rolled into One S1.3b Angle Relationships
Reproducibles* At the end of TE-UPI	R1-1 Triangles [1.3] (optional, 1 page for up to 8 students) R1-2 Angles Cards [1.3] (1 per pair/group) R1-3 Angle Facts Related to Triangles Cards [1.3] (1 per pair/group) R1-4 Match and Compare Sort Cards: Plane and Solid Figures [Review] (1 per pair/group)
Prepare Ahead Use Activity Routines files in General Resources at the start of the year	• Internet access or a geometric volume set [1.2] Ask students to bring empty toilet paper rolls to class [1.3] Cut triangles of different sizes ahead of time, or use R1-1 [1.3] See Activity Routines in Program Information for directions for the *MathLinks* Rubric, Poster Problems, Match and Compare Sort, and READY-X [1.1, 1.2, 1.3, Review]
Other Resources on the Teacher Portal Watch Getting Started Videos in General Resources at the start of the year	• Getting Started Videos and Resources - General Resources • Skill Boosters - Teacher Access page (fractions) nes - Teacher Access page (What's My Angle?, READY-X,

PLANNING FOR DIFFERENT USERS

Student Packet (SP)

Unit 1 component options for those who support students:

For teachers	• Teacher Edition (this document) • Teacher Portal (Unit Resources, General Resources) • Program Information
For substitutes	• SP (Practice 1 – 4 may be completed independently any time after instruction; Spiral Review; Vocabulary Review) • Extra Problems • *MathLinks* Puzzles / Games
For parents	• Resource Guide • Parent Letter (English and Spanish)

Print copy and on the Portal

Created by Carole Greenes for *MathLinks* users; on the Portal; not linked to any course

In addition to unit-by-unit Parent Letters, the public website also has an introductory letter.

Unit 1 component options to use with all students (all available on the Teacher Portal):

- SP (Word Bank, (
  Activity Routines,
  self-monitoring, jc
- Student Packet T
- Extra Problems (
- Essential Skills (just-in-time review)
- Math Talks (whole-class discourse)
- Nonroutine Problems (enrichment)
- Tasks (multi-part problems)
- Projects (authentic multi-day experiences)
- Technology Activities (variety)
- *MathLinks* Puzzles / Games (fun challenges)

Unit 1 component options for particular subgroups of students:

For English learners see pgs x-xi for specific strategies	• SP Text File for Translation • SP features for language development (Word Bank, Vocabulary Review, consistent structure for reading and writing, grouping opportunities for speaking and listening) • SP Routines for language development (rubric-worthy problems with the *MathLinks* Rubric, Poster Problems, Match and Compare Sort) • Math Talks for speaking and listening
For struggling learners see pgs x-xi for specific strategies	• SP features for math confidence (Getting Started, Review including Spiral Review, Word Bank, Vocabulary Review, consistent structure, g • SP Activity Routines for math confidence (rubric-worthy s Rubric, Poster Problems, Match and Compare Sort, RE • Essential Skills for just-in-time intervention • Extra Problems (by lesson) for practice, review, or assessment • Skill Boosters (Fraction Multiplication and Division)
For enrichment and advanced learners	• SP enrichment (see page xiv for specific options) • SP Activity Routines for enrichment (rubric-worthy problems with the *MathLinks* Rubric, Match and Compare Sort) • Nonroutine Problems (including problems form the Math Olympiad) • Technology Activities for variety • Projects for applications

Use a translation app

A skills practice routine; on the Portal; not linked to any course

Activity Routines recur throughout a course. Use the introductory activities in General Resources first.

MATH BACKGROUND

Why Does a Circle Have 360 Degrees?

Lack of sources makes it impossible to determine how various enumeration systems originated. There has been a great deal of speculation about why various systems were devised and how they interacted.

We know virtually nothing about the systems of enumeration used by the Sumerian civilization in Mesopotamia (before say 2200 BCE). However, we have extensive evidence of the systems of enumeration used by the subsequent old Babylonian civilization, going back past the Code of Hammurabi (1800 BCE). The evidence comes from excavations of mathematical cuneiform tablets with extensive records of warehouse inventories and also astronomical observations and calculations.

The two most important features of Babylonian mathematics are:

1. a place value system of recording numbers, and
2. use of a base-60 system of enumeration.

The use of a base-60 system is convenient because 60 has so many divisors. One theory about how this system evolved is based on currency conversions. In antiquity there were numerous currency systems, with different bases (think of pounds, shillings, and pence). It might have been natural for some merchants in trading centers to adopt a unit that would facilitate conversion rates among various currencies.

Whatever the origin of the base-60 system, the choice was serendipitous in the sense that it facilitated many mathematical calculations, such as calculations of reciprocals. In the golden era of Mesopotamian astronomy (final four centuries BCE), the Babylonian astronomers had a big advantage over the Egyptians, who were tied to a system of unit fractions that did not facilitate the intricate calculations required by astronomers.

The number 360 of degrees in a circle is a relatively recent development, stemming from the golden age of Mesopotamian astronomy. One explanation for the number 360 is that it might have been natural to approximate a circle (think in terms of the constellations through which the moon and planets pass) by chords that are sides of a regular hexagon composed of 6 equilateral triangles, thereby dividing the skies into 6 sectors. In a base-60 system, it would be natural to divide further the central angle of each triangle into 60 degrees, thereby leading to a total of 6 times 60, or 360, degrees in the circle. Further subdivision leads to 60 minutes in a degree, and 60 seconds in a minute.

A convenient property of the number 360 is that it is close to the number of days in a calendar year. The sun advances across the firmament by close to $\frac{1}{360}$ of a turn each day. The Egyptian calendar, claimed to have been adopted before 4000 BCE, divided the year into 12 months of 30 days for a total of 360 days, plus 5 extra days added to the end of the cycle. This scheme was modified by the Romans (Julian calendar of 12 months with a total of 365 days), and it was fine-tuned by Pope Gregory XIII (Gregorian calendar, with a 400-year cycle of prescribed leap years).

Math Background includes adult-level information and explanations written by our PhD mathematicians.

Angle Measurement

In Euclidean geometry, an angle is a geometric shape formed by two (distinct) rays in a half-plane that share a common endpoint (the vertex of the angle). Through a limiting process, it is possible to assign degree measures to angles (in a half-plane) so that:

1. the measure of an angle is a positive number between 0 degrees and 180 degrees;
2. a right angle has measure 90 degrees;
3. when we bisect an angle, we get adjacent angles whose measures are both half of the original angle; and
4. when we adjoin two adjacent angles in a half-plane, we get an angle whose measure is the sum of the measures of the two angles.

The degree measure of an angle can be expressed in terms of the length of the minor circular arc cut out by the angle on a circle centered at its vertex. In fact, the degree measure of the angle is 360 times the fraction of the length of the circle cut out by the angle.

Example: Since a right angle cuts out a quarter of a circular arc centered at the vertex, its degree measure is a quarter of 360, or 90 degrees.

Closely related to the measure of Euclidean angles is the angle of rotation of a ray as it turns about its vertex. If we rotate a ray through $\frac{1}{360}$ of a circle in a counterclockwise direction, we create an angle that measures 1 degree. We say that the angle of rotation is 1 degree. Similarly, if we rotate a ray counterclockwise through $\frac{2}{360}$ of a circle, we create an angle of measure 2 degrees, and we say that the angle of rotation is 2 degrees. And so on. Continuing in this fashion, we are led to angles of rotation of more than 180 degrees.

Example: The counterclockwise rotation of a ray through $\frac{3}{4}$ of a circle corresponds to an angle of rotation of 270 degrees (270 = $\frac{3}{4}$ times 360). The counterclockwise rotation of a ray through one full circle, bringing it back to its starting position, corresponds to an angle of rotation of 360 degrees (360 = 1 times 360). The counterclockwise rotation of a ray through two full circles (twice around), bringing it back to its starting position, corresponds to an angle of rotation of 720 degrees (720 = 2 times 360).

The angle of rotation can also be negative. A negative angle of rotation corresponds to rotating the ray in the negative direction, that is, in the clockwise direction.

Example: If we rotate the ray a quarter of a turn clockwise, we create a right angle, but since the sense of the rotation is “backwards,” the angle of rotation is -90 degrees instead of +90 degrees.

Unlike the degree measures in Euclidean geometry, which run between 0 and 180 degrees, the angles of rotation run the entire number line, from $-\infty$ to $+\infty$.

A fundamental property of angles of rotation is the angle addition principle: The result of rotating a ray by c degrees, and then rotating it by d degrees, is a rotation of the ray by $c + d$ degrees.

Example: A rotation of a ray by 90 degrees, followed by another rotation of the ray by 90 degrees, results in a rotation of the ray by 180 degrees (90 + 90 = 180). The ray ends up pointing in the opposite direction of its initial position.

The Parallel Postulate

The parallel postulate is historically the most interesting postulate in Euclid's *Elements*. There are many equivalent formulations of the parallel postulate. Euclid's original formulation of the postulate is rather complicated:

> If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side with the angles less than the two straight angles.

Recall that two lines in the plane are parallel if they do not meet. Perhaps the most popular textbook formulation of the parallel postulate is the following:

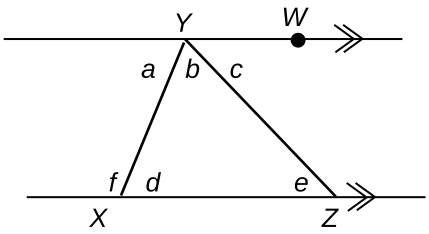
> Given a straight line and a point not on it, there exists one and only one straight line through the point and parallel to the given line.

Mathematicians tried for centuries without success to prove that the parallel postulate is a consequence of Euclid's other axioms. Finally, in the 1800's, through the work of a several mathematicians (Gauss, Bolyai, Lobachevsky, Beltrami), a geometry was discovered that satisfies all of Euclid's other axioms, but for which the parallel postulate fails. That geometry is called hyperbolic geometry.

In Euclidean geometry, the sum of the measures of the angles of a triangle is equal to 180° (a straight angle). It turns out that in hyperbolic geometry, the sum of the measures of the angles of a triangle is always less than 180°. In contrast, in spherical geometry (which does not satisfy all of Euclid's other postulates) the sum of the measures of the angles of a triangle is always greater than 180°.

Angle Sums in Triangles

Here are two important facts about angle sums in triangles.
They can be proved based on the properties of angles formed when a transversal cuts two parallel lines.



1. The sum of the measures of the angles in a triangle is equal to 180 degrees.

   Symbolically: $|\angle b| + |\angle d| + |\angle e| = 180°$

Statements	Reason
There is a line $\overleftrightarrow{WY}$ passing through Y and parallel to $\overleftrightarrow{XZ}$.	Parallel postulate
$|\angle a| + |\angle b| + |\angle c| = 180°$	Sum of measures of angles on a straight line equals 180°.
$|\angle a| = |\angle d|$ and $|\angle c| = |\angle e|$	If two lines are parallel, then alternate interior angles have equal measures.
$|\angle b| + |\angle d| + |\angle e| = 180°$	Substitution.

2. The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

   Symbolically: $|\angle b| + |\angle e| = |\angle f|$

Statements	Reason
$180° = |\angle f| + |\angle d|$	The sum of the measures of supplementary angles is 180 degrees.
$|\angle b| + |\angle d| + |\angle e| = |\angle f| + |\angle d|$	Substitution. The sum of the measures of the angles in a triangle is equal to 180 degrees.
$|\angle b| + |\angle e| = |\angle f|$	Addition property of equality.

TEACHING TIPS

Applying Standards for Mathematical Practice (SMP)

Here is an abbreviated version of the SMPs and some ways they are applied in this unit.

SMP1	**Make sense of problems and persevere in solving them.** • Understand a problem and look for entry points • Consider simpler or analogous problems • Monitor progress and alter solution course as needed • Make connections between multiple representations • Check answers with a different method	[1.0, 1.1] The opening problem, **Paper Solids**, requires using volume of solids knowledge from previous grades and trying to extend it to a new concept, the volume of a cylinder. Students will revisit this problem in **Practice 2**. [Review] When solving **A Big Puzzle**, students must identify given information and then find an appropriate entry point to solve the problem.
SMP2	**Reason abstractly and quantitatively.** • Use numbers and quantities flexibly in computations • Attend to the meaning of quantities • Decontextualize a problem using symbols, manipulate them, and then interpret based on the context	[1.0, 1.1, 1.2, Review] Students may demonstrate computational flexibility as they find volumes of solids. [1.2] Students use data in formulas to find the volume of various ice cream cones, and interpret the results. [1.3] Students find the value of *n* using algebra, and use the value to determine measures of angles.
SMP3	**Construct viable arguments and critique the reasoning of others.** • Use assumptions, definitions, established results, examples, and counter examples to analyze an argument and discuss its merits or flaws • Make and test conjectures based on evidence • Analyze situations by breaking them into cases • Understand and analyze the approaches of others	[1.1] Students critique the reasoning of two students who state different approaches for finding the volume of a cylinder. [1.1, 1.2, 1.3] Given various assumptions and observations, students create plausible arguments for volume formulas and angle relationships.
SMP4	**Model with mathematics.** • Attach meaningful mathematics to everyday problems and questions of interest • Make reasonable assumptions and approximations to simplify a situation • Identify quantities, use mathematical tools (such as multiple representations, formulas, equations) to analyze relationships • Interpret results and draw conclusions in the context of the situation	[1.0] Students explore the volumes of shapes created from paper, make predictions about the shape that will have greatest volume, and draw conclusions. [1.1] In **A Coin Problem**, students use formulas to compare stacks of coins. [1.2] In **Ice Cream Cones**, students use data to determine which cone holds more ice cream and which is the better value.

Abbreviated descriptions of the Standards for Mathematical Practice appear in every unit.

Specific examples from the Student Packet for this unit.

Applying Standards for Mathematical Practice (SMP) Continued		
SMP5	**Use appropriate tools strategically.** • Select and use tools strategically (and flexibly) to visualize, explore, and compare information • Use technological tools and resources to solve problems and deepen understanding	[1.2] Experiments with pouring water and/or viewing internet videos helps students understand volume formulas for the cone and sphere. [1.0, 1.1, 1.3] Students use paper to create solids and to explore angle measures. [1.0, 1.3] Students use tools to measure geometric figures. [All Lessons] Students use calculators to assist with applying formulas as they learn concepts and solve problems.
SMP6	**Attend to precision.** • Calculate accurately and efficiently • Explain thinking using mathematical vocabulary • Use symbols appropriately • Specify units of measure	[1.1, 1.2] Students use pi in its exact form (π) or as a rational approximation. [All Lessons] Extensive vocabulary is introduced carefully so that students will be able to use and apply it.
SMP7	**Look for and make use of structure.** • Recognize the structure of a symbolic representation and generalize it • See complicated objects as composed of chunks of simpler object	[1.1] Students observe the relationship between a prism and a cylinder to establish the formula for the volume of a cylinder. [1.2] Students observe the cylinder-cone and cylinder-sphere relationships to establish the volume formulas. [1.3] Students use their knowledge about a straight angle to informally establish angle relationships in triangles and for parallel lines.
SMP8	**Look for and make use of repeated reasoning.** • Identify repeated calculations and patterns • Generalize procedures based on repeated patterns or calculations • Find shortcuts based on repeated patterns or calculations	[1.2] Students apply similar reasoning strategies to arrive at the formulas for a cone and a sphere. [1.3] After establishing that the number of degrees in a triangle (or straight angle) is 180°, students may use this fact to find angle measures without using a protractor.

Strategies to Support Different Learners

Classrooms typically include students with different learning styles and needs. Here are some specific ways that *MathLinks* supports special populations. Strategies essential to the academic success of English learners are noted with a star (*). See Universal Design for Learning in Program Information for more details.

	General Examples	***MathLinks* Examples**
Know your Learner	✓ Understand student attributes that support or interfere with learning ✓ Determine preferred learning and interaction styles ✓ Assess student knowledge of prerequisite mathematics content ✓ Check for understanding continuously ✓ Provide differentiation opportunities for intervention to reach more learners ✓ Encourage students to write about their attitudes and feelings towards math ✓ Use contexts that link to students' cultures*	Built into the *MathLinks* Design: SP: Getting Started, Spiral Review, Monitor Your Progress, Unit Reflection TE: References to Journals UR: Extra Problems, Essential Skills, Projects OR: Skill Boosters, Assessment Options [All Lessons] *MathLinks* TE's provide many suggestions for good journal prompts to help informally assess student understanding of concepts. When students write to you, use the opportunity to get to know them better, and their attitudes and experiences about learning mathematics. [1.1, 1.2, 1.3] Do not limit special needs students to skill pages alone. Scaffolding support within lessons allows for students to develop conceptual understanding at different paces and in different ways. Contexts, visuals, and manipulatives are vital in providing this support. [1.0, expr contexts in this unit.
Increase Academic Language through Mathematics	✓ Provide opportunities for students to read, write, speak, and listen ✓ Explain the academic vocabulary needed to access mathematical ideas, providing both examples and non-examples ✓ Use strategically organized groups that attend to language needs* ✓ Use rich mathematical contexts and sophisticated language to help ELs progress in their linguistic development* ✓ Use cognates and root words (when appropriate) to link new math terms to students' background knowledge*	Built into the *MathLinks* Design: SP: Word Bank, Vocabulary Review, Student Resources TE: Grouping suggestions, References to Journals, Suggested problems for the *MathLinks* Rubric UR: Math Talks OR: Critique student work on Slide Decks [1.1, 1.2, 1.3] The study of geometry includes extensive vocabulary that contain Latin roots. Ask students who speak Spanish or other Latin-based languages to share connections.* [1.1, 1.2, 1.3] Create word walls and flash cards to help students build geometry vocabulary.*

Many strategies are consistently in components that are built into the MathLinks design

A "*" indicates a strategy for English learners.

Components cited: Student Packet (SP), Teacher Edition (TE), Unit Resources (UR), Oth

These General Examples appear in every unit

Specific examples from the Student Packet for this unit

Strategies to Support Different Learners (Continued)

	General Examples	***MathLinks* Examples**
Increase Comprehensible Input	✓ Link concepts to past learning ✓ Make concepts meaningful through hands-on activities, visuals, demonstrations, and color-coding ✓ Use a think-aloud strategy to model appropriate thinking processes and academic language use ✓ Use graphic organizers to help students record information and data, see patterns, and generalize them ✓ Use multiple representations (pictures, numbers, symbols, words, contexts) of math ideas to create meaning and make connections ✓ Strategically sequence and scaffold to make mathematics accessible ✓ Simplify written instructions, rephrase explanations, and use verbal and visual clues*	Built into the *MathLinks* Design: SP: Structured workspace TE: Slide Deck Alternatives, Reproducibles, Materials OR: Slide Decks [1.2] The pouring water and/or internet video demonstrations provide visual plausibility for cone and sphere volume formulas.* [1.3] Use gestures and demonstrations to accompany hands-on experiments for angle sums in triangles and properties of parallel lines.* [1.0, 1.1, 1.2, 1.3] There is no need to create templates. Structured workspace allows students to organize work as they develop concepts. [1.0, 1.1, 1.2, 1.3] Visuals and hands-on activities are used in all lessons to help students make connections between abstract mathematical ideas and vocabulary.*
Promote Student Interaction	✓ Use flexible group configurations that support content objectives ✓ Use strategies and activities that promote teacher/student and student/student interactions (e.g., think-pair-share, Poster Problems) ✓ Encourage elaborate responses through questioning ✓ Allow processing time and appropriate wait time, recognizing the importance of the different requirements for speaking, reading, and writing in a new language* ✓ Allow alternative methods to express mathematical ideas (e.g., visuals, students' first language)*	Built into the *MathLinks* Design: SP: Lesson and Review activities TE: References for Journals, Suggested problems for the *MathLinks* Rubric UR: Math Talks, various games and puzzles OR: Slide Decks, Activity Routines [1.0] Consider groups of 3 for the paper folding experiment. This is a natural place to establish a collaborative environment and save paper in the process! [Review] **Poster Problems** occur in every unit. Taking time to establish this routine now will pay off all year long.

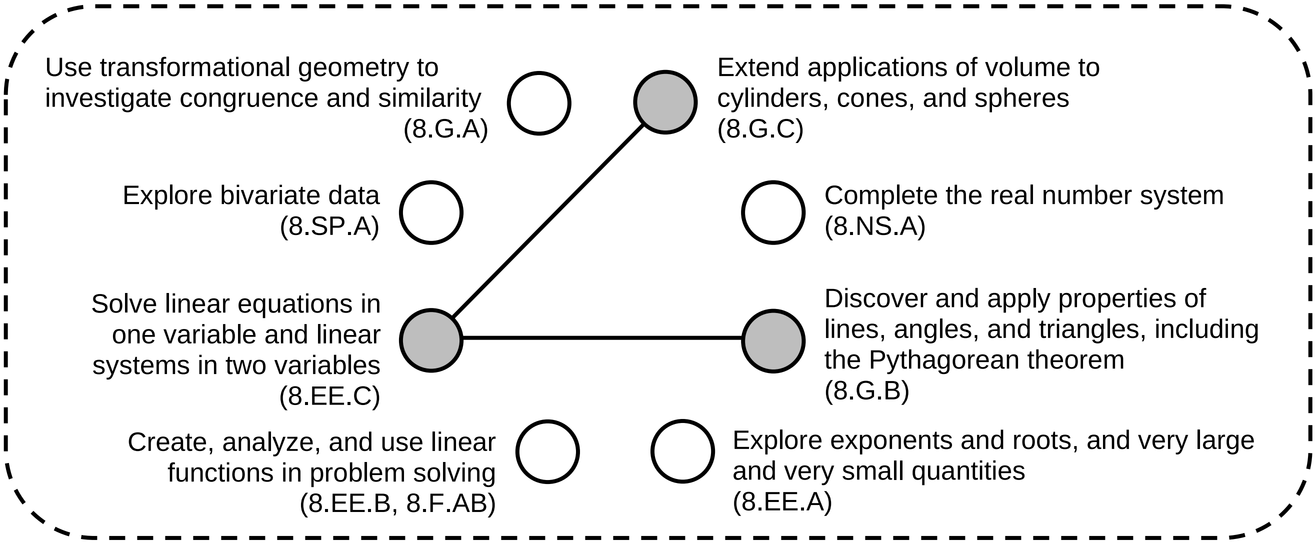
Components cited: Student Packet (SP), Teacher Edition (TE), Unit Resources (UR), Other Resources (OR)

Strategies to Support Different Learners combines the Universal Design for Learning (UDL) with other research-based strategies that are proven successful for a wide range of learners, especially those with special needs and English learners. See program Information for more details and a bibliography of cited references and resources for inspiration.

Big Ideas and Connections

The Center for Mathematics and Teaching is dedicated to igniting and nurturing passion for mathematics in middle school students. We see the classroom as a place of joy and wonder, collaboration and purpose, perseverance and empowerment. We want all students to succeed in mathematics, as they explore its beauty in patterns, concepts, connections, and applications.

MathLinks: Grade 8 is organized around eight big ideas. This graphic provides a snapshot of the ideas in Unit 1 and their connections to each other.



These ideas build on past work and prepare students for the future. Some of these include:

Prior Work

- Use a protractor (4.MD.C)
- Solve problems involving angle measures (7.G.AB)
- Find areas of triangles and quadrilaterals (6.G.A, 7.G.AB)
- Find volumes of prisms (6.G.A, 7.G.B)
- Find circumference and area of circles (7.G.B)
- Solve problems using numerical and algebraic expressions and equations (6.EE.AB, 7.EE.AB)

What's Ahead

- Analyze and solve linear equations (8.EE.C, HS)
- Solve problems involving surface area and volume (HS)
- Explain volume formulas and use them to solve problems (HS)
- Connections made to:
  - work from previous courses or from earlier in the grade (above)
  - work later in grade or to future courses (to the right)

The "Big Idea wheel" helps teachers and students see connections among mathematical ideas within a unit. Big Ideas also ensure a cohesive and efficient design program. The Big Idea wheel also appears on the Reelection page of every Student Packet. The *MathLinks* version was inspired by the work of Randy Charles and Jo Boaler (see the References section of Program Information, Portal version only). It consists of Domains from the Common Core State Standards.

Appears in every TE-UPI

Students May Wonder…

Why does a circle have 360°? See **Math Background** for this information. Students may also be interested to know that sometimes a circle is measured in “radians”, and one rotation of a circle is 2π. This makes calculations related to circumference of a circle easier, and is something they will learn more about in high school.

Is it possible to have a triangle whose interior angles sum to more than 180º? Yes! This is the case with spherical geometry. See **Math Background** for more information about the parallel postulate and spherical geometry.

Appears in every TE-UPI

Developing Language Skills through *MathLinks*

Language (reading, writing, speaking, and listening) helps students communicate math ideas and understand concepts. Here are some language examples in Unit 1.

Language Objectives

Student will:

(Lesson 1) Explain (writing or speaking) how a rectangular prism and a cylinder are the same and different and how their volume formulas relate to one another.

(Lesson 2) Explain (writing or speaking) how cylinders relate to cones.

(Lesson 3) Describe the differences between different types of angles using pictures and words. For example, how alternate interior and alternate exterior angles are different.

(Review) Collaborate in discussion on all activities with partners. Student groupings should reflect a safe environment for expressing ideas orally and in writing.

Group Discussions to Promote Reading, Listening, and Speaking (2+)

Critique reasoning situations appear on Slide Decks S1.1 and S1.3b. Use a partner strategy such as “turn and talk” to encourage students to respectfully critique the reasoning of other students. While walking around the room, encourage students to elaborate on their critiques.

On Angles (pg 12), provide time for pairs or small groups to work together to match vocabulary with pictures and definitions.

Practice 3 (pg 14) uses many new vocabulary terms. Ask students to share connections they see with these new terms.

All of the Review activities are designed for collaboration. Expect 100% participation and plenty of discussion between partners in pairs, trios, or groups of four (depending upon the activity). After written answers are made, consider groups sharing out with other groups, or with the whole class.

Journal Ideas to Promote Writing

(Explaining Concepts) Journal ideas appear on pages 6, 12, and 15.

(New Language) In Lesson 1 we found volume of a cylinder and in Lesson 2 we found volume of a cone and a sphere. Explain in your own words how these three figures are the same and how they are different.

(Language in the Real World) Describe figures in the real world (not already used in this unit) that remind you of cylinders, cones, and spheres.

Appears in every TE-UPI

Enrichment and Challenges for Advanced Learners

MathLinks: Grade 8 materials provide multiple opportunities for advanced students to investigate grade-level mathematics at a higher level of complexity without doing more work than their peers.

Within this Student Packet, here are some pages that have accessible entry points and opportunities for extensions (low floors / high ceilings).

- Paper Solids (pg 1) with Practice 2: Extend your Thinking (pg 5)
- Two Investigations Rolled Into One (pg 11) Challenge students with the question: ***How many degrees would be in the interior angles of a polygon if it had 4 sides? 5 sides? 6 sides? 7 sides? n-sides?***

Every student does not need to do every problem in a Student Packet. For those who are ready, challenge them with these pages, while other students might do unfinished work, Spiral Review, or more practice problems as needed.

- A Coin Problem (pg 6)
- Ice Cream Cones (pg 9)
- Angle Facts Related to Triangles (pg 15)

Consider speeding up instruction and skipping some Practice and Spiral Review. See also Planning for Different Users (TE, pg iii) and Enrichment for Advanced Learners and Those with Undiscovered Hidden Talents (Program Information → Universal Design for Learning) for more ideas.

Pi (π)

The choice of the approximate value to use for π depends on the context of and numbers in a problem, and the directions given. Teachers may want to guide students to leave answers in terms of π when solving basic mathematical problems (e.g., find the volume of a cylinder), but use an approximation of π when solving real life problems (find the volume of the soup can). To give students added practice, teachers may ask for solutions to problems in both forms.

Teaching Tips later in the TE-UPI are about specific instructional strategies. See above and also the following page.

Lines and Angles

Students begin to learn about lines and angles in 4th grade, according to the Common Core State Standards, and substantial work on this topic occurs in 7th grade.

4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement

4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, (e.g., by using an equation with a symbol for the unknown angle measure).

7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multistep problem to write and solve simple equations for an unknown angle in a figure.

Be aware that students may have forgotten these prior experiences. Some review is embedded in lessons. Use Unit Resources for additional review work if needed.

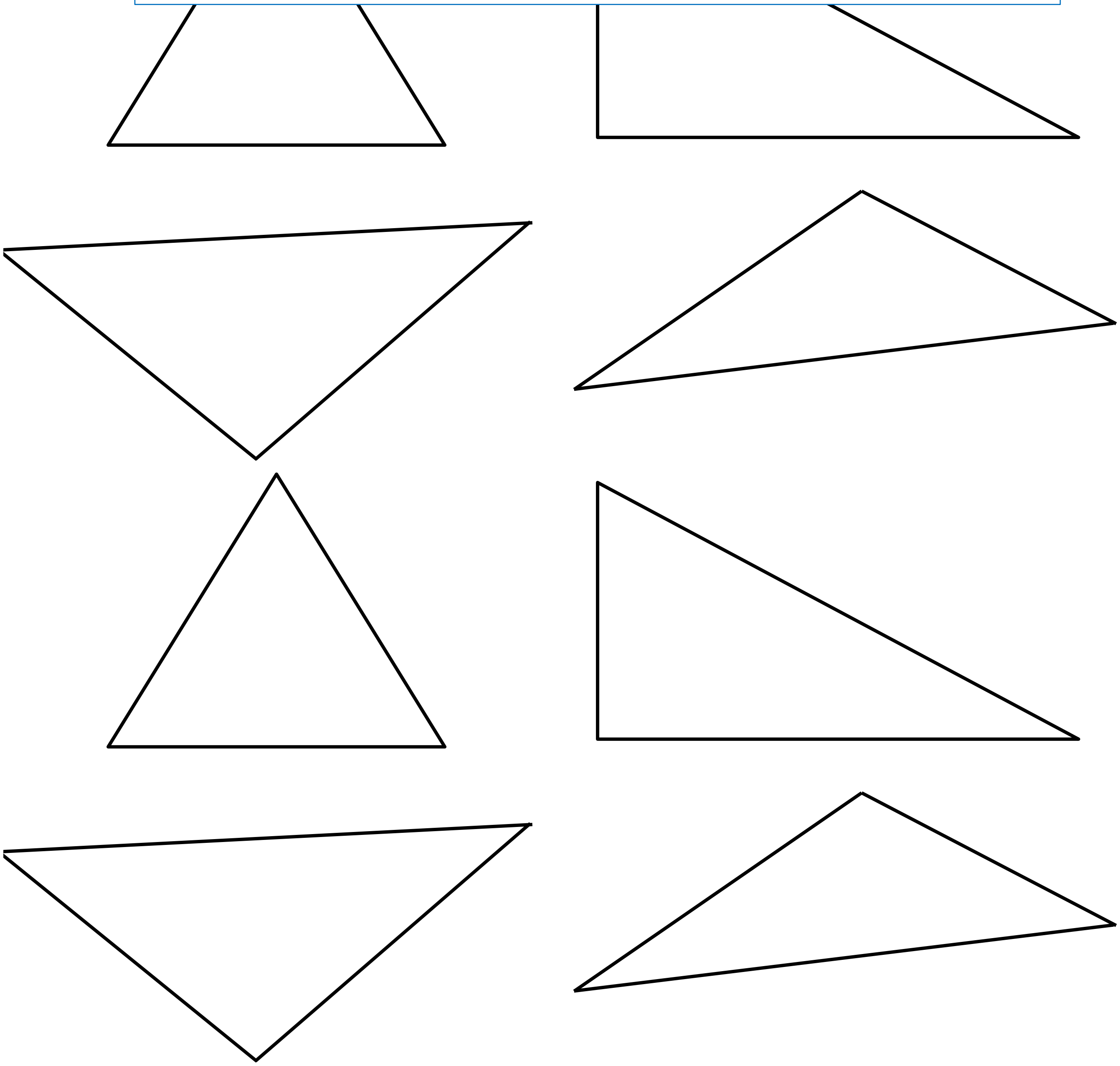
Conventions when Drawing Diagrams

In geometry, we use diagrams to help interpret a problem. Typically, students may assume the relative position and shapes of objects in diagrams. For example, we will assume that straight lines are, in fact, straight. Students should be advised that, unless otherwise noted, a drawing such as an angle or triangle is not necessarily to scale, nor can it be used to estimate measurements.

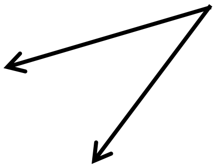
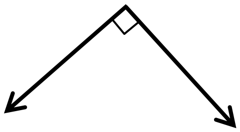
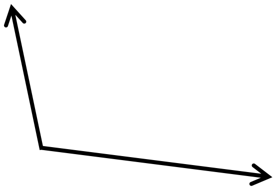
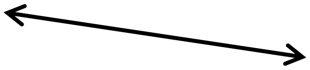
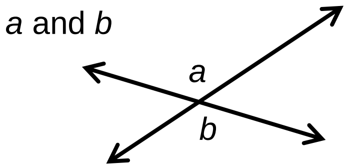
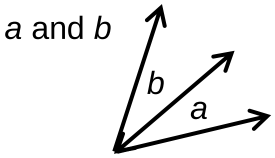
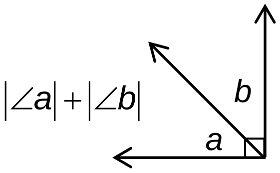
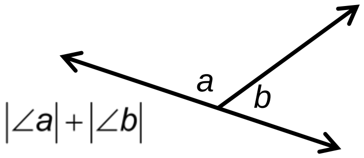
REPRODUCIBLES

R1-1 TRIANGLES

Reproducibles are typically for games, card sorts, templates, and manipulatives. They are listed on the TE-UPI Unit Planning page (always page ii) with details of where they are used and how many copies are needed. There is also a reminder on the TE-AK page where they are needed, though sometimes they are listed as optional.



R1-2 ANGLES CARDS

A Acute angle	**R** Less than 90º	**K** 
B Right angle	**O** Exactly 90º	
C Obtuse angle	**X** greater than 90º	**N** 
D Straight angle	**U** Exactly 180º	
E Vertical angles	**L** Share a vertex, but not a side	**T** 
F Adjacent angles	**V** Share a vertex and a side	
G Complementary angles	**Q** Sum equals 90º	**P** 
H Supplementary angles	**Z** Sum equals 180º	
M  	**S**  	**W** 
Y  		

R1-3 ANGLE FACTS RELATED TO TRIANGLES CARDS

G On a diagram, two **parallel lines** are signified by the use of arrows.	**M** On a diagram, two **angles with the same measure** are signified by the use of arcs.
R The sum of the angles along a straight line is 180°. (**a straight angle**)	**L** On a diagram, a 90° angle is signified by a small square. (**a right angle**)
J When two parallel lines are cut by a transversal, **alternate interior angles** have the same measure.	**P** When two parallel lines are cut by a transversal, **alternate exterior angles** have the same measure.
Q When two parallel lines are cut by a transversal, **corresponding angles** have the same measure.	**H** If $a = b$ and $b = c$, then $a = c$. **Substitution Property**
N If $a = b$, then $a - c = b - c$. **Subtraction Property of Equality**	**K** If $a = b$, then $ac = bc$. **Multiplication Property of Equality**

R1-4 MATCH AND COMPARE SORT CARDS: PLANE AND SOLID FIGURES

I △ TRANSVERSAL	I ○ PARALLEL LINES
II △ CONE	II ○ CYLINDER
III △ CORRESPONDING ANGLES	III ○ ALTERNATE INTERIOR ANGLES
IV △ ALTERNATE EXTERIOR ANGLES	IV ○ EXTERIOR ANGLE OF A TRIANGLE
A △ ✓ Angles are formed when lines are cut by a transversal. These are the pairs of angles that are “outside” the lines on opposite sides of the transversal.	A ○ ✓ an angle formed when a side of a triangle is extended ✓ this angle is adjacent to an interior angle of a triangle
B ✓ Angles are formed when lines are cut by a transversal. These are the pairs of angles that are in the same relative position.	B ○ ✓ Angles are formed when lines are cut by a transversal. These are the pairs of angles that are “inside” the lines on opposite sides of the transversal.
C △ ✓ has a circular base and an apex ✓ a common example of this holds ice cream	C ○ ✓ two lines in a plane that never intersect ✓ a common real-life example is the two stripes of a crosswalk
D △ ✓ a line that intersects two or more other lines	D ○ ✓ has a two parallel circular base. ✓ a common example of this holds soup, beverages, vegetables, etc.

Name ______________________________ Period __________ Date ______________

UNIT 1
ANSWER KEY

The start of TE-AK

Math Links
GRADE 8

PLANE AND SOLID FIGURES

For student self- assessment after each lesson → Monitor Your Progress

Materials   Grouping   Reproducibles   Slide Deck   Journal Idea

Look for these icons at the bottom of many TE-AK pages for instructional suggestions.

Parent (or Guardian) signature ______________

Parent Support letters are available on the Teacher Portal AND on the public website for those who help your students at home.

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

alternate exterior angles	alternate interior angles
cone	corresponding angles
cylinder	exterior angle of a triangle
parallel transversal	sphere

When a vocabulary word first comes up in context, take the time to support students in writing something that is meaningful to them, whether it's an explanation of the vocabulary in their own words, an example, and/or a picture.

This is the first of many annotations you'll see in red comic sans. This one is meant as a timely teaching note.

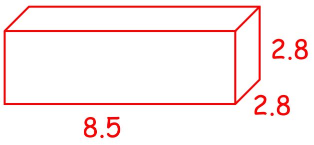
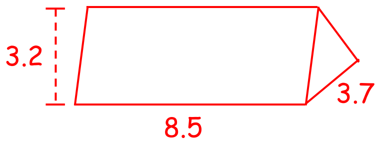
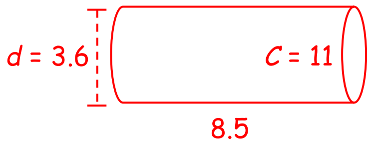
OPENING PROBLEM: PAPER SOLIDS

[SMP 1, 2, 4, 5, 6]

Lesson 1 addresses volume of a cylinder. Sketch 3 is exploratory.

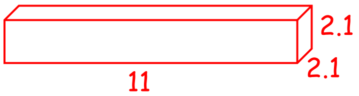
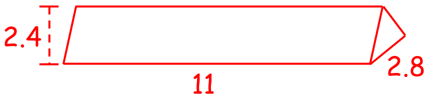
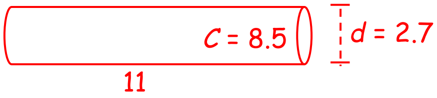
Measurements and rounding may vary (work done in 10ths of inches below).

Follow your teacher's directions for (1) – (4) for each solid.

(1)	**Sketch #1:**  	**Sketch #2:**  	**Sketch #3:** 
(2)	***Name of solid:*** Square (rectangular) prism	***Name of solid:*** Triangular prism	***Name of solid:*** Cylinder
(3)	***Area of Base:*** Area = (length) • (width) A = (2.8)(2.8) = 7.8 in^2	***Area of Base (h ≈ 3.2 in):*** Area = $\frac{1}{2}$ • (base) • (height)	***Area of Base (d ≈ 3.6 in):*** Area = π • (radius)^2 A = (3.14)(1.8)^2 = 10.2 in^2
(4)	***Volume (h = 8.5 i*** Volume = (Base are V = (7.8)(8.5) = 66.		***in):*** rea) • (height) 86.7 in^3

Every unit begins with an opening problem to motivate learning and pique interest. Some are wrapped up within a class hour. Many require more concepts and skills than students have at the beginning of a unit, so are revisited later. When to revisit is always stated directly in Lesson Notes or on a student page. This one is referenced on page 5.

Recording space for **Practice 2**.

Sketch #4:	**Sketch #5:**	**Sketch #6:**
 	 	
Area of Base: Area = $\ell\ w$ A = (2.1)(2.1) = 4.4 in^2	***Area of Base (h ≈ 2.4 in):*** Area = $\frac{1}{2}$ $b\ h$ A = $\frac{1}{2}$ • (2.8)(2..4) = 3.4 in^2	***Area of Base (d ≈ 2.7 in):*** Area = π r^2 A = (3.14)(1.4)^2 = 6.2 in^2
Volume (h = 11 in): V	***Volume (h = 11 in):***	***Volume (h = 11 in):*** = 68.2 in^3

Here are some helpful icons. You'll need a few simple materials. Slide Deck 1.0 is available to use (or its alternative page), with corresponding Lesson Notes. During instruction, it's suggested that students work individually or in trios.

Scratch paper, tape, ruler

S1.0

1 or 3

LESSON NOTES S1.0: PAPER SOLIDS

On slides, blue italic text suggests discussion; blue numl

Review Slide Decks (or the alternative page(s)) and Lesson Notes prior to instruction. These notes are not intended as a script.

In previous grades, students studied areas of common p
prisms. This activity revisits that work and challenges students to think about how to find the volume of a cylinder. Some students may reason that the volume formula, $V = Bh$ (volume equals the area of the base times the height) will hold for cylinders. Students will revisit this activity near the end of Lesson 1.

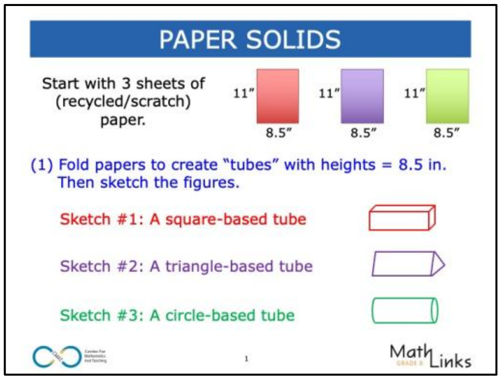
If this activity is done individually, students will need 3 sheets of paper. If done in groups of 3, each group will need 3 sheets of paper, and students can divide up the folding tasks.

- Slide 1: Use scratch paper. Ask students to fold each sheet of paper to create a tube, each of which will become the
lateral faces of solids w
each tube will be 8.5 in
together. Share folding

We think the ***bold italic*** questions are good to ask. Modify them to suit your students' needs.

  How did you fold your paper to make the square-based tube? In quarters on the 11-inch side ***The triangular based tube?*** In thirds ***The circular based tube?*** No folds.

  For (1), students sketch their models.



PAPER SOLIDS

Start with 3 sheets of (recycled/scratch) paper.

(1) Fold papers to create "tubes" with heights = 8.5 in. Then sketch the figures.

Sketch #1: A square-based tube

Sketch #2: A triangle-based tube

Sketch #3: A circle-based tube

- Slide 2: Explain that if the tubes had bases, they would be called "solids." For (2), students write the names and dimensions of the solids.

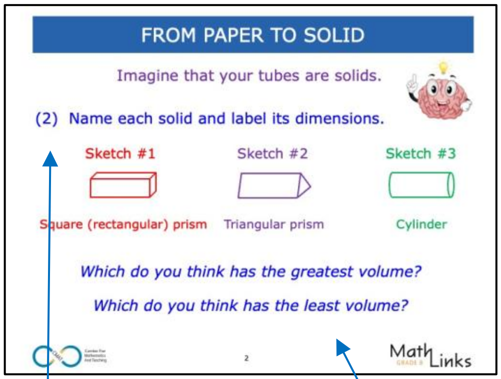
  What is the name of each solid you built for (1)? Square (rectangular) prism, triangular prism, and cylinder.

  What is the shape of each base? A square, a triangle, and a circle

  What are the lengths of the sides of each base? Square: 2.8 inches; Triangle: 3.7 in; Circle: no sides

  What is the height of each prism? All are 8.5 inches.

  Which of your solids do you think has the greatest volume? The least volume? Encourage discussions and reasoned explanations. This may not be obvious just by inspection. The cylinder has the greatest and the triangular prism the least.



FROM PAPER TO SOLID

Imagine that your tubes are solids.

(2) Name each solid and label its dimensions.

Square (rectangular) prism | Triangular prism | Cylinder

Which do you think has the greatest volume?

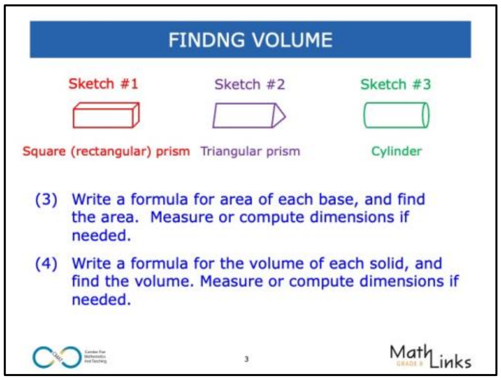
Which do you think has the least volume?

Discussion questions are always in italics in this blue color.

The same blue is used for numbered problems that require student writing.

LESSON NOTES S1.0: PAPER SOLIDS

Continued



FINDNG VOLUME

(3) Write a formula for area of each base, and find the area. Measure or compute dimensions if needed.

(4) Write a formula for the volume of each solid, and find the volume. Measure or compute dimensions if needed.

- Slide 3: For (3), students find the area of each base. Use the work and discussion time to informally assess student recall and understanding of area and volume. Entertain conjectures about what to do with the triangular prism and cylinder.

  Do you have all the dimensions that you need to find the area of the square base? Yes.

  Do you have all the dimensions that you need to find the area of the triangular base? No. Students will need to measure the height in order to find the area of the base. (This computation requires the Pythagorean theorem, which will be studied in Unit 2).

  Do you have all the dimensions that you need to find the area of the circle? Yes. If students recall from 7th grade how to find the radius given the circumference, then they can perform the computations. If they do not recall this, either review this topic or suggest they measure in order to find the diameter/radius.

  For (4), students write a formula for each solid and find the volume. Volume of a cylinder is the topic of lesson 1, but some students may conjecture correctly that it is computed by finding the area of the base (a circle) and multiplying by the height of the cylinder.

  Note that the height (h) in these formulas represents the height of the solid, not the height of the triangle (as in problem 3). This is determined by context, but also shows why it is important to identify variables.

  Students will revisit this page and dig deeper on **Practice 2: Extend Your Thinking**.

SLIDE DECK ALTERNATIVE S1.0: PAPER SOLIDS

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slide 1

(1) Fold the papers to create "tubes." Then sketch the

- Sketch #1: A square-based tube with height = 8.
- Sketch #2: A triangle-based tube with height =
- Sketch #3: A circle-based tube with height = 8.

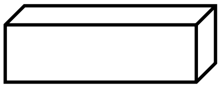
As an alternative to using the slides, some teachers project this page on a white board or use a document camera.

Note that all of the important parts of the slide decks are included on the alternative page.

Slides 2

(2) Name each solid and label its dimensions.

Sketch #1



Square (rectangular) prism

Sketch #2

Triangular prism

Sketch #3

Cylinder

Which do you think has the greatest volume?

Which do you think has the least volume?

Slide 3

(3) Write a formula for the area of each base, and find the area. Measure or compute the dimensions if needed.

(4) Write a formula for the volume of each solid, and find the volume. Measure or compute the dimensions if needed.

VOLUME OF CYLINDERS

We will develop the formula for the volume of a cylinder and use it to solve problems.
[8.G.9; SMP1, 2, 3, 4, 5, 6, 7]

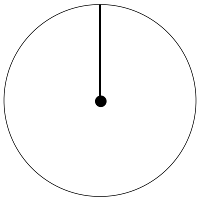
GETTING STARTED

Each lesson begins with this gray box containing a lesson summary and Common Core content and practice standards.

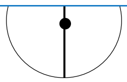
Getting Started problems follow and are intended to review or preview important content for the lesson.

Find the area of each (sures.

1. r = 6 mm (Use π = ns of π.)

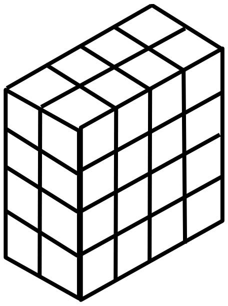


113.04 mm^2



16 π mm^2

Consider the rectangular prism pictured to the right to complete the following.



3. How many squares are in the rectangular base (B) of this prism (either top or bottom)? __8__

4. How many cubes are in the top (or bottom) horizontal "layer" of this rectangular prism? __8__

5. How many horizontal layers are there? __4__

6. How many cubes are there in all (the total volume)? __32__

7. Write the length, width, and height of this prism: ℓ = __2__, w = __4__, h = __4__

8. Write a formula to find the volume of a rectangular prism using ℓ, w, and h:

V = __ℓwh__

9. Write a second version of this formula using B as the area of the base:

V = __Bh__

Language used in directions and problems is precise and consistent from unit to unit.

VOLUME OF A CYLINDER

[SMP2, 3, 6, 7]

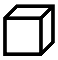
Follow your teacher's directions for (1) – (7). Use π = 3.14 as needed.

(1) ***List some common objects that are shaped like cylinders.***
Some possible answers:
Cans of food or drinks, candles, paper towel / toilet paper rolls, cups and glasses, coins, etc.

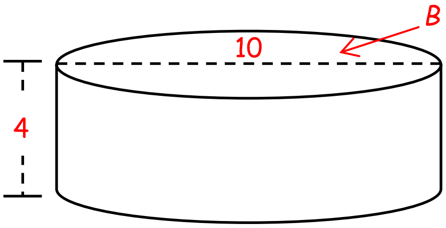
(2) ***Fill in the blanks for these common units of measurement.***

— If this is 1 unit of length,

□ then this is 1 square unit of area,

 and this is 1 cubic unit of volume.

(3) ***Label measurements for the height and diameter. Then find the area of the base (B).***
$B = (\pi r^2) = (3.14)(5^2) = 78.5\ u^2$



(4) ***Find the number of cubic units in the single layer.***
$(78.5)(1) = 78.5\ u^3$

(5) ***Find the total number of cubic units in the cylinder.***
$(78.5)(4) = 314\ u^3$

(6) ***Write a volume formula for a cylin***
$V = Bh$

(7) ***Substitute the circle area formula***
formula.
$V = (\pi r^2)h$

Notice the uncluttered, structured work space. Most *MathLinks* teachers do not require students to take traditional notes, since they are doing and discussing mathematics constantly, and all of the information students need is contained within the Student Packet.

Find the volume of each cylinder described below.

8. Use π = 3.14, h = 10 cm, d = 4 cm 125.6 cm^3	9. Use $\pi = \frac{22}{7}$, $h = \frac{7}{8}$ in, $r = \frac{1}{2}$ in $\frac{11}{16}$	10. Leave in terms of π, h = 6 mm, $C = 24\pi$ mm

A reminder that vocabulary should be recorded on page 0. New vocabulary is always underlined the first time it appears.

11. Record the meaning of <u>cylinder</u> in **My Word Bank**.

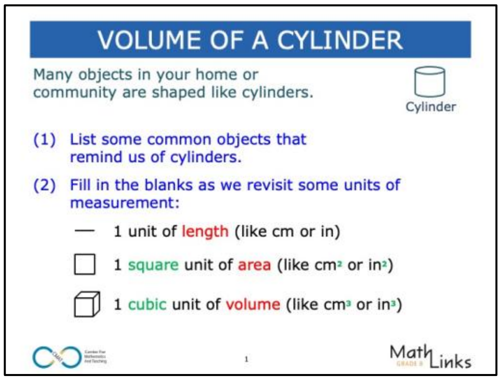
LESSON NOTES S1.1: VOLUME OF A CYLINDER

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

Students are guided to derive the formula for volume of a cylinder based upon prior knowledge of volume of a prism. We use B to represent the area of the base of a solid. Although exact numerical measurements for circular objects are not possible because π is an irrational number, some problems will direct students to use an exact value to represent π.

- Slide 1: Students are likely familiar with everyday objects that are cylindrical (or close approximations to them). For (1), they list some of these objects that come to mind. Share student lists.

  For (2), remind students of the difference between length, area, and volume measurements. Discuss notation, and continue to address it as needed.



VOLUME OF A CYLINDER

Many objects in your home or community are shaped like cylinders.

(1) List some common objects that remind us of cylinders.

(2) Fill in the blanks as we revisit some units of measurement:

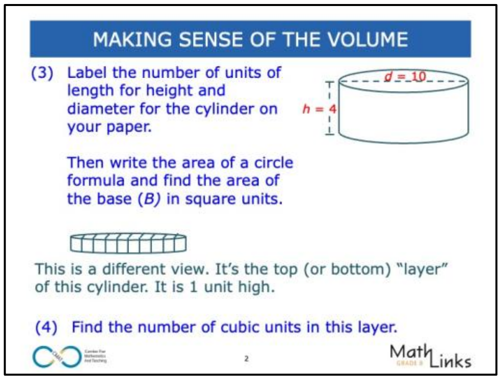
— 1 unit of length (like cm or in)

☐ 1 square unit of area (like cm^2 or in^2)

1 cubic unit of volume (like cm^3 or in^3)

- Slide 2: For (3), ask students to label the important measurements on the cylinder, and then find the area of the circular base.

  Reveal the top (or bottom) layer of the cylinder. For (4), students write the volume of this layer. They may not notice that they are actually using the volume of a cylinder formula to do this (*Bh* for *h* = 1).



MAKING SENSE OF THE VOLUME

(3) Label the number of units of length for height and diameter for the cylinder on your paper.

Then write the area of a circle formula and find the area of the base (*B*) in square units.

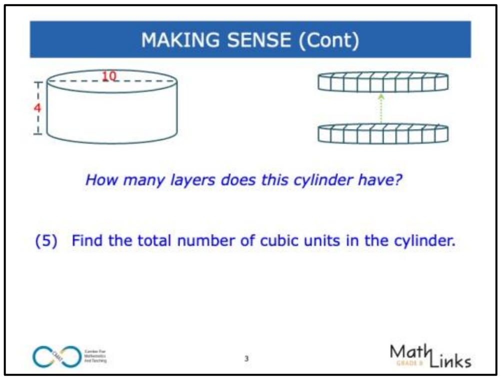
This is a different view. It's the top (or bottom) "layer" of this cylinder. It is 1 unit high.

(4) Find the number of cubic units in this layer.

- Slide 3: Continue to develop the volume formula.

  How many layers does this cylinder have? 4, which is the height of the cylinder.

  For (5), students multiply the number of cubes in the single layer times 4. They are again applying the volume of a cylinder formula.



MAKING SENSE (Cont)

How many layers does this cylinder have?

(5) Find the total number of cubic units in the cylinder.

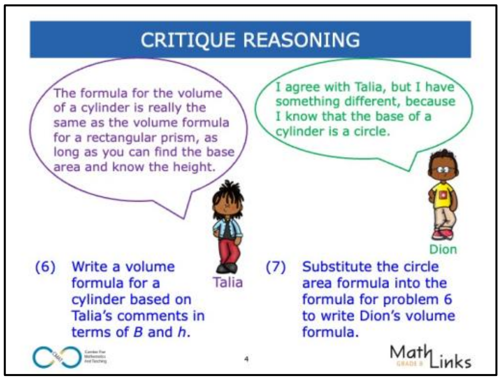
LESSON NOTES S1.1: VOLUME OF A CYLINDER

Continued

- Slide 4: Use the strategies from Talia and Dion to spark a discussion about two different ways to write the volume formula.

  For (6), students should arrive at the general $V = Bh$.

  For (7), students use substitution to establish $V = \pi r^2 h$.



CRITIQUE REASONING

The formula for the volume of a cylinder is really the same as the volume formula for a rectangular prism, as long as you can find the base area and know the height.

I agree with Talia, but I have something different, because I know that the base of a cylinder is a circle.

Talia

Dion

(6) Write a volume formula for a cylinder based on Talia's comments in terms of *B* and *h*.

(7) Substitute the circle area formula into the formula for problem 6 to write Dion's volume formula.

4

MathLinks

Numbered problems not in parentheses (such as 8 – 10 in this activity) are not referenced in the slide deck, and are intended for practice individually or in groups.

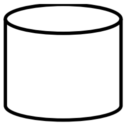
The students in our "*MathLinks* class" appear on the front of every Student Packet in grades 6 – 8. They portray a wide range of cultures and qualities. These students often share their thinking, their work, or their errors on slides, which is intended to stimulate class discussions. They typically do not appear in Student Packets to minimize clutter and maximize work space on student pages.

SLIDE DECK ALTERNATIVE S1.1: VOLUME OF A CYLINDER

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slide 1

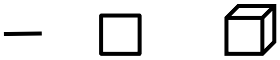
Many objects in your home or community are shaped like cylinders.



Cylinder

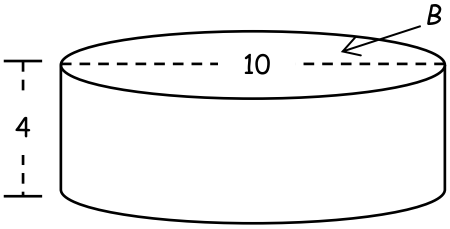
(1) List some common objects that remind us of cylinders.

(2) Fill in the blanks for these units of measurement:



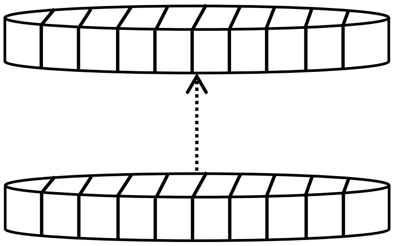
Slides 2 – 3

(3) Write in the height (*h*) and diameter (*d*) of this cylinder. Use the circle formula and find the area of the base, *B*.



This is a "layer" view of the cylinder.

(4) Find the number of cubic units in one layer.



Slide 4

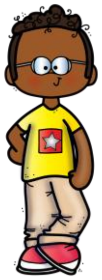
How many layers does our cylinder have?

(5) Find the total number of cubic units in the cylinder.

Talia

The formula for the volume of a cylinder is really the same as the volume formula for a rectangular prism, as long as you can find the base area and know the height.

I agree with Talia, but I have something different because I know that the base of a cylinder is a circle.



Dion

A *MathLinks* class critique reasoning opportunity. (SMP3)

(6) Write a volume formula for a cylinder based on Talia's comments in terms of *B* and *h*.

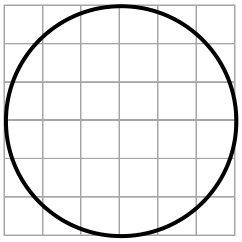
(7) Substitute the circle area formula into the formula for problem 6 to write Dion's volume formula.

Practice follows the introduction of a concept.

PRACTICE 1

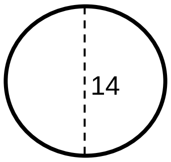
Find the volume of each cylinder described below.

1. The base is pictured below.
   Each small square is 1 square unit.
   The height is 10 units.
   Use π = 3.14.



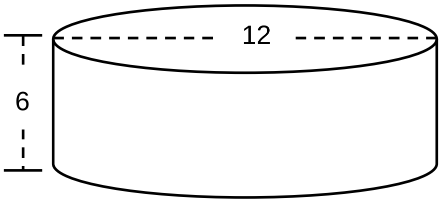
282.6 u^3

2. The base is pictured below.
   The diameter is given in millimeters.
   The height is 20 mm.
   Use π = 3.14.



3,077.2 mm^3

3. Units given in centimeters.
   Use π = 3.14.



678.24 cm^3

4. The base radius is 6 feet. The height is 30 feet. Leave in terms of π.

1,080π ft^3

Watch for (4): Do students understand the direction "leave in terms of π?"

5. The base radius is $3\frac{1}{2}$ cm. The height is 4 cm. Use $\pi = \frac{22}{7}$.

A "watch for" is a timely formative assessment heads-up where many students traditionally have misconceptions, make mistakes, or struggle with concepts.

Find the height of each cylinder described below.

6. A cylinder with circumference of 62.8 inches has volume = 628 cubic inches.
   Use π = 3.14.

2 in

7. A cylinder with diameter = 8 cm has volume = 400 cubic cm.
   Write in terms of π and use π = 3.14.

$\frac{25}{\pi}$ cm and 7.96 cm

Appropriate for independent practice or for partners.

Calculator

1+

PRACTICE 2: EXTEND YOUR THINKING

The *MathLinks* Rubric: See Activity Routines on the Teacher Portal for directions. [SMP1, 4]

In the opening prob
from an 8.5 × 11-inc

See Program Information and the Teacher Portal (General Resources → Activity Routines) for more information and to prepare for using The *MathLinks* Rubric.

1. Go back to **Paper Solids** and update or correct your work if needed.

2. Suppose you created models for a square prism, triangular prism, and cylinder from an 8.5 × 11-inch piece of paper with a height of 11 inches. Which of the six models do you think would have the greatest volume? Why?
   Predictions will vary.

Revisiting contexts is common when continuing concept development.

3. In the space provided on **Paper Solids**, sketch the following with 11-inch heights:
   Answers are on Page 1.
   - Sketch #4, a square prism,
   - Sketch #5, a triangular prism, and
   - Sketch #6, a cylinder.

   Then find the areas of the bases and the volumes.

4. Write conclusions based on your work. Compare volumes based on the height or shape of the base. Include which has the greatest volume and the least volume in your explanation.
   Of the three solids with the same height, the cylinder has the greatest volume, and the triangular prism the least. Of the two different heights, the shorter solids have the greater volumes. So, the short cylinder (#3) has the greatest volume of all six solids, and the tall triangular prism (#5) has the least.

5. A soup can is measured and found to have a radius of about 3.7 cm and a height of about 7.3 cm. The label on the can lists the volume as 310.52 mL. Is this a reasonable volume of soup? Explain. (1 cubic cm is equivalent to 1 mL.)
   Yes, this is reasonable. The volume of the can is 313.8 cm^3. The slight difference may be because the contents are not filled to the top.

A COIN PROBLEM

[SMP2, 4, 5, 6]

	Dime	**Quarter**
Weight (grams)	2.27	5.67
Thickness (mm)	1.35	1.75
Diameter (mm)	17.9	24.26

To the right is some information about coins.

Suppose you had a $10 stack of dimes and a $10 stack of quarters.

1. Make some predictions: Predictions will vary.

   - Which stack will have the greatest weight?
   - Which stack will have the greatest height?
   - Which stack will have the greatest volume?

2. Compute the weight, height, and volume for the $10 stacks of coins.

	weight	**height**	**volume**
dime	227 g	135 mm	33,955.45 mm^3
quarter	226.8 g	70 mm	32,340.69 mm^3

3. Compare your results to your predictions. Were there any surprises?
   Conclusions will vary. Some may be surprised that the weights and volumes of a $10 stack of dimes and $10 stack of quarters are about the same.

Problem 3 is suggested as possible journal idea.

Also a reminder at the end of the lesson to ask students to go back to the Student Packet front cover for self-assessment

Calculator

1+

Journal Problem 3
Monitor Your Progress

VOLUME OF CONES AND SPHERES

We will develop formulas for the volume of a cone and a sphere and apply them to solve problems.
[8.G.9; SMP2, 3, 4, 3, 5, 6, 7, 8]

GETTING STARTED

1a. Write the formula for the volume of a cylinder in words and using symbols.

Volume = (Base area)(height) = *Bh*

$V = \pi r^2 h$

1b. Find the volume of a cylinder with a height of 20 cm and diameter of 14 cm. Express an exact answer in terms of π and an approximate answer using $\pi = \frac{22}{7}$.

$V = \pi r^2 h$

$= \pi \bullet (7)^2 \bullet (20)$

$= 980\pi \text{ cm}^3$

or 3080 cm^3

2a. Write the formula for the circumference of a circle in words and using symbols.

Circumference = π (diameter)

$C = \pi d$

2b. Find the diameter of a circle with a circumference of 40 in. Express an exact answer in terms of π and an approximate answer using $\pi = 3.14$.

$C = \pi d$

$40 = \pi d$

$\frac{40}{\pi} = d$

or 12.7 in

Calculator

1+

VOLUME OF A CONE AND A SPHERE

[SMP2, 3, 5, 6, 7, 8]

Follow your teacher's directions for (1) – (5). Use π = 3.14 as needed.

(1) ***List some common objects that are shaped like cones.***
Some possible answers:
Traffic cones, ice cream cones/waffle cones, funnels, megaphones, etc.

(2) ***List some common objects that are shaped like spheres.***
Some possible answers:
Balls like baseballs, tennis balls, and basketballs; planets; fruit like oranges, grapefruits, and grapes; etc.

(3) ***Predict the order from greatest to least of the volume of a cylinder, cone, and sphere with all the same diameters and heights.*** Predictions may vary.

(4) ***Write a sentence that relates the volume of a cylinder and cone. Then derive the formula for the volume of a cone.***
The volume of the cone is one-third the volume for the cylinder. The volume of the cylinder is 3 times the volume of the cone.

$V_{cylinder} = \pi r^2 h$

$V_{cone} = \left(\frac{1}{3}\right) \bullet V_{cylinder}$

$$\boxed{V_{cone} = \frac{1}{3}\pi r^2 h}$$

(5) ***Write a sentence that relates the volume of a cylinder and sphere. Then derive the formula for volume of a sphere.***
The volume of the sphere is two-thirds the volume for the cylinder. The volume of the cylinder is 1.5 times the volume of the sphere.

$V_{cylinder} = \pi r^2 h$

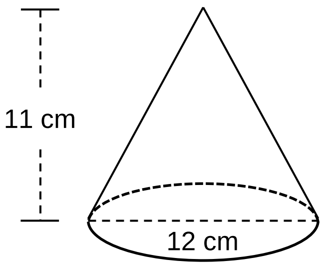
$V_{sphere} = \left(\frac{2}{3}\right) \bullet V_{cylinder}$

$V_{sphere} = \frac{2}{3}\pi r^2 h = \frac{2}{3}\pi r^2 \bullet d;\ [d = 2r]$

$$\boxed{V_{sphere} = \frac{4}{3}\pi r^3}$$

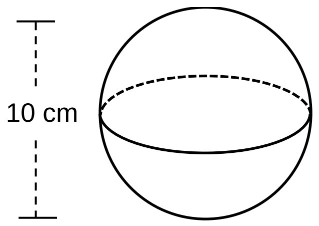
Find the volume of each figure below. Use π = 3.14. Round appropriately.

6.



414.5 cm^3

7.



523.3 cm^3

8. Record the meanings of <u>cone</u> and <u>sphere</u> in **My Word Bank**.

Internet access of cylinder set, water, towels

S1.2

2+

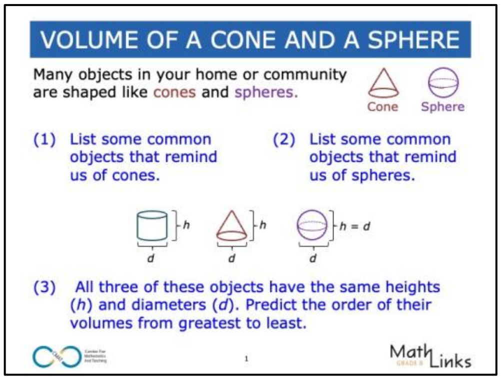
LESSON NOTES S1.2: VOLUME OF A CONE AND A SPHERE

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

To guide students to derive the formulas for volume of a cone and a sphere based upon prior knowledge of volume of a cylinder, a demonstration is required. Teachers either need to use a plastic set of these geometric figures, or show a video. For video choices, search online for "water pouring video for deriving the formula for volume of sphere" and "water pouring video for deriving the formula for volume of cone" and choose the preferred option.

- Slide 1: Students are familiar with everyday objects that are conical and spherical (or close approximations to them). For (1) and (2), students list some of these objects. Share student lists.

  For (3), ask students to predict the order from greatest to least of the volumes of cones, spheres, and cylinders with the same diameters and heights. Discuss predictions.



VOLUME OF A CONE AND A SPHERE

Many objects in your home or community are shaped like cones and spheres.

(1) List some common objects that remind us of cones.

(2) List some common objects that remind us of spheres.

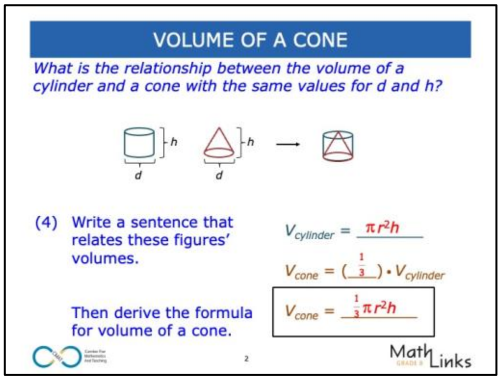
(3) All three of these objects have the same heights (*h*) and diameters (*d*). Predict the order of their volumes from greatest to least.

Questioning suggestions support students with problem comprehension, which is especially important for struggling learners and English learners.

- S for a cone.

  How do you think the volumes of the cylinder and cone compare? Answers may vary. The cylinder's volume is more than the cone's volume. Follow up with: ***How many times greater do you think the cylinder's volume is?***

  For (4), students write their observation. Then continue the prompts to derive the volume of a cone formula. Provide answers as needed.



VOLUME OF A CONE

What is the relationship between the volume of a cylinder and a cone with the same values for d and h?

(4) Write a sentence that relates these figures' volumes.

$V_{cylinder} = \underline{\pi r^2 h}$

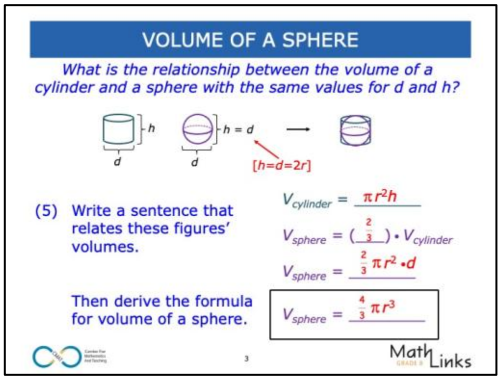
$V_{cone} = (\underline{\tfrac{1}{3}}) \cdot V_{cylinder}$

Then derive the formula for volume of a cone.

$\boxed{V_{cone} = \underline{\tfrac{1}{3}\pi r^2 h}}$

- Slide 3: Use this slide in the same way as the slide 2 to derive the formula for the volume of a sphere. For (5), there is an extra line in the derivation because students must make a substitution for height (*h* = *d* = 2*r*) so that volume is expressed in terms of the radius length.

  Revisit predictions from (3). For the same *h* & *d* values, the order from least to greatest for volumes in cylinder, sphere, cone.



VOLUME OF A SPHERE

What is the relationship between the volume of a cylinder and a sphere with the same values for d and h?

(5) Write a sentence that relates these figures' volumes.

$V_{cylinder} = \underline{\pi r^2 h}$

$V_{sphere} = (\underline{\tfrac{2}{3}}) \cdot V_{cylinder}$

$V_{sphere} = \underline{\tfrac{2}{3}\pi r^2 \cdot d}$

Then derive the formula for volume of a sphere.

$\boxed{V_{sphere} = \underline{\tfrac{4}{3}\pi r^3}}$

SLIDE DECK ALTERNATIVE S1.2: VOLUME OF A CONE AND A SPHERE

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

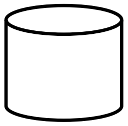
Slide 1

Many objects in your home or community are shaped like cones and spheres.

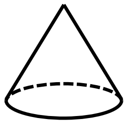
(1) List some common objects that remind us of cones.

(2) List some common objects that remind us of spheres.

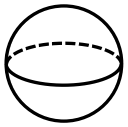
(3) All three of the objects to the right have the same height (h) and the same diameter (d). Note that for the sphere, $h = d$. Predict the order of their volumes from greatest to least.



Cylinder



Cone



Sphere

Slide 2

(4) How are the volume of the cone and cylinder related?
Derive the formula for volume of a cone:

$V_{cylinder} = \_\_\_\_\_\_\_\_\_\_\_\_$

$V_{cone} = (\_\_\_\_\_) \bullet V_{cylinder} \rightarrow$ $\boxed{V_{cone} = \_\_\_\_\_\_\_\_\_\_\_\_}$

Slide 3

(5) How are the volume of the sphere and cylinder related?
Derive the formula for volume of a sphere:

$V_{cylinder} = \_\_\_\_\_\_\_\_\_\_\_\_$

$V_{sphere} = (\_\_\_\_\_) \bullet V_{cylinder}$

$V_{sphere} = \_\_\_\_\_\_\_\_\_\_\_\_$ $\boxed{V_{sphere} = \_\_\_\_\_\_\_\_\_\_\_\_}$

ICE CREAM CONES

The *MathLinks* Rubric: See Activity Routines on the Teacher Portal for directions. [SMP2, 4, 5, 6]

An ice cream store has two different kinds of cones. For a single scoop, they fill the cone with ice cream and then put a dome (half sphere) of ice cream on the top. Below are the dimensions and prices for one scoop. *Remember that $d = 2r$.

	Height	**Top Diameter***	**Bottom Diameter***	**Cost**
Sugar Cone	$4\frac{5}{8}$ inches	2 inches	0 inches	\$3.50
Cake Cone	3 inches	$2\frac{1}{2}$ inches	$1\frac{1}{2}$ inch	\$3.50

Sugar Cone

Cake Cone

1. Predict which option you think will have the most ice cream: Answers will vary.

Watch for (2): Are students using the correct volume formulas?

2. Rank the amount of ice cream from least to greatest. Show formulas and substitutions.

Sugar Cone	**Cake Cone**
$V_{(cone)} = \frac{1}{3}\pi r^2 h = \frac{1}{3}(3.14)(1)^2(4.625) = 4.8\text{ in}^3$ $V_{(dome)} = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{1}{2}\left(\frac{4}{3}\right)(3.14)(1)^3 = 2.1\text{ in}^3$ TOTAL ICE CREAM = 6.9 in^3	Assume two cylinders. The bottom has twice the height as the top. $V_{(bottom)} = \pi r^2 h = (3.14)(0.75)^2(2) = 3.5\text{ in}^3$ $V_{(top)} = \pi r^2 h = (3.14)(1.25)^2(1) = 4.9\text{ in}^3$ $V_{(dome)} = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{1}{2}\left(\frac{4}{3}\right)(3.14)(1.25)^3 = 4.1\text{ in}^3$ TOTAL ICE CREAM = 12.5 in^3 This cone l

Instructional idea:
For problems that require students to explain their reasoning, consider doing some of these as pair-share activities. Discussing them first may improve the quality of written responses.

SMP 5 is all about choosing tools wisely. Helping students learn when to use a calculator is important. On this page, cumbersome arithmetic may get in the way of concept development and problem solving goals.

3. Which is the best buy? Which would you choose? Explain your reasoning.

Sugar cone unit price: $\frac{3.5}{6.9}$ = \$0.51 per in^3

Cake cone unit price: $\frac{3.5}{12.5}$ = \$0.28 per in^3

For ice cream quantity, the better value is the cake cone. But if a sugar cone is preferred to having more ice cream, then the sugar cone could be a better choice.

LINES, ANGLES, AND TRIANGLES

We will establish facts about angles in the interior and on the exterior of a triangle. We will introduce vocabulary and facts related to angles formed when two parallel lines are intersected by another line. We will use properties of parallel lines to solve problems.

[8.G.5; SMP2, 3, 5, 6, 7, 8]

GETTING STARTED

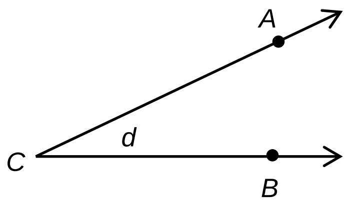


Figure 1

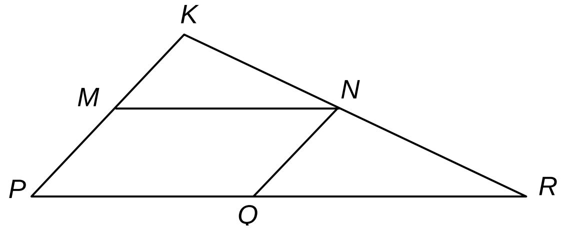


Figure 2

Refer to Figure 1 for problems 1 – 2.

1. Which one of the labeled points represents the vertex of the angle? C
2. Circle all the names below that can correctly be used to name this angle.

$\angle A$ $\angle B$ $\angle ABC$ [✓ $\angle ACB$]

[✓ $\angle C$] [✓ $\angle d$] $\angle BAC$ [✓ $\angle BCA$]

Refer to Figure 2 for problems 3 – 5.

3. Name three different triangles that appear to be scaled copies of one another.
   $\triangle KMN$, $\triangle KPR$, $\triangle NQR$
4. Name one quadrilateral that appears to be a parallelogram.
   Quadrilateral PMNQ
5. Name one quadrilateral that is not a parallelogram. Some possible answers: Quadrilaterals PMNR, PKNQ

We will use absolute value notation for measures of geometric objects.

Measure of $\angle d \rightarrow |\angle d|$

Use a protractor to find angle measures in degrees for the figures above.

6. $\lvert\angle d\rvert$ 28°	7. $\lvert\angle P\rvert$ 45°	8. $\lvert\angle KMN\rvert$ 45°
9. $\lvert\angle PQN\rvert$ 135°	10. $\lvert\angle PRK\rvert$ 25°	11. $\lvert\angle PKR\rvert$ 110°

TWO INVESTIGATIONS ROLLED INTO ONE

[SMP3, 5, 6, 7]

Follow your teacher's directions. Measurements may vary due to brand used.

(1) ***Measure or compute the height, diameter, and circumference of a toilet paper roll to the nearest one-fourth inch.***

h is about 4 or 4.5 in;

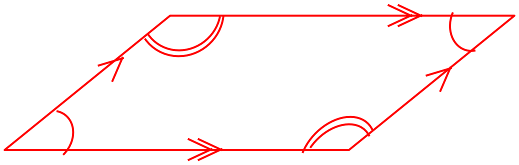
d is about 1.5 to 1.75 in;

C is about 4.7 to 5.5 in

(2) – (4) ***Name the figure, sketch it, record length and angle measures, and state the properties of sides and angles.***

Name: parallelogram

Sketch with dimensions:
Measures may vary.



Properties of sides - important facts:
Opposite sides are the same length.
Opposite sides are parallel.

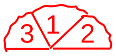
Properties of angles - important facts:
Opposite angles are the same measure.
Adjacent angles are supplementary.

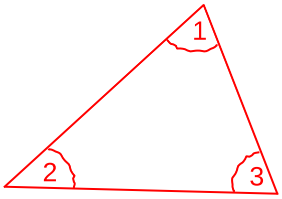
(5) ***Start with a triangle. Tear the three corners and rearrange. Sketch a diagram. Why does the diagram suggest that the sum of the interior angles is 180°?***

A straight angle measures 180°.

The sum of the measures of the interior angles in a triangle is 180°.

Sketch:





The angles form a half rotation of a circle (a straight angle).

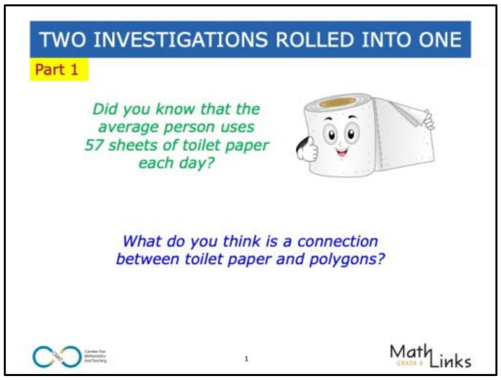
LESSON NOTES S1.3a: TWO INVESTIGATIONS ROLLED INTO ONE

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

This is a two-part investigation of properties of geometric figures. In Part 1, students explore with an unrolled toilet paper roll to see the relationships among angles and sides of a parallelogram. In Part 2, they discover facts about the angle sums of triangles. Helpful geometric facts and notation are included. One toilet paper roll per group is recommend. Ask students to bring them to class prior to this lesson.

- Slide 1: Begin Part 1. Create interest with this toilet paper question. Encourage students to be creative with the connection question, but limit the discussion time.

  Did you know that the average person uses 57 sheets of toilet paper each day? What do you think is a connection between toilet paper and polygons? Typically, each sheet is a square, and multiple connected sheets are rectangles.



TWO INVESTIGATIONS ROLLED INTO ONE

Part 1

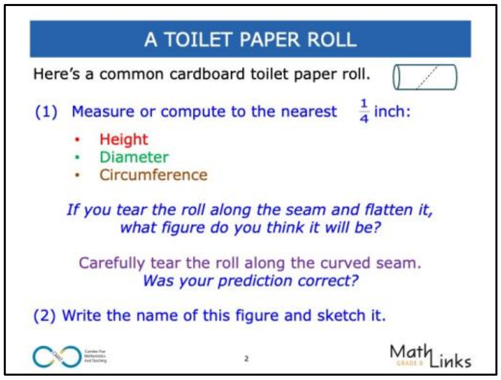
Did you know that the average person uses 57 sheets of toilet paper each day?

What do you think is a connection between toilet paper and polygons?

- Slide 2: For (1), students measure the height and diameter of a toilet paper roll and calculate the circumference of its base.

  Predict the figure formed when the roll is torn along its seam and unfolded. Students may be surprised to learn it's a parallelogram.

  For (2) students write the name of the figure and make a sketch.



A TOILET PAPER ROLL

Here's a common cardboard toilet paper roll.

(1) Measure or compute to the nearest $\frac{1}{4}$ inch:

- Height
- Diameter
- Circumference

If you tear the roll along the seam and flatten it, what figure do you think it will be?

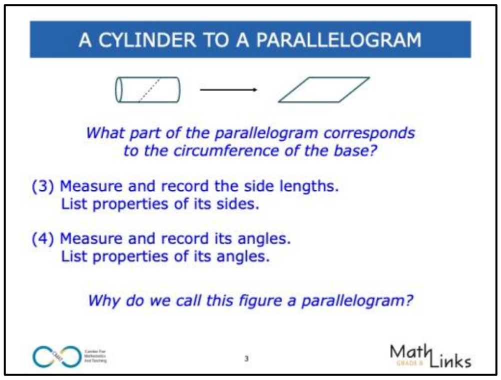
Carefully tear the roll along the curved seam. *Was your prediction correct?*

(2) Write the name of this figure and sketch it.

- Slide 3: ***What part of the parallelogram corresponds to the circumference of the base***? With the roll in hand, students will see that it's the shorter, sides of the parallelogram.

  For (3) and (4), students measure and list properties of the parallelogram's sides and angles.

  Why do we call this figure a parallelogram? Opposite sides are parallel. More questions about this follow on the next slide.



A CYLINDER TO A PARALLELOGRAM

What part of the parallelogram corresponds to the circumference of the base?

(3) Measure and record the side lengths. List properties of its sides.

(4) Measure and record its angles. List properties of its angles.

Why do we call this figure a parallelogram?

LESSON NOTES S1.3a: TWO INVESTIGATIONS ROLLED INTO ONE

Continued

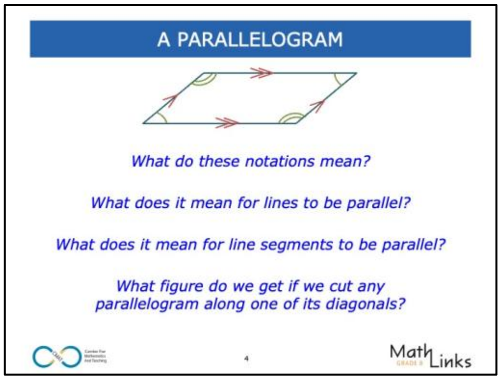
- Slide 4: Use the picture of the parallelogram to generate discussion.

  What do these notations mean? They indicate parallel lines and angles with equal measures.

  What does it mean for lines to be parallel? In a plane, parallel lines will never meet.

  What does it mean for line segments to be parallel? Though it may seem like the same question to students, it is not. Segments that lie along parallel lines are parallel. Ask students to draw two segments that DO NOT meet but are also NOT parallel.

  What figure is created by cutting along the diagonal of a parallelogram? A triangle.



A PARALLELOGRAM

What do these notations mean?

What does it mean for lines to be parallel?

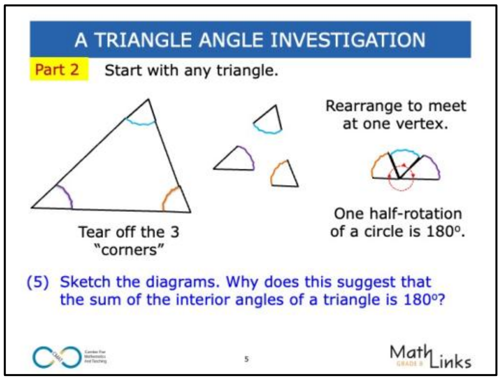
What does it mean for line segments to be parallel?

What figure do we get if we cut any parallelogram along one of its diagonals?

- Slide 5: Begin Part 2. Distribute R1-1 if desired, or ask students to cut out a triangle. Use the slide to guide this classic experiment which suggests the sum of the interior angles of a triangle are equal to a straight angle. Make sure students know that these “arcs” do not indicate equal measures; they suggest where to tear. Connect the full rotation of a circle (360°) to a familiar activity, such as skateboarding.

  How many interior angles does a triangle have? 3
  What is the sum of these angles? 180°
  How do we know? Though not a formal proof, this tearing exploration with different triangles makes this conjecture and generalization plausible.

  For (5), students sketch and explain the reasoning for the process.



A TRIANGLE ANGLE INVESTIGATION

Part 2 Start with any triangle.

Rearrange to meet at one vertex.

Tear off the 3 “corners”

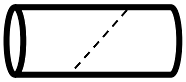
One half-rotation of a circle is 180º.

(5) Sketch the diagrams. Why does this suggest that the sum of the interior angles of a triangle is 180º?

SLIDE DECK ALTERNATIVE S1.3a:
TWO INVESTIGATIONS ROLLED INTO ONE

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

a common cardboard
toilet paper roll



Slides 1 – 4 (Part 1)

Did you know that the average person uses 57 sheets of toilet paper each day?
What do you think is a connection between toilet paper and polygons?

(1) Measure or compute the height, diameter, and circumference of a toilet paper roll to the nearest one-fourth inch.

If you were to tear the roll along the dotted seam and flatten it out,
what figure do you think it will be?

Carefully tear the roll along the seam.
Was your prediction correct?

(2) Write the name of this figure and sketch it.

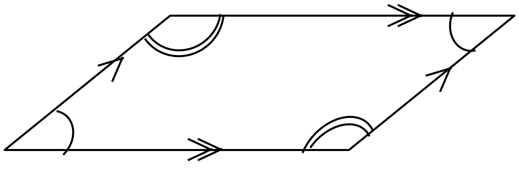
What part of the parallelogram corresponds to the circumference of the base?

(3) Measure and record the side lengths. List properties of its sides.

(4) Measure and record its angles. List properties of its angles.

Why do we call this figure a parallelogram?

What do these notations mean?



What does it mean for lines to be parallel?

What does it mean for line segments to be parallel?

What figure do we get if we cut any parallelogram
along one of its diagonals?

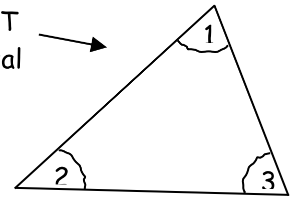
SLIDE DECK ALTERNATIVE S1.3a: TWO INVESTIGATIONS ROLLED INTO ONE

Continued

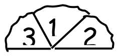
Slide 5 (Part 2)

Try this investigation

- Start with any triangle and tear off the "corners."
- Rearrange the angles to meet at one vertex.



These "arcs" are places to tear; They're NOT showing equal measure.



(5) Sketch the diagrams. Why does this suggest that the sum of the interior angles of a triangle is 180°?

ANGLES

1. Your teacher will give you some cards. Match vocabulary to descriptions and pictures.

Vocabulary	A	B	C	D	E	F	G	H
Description	R	O	X	U	L	V	Q	Z
Picture	K	N	T	P	M	S	W	Y

2. Which card(s) were difficult to match up? Explain.

Answers will vary. Adjacent angles could also have been W ot Y, but they were needed for other clues.

3. Find the missing angle measures in the figure below.

a 90°

b 40°

c 140°

d 40°

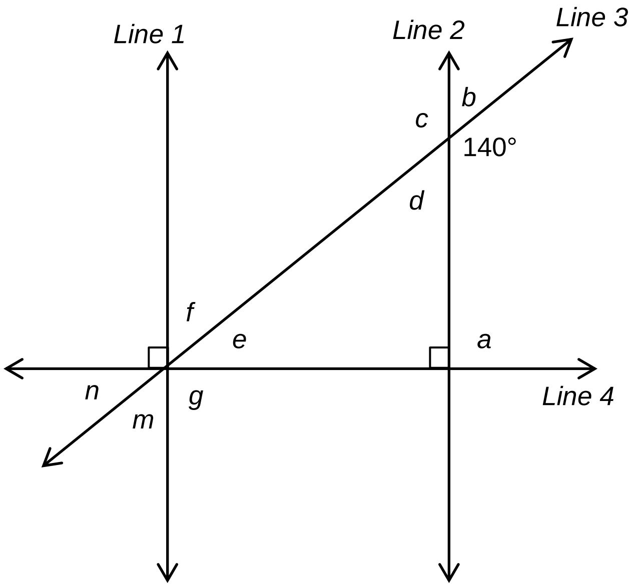
e 50°

f 40°

g 90°

m 40°

n 50°



4. Refer back to your measurements in **Getting Started** for this lesson.

Does $|\angle P| + |\angle K| + |\angle R| = 180$? Explain.

The sum should be 180°, because the sum of the angles in a triangle is 180°. Measurements with a tool like a protractor are typically not exact, which may account for any discrepancy.

ANGLE RELATIONSHIPS

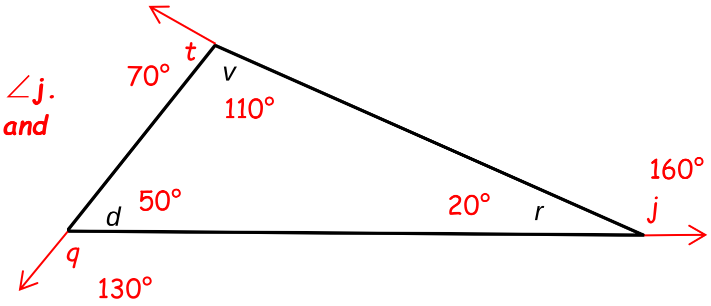
[SMP3, 6, 7, 8]

Follow your teacher's directions for (1) – (5).

(1) ***Record given angle measures. Find $|\angle v|$.***

(2) ***Extend a side to form an <u>exterior angle</u>, $\angle j$. Find $|\angle j|$. How does it compare to $|\angle d|$ and $|\angle v|$?***

$|\angle j|$ equals the sum of those two angles.



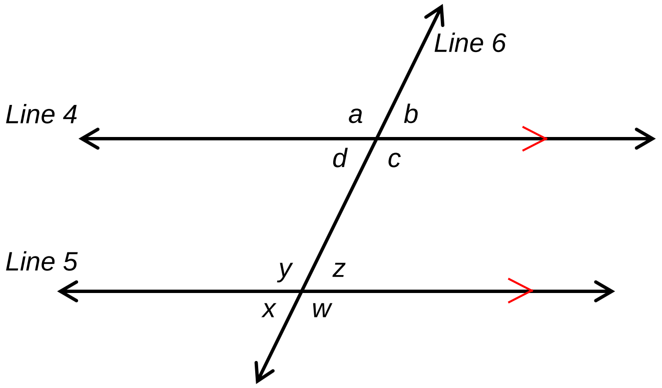
(3) ***Draw two more exterior angles, $\angle t$ and $\angle q$, find their measures, and compare to non-adjacent interior angles of the triangle.***

$|\angle t| = |\angle r| + |\angle d|$ and $|\angle q| = |\angle v| + |\angle r|$ This makes sense because the sum of the measures o two angles that form a line = 180° and the sum of the measures of three angles in a triangle = 180°.

(4) ***Which angles appear to be in the same relative position as $\angle e$? $\angle f$? $\angle g$? $\angle h$?***

Angles n, p, k, and m, respectively.

(5) ***Mark parallel lines. Measure the eight angles and record these values.***



$|\angle a| =$ 115° $|\angle b| =$ 65°

$|\angle c| =$ 115° $|\angle d| =$ 65°

$|\angle w| =$ 115° $|\angle x| =$ 65°

$|\angle y| =$ 115° $|\angle z| =$ 65°

6. List all pairs of vertical angles that have the same measures.

$\angle a$ and $\angle c$ $\angle b$ and $\angle d$ $\angle y$ and $\angle w$ $\angle z$ and $\angle x$

7. List all pairs of corresponding angles that have the same measures.

$\angle a$ and $\angle y$ $\angle b$ and $\angle z$ $\angle d$ and $\angle x$ $\angle c$ and $\angle w$

8. List all pairs of alternate interior angles that have the same measures.

$\angle d$ and $\angle z$ $\angle c$ and $\angle y$

9. List all pairs of alternate exterior angles that have the same measures.

$\angle a$ and $\angle w$ $\angle b$ and $\angle x$

10. Record the meanings of <u>parallel</u> and <u>transversal</u> in **My Word Bank**.

LESSON NOTES S1.3b: ANGLE RELATIONSHIPS

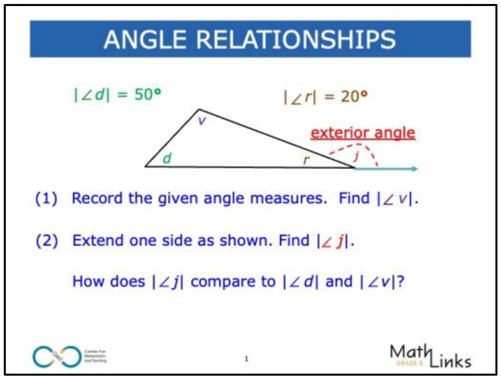
On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

Students learn facts about exterior angles of a triangle, and about the angles created when parallel lines are cut by a transversal. A lot of new vocabulary is introduced here.

- Slide 1: For (1), students record the given angle measures and find the measure of the third angle in the triangle.

  Before revealing (2), ask: ***What is the measure of a straight angle?*** 180°.

  For (2), students extend a side of the triangle, find the measure of the exterior angle, and explain that this angle is equal to the sum of the non-adjacent (remote interior) angles in the triangle.



ANGLE RELATIONSHIPS

$|\angle d| = 50°$

$|\angle r| = 20°$

(1) Record the given angle measures. Find $|\angle v|$.

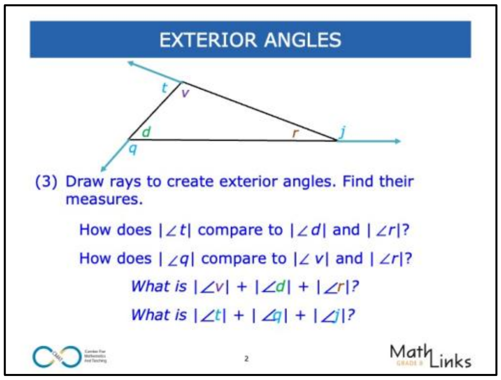
(2) Extend one side as shown. Find $|\angle j|$.

How does $|\angle j|$ compare to $|\angle d|$ and $|\angle v|$?

- Slide 2: Reveal two different exterior angles to the triangle and pose (3).

  Why does this relationship hold? The exterior angle and its adjacent interior angle form a straight angle, which is 180°. The sum of the angles of a triangle is also 180°. By substitution, an exterior angle is equal to the sum of the two non-adjacent angles of a triangle.

  Follow up with the questions about the sums of the interior angles (180°), and the sums of the exterior angles (360°).



EXTERIOR ANGLES

(3) Draw rays to create exterior angles. Find their measures.

How does $|\angle t|$ compare to $|\angle d|$ and $|\angle r|$?

How does $|\angle q|$ compare to $|\angle v|$ and $|\angle r|$?

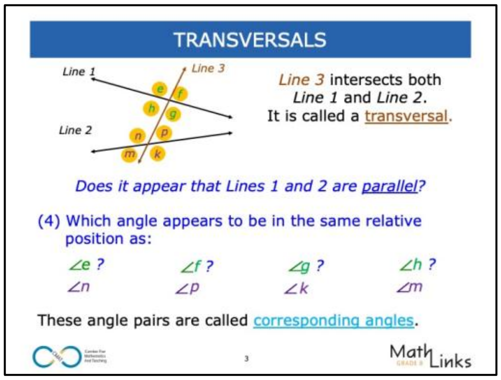
What is $|\angle v| + |\angle d| + |\angle r|$*?*

What is $|\angle t| + |\angle q| + |\angle j|$*?*

- Slide 3: Discuss the picture of the two lines cut by a transversal. These lines are intentionally NOT parallel. ***How do we know that these lines are not parallel?*** If extended toward the right, they would eventually intersect.

  Pose (4) and discuss the meaning of corresponding angles. ***Which two angles are in the "upper left" positions?*** $\angle e$ and $\angle n$. ***Do these have equal measures?*** No.

  Follow up with questions about known angle relationships. For example, the measures of supplementary angles total 180° (e.g., $|\angle e|+|\angle h| = 180°$) and the measures of vertical angles are equal (e.g., $|\angle e| = |\angle g|$).



TRANSVERSALS

Line 3 intersects both *Line 1* and *Line 2*. It is called a transversal.

Does it appear that Lines 1 and 2 are parallel?

(4) Which angle appears to be in the same relative position as:

$\angle e$?	$\angle f$?	$\angle g$?	$\angle h$?
$\angle n$	$\angle p$	$\angle k$	$\angle m$

These angle pairs are called corresponding angles.

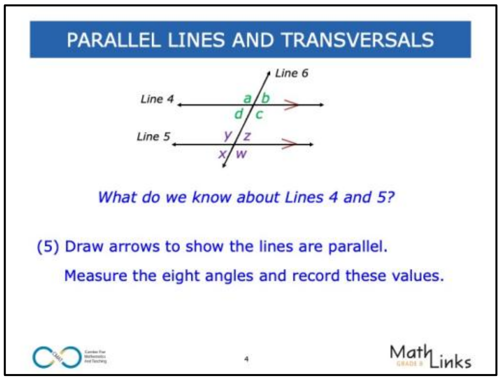
LESSON NOTES S1.3b: ANGLE RELATIONSHIPS

Continued

- Slide 4: ***How do we know lines 4 and 5 are parallel?*** The arrows indicate that these lines are parallel.

  Pose (5). Ask students to check measurements with each other.

  Did we all get the same measurements? Possibly not, but they should be close. All measurement tools, like rulers and protractors, give approximations



PARALLEL LINES AND TRANSVERSALS

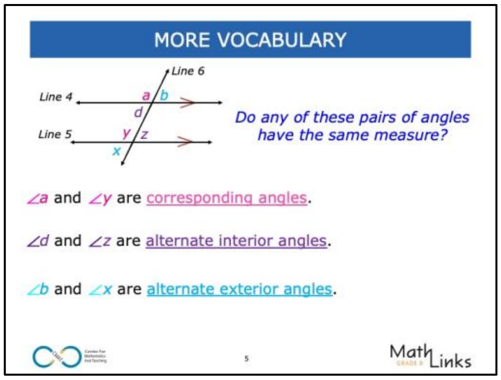
What do we know about Lines 4 and 5?

(5) Draw arrows to show the lines are parallel.

Measure the eight angles and record these values.

- Slide 5: Pose each statement one at a time for discussion. Again, using informal language, alternate interior angles are on opposite sides of the transversal and inside the parallel lines, and alternate exterior angles are on opposite sides of the transversal and outside the parallel lines.

  Do any of these angle pairs have the same measures? Allow time for students to check the measurements written on their papers as needed. Yes, for all three.



MORE VOCABULARY

Do any of these pairs of angles have the same measure?

$\angle a$ and $\angle y$ are corresponding angles.

$\angle d$ and $\angle z$ are alternate interior angles.

$\angle b$ and $\angle x$ are alternate exterior angles.

- Slide 6: To reinforce concepts and vocabulary, call attention to pairs of angles (by color), and reveal relationships one at a time.

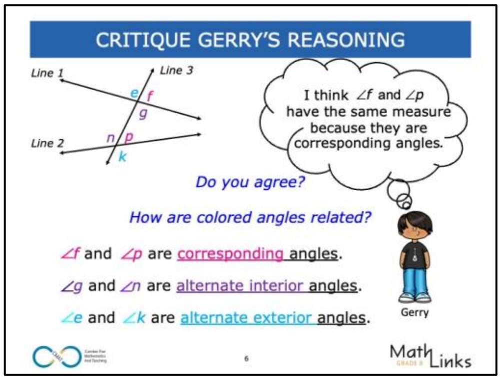
  How are the pink angles related? They are corresponding angles.

  How are the purple angles related? They are alternate interior angles.

  How are the blue angles related? They are alternate exterior angles.

  Ask students to critique the reasoning of Gerry.

  What is flawed in this thinking? The lines are not parallel, so we cannot conclude that the angles have the same measurement.



CRITIQUE GERRY'S REASONING

Do you agree?

How are colored angles related?

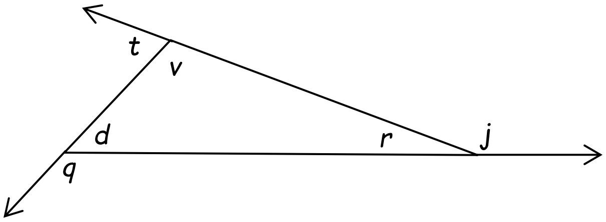
$\angle f$ and $\angle p$ are corresponding angles.

$\angle g$ and $\angle n$ are alternate interior angles.

$\angle e$ and $\angle k$ are alternate exterior angles.

SLIDE DECK ALTERNATIVE S1.3b: ANGLE RELATIONSHIPS

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.



Slides 1 – 2

(1) If $|\angle d| = 50°$ and $|\angle r| = 20°$, find $|\angle v|$.

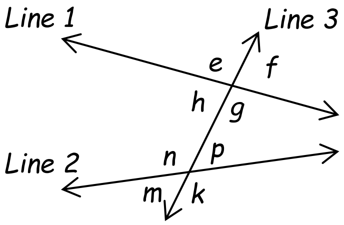
(2) $\angle j$ is called an <u>exterior angle</u> to the triangle.
Find its measure. How does this measure compare to $|\angle d|$ and $|\angle v|$?

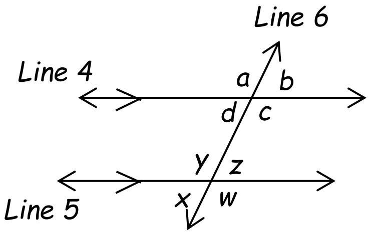
(3) Find $|\angle t|$ and compare it to $|\angle d|$ and $|\angle r|$.

Find $|\angle q|$ and compare it to $|\angle v|$ and $|\angle r|$.

What is $|\angle v| + |\angle d| + |\angle r|$?
What is $|\angle t| + |\angle q| + |\angle j|$?

Slides 3 – 4



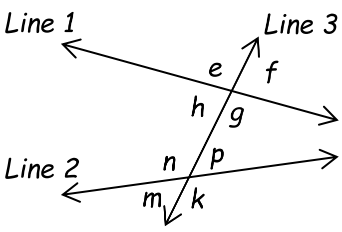


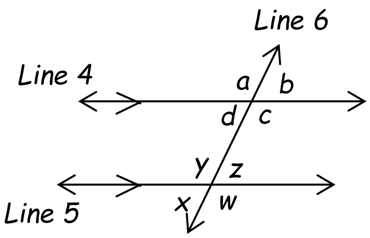
Does it appear that Lines 1 and 2 are <u>parallel</u>?	*What do we know about Lines 4 and 5 in this figure?*
(4) Which angles appears to be in the same relative positions as $|\angle e|$, $|\angle f|$, $|\angle g|$, and $|\angle h|$? These angle pairs are called <u>corresponding angles</u>.	(5) Measure the eight angles in the figure and record these values in the spaces provided.

SLIDE DECK ALTERNATIVE S1.3b: ANGLE RELATIONSHIPS

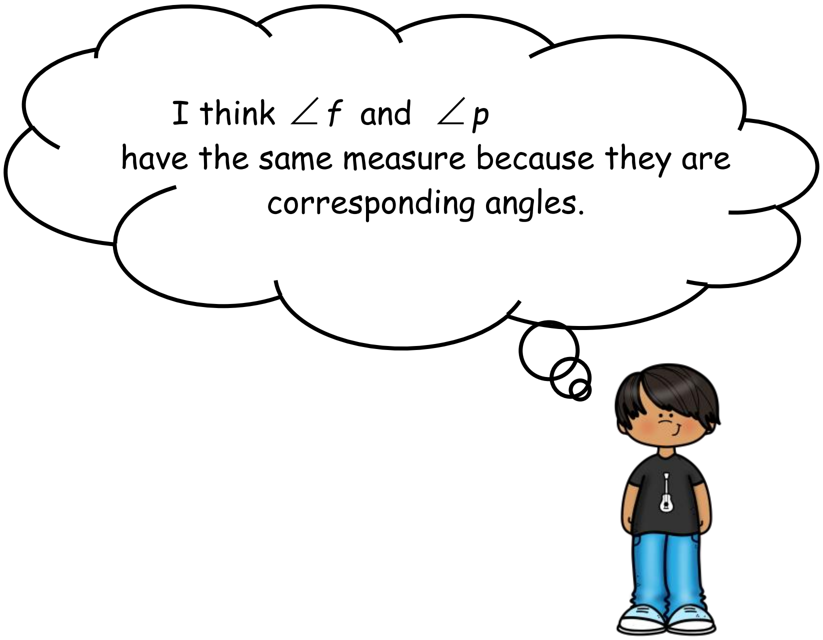
Continued

Slides 5 – 6





In each figure:	**Figure A**	**Figure B**
Name a pair of corresponding angles:	$\angle f$ and $\angle p$	$\angle a$ and $\angle y$
Name pair of alternate interior angles:	$\angle g$ and $\angle n$	$\angle d$ and $\angle z$
Name A pair of alternate exterior angles:	$\angle e$ and $\angle k$	$\angle b$ and $\angle x$



Critique Gerry's thinking.

PRACTICE 3

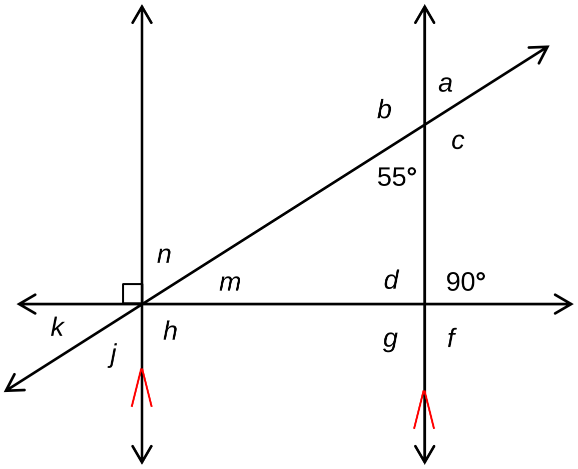
Use "What's My Angle" (Portal access → Puzzles and Games) for more practice and fun. [SMP8]

Watch for (all): Are students keeping track of the different types of angle relationships?

1. If two lines are cut by a transversal, and the three statements below are true, what do we know about the two lines? They are parallel.

   - corresponding angles have equal measure
   - alternate interior angles have equal measure
   - alternate exterior angles have equal measure

2. In the figure to the right, assume lines that appear to be parallel are parallel. Label the parallel lines using arrow notation. Then find the measures of all labeled angles.



$|\angle a|$ = 55° $|\angle b|$ = 125° $|\angle c|$ = 125° $|\angle d|$ = 90°

$|\angle f|$ = 90° $|\angle g|$ = 90° $|\angle m|$ = 35° $|\angle n|$ = 55°

$|\angle h|$ = 90° $|\angle j|$ = 55° $|\angle k|$ = 35°

Name one pair of each of the following types of angles from the figure above.

Some possible answers:

3. Acute, vertical angles $\angle n$ and $\angle j$	4. Right, vertical angles $\angle d$ and $\angle f$	5. Obtuse, vertical angles $\angle b$ and $\angle c$
6. Form a straight angle $\angle d$ and $\angle g$	7. Corresponding angles with the same measures $\angle a$ and $\angle n$	8. Alternate exterior angles with the same measures $\angle a$ and $\angle j$
9. Alternate interior angles with the same measures $\angle d$ and $\angle h$	10. Adjacent, supplementary angles $\angle a$ and $\angle b$	11. Vertical, supplementary angles $\angle d$ and $\angle f$

12. When do corresponding angles have equal measures?
    When parallel lines are cut by a transversal.

13. For the triangle in the figure above, name two exterior angles whose measures are equal to the sum of $|\angle m| + |\angle d|$. $\angle b$ and $\angle c$

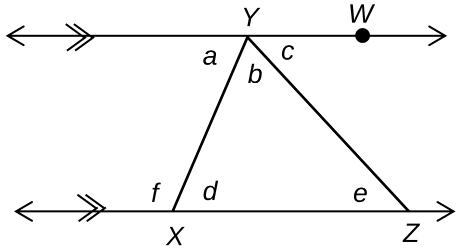
14. Record the meanings of corresponding angles, alternate interior angles, alternate exterior angles, and exterior angle of a triangle in **My Word Bank**.

ANGLE FACTS RELATED TO TRIANGLES

The *MathLinks* Rubric: See Activity Routines on the Teacher Portal for directions. [SMP3, 6]

Use the cards given to you by your teacher and the figure to the right to establish two important facts about angles related to triangles.

Match each fact statement 1 – 8 below with a fact card. Some cards may be used more than once, and some not at all.



	Statement	Card
1.	$\overleftrightarrow{WY}$ is parallel to $\overleftrightarrow{XZ}$	G
2.		R
3.	$\lvert\angle a\rvert = \lvert\angle d\rvert$	J
4.	$\lvert\angle c\rvert = \lvert\angle e\rvert$	J
5.	$\lvert\angle d\rvert + \lvert\angle b\rvert + \lvert\angle e\rvert = 180°$	H
6.	$\lvert\angle f\rvert + \lvert\angle d\rvert = 180°$	R
7.	$\lvert\angle b\rvert + \lvert\angle d\rvert + \lvert\angle e\rvert = \lvert\angle f\rvert + \lvert\angle d\rvert$	H
8.	$\lvert\angle b\rvert + \lvert\angle e\rvert = \lvert\angle f\rvert$	N

Card sorts throughout the course add to variety, engagement, and collaborative support.

9. Which statement (problem) establishes that:

The sum of the measures of the interior angles of a triangle equals 180°.

Problem 5

10. Which statement (problem) establishes that:

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

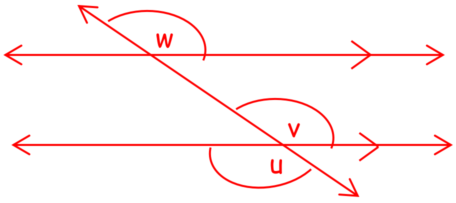
Problem 8

11. Choose any three **unused** cards so that you can draw a figure that incorporates all three of them. Describe your examples. Examples will vary. One possible answer:

Cards chosen: M, P, and Q

Bold words on cards: angles with the same measure; alternate exterior angles; corresponding angles

Figure and descriptions:



Since there are two parallel lines cut by a transversal:

- angles *w* and *v* have the same measure because they are corresponding angles;
- angles *w* and *u* have the same measure because they are alternate exterior angles;
- so, angles *w*, *v*, and *u* all have the same measure (notice the single arcs on each)

PRACTICE 4

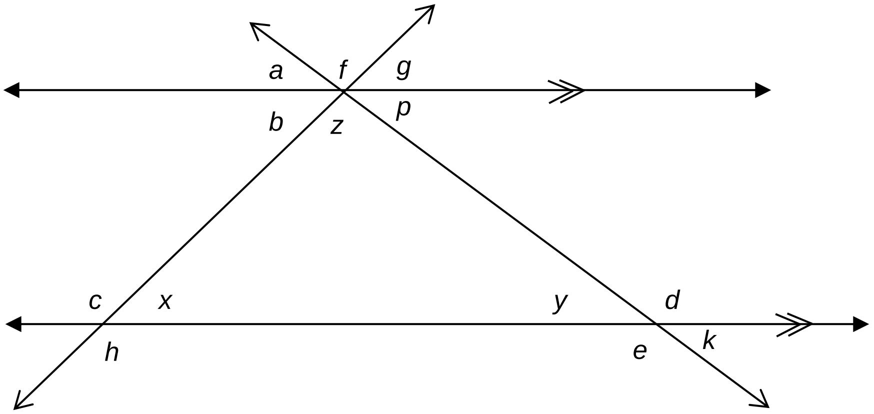
[SMP2]

Use this figure for the problems below.
Write important information into the figure as needed.

Let $\lvert\angle x\rvert = n°$

$\lvert\angle y\rvert = (n - 10)°$

$\lvert\angle z\rvert = (2n + 10)°$



1. Find n.
   $n = 45$

2. Find the measures of each angle in the triangle.

$\lvert\angle x\rvert$	45°	$\lvert\angle y\rvert$	35°	$\lvert\angle z\rvert$	100°

3. The vertical angle to $\angle z$ is $\angle f$ and it measures 100°.

4. An angle adjacent to $\angle x$ is $\angle c$ or $\angle h$ and it measures 135°.

5. The alternate interior angle to $\angle x$ is $\angle b$ and it measures 45°.

6. The measure of $\angle h$ is 135° and it is an alternate exterior angle to the sum of angles a and f.

7. The measure of $\angle c$ is 135° and it corresponds to the sum of angles a and f.

8. The angle that corresponds to $\angle k$ is $\angle p$ and it measures 35°.

9. The exterior angle of the triangle that is adjacent to $\angle y$ is $\angle d$ and it measures 145°.

10. The two angles in the interior of the triangle that have the same sum as $\angle c$ are angles y and z.

11. Corresponding angles in the figure above have the same measure. Under what condition do corresponding angles NOT have the same measure?
    When the given lines that are cut by the transversal are NOT parallel.

REVIEW

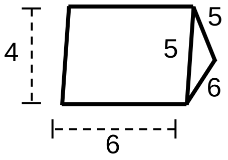
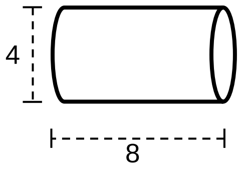
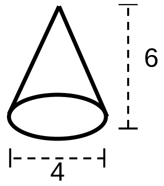
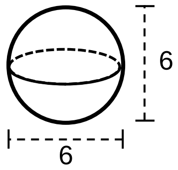
POSTER PROBLEMS: PLANE AND SOLID FIGURES

See Activity Routines on the Teacher Portal for directions. [SMP2, 6]

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _______.
- Each group will have a different colored marker. Our group marker is _________.

Part 2: Do the problems on the posters by following your teacher's directions.

Poster 1 (or 5)	**Poster 2 (or 6)**	**Poster 3 (or 7)**	**Poster 4 (or 8)**
			

Use $\pi = 3.14$ and round results to two decimal places as needed.

A. Write the name of the solid figure and its volume formula. If a se exists, include it.

B. Find the volume of the solid.

C. Find the volume if the height of the solid is doubled.

D. Find the volume of the solid if ALL given measures on the figure

Part 3: Return to your seats. Work with your group, and show all wor

Compare the three volume measures in parts B – D for your "start p you notice below. Be ready to share with the class.

For posters 1, 2, 3 → C is twice B, and D is 8 times B. This occurs because wl doubled, the total volume is increased by a factor of $2 \times 2 \times 2$ or 2^3.

For the sphere, this pattern only holds if diameter is doubled in "both" direc shape is not a sphere.

Poster Problems is an Activity Routine that appears in the Review section in every unit in grades 6 – 8. It is an engaging way for students to solve problems and analyze work together. When teachers watch and listen, they are able to gauge student understanding and identify areas for review or reteaching. By establishing classroom norms, all students will find a safe place to contribute ideas and collaborate with peers.

Chart paper or board space, markers

4

POSTER PROBLEMS: PLANE AND SOLID FIGURES

Answer Key

Poster	**A**	**B**	**C**	**D**
1, 5	Triangular prism $V = Bh$ $V = \frac{1}{2}(b)$(triangle height)(prism height)	72 u^3	144 u^3	576 u^3
2, 6	Cylinder $V = Bh$ $V = \pi r^2 h$	100.48 u^3	200.96 u^3	803.84 u^3
3, 7	Cone $V = \frac{1}{3}Bh$ $V = \frac{1}{3}\pi r^2 h$	25.12 u^3	50.24 u^3	200.96 u^3
4, 8	Sphere $V = \frac{4}{3}\pi r^3$	113.04 u^3	*904.32 u^3	904.32 u^3

* Height is used, but a sphere has a diameter, and if only one “dimension” is doubled it no longer is a sphere, so this solution assumes that if height is doubled, then that means the same as diameter is doubled, and then it is still a sphere. Interpretations may differ.

MATCH AND COMPARE SORT: PLANE AND SOLID FIGURES

See Activity Routines on the Teacher Portal for directions. [SMP3, 6]

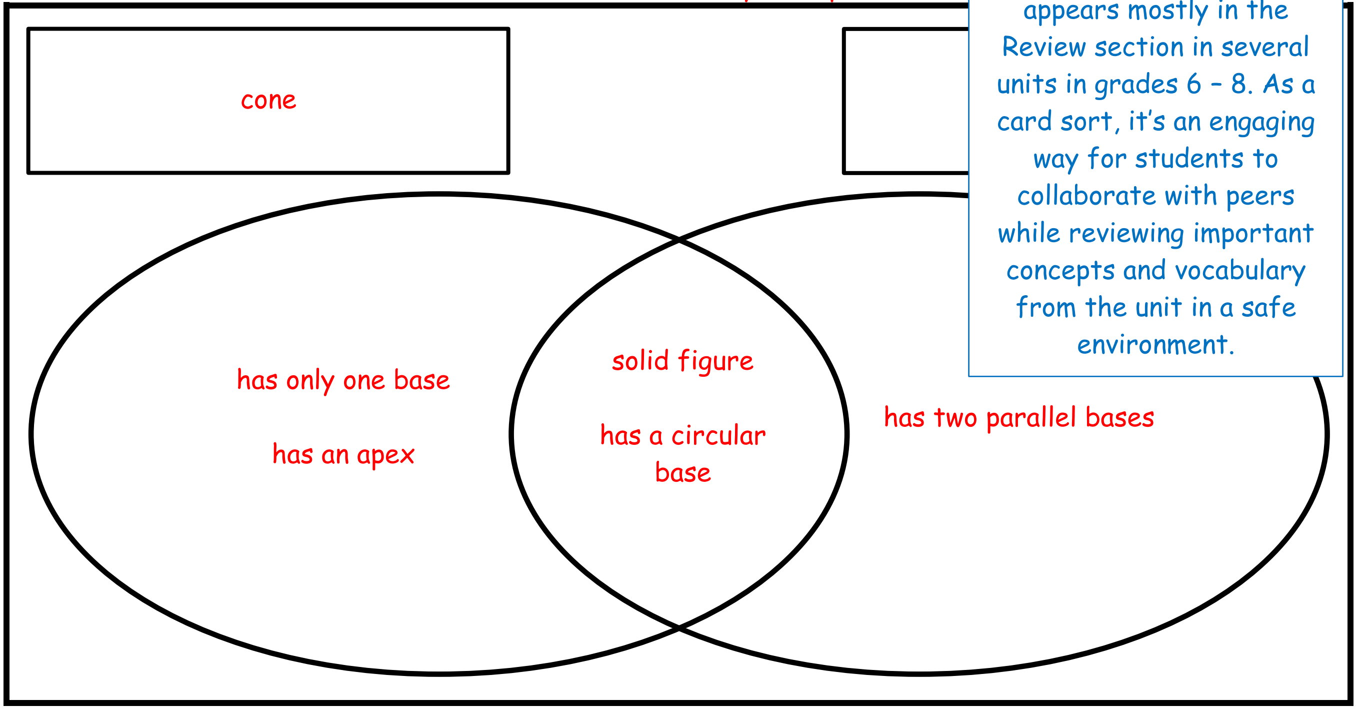
Your teacher will give you some cards. Cut them out.

1. Individually, match words with descriptions. Record results.

Card set △			Card set ○		
Card number	word	Card letter	Card number	word	Card letter
I	transversal	D	I	parallel lines	C
II	cone	C	II	cylinder	D
III	corresponding angles	B	III	alternate interior angles	B
IV	alternate exterior angles	A	IV	exterior angle of a triangle	A

2. Partners, choose a pair of numbered matched cards and record t same and those that are different. Answers will vary. One possible ans

Match and Compare Sort is an Activity Routine that appears mostly in the Review section in several units in grades 6 – 8. As a card sort, it's an engaging way for students to collaborate with peers while reviewing important concepts and vocabulary from the unit in a safe environment.



3. Partners, choose another pair of numbered matched cards and discuss the attributes that are the same and those that are different.

A BIG PUZZLE

[SMP1, 8]

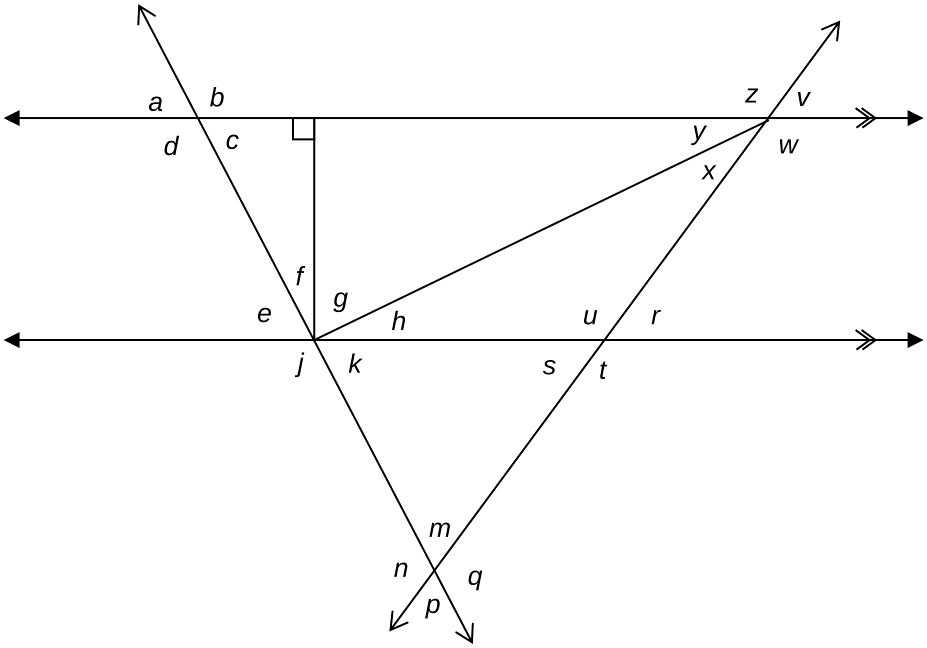
Use the figure below for all problems. For problems 1 – 8, name an angle that is:

Answers may vary.

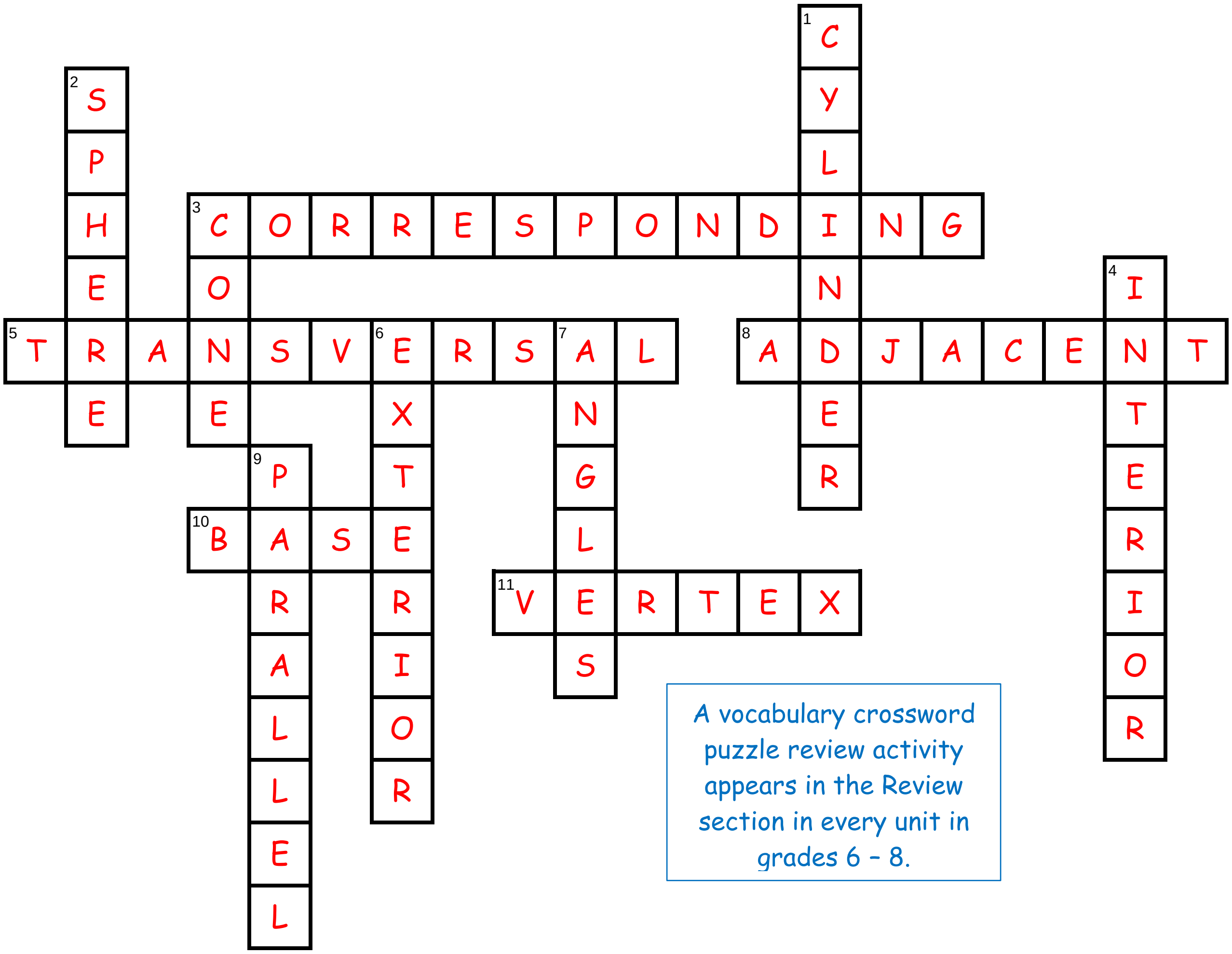
1. adjacent to $\angle m$. $\angle q$ or $\angle n$	2. vertical to $\angle r$. $\angle s$	3. supplementary to $\angle m$. $\angle q$ or $\angle n$
4. corresponding to $\angle z$. $\angle u$	5. complementary to $\angle e$. $\angle f$	6. alternate interior to $\angle u$. $\angle w$
7. alternate exterior to $\angle j$. $\angle b$	8. an exterior angle of a triangle and equal to $\lvert\angle h\rvert + \lvert\angle x\rvert$. $\angle r$ or $\angle s$	

Find the angle measures in any order. Notice that $|\angle a|$, $|\angle h|$ and $|\angle z|$ are given.

9. $|\angle a|$ = 60°
10. $|\angle b|$ = 120°
11. $|\angle c|$ = 60°
12. $|\angle d|$ = 120°
13. $|\angle e|$ = 60°
14. $|\angle f|$ = 30°
15. $|\angle g|$ = 60°
16. $|\angle h|$ = 30°
17. $|\angle j|$ = 120°
18. $|\angle k|$ = 60°
19. $|\angle m|$ = 65°
20. $|\angle n|$ = 115°
21. $|\angle p|$ = 65°
22. $|\angle q|$ = 115°
23. $|\angle r|$ = 55°
24. $|\angle s|$ = 55°
25. $|\angle t|$ = 125°
26. $|\angle u|$ = 125°
27. $|\angle v|$ = 55°
28. $|\angle w|$ = 125°
29. $|\angle x|$ = 25°
30. $|\angle y|$ = 30°
31. $|\angle z|$ = 125°



VOCABULARY REVIEW



A vocabulary crossword puzzle review activity appears in the Review section in every unit in grades 6 – 8.

Across

3 Angles in the same relative location

5 A line that intersects two or more other lines

8 Two angles that share a common side

10 One of the parallel faces of a cylinder or prism

11 The point where two rays of an angle meet

Down

1 Solid with two parallel circular bases

2 Tennis ball is an example

3 Solid with circular base and an apex

4 Alternate angles on the inside of lines on opposite sides of a transversal

6 Angle adjacent to a triangle’s interior angle

7 Measured in degrees

9 Lines that never meet

SPIRAL REVIEW

See Activity Routines on the Teacher Portal for directions. For more challenging READY-X puzzles for advanced learners, see Puzzles and Games in the Teacher Portal.

1. **READY-X.** Solve for the values of R, E, A, indicated at the end of each row and colum

The READY-X and Alge-Grid Activity Routines alternate in Spiral Review for every 8th grade Student Packet to help address basic grade level algebra fluency goals.

One or more Spiral Review pages appear in every Student Packet. These have been carefully designed to distribute practice so that important mathematical ideas remain fresh for students. See Program Information charts for more details about when topics are reviewed. Note that this is one place where we list *topics* that are reviewed, *not standards*.

COLU

ROWS

X			
A			
Y	X	D	**14**
E	R	D	**17**
14	**14**	**36**	

R = 5 E = 3 A = 6 D = 9 Y = 1 X = 4

2. Solve each equation below.

a. $4 - 6a = 22$ $a = -3$	b. $36 = 3(7 - x)$ $x = -5$

SPIRAL REVIEW

Continued

3. Write an expression for each word description.

a. 3 more than 4 times a number $4n + 3$

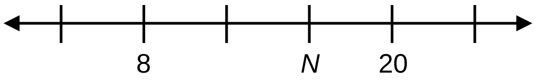
b. The difference of twice a number and 20 $2n - 20$

c. 2 less than the sum of a number and 8 $(n + 8) - 2$

d. The quotient of a number and the sum of 5 and 7 $\frac{n}{5 + 7}$

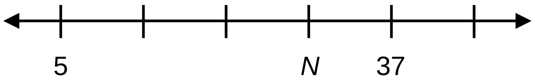
4. Find the value of *N*.

a.



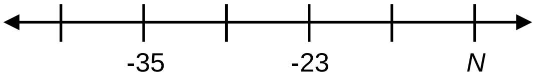
16

b.



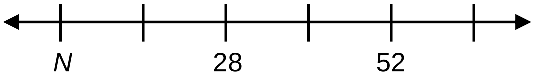
29

c.



-11

d.



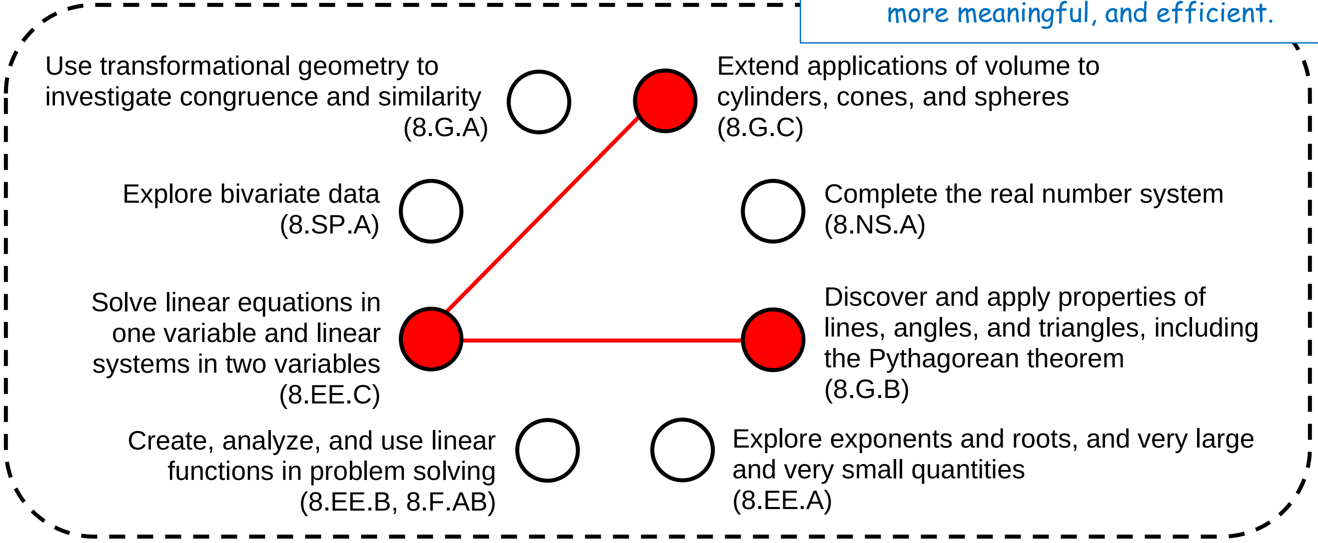
4

REFLECTION

Identifying Big Ideas and their connections helps students view mathematics as a cohesive and connected body of knowledge. It also makes learning more meaningful, and efficient.

Answers will vary. Some possible answers:

1. **Big Ideas**. Shade all circles that describe big ideas in connections that you noticed. See Teaching Tips for conn



Give an example from this unit of one of the connections above.

Used algebra equations to solve volume problems.

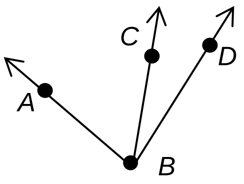
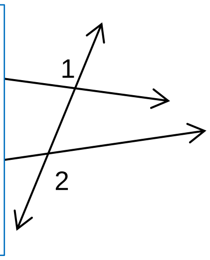
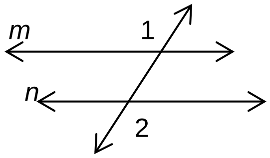
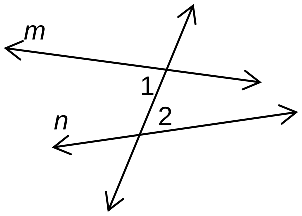
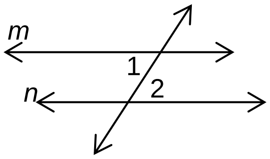
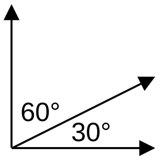
2. **Unit Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before or something you would still like to work on.

3. **Mathematical Practice.** Explain how the structure of a previously learned concept helped you learn a new one [SMP7]. Then circle one more SMP on the back of this packet that you think was addressed in this unit and be prepared to share an example.
   - The volume of a prism leads directly to the volume of a cylinder, which in turn leads to both the volume of a cone and a sphere.
   - Finding missing angle measures in figures requires previously learned concepts about angle relationships (complementary, supplementary, adjacent, and vertical angles).

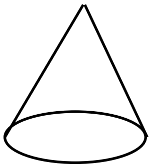
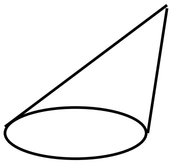
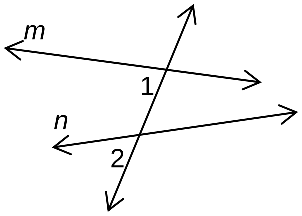
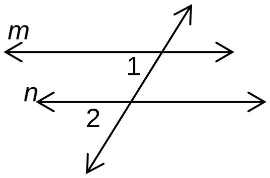
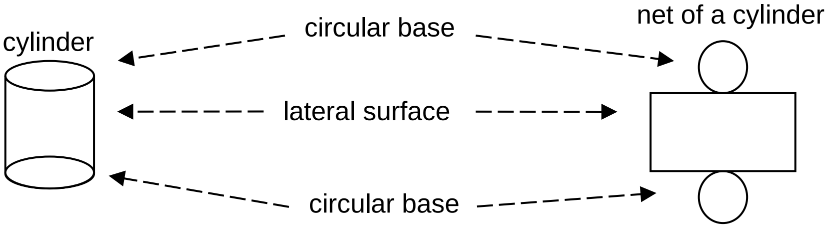
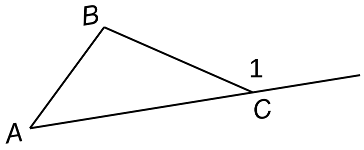
4. **Making Connections.** Describe something new that you learned about shapes and space.
   The formulas for cylinders, cones, and spheres are all related. There is a connection between a line that is 180° and a triangle whose angles measure 180°.

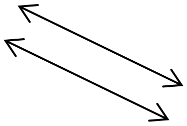
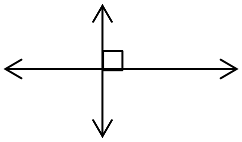
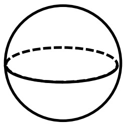
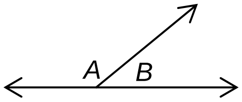
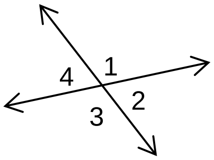
This section is available on the public website for parents and others who support students.

STUDENT RESOURCES

Word or Phrase	Definition
adjacent angles	Two angles are <u>adjacent</u> if they have the same vertex and share a common ray, and they lie on opposite sides of the common ray. ∠*ABC* and ∠*CBD* are adjacent angles. 
alternate exterior angles	When two lines in a plane are cut by a transversal, two angles on opposite sides of the transversal and outside the two lines are referred to as <u>alternate exterior angles</u>. When parallel lines are cut by a transversal, alternate exterior angles have the same measure. Line *m* is not parallel to line *n*.  ∠1 and ∠2 are alternate exterior angles. Line *m* is parallel to line *n*.  ∠1 and ∠2 are alternate exterior angles. $\|\angle 1\| = \|\angle 2\|$
alternate interior angles	When two lines in a plane are cut by a transversal, two angles on opposite sides of the transversal and between the two lines are referred to as <u>alternate interior angles</u>. When parallel lines are cut by a transversal, alternate interior angles have the same measure. Line *m* is not parallel to line *n*.  ∠1 and ∠2 are alternate interior angles. Line *m* is parallel to line *n*.  ∠1 and ∠2 are alternate interior angles. $\|\angle 1\| = \|\angle 2\|$
complementary angles	Two angles are <u>complementary</u> if the sum of their measures is 90°. Two angles that measure 30° and 60° are complementary. 

This first part of Student Resources includes precise definitions. When they first arise in a unit, teachers help their students unpack these definitions and record them in My Word Bank using their own words, examples, and/or diagrams.

Word or Phrase	Definition
cone	A circular cone is a figure in space consisting of a circle in a plane (called the base of the cone), a point off the plane (called the vertex of the cone), and all the straight line segments joining the vertex to the base. If the line joining the vertex of the cone to the center of its base is perpendicular to the base, the cone is a right circular cone. Otherwise it is an oblique circular cone.  right circular cone  oblique circular cone
corresponding angles	When two lines in a plane are cut by a transversal, two angles that appear on the same side of the transversal in the same relative location are referred to as corresponding angles. When parallel lines are cut by a transversal, corresponding angles have the same measure. Line *m* is not parallel to line *n*.  $\angle 1$ and $\angle 2$ are corresponding angles. Line *m* is parallel to line *n.*  $\angle 1$ and $\angle 2$ are corresponding angles. $\|\angle 1\| = \|\angle 2\|$
cylinder	A (right circular) cylinder is a figure in three-dimensional space that has two parallel circular bases. These circles are connected by a curved surface, called the lateral surface, which is a “rolled up” rectangle. Most soup cans have the shape of a right circular cylinder. 
exterior angle of a triangle	An exterior angle of a triangle is an angle formed by a side of the triangle and an extension of its adjacent side. $\angle 1$ is an exterior angle of ΔABC. 

Word or Phrase	Definition
parallel	Two lines in a plane are <u>parallel</u> if they do not meet. Two line segments in a plane are parallel if the lines they lie on are parallel. 
perpendicular	Two lines are <u>perpendicular</u> if they intersect at right angles. 
sphere	A <u>sphere</u> is a closed surface in three-dimensional space consisting of all points at a fixed distance (the radius) from a specified point (the center). 
supplementary angles	Two angles are <u>supplementary</u> if the sum of their measures is 180°. Any two right angles are supplementary, because the sum of their measures is 90° + 90° = 180°. Angles *A* and *B* are supplementary because they determine a straight line, or 180°. 
transversal	A <u>transversal</u> is a line that passes through two or more other lines.
vertical angles	Two angles are <u>vertical angles</u> if they are the opposite angles formed by a pair of intersecting lines. When two lines intersect at a point, they form two pairs of vertical angles with vertex at the point. $\angle 1$ and $\angle 3$ are vertical angles. $\angle 2$ and $\angle 4$ are vertical angles. 

Some Properties of Equality

This next part of Student Resources includes examples and explanations for use in class or at home.

Properties of equality govern the manipulation of equations (mathematical ser

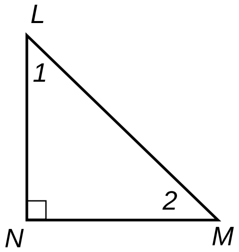
For any three numbers *a*, *b*, and *c*:

- ✓ Addition property of equality
  (Subtraction property of equality)
  If $a = b$ and $c = d$, then $a + c = b + d$.
- ✓ Multiplication property of equality
  (Division property of equality)
  If $a = b$ and $c = d$, then $ac = bd$
- ✓ Reflexive property of equality
  $a = a$
- ✓ Symmetric property of equality
  If $a = b$, then $b = a$
- ✓ Transitive property of equality
  (Substitution property)
  If $a = b$, and $b = c$, then $a = c$

Geometry Notation

Here are some geometry notations used in these lessons.

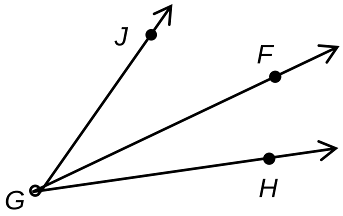
- Points are named by capital letters.
- The symbol for triangle is Δ.
- The symbol for angle is $\angle$.
- Absolute value signs are used to denote nonnegative quantities that measure the "size" of something, such as length or angle measure.



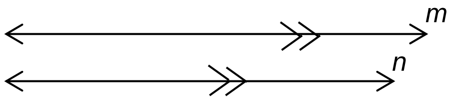
The measure of an angle called $\angle N$ is denoted by $\left|\angle N\right|$. The small square at N indicates that $\angle LNM$ is a right angle, that is, that $\left|\angle LNM\right|$ = 90°.

In naming a triangle, vertices may be listed in either a clockwise or counter - clockwise direction. For example, the triangle may be named ΔLMN or ΔLNM.

In naming an angle, vertices may be listed in either a clockwise or counterclockwise direction. In the triangle above, the angle at the top can be denoted by $\angle NLM$, $\angle MLN$, $\angle L$ or $\angle 1$.



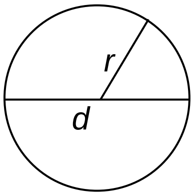
The pair of adjacent angles to the right are $\angle FGJ$ and $\angle HGF$. Using $\angle G$ to name an angle is unclear They share the common ray $\overrightarrow{GF}$.
The two adjacent angles together form the angle $\angle JGH$.



The arrows on the lines m and n indicate that they are parallel.

Formulas for Circles

Let r = radius of a circle.
Let d = diameter of a circle.



Circumference: $C = \pi d$ or $C = 2\pi r$

Area: $A = \pi r^2$

Volume Formulas

Here are some volume formulas from this unit.

Volume of a Rectangular Prism

Let ℓ = length and w = width of rectangular base.

$$V = Bh$$

Area of base (B) = ℓw

Therefore, $V = \ell wh$

Volume of a Cylinder

Let r = radius of the circular base.

$$V = Bh$$

Area of base (B) = πr^2

Therefore, $V = \pi r^2 h$

Volume of a Cone

Through experimentation, observe that the volume of a cone is $\frac{1}{3}$ of the volume of a cylinder with the same height and base.

Let r = radius of the circular base

$$V = \frac{1}{3}Bh$$

Area of base (B) = πr^2

Therefore, $V = \frac{1}{3}\pi r^2 h$

Volume of a Sphere

Through experimentation, observe that the volume of a sphere is $\frac{2}{3}$ of the volume of a cylinder whose diameter and height are the same as the diameter of the sphere. Use substitution to derive the formula of a sphere.

Let r = radius of the sphere and cylinder

Then height (h) of cylinder = $2r$
Volume of cylinder = $\pi r^2 (2r) = 2\pi r^3$

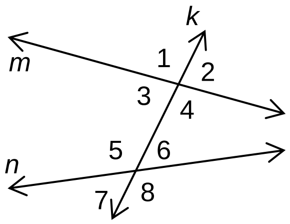
Observe that volume of sphere is $\frac{2}{3}$ of the volume of a cylinder.

Therefore, $V_{sphere} = \frac{2}{3} \bullet 2\pi r^3 = \frac{4}{3}\pi r^3$

Transversals and Parallel Lines

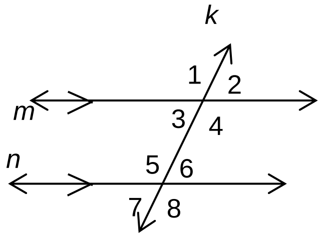
In this figure, line *k* is a transversal. Lines *m* and *n* are NOT parallel.

When two lines in a plane are cut (crossed) at two points by a transversal, eight angles are created. Some of these pairs of angles have special names.



corresponding angles		**alternate interior angles**	**alternate exterior angles**
$\angle 1$ and $\angle 5$	$\angle 2$ and $\angle 6$	$\angle 3$ and $\angle 6$	$\angle 1$ and $\angle 8$
$\angle 3$ and $\angle 7$	$\angle 4$ and $\angle 8$	$\angle 4$ and $\angle 5$	$\angle 2$ and $\angle 7$

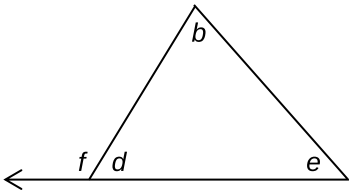
Here are three important properties of the angles formed when a transversal cuts two parallel lines.



1. If two parallel lines are cut by a transversal, then alternate interior angles have the same measure.
   Example: $|\angle 3| = |\angle 6|$ and $|\angle 4| = |\angle 5|$
2. If two parallel lines are cut by a transversal, then alternate exterior angles have the same measure.
   Example: $|\angle 1| = |\angle 8|$ and $|\angle 2| = |\angle 7|$
3. If two parallel lines are cut by a transversal, then corresponding angles have the same measure.
   Example: $|\angle 2| = |\angle 6|$ and $|\angle 4| = |\angle 8|$

Interior and Exterior Angles in Triangles

Here are two important facts about angle sums in triangles.



1. The sum of the measures of the angles in a triangle is equal to 180°.

   $|\angle d| + |\angle b| + |\angle e| = 180°$

2. The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

   $|\angle b| + |\angle e| = |\angle f|$

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
8.G.A	**Understand congruence and similarity using physical models, transparencies, or geometry software.**
8.G.5	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, ~~and the angle-angle criterion for similarity of triangles~~. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
8.G.C	**Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**
8.G.9	Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

STANDARDS FOR MATHEMATICAL PRACTICE	
SMP1	Make sense of problems and persevere in solving them.
SMP2	Reason abstractly and quantitatively.
SMP3	Construct viable arguments and critique the reasoning of others.
SMP4	Model with mathematics.
SMP5	Use appropriate tools strategically.
SMP6	Attend to precision.
SMP7	Look for and make use of structure.
SMP8	Look for and express regularity in repeated reasoning.

The final page of a Student Packet lists all of the Common Core content and practice standards addressed in the unit. Note that some standards develop over time and are included in multiple units.

Math is a drama queen. It can't seriously have that many problems.

3.14% of all sailors are pi rates.

Do you know why the two 4s didn't go to the cafeteria for lunch? They already 8!

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