STUDENT RESOURCES

Word or Phrase	Definition
conjecture	A conjecture is a statement that is proposed to be true, but has not been proven to be true nor to be false. After creating a table of sums of odd numbers such as 1 + 3 = 4, 1 + 5 = 6, 5 + 7 = 12, 3 + 9 = 12, etc., we may make a conjecture that the sum of any two odd numbers is an even number. This conjecture can be proven to be true.
cube of a number	The cube of a number n is the number $n^3 = n \bullet n \bullet n$. The cube of -5 is $(-5)^3 = (-5)(-5)(-5) = -125$.
cube root	The cube root of a number n is the number whose cube is equal to n. That is, the cube root of n is the value of x such that $x^3 = n$. The cube root of n is written $\sqrt[3]{n}$. The cube root of -125 is $\sqrt[3]{-125} = -5$, because $(-5)^3 = (-5)(-5)(-5) = -125$.
exponent notation	The exponent notation b^n (read as " b to the power n ") is used to express n factors of b. The number b is the base, and the natural number n is the exponent. Exponent notation is extended to arbitrary integer exponents by setting $b^0 = 1$ and $b^{-n} = \frac{1}{b^n}$. $2^3 = 2 \bullet 2 \bullet 2 = 8$ (the base is 2 and the exponent is 3) $3^2 \bullet 5^3 = 3 \bullet 3 \bullet 5 \bullet 5 \bullet 5 = 1{,}125$ (the bases are 3 and 5) $2^0 = 1$ $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
radical expression	A radical expression is an expression involving a root, such as a square root. $\sqrt{20}$ and $5\sqrt{3}$ are radical expressions.
scientific notation	Scientific notation for a positive number represents the number as a product of a decimal between 1 and 10 and a power of 10. It is typically used to write either very large numbers or very small numbers. In scientific notation, the number 245,000 is written as 2.45×10^5. In scientific notation, the number 0.0063 is written as 6.3×10^{-3}.

Numbers Squared and Cubed

Why do we say that a number raised to the second power is “squared”? The reason has to do with the area formula for squares. The area of a square of side length s is given by

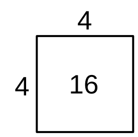
$$\text{area} = s \bullet s = s^2 .$$

A square with side length 4 units has area “4 squared” = 4^2 = 16 square units.

What about “square root” – where does that term come from?

Here the reason is that a “root” can also refer to the solution of an equation. A “square root” has to do with finding the side length of a square of a given area; that is, of solving the equation $s^2 = A$. For a given area A, the side length s of the square with area A is side length = $s = \sqrt{A}$ = “square root of A.”

A square with area 16 square units has side length $\sqrt{16}$ = 4 units.



$\longrightarrow$ $4^2 = 16$ and $\sqrt{16} = 4$

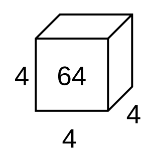
Why do we say that a number raised to the third power is “cubed”? In this case, the answer has to do with the volume formula for cubes. The volume of a cube with side length s is given by

$$\text{volume} = s \bullet s \bullet s = s^3 .$$

A cube with side length 4 units has volume “4 cubed” = 4^3 = 64 cubic units.

In turn, a “cube root” has to do with finding the side length of a cube of a given volume, that is, of solving the equation $s^3 = V$. For a given volume V, the side length s of the cube with volume V is side length = $s = \sqrt[3]{V}$ = “cube root of V.”

A cube with volume 64 cubic units has side length $\sqrt[3]{64}$ = 4 units.



$\longrightarrow$ $4^3 = 64$ and $\sqrt[3]{64} = 4$

Although we assume here that V is positive, the cube root of a negative number can be found by solving the equation, $s^3 = V$. The square root of a negative number is not a real number.

Squaring a number and finding the square root of a number are inverse operations. Similarly, cubing a number and finding the cube root of a number are inverse operations.

Three Facts and Three Rules for Exponents		
Definitions and Rules		**Example**
Meaning of positive exponent:	$x^m = x \bullet x \bullet ... \bullet x$ (m factors)	$3^4 = 3 \bullet 3 \bullet 3 \bullet 3$ (4 factors of 3)
Fact about zero as an exponent:	$x^0 = 1, \ x \neq 0$	$3^0 = 1$, (0^0 is not defined)
Fact about a negative exponent:	$x^{-a} = \frac{1}{x^a}, \ x \neq 0$	$3^{-2} = \frac{1}{3^2}$, (0 cannot be in the denominator because division by 0 is not defined)
Product rule for exponents:	$x^a \bullet x^b = x^{a+b}$	$3^2 \bullet 3^1 = 3 \bullet 3 \bullet 3 = 3^{2+1} = 3^3$
Power rule for exponents:	$(x^a)^b = x^{a \bullet b}$	$(3^2)^3 = 3^2 \bullet 3^2 \bullet 3^2 = 3^{3 \bullet 2} = 3^6$
Quotient rule for exponents:	$\frac{x^a}{x^b} = x^{a-b}, \ x \neq 0$	$\frac{3^4}{3^6} = \frac{3 \bullet 3 \bullet 3 \bullet 3}{3 \bullet 3 \bullet 3 \bullet 3 \bullet 3 \bullet 3} = 3^{4-6} = 3^{-2}$, (0 cannot be in the denominator because division by 0 is not defined)
The three rules above apply to expressions with the same base numbers. For example: $5^3 \bullet 4^2 = (5 \bullet 5 \bullet 5) \bullet (4 \bullet 4)$, and the product rule does not apply.		

Making Sense of Zero and Negative Exponents			
These patterns show that the definitions for zero and negative exponents are reasonable.			
Pattern: Divide by 2	**Result of the division**	**Pattern as a product**	**Pattern in exponent form**
	Start with 8	$2 \bullet 2 \bullet 2$	2^3
$8 \div 2$	4	$2 \bullet 2$	2^2
$4 \div 2$	2	2	2^1
$2 \div 2$	1	1	2^0
$1 \div 2$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2^1}$ or 2^{-1}
$\frac{1}{2} \div 2$	$\frac{1}{4}$	$\frac{1}{2 \bullet 2}$	$\frac{1}{2^2}$ or 2^{-2}
$\frac{1}{4} \div 2$	$\frac{1}{8}$	$\frac{1}{2 \bullet 2 \bullet 2}$	$\frac{1}{2^3}$ or 2^{-3}

Scientific Notation				
Given number	**Related decimal between 1 and 10**	**Power of 10**	**Number in scientific notation**	**Reasoning**
120,000,000	1.2	10^8	1.2×10^8	The given number is 10^8 times 1.2; adjust place values by multiplication.
0.0000345	3.45	10^{-5}	3.45×10^{-5}	3.45 is 10^5 times the given number; adjust place values by multiplication.

Some of the benefits of scientific notation:

(1) Scientific notation is useful for writing numbers with very large or very small values in a compact way.

(2) The power of 10 gives an immediate clue to the relative size of the number.

Error alert!

When comparing numbers in scientific notation such as 2.5×10^{12} and 8.76×10^8, a common mistake is to focus on the fact that 8.76 > 2.5. **Focus on the exponent!**

$$2.5 \times 10^{12} = 2{,}500{,}000{,}000{,}000$$
$$8.76 \times 10^8 = 876{,}000{,}000$$