STUDENT RESOURCES

Word or Phrase	Definition
slope-intercept form	The slope-intercept form of the equation of a line is the equation $y = mx + b$, where m is the slope of the line, and b is the y-intercept of the line. The equation $y = 2x + 3$ determines a line with slope 2 and y-intercept 3.
solution to an equation	A solution to an equation involving variables consists of values for the variables which, when substituted, make the equation true. The value $x = 8$ is a solution to the equation $10 + x = 18$. If we substitute 8 for x in the equation, the equation becomes true: $10 + 8 = 18$.
solve an equation	To solve an equation refers to finding all values for the variables in the equation that, when substituted, make the equation true. Values that make an equation true are called solutions to the equation. To solve the equation $2x = 6$, one might think "two times what number is equal to 6?" Since $2(3) = 6$, the only value for x that satisfies this condition is 3. Therefore 3 is the solution.
substitution	Substitution refers to replacing a value or quantity with an equivalent value or quantity. If $y = x + 5$, and we know that $x = 3$, then we may use substitution to rewrite the first equation to get $y = 3 + 5$. If $y = x + 10$, and we know also that $y = 2x + 4$, then we may use substitution to write one equation in x to get $x + 10 = 2x + 4$.
system of linear equations	A system of linear equations is a set of two or more linear equations in the same variables. An example of a system of linear equations in x and y: $\begin{cases} x + y = 1 \\ x + 2y = 4 \end{cases}$

Properties of Arithmetic

Properties of arithmetic govern the manipulation of expressions (mathematical phrases).
For any three numbers *a*, *b*, and *c*:

- ✓ Associative property of addition
  $a + (b + c) = (a + b) + c$
- ✓ Commutative property of addition
  $a + b = b + a$
- ✓ Additive identity property
  (addition property of 0)
  $a + 0 = 0 + a = a$
- ✓ Additive inverse property
  $a + (-a) = -a + a = 0$
- ✓ Associative property of multiplication
  $a \bullet (b \bullet c) = (a \bullet b) \bullet c$
- ✓ Commutative property of multiplication
  $a \bullet b = b \bullet a$
- ✓ Multiplicative identity property
  (multiplication property of 1)
  $a \bullet 1 = 1 \bullet a = a$
- ✓ Multiplicative inverse property
  $a \bullet \frac{1}{a} = \frac{1}{a} \bullet a = 1 \quad (a \neq 0)$
- ✓ Distributive property relating addition and multiplication
  $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for any three numbers *a*, *b*, and *c*.

Properties of Equality

Properties of equality govern the manipulation of equations (mathematical sentences).
For any three numbers *a*, *b*, and *c*:

- ✓ Addition property of equality
  (Subtraction property of equality)
  If $a = b$ and $c = d$, then $a + c = b + d$
- ✓ Multiplication property of equality
  (Division property of equality)
  If $a = b$ and $c = d$, then $ac = bd$
- ✓ Reflexive property of equality: $a = a$
- ✓ Symmetric property of equality: If $a = b$, then $b = a$
- ✓ Transitive property of equality: If $a = b$, and $b = c$, then $a = c$

A Strategy for Solving Equations with Rational Coefficients	
Equations with rational number coefficients may be solved the same way that equations with integer coefficients are solved using properties of arithmetic and equality. However, many people prefer to rewrite the equation without fractions or decimals before solving it. This can be accomplished by: • Determining a common multiple for all denominators in the equation. • Multiplying both sides of the equation by that common multiple. What remains will be an equation to solve with integer coefficients.	
$\frac{2}{3}(x-1) = \frac{1}{6}(x+2)$ $6\left[\frac{2}{3}(x-1)\right] = 6\left[\frac{1}{6}(x+2)\right]$ $4(x-1) = x+2$	A common multiple of 3 and 6 is 6. Here we multiply both sides of the equation by 6 (multiplication property of equality). The result is an equation with integer coefficients.
$0.1x + 0.25(33-x) = 5.1$ $100\,[0.1x + 0.25(33-x)] = 100\,[5.1]$ $10x + 25(33-x) = 510$	This equation includes tenths ($\frac{1}{10}$) and hundredths ($\frac{1}{100}$). A common multiple of these denominators is 100. Here we multiply both sides of the equation by 100. The result is an equation with integer coefficients.

Solving a System of Linear Equations by Substitution

While there may be multiple ways to use substitution to solve a system of equations, this is one way that has been demonstrated in this unit.

1. Write both equations in slope-intercept form.

$$\begin{cases} 2x + y = 5 \quad \rightarrow \quad y = -2x + 5 \\ x + 2y = 4 \quad \rightarrow \quad 2y = -x + 4 \quad \rightarrow \quad y = -\frac{1}{2}x + 2 \end{cases}$$

2. Use the substitution property. Since the right side of both equations are equal to the same thing, namely *y*, the two expressions in *x* must be equal to each other.

   Write one equation and solve for *x*.

$$-2x + 5 = -\frac{1}{2}x + 2$$
$$x = 2$$

3. Substitution this *x*-value into either equation to obtain the *y*-value.

$$y = -2(2) + 5$$
$$y = 1$$

Solution to the system: (2, 1)

For this example, another substitution approach would be to write the first one equation in slope-intercept form and substitute the expression for *y* in the second equation. Using this approach:

$$x + 2(-2x + 5) = 4$$
$$x = 2$$

Solving a System of Linear Equations by Elimination

Elimination, which applies the multiplication property of equality and the addition property of equality, is another method for solving a system of equations. Here is an example.

Example: Solve this system of equations by elimination.
(We will number each equation with brackets to keep track of them.)

$$\begin{cases} 2x + y = 5 & [1] \\ x + 2y = 4 & [2] \end{cases}$$

1. Use the multiplication property of equality. Multiply both sides of one (or both) equations by some number that will make one of the variable expressions in each equation opposites of each other. In this case, we might multiply both sides of the first equation by -2.

$$-2(2x + y) = -2(5) \rightarrow -4x - 2y = -10 \quad [3]$$

2. Use the addition property of equality. Add expressions on each side of the equation together. Solve.

$$\begin{array}{rcll} -4x - 2y &=& -10 & [3] \\ \underline{x + 2y} &=& \underline{4 \quad\quad [2]} & \\ -3x &=& -6 & \\ x &=& 2 & \end{array}$$

3. Substitute into one of the original equations to find y.

$$[1] \quad 2x + y = 5 \rightarrow 2(2) + y = 5$$

$$y = 1$$

4. Substitute into the other original equation to check.
$[2] \quad x + 2y = 4 \quad 2 + 2(1) = 4$ (true)

A Strategy for Organizing Problem Solving Work Involving Equations

Many algebra problems can be solved with these steps. Here are the steps and an example.

> Talia's coin jar contains nickels and quarters. There are 38 coins in all. The total value of jar is $5.30. Find the coins in the jar.

• Identify the variable(s)	Let n = the number of nickels Let q = the number of quarters
• Write the equation(s)	$n + q = 38 \rightarrow q = 38 - n$ $0.05n + 0.25q = 5.30$
• Solve the equation(s)	By substitution: $0.05n + 0.25(38 - n) = 5.30$ $n = 21$ and $q = 17$
• Answer the question(s)	There are 21 nickels and 17 quarters
• Interpret the solution in the problem	Money makes sense: 21($0.05) + 17($0.25) = $5.30