STUDENT RESOURCES

Word or Phrase	Definition
converse of the Pythagorean theorem	The converse of the Pythagorean theorem states that if the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle. See Pythagorean theorem. If the lengths of the sides of a triangle are 3, 4, and 5 units respectively, then the triangle is a right triangle, because $3^2 + 4^2 = 5^2$.
hypotenuse	The hypotenuse of a right triangle is the side of the triangle opposite the right angle. It is the longest side in a right triangle.
integers	The integers are the whole numbers and their opposites. They are the numbers 0, 1, 2, 3, … and -1, -2, -3, … .
irrational numbers	Irrational numbers are real numbers whose decimal expansions continue infinitely without continuously repeating the same block of digits. The irrational numbers are the real numbers that are not rational. $\sqrt{2}$, , and 0.101001000100001... are irrational numbers and cannot be written as quotients of integers.
legs	The legs of a right triangle are the two sides of the triangle adjacent to the right angle.
natural numbers	The natural numbers are the numbers 1, 2, 3, … .Natural numbers are also referred to as counting numbers.
perfect square	A perfect square, or square number, is a number that is the square of a natural number. The area of a square with a natural number side-length is a perfect square. The perfect squares are $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, $16 = 4^2$, $25 = 5^2$, … .
Pythagorean theorem	The Pythagorean theorem states that the sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse. See converse of the Pythagorean theorem. $a^2 + b^2 = c^2$ If the lengths of the legs of a right triangle are 5 and 12 units respectively, then the hypotenuse has length 13 units, because $13^2 = 5^2 + 12^2$.
radical expression	A radical expression is an expression involving a root, such as a square root. In a radical expression, the symbol $\sqrt{\ }$ is called a radical sign, and the number under the radical sign is called the radicand. $\sqrt{5}$ is a radical expression. The radicand is 5.

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rational numbers	Rational numbers are quotients of integers. Rational numbers can be expressed as $\frac{m}{n}$, where m and n are integers and $n \neq 0$. $\frac{3}{5}$ is rational because it *is* a quotient of integers. 4, $2\frac{1}{3}$, 0.7, and $0.\overline{25}$ are rational numbers because they *can be* expressed as quotients of integers ($4 = \frac{4}{1}$; $2\frac{1}{3} = \frac{7}{3}$; $0.7 = \frac{7}{10}$; $0.\overline{25} = 0.2525\ldots = \frac{25}{99}$). $\sqrt{2}$ and π are NOT rational numbers. They *cannot be* expressed as a quotient of integers.
real numbers	Real numbers refer to the rational numbers and irrational numbers together. Each real number has a decimal name (address) locating it on the real number line.
repeating decimal	A repeating decimal is a decimal that ends in repetitions of the same block of digits. A "repeat bar" can be placed above the digits that repeat. A terminating decimal is regarded as a repeating decimal that ends in all zeros. Repeating decimals represent rational numbers. $\frac{2}{9} = 0.22222\ldots = 0.\overline{2}$ $\frac{2}{11} = 0.181818\ldots = 0.\overline{18}$ } these repeating decimals do NOT terminate $\frac{1}{2} = 0.50000\ldots = 0.5\overline{0} = 0.5$ $\frac{3}{4} = 0.750000\ldots = 0.75\overline{0} = 0.75$ } these repeating decimals do terminate
square of a number	The square of a number is the product of the number with itself. The square of 5 is 25, since $5^2 = (5)(5) = 25$. The square of -5 is also 25, since $(-5)^2 = (-5)(-5) = 25$. This is different than $-5^2 = -(5)(5) = -25$.
square root	A square root of a number n is a number whose square is equal to n, that is, a solution of the equation $x^2 = n$. The positive square root of a number n, written $\sqrt{n}$, is the positive number whose square is n. Except where otherwise noted, the term "the square root of *n*" refers to the positive square root. $\sqrt{25} = 5$, because $5^2 = (5)(5) = 25$
terminating decimal	A terminating decimal is a repeating decimal whose digits are eventually a repeating 0 from some point on. The final 0's in the expression for a terminating decimal are usually omitted. 4.6200000… = 4.62. It is a terminating decimal with value $4 + \frac{6}{10} + \frac{2}{100}$.
whole numbers	The whole numbers are the natural numbers together with 0. They are the numbers 0, 1, 2, 3, … .

Numbers Squared

Why do we say that a number raised to the second power is “squared”? The reason has to do with the area formula for squares. The area of a square of side length s is given by

$$\text{area} = s \bullet s = s^2.$$

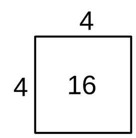
A square with side length 4 units has area “4 squared” = $4^2 = 16$ square units.

What about “square root” – where does that term come from?

Here the reason is that a “root” can also refer to the solution of an equation. A “square root” has to do with finding the side length of a square of a given area; that is, of solving the equation $s^2 = A$. For a given area A, the side length s of the square with area A is

side length = $s = \sqrt{A}$ = “square root of A.”

A square with area 16 square units has side length $\sqrt{16}$ = 4 units.



→ $4^2 = 16$ and $\sqrt{16} = 4$

Square Roots: Estimates Versus Exact Value

Q: What is the square root of 9? *A*: We know that $3^2 = 9$, so $\sqrt{9} = 3$.

Q: What is the square root of 7? *A*: We know of no rational number that, when squared, is equal to 7.

Using the square root function on a simple calculator, we get an approximation to several decimal places: $\sqrt{27} \approx 5.196152$, and then by multiplication: $(5.196152)^2 = (5.196152)(5.196152) = 26.9999956$.

Find another approximation for $\sqrt{27}$ using a calculator with greater capacity, then square that number, and it will still not be exactly equal to 27.

We know that $\sqrt{27}$ is an irrational number and its decimal expansion is infinite with no block of digits that repeats.

So how do we write $\sqrt{27}$? The only way to write it exactly is to leave it in square root form.

If we choose to approximate $\sqrt{27}$, the simplest way may be to state which two consecutive integers it is between. We know that $5^2 = 25$ and $6^2 = 36$, and we also know that 27 is between 25 and 36, so $\sqrt{27}$ is between 5 and 6.

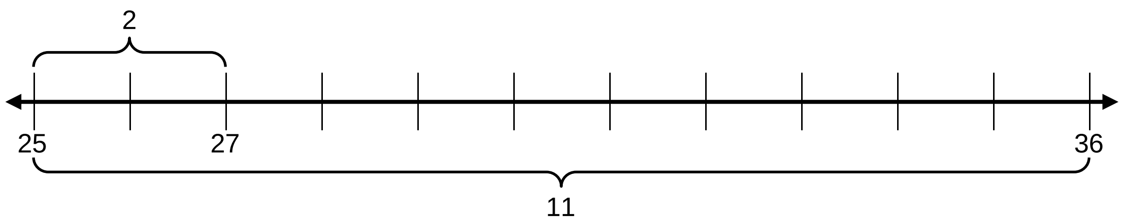
See the next box for an estimation method from lesson 2.1 that is more accurate.

Estimating Square Roots with Greater Accuracy: Linear Interpolation

The following two strategies may be helpful for square root estimation. Note: the method illustrated below is referred to as "linear interpolation."

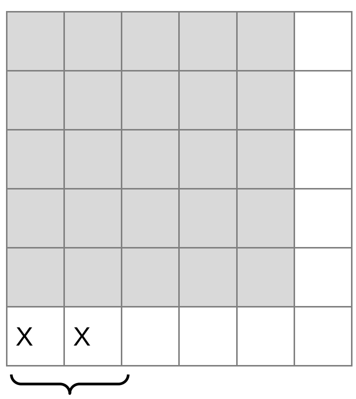
Find an estimate of $\sqrt{27}$ as a mixed number and as a whole number with decimal remainder.

Strategy 1: Since $\sqrt{27}$ is between $\sqrt{25}$ and $\sqrt{36}$ (25 and 36 are perfect squares), provide a "magnified" portion of a number line from 25 to 36.



The distance from 25 to 27 is $\frac{2}{11}$ of the distance from 25 to 36. Hence, the distance from $\sqrt{25}$ to $\sqrt{27}$ should be about $\frac{2}{11}$ of the distance from $\sqrt{25}$ to $\sqrt{36}$. In other words, the distance from 5 to $\sqrt{27}$ should be about $\frac{2}{11}$ of the distance from 5 to 6. Thus $\sqrt{27} \approx 5\frac{2}{11}$.

Strategy 2: Since 25 and 36 are perfect squares, on grid paper, draw a 5 x 5 square inside a 6 x 6 square as illustrated below.



The two X's represent the 26th and 27th unit squares.

- The larger square is $6 \times 6 = 6^2 = 36$ unit squares.
- The smaller, shaded square inside is $5 \times 5 = 5^2 = 25$ unit squares.

So, $\frac{27-25}{36-25} = \frac{2}{11}$

Therefore, $\sqrt{27}$ is about $5\frac{2}{11}$. (calculator check: $5\frac{2}{11} = 5.1\overline{8}$ and $\sqrt{27} \approx 5.196$)

A Clever Procedure: Writing a Repeating Decimal as a Quotient of Integers

Any repeating decimal can be written as a quotient of integers. Therefore, all repeating decimals are rational. The following algebraic idea is used to change a repeating decimal to a quotient of integers.

Example 1: Change $0.1\overline{6} = 0.16666\ldots$

$10x = 1.66666\ldots$ (2)

Let $\boldsymbol{x} = \mathbf{0.16666\ldots}$ **(1)**

$9x = 1.5$ (3)

$x = \frac{1.5}{9} = \frac{15}{90} = \frac{1}{6}$ (4)

- Notice that step 2 is above step 1
- The "trick" is to multiply both sides of the equation in step 1 by a power of 10 that will "line-up" the repeating portion of the decimal.
- Subtract the expressions in step 1 from step 2. This results in a step 3 equation that has a terminating decimal. Solve for x in step 4 and simplify the result so it is a quotient of integers

Example 2: Change $0.\overline{7} = 0.77777\ldots$

$10x = 7.777777\ldots$ (1)

Let $\boldsymbol{x} = \mathbf{0.7777777\ldots}$ **(2)**

$9x = 7.000000\ldots$ (3)

$x = \frac{7}{9}$ (4)

Ask yourself:

How many digits are repeating?
one

What do I multiply both sides by?
10 (a power of 10 with one zero)

Example 3: Change $0.\overline{45} = 0.454545\ldots$

$10x = 45.454545\ldots$ (1)

Let $\boldsymbol{x} = \mathbf{0.45454545\ldots}$ **(2)**

$99x = 45.00000\ldots$ (3)

$x = \frac{45}{99} = \frac{15}{33} = \frac{5}{11}$ (4)

Ask yourself:

How many digits are repeating?
two

What do I multiply both sides by?
100 (a power of 10 with two zeros)