STUDENT RESOURCES

<table>
<tr><th>Word or Phrase</th><th>Definition</th></tr>
<tr><td>coefficient</td><td>A coefficient is a number or constant factor in a term of an algebraic expression.

In the expression $3x + 5$, 3 is the coefficient of the term $3x$, and 5 is the constant term.</td></tr>
<tr><td>dependent variable</td><td>A dependent variable is a variable whose value is determined by the values of the independent variables. See independent variable.</td></tr>
<tr><td>function</td><td>A function is a rule that assigns to each input value exactly one output value.

For $y = 3x + 6$, any input value, say $x = 10$, has a unique output value, in this case $y = 36$.

For $y = x^2 + 1$, $x = 2$ has the unique output value $y = 2^2 + 1 = 5$.</td></tr>
<tr><td>graph of a function</td><td>The graph of a function is the set of all ordered pairs (x, y) where y is the output for the input value x. If x and y are real numbers, then we can represent the graph of a function as points in the coordinate plane.</td></tr>
<tr><td>independent variable</td><td>An independent variable is a variable whose value may be specified. Once specified, the values of the independent variables determine the values of the dependent variables.

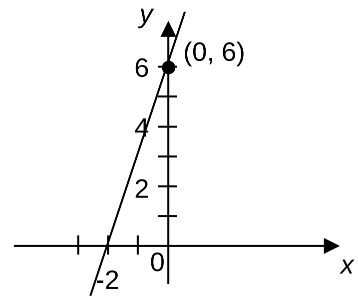
For the equation $y = 3x$, y is the dependent variable and x is the independent variable. We may assign a value to x. The value assigned to x determines the value of y.</td></tr>
<tr><td>input-output rule</td><td>An input-output rule for a sequence of values is a rule that establishes explicitly an output value for each given input value.

<table>
<tr><td>input value (x)</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>x</td></tr>
<tr><td>output value (y)</td><td>1.5</td><td>3</td><td>4.5</td><td>6</td><td>7.5</td><td>$1.5x$</td></tr>
</table>

In the table above, the input-output rule could be $y = 1.5x$. To get the output value, multiply the input value by 1.5. If $x = 100$, then $y = 1.5(100) = 150$.</td></tr>
<tr><td>proportional</td><td>Two variables are proportional if the values of one are the same constant multiple of the corresponding values of the other. The variables are said to be in a proportional relationship, and the constant is referred to as the constant of proportionality.

If Wrigley eats 3 cups of kibble each day, then the number of cups of kibble is proportional to the number of days. If x is the number of days, and y is the number of cups of kibble, then $y = 3x$. The constant of proportionality is 3.</td></tr>
<tr><td>unit rate</td><td>The unit rate associated with a ratio $a : b$ of two quantities a and b, $b \neq 0$, is the number $\frac{a}{b}$, to which units may be attached. This is sometimes referred to as the value of the ratio.

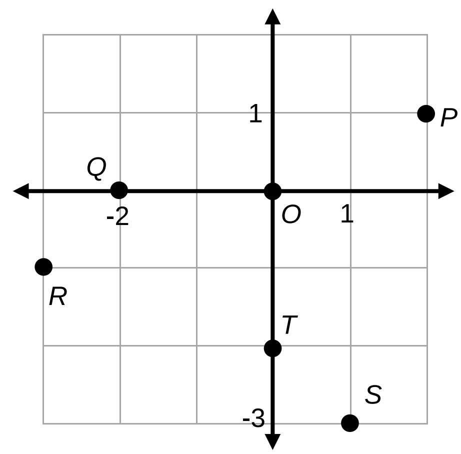
The ratio of 40 miles for every 5 hours has a unit rate of 8 miles per hour.</td></tr>
</table>

Word or Phrase	Definition
y-intercept	The <u>*y*-intercept</u> of a line is the *y*-coordinate of the point at which the line crosses the *y*-axis. It is the value of y that corresponds to $x = 0$. The *y*-intercept of the line $y = 3x + 6$ is 6. If $x = 0$, then $y = 6$. 

The Coordinate Plane

A coordinate plane is determined by a horizontal number line (the *x*-axis) and a vertical number line (the *y*-axis) intersecting at the zero on each line. The point of intersection (0, 0) of the two lines is called the origin. Points are located using ordered pairs (*x*, *y*).

- The first number (*x*-coordinate) indicates how far the point is to the right or left of the *y*-axis.
- The second number (*y*-coordinate) indicates how far the point is above or below the *x*-axis.



Point, coordinates, and interpretation

O (0, 0) → This is the intersection of the axes (origin).

P (2, 1) → start at the origin, move 2 units right, then 1 unit up

R (-3, -1) → start at the origin, move 3 units left, then 1 unit down

S (1, -3) → start at the origin, 1 unit right, then 3 units down

Q (-2, 0) → start at the origin, move 2 units left, then 0 units up or down

T (0, -2) → start at the origin, 0 units right or left, then 2 units down

Functions

Some ways to represent rules in mathematics are input-output tables, mapping diagrams, ordered pairs, equations, and graphs.

Examples that are Functions

Input-Output Table

x **input**	***y*** **output**
1	1
3	3
5	5
7	7
9	9

This table lists input values with unique output values.

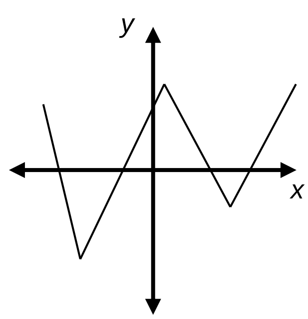
Ordered Pairs

(0, 2), (1, -2), (2, 2), (3, -2)

In this set of ordered pairs, each input value is assigned to a unique output value. Note that different input values may be assigned the same output value. In this example, both 1 and 3 are assigned the output value -2.

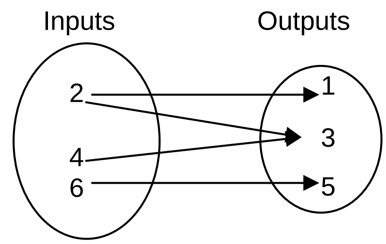
Graph

This graph represents a function because every vertical line through it intersects at most one point of the graph. In other words, each possible *x*-value corresponds to a unique *y*-value.



Examples that are NOT Functions

Mapping Diagram



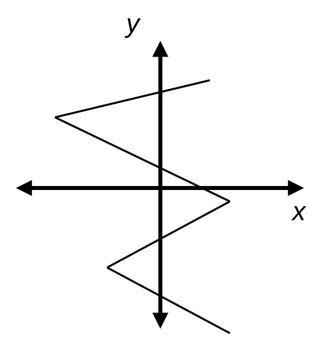
This mapping diagram is not a function. It is not permissible for the same input value (in this case 2) to be assigned two different output values. However, all other input-output mappings above are fine.

Equation (with Ordered Pairs)

Consider the set of pairs (x, y) that satisfy $x = y^2$, such as (0, 0), (25, 5), and (25, -5). Since the input value, $x = 25$, corresponds to two different output values ($y = 5$ and $y = -5$), the y-values are not a function of the x values.

Graph

This graph does not represent a function because some vertical lines (for example, the *y*-axis) intersect the graph in more than one point. In other words, some *x*-values correspond to more than one *y*-value.



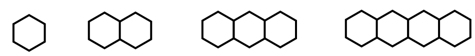
Using Multiple Representations to Describe Linear Functions

Here are four representations commonly used to approach a math problem:

- Numbers (numerical approach, as by making a table)
- Pictures (visual approach, as with a picture or graph)
- Symbols (approaching the problem using algebraic symbols)
- Words (verbalizing a solution, orally or in writing)

Each approach may lead to a valid solution. Collectively they should lead to a complete and comprehensive solution, one that is readily accessible to more people and that provides more insight.

Example 1: Describe this pattern of hexagons using numbers, pictures, words, and symbols.



Numbers

Step #	**number of segments**	**Breaking apart numbers sometimes helps you see an input-output rule.**	
1	6	6	= 6 + (0)5
2	11	6 + 5	= 6 + (1)5
3	16	6 + 5 + 5	= 6 + (2)5
4	21	6 + 5 + 5 + 5	= 6 + (3)5
5	26	6 + 5 + 5 + 5 + 5	= 6 + (4)5
n	$5n + 1$	$5n + 1$	$= 6 + (n - 1)5$

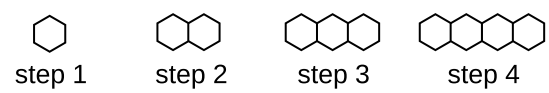
Words

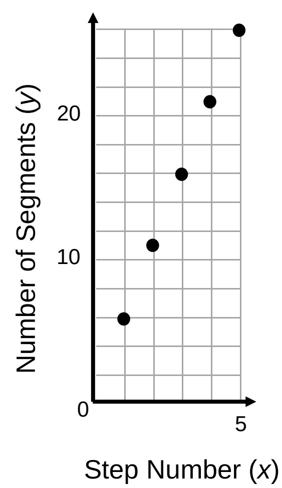
One way to describe the hexagonal pattern is to start with 6 segments and add 5 more segments at each subsequent step. Notice that the number of 5's added at each step is equal to 1 less than the step number.

Symbols

A rule for finding the number of segments at step n is $6 + (n - 1)5$, which can be simplified to $5n + 1$.

Pictures





Note: we consider a graph to be a picture.

Using Multiple Representations to Describe Linear Functions (Continued)

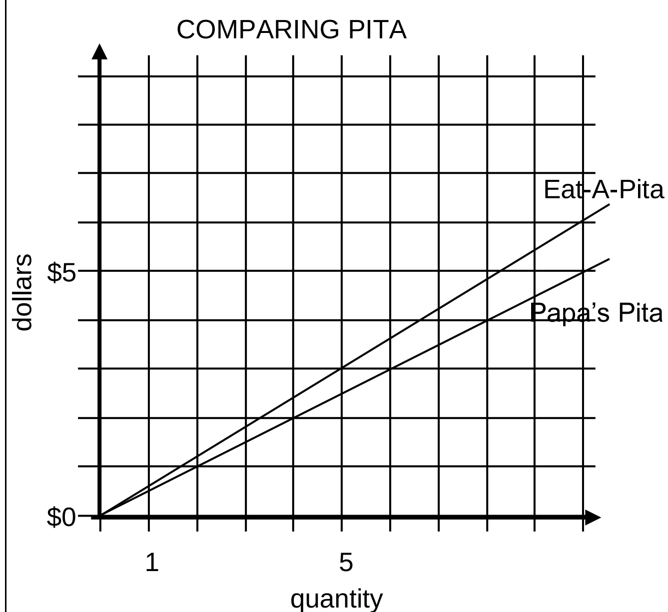
Example 2: At Papa's Pitas, 2 pitas cost $1.00. At Eat-A-Pita, 5 pitas cost $3.00. Assuming a proportional relationship between the number of pitas and their cost, use multiple representations to explore which store offers the better buy for pitas.

Numbers (make a table)

PAPA'S PITAS	
# of pitas (x)	cost (y)
2	$1.00
4	$2.00
6	$3.00
8	$4.00
10	$5.00

EAT-A-PITA	
# of pitas (x)	cost (y)
5	$3.00
10	$6.00
15	$9.00
20	$12.00
25	$15.00

Pictures (make a graph)



Words (write sentences)

Based on the table, Papa's Pitas is the better buy.

At Papa's Pitas, you get 6 pitas for $3.00.
This means the unit price (cost for one pita) is $0.50.

At Eat-A-Pita you only get 5 pitas for $3.00. This means the unit price (cost for one pita) is $0.60.

Symbols (write equations to relate the number of pitas to cost)

PAPA'S PITAS $y = 0.5x$

EAT-A-PITA $y = 0.6x$

Notice that $0.50 is the cost of one pita at Papa's Pita. This corresponds to the point (1, 0.5) on the graph.

Notice that $0.60 is the cost of one pita at Eat-A-Pita. This corresponds to the point (1, 0.6) on the graph.

The equations above are both in the form $y = mx$. This equation form represents a proportional relationship because y is a constant multiple of x. Graphs of equations in this form are always lines going through the origin. They will be explored more in the next unit and contrasted with equations in the form $y = mx + b$.