FOCUS, COHERENCE, AND RIGOR

A group of educators at Rutgers University led by Dr. Roberta Shorr (Young, 2009) conducted a seven-year study on problem-solving with minority and low-income students in low-performing schools. They found that giving conceptually challenging problems to students caused frustration, but at the same time, students were engaged and motivated. Working through the frustration in an emotionally safe environment, students gained "satisfaction, pride, and a willingness to work harder next time." This work also resulted in higher standardized test scores (e.g., average scores for Newark fourth graders rose from 45 to 79 percent).

At the Center for Mathematics and Teaching, we know that all students have the potential to achieve in mathematics, we believe that the development of mathematics should reflect the connectedness of Big Ideas into a coherent whole, and we make mathematics inviting and inclusive to more students. One of the ways we put these beliefs and intentions into action is by attending to focus, coherence, and rigor in program development.

FOCUS

According to Achieve the Core (2014), "Not all content in a given grade is emphasized equally in the Standards. Some clusters require greater emphasis than others based on the depth of ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness."

Achieve the Core organizes mathematics clusters into three categories: major clusters, supporting clusters, and additional clusters. This table identifies the clusters for 8th grade and their alignment with *MathLinks* lessons.

GRADE 8: FOCUS IN *MATHLINKS* BASED ON MAJOR, MINOR, AND SUPPORTING CLUSTERS

CLUSTER	Standards	Lesson Focus	Additional Lessons (L) Spiral Review (SR)
Major Clusters			
• 8.EE.A Work with radicals and integer exponents	8.EE.1-4	3.1, 3.2, 3.3, 10.1, 10.2, 10.3	SR: 2,4,6,7,8,10
• 8.EE.B Understand the connections between proportional relationships, lines, and linear equations.	8.EE.5-6	4.1, 4.3, 5.1, 5.2, 5.3, 10.3	SR: 2,3,9
• 8.EE.C Analyze and solve linear equations and pairs of simultaneous linear equations	8.EE.7-8	7.1, 7.2, 7.3, 8.1, 8.2, 8.3	SR: 1,2,3,4,5,6,7,8,9,10
• 8.F.A Define, evaluate, and compare functions	8.F.1-3	4.1, 4.2, 4.3, 5.2, 5.3, 8.2, 8.3	L: 7.1, 9.1, 10.3
• 8.F.B Use functions to model relationships between quantities.	8.F.4-5	4.1, 4.2, 4.3, 5.1, 5.2, 5.3, 6.2, 8.3	L: 7.1 SR: 8
• 8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software.	8.G.1-5	1.3, 9.1, 9.2, 9.3	L: 10.1 SR: 10
• 8.G.B Understand and apply the Pythagorean theorem.	8.G.6-8	2.2, 10.3	L: 9.1, 9.3, 10.1, 10.2,10.3 SR: 4,7,8
Supporting Clusters			
• 8.NS.A Know that there are numbers that are not rational, and approximate them by rational numbers.	8.NS.1-2	2.1, 2.3	SR: 5,7,9
• 8.SP.A Investigate patterns of association in bivariate data.	8.SP.1-4	6.1, 6.2, 6.3	SR: 8,9

Functions Number Sense Expressions and Equations Statistics and Probability Geometry

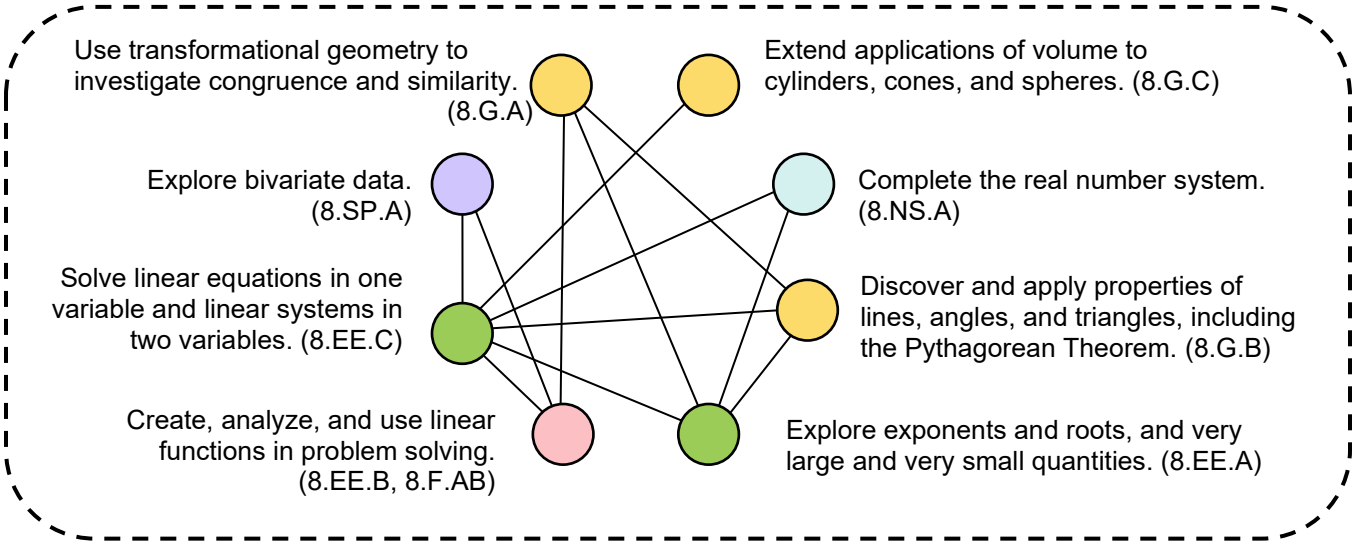
MathLinks intentionally avoids the trap of a "mile wide and inch deep" curriculum that uses standards as a checklist. Rather, we focus students on understanding grade-level mathematics deeply, connecting Big Ideas (see next page), and offering students problems, tasks, and investigations worthy of engagement.

COHERENCE

According to the 2023 California Mathematics Framework, “instructional materials should primarily involve tasks that invite students to make sense of big ideas, elicit wondering in authentic contexts, and necessitate mathematics. Big ideas in math are central to the learning of mathematics, link numerous mathematical understandings into a coherent whole, and provide focal points for students’ investigations.”

Inspired by the work of Jo Boaler (2018) and based on the Content Domains, the *MathLinks* team synthesized the work of the 8th grade into eight Big Ideas. These ideas and their connections within the entire program are shown here.

Grade 8: Big Ideas and Connections



The work in *MathLinks* is organized into 10 units. An expanded dive into the specific Big Ideas, connections, and progressions across grades are included in the front section of each unit of the Teacher Edition. Toward the end of each Student Packet, students identify the Big Ideas within the unit and reflect upon how they are connected to each other.

RIGOR

Achieve the Core (2015) states, “a rigorous math program will pursue with equal intensity conceptual understanding, procedural skill and fluency, and applications.” We now examine the *MathLinks*: Grade 8 program through this lens.

Conceptual Understanding in *MathLinks*

Conceptual understanding, the bedrock of a *MathLinks* course, frequently drives the other two components of rigor. It is a *MathLinks* philosophy to make sure all students have the opportunity to make meaning for every concept presented; we focus on the conceptual development of Big Ideas in depth and make them plausible through investigations, activities, and practice. This is commonly done throughout lessons in all units, oftentimes with the help of teacher Lesson Notes and Slide Decks (or Slide Deck Alternatives).

The table below identifies some examples of concept development in *MathLinks:* Core Grade 8 (unit number and lesson number in parentheses). Typically, these lessons include guided instruction with a slide deck, lesson notes, class discussions, or group work. Opportunities for independent work within a Student Packet appear as Practice pages within lessons, in the Review section as activities, and as Spiral Reviews in subsequent units. Unit Resources on the Teacher Portal also contain Extra Problems, Tasks, and Projects to support conceptual development.

GRADE 8: EXAMPLES OF CONCEPT DEVELOPMENT IN *MATHLINKS*	
CLUSTER DESCRIPTION	**(LESSON NUMBER) CONCEPT DEVELOPMENT ACTIVITY**
Major Clusters	
• 8.EE.A (EE 1-4) Work with radicals and integer exponents	(3.1) Investigating Two Exponent Patterns
• 8.EE.B (EE 5-6) Understand the connections between proportional relationships, lines, and linear equations.	(5.1) The Meaning of Slope (5.1) The Slope Formula
• 8.EE.C (EE 7-8) Analyze and solve linear equations and pairs of simultaneous linear equations	(7.1) Using Substitution (100-Mile Walking Challenge) (7.2) Solving Equations with Balance 1, 2
• 8.F.A (F 1-3) Define, evaluate, and compare functions	(4.1) The Pool Problem
• 8.F.B (F 4-5) Use functions to model relationships between quantities.	(4.1) Saving vs. Spending
• 8.G.A (G 1-5) Understand congruence and similarity using physical models, transparencies, or geometry software.	(9.0) Slides, Turns, and Flips (10.0) A Rubber Band Investigation
• 8.G.B (G 6-8) Understand and apply the Pythagorean theorem.	(2.2) A Famous Theorem
Supporting and Additional Clusters	
• 8.NS.A (NS 1-2) Know that there are numbers that are not rational, and approximate them by rational numbers.	(2.1) A Radical Investigation (2.3) How Can 0.999… = 1?
• 8.SP.A (SP 1-4) Investigate patterns of association in bivariate data.	(6.1) Association and Causation
• 8.G.C (G 9) Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	(1.2) Volume of a Cone and a Sphere

Functions Number Sense Expressions and Equations Statistics and Probability Geometry

Applications in *MathLinks*

Problem-solving is an important driver of instruction within *MathLinks* courses. In *MathLinks*, we include engaging mathematical problems and applications with accessible entry points for all students, multiple approaches or solutions, and extensions to challenge and enrich. All units begin with an Opening Problem, which introduces a concept or establishes a need to know. In many cases, students require more instruction throughout the unit before they are fully prepared to bring the problem to its conclusion. Substantial problems exist throughout the units as well.

The table below identifies some examples of mathematical problems and applications in *MathLinks:* Grade 8 (unit number and lesson number in parentheses). Some of these problems include guided instruction with a slide deck and lesson notes. Others are appropriate for independent work. Unit Resources on the Teacher Portal (e.g., Tasks, Projects) also contain mathematical problems and applications.

GRADE 8: EXAMPLES OF PROBLEM-SOLVING IN *MATHLINKS*	
CLUSTER DESCRIPTION	**(LESSON NUMBER) MATHEMATICAL PROBLEMS AND APPLICATIONS**
Major Clusters	
• 8.EE.A (EE 1-4) Work with radicals and integer exponents	(2.0, 2.2) A Rectangle Paradox (2.2) The Club and the Box
• 8.EE.B (EE 5-6) Understand the connections between proportional relationships, lines, and linear equations.	(5.0, 5.3) The Rope Investigation, The Rope Revisited (5.3) Rectangle Paradox: A Fresh Look (7.1) Practice 3 (Skateboard problem)
• 8.EE.C (EE 7-8) Analyze and solve linear equations and pairs of simultaneous linear equations	(8.3) Watering Cans
• 8.F.A (F 1-3) Define, evaluate, and compare functions	(4.1) The Pool Problem
• 8.F.B (F 4-5) Use functions to model relationships between quantities.	(7.1) Using Substitution (100-Mile Walking Challenge) (8.3) Training for a Marathon
• 8.G.A (G 1-5) Understand congruence and similarity using physical models, transparencies, or geometry software.	(9.3) Swimming at the River
• 8.G.B (G 6-8) Understand and apply the Pythagorean theorem.	(2.0, 2.2) A Rectangle Paradox (2.2) The Club and the Box (10.3) And Finally… A Mathematical Surprise!
Supporting and Additional Clusters	
• 8.NS.A (NS 1-2) Know that there are numbers that are not rational, and approximate them by rational numbers.	(2.3) A Rational Numbers Investigation (3.2) What in the World?, Practice 7 (problem 10)
• 8.SP.A (SP 1-4) Investigate patterns of association in bivariate data.	(6.1, 6.2) Practice 2, Obesity Rates by State (6.3) Two-Way Tables
• 8.G.C (G 9) Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	(1.0, 1.1) Paper Solids, Practice 2 (1.1) A Coin Problem

Functions **Number Sense** **Expressions and Equations** **Statistics and Probability** **Geometry**

Procedural Skill and Fluency in *MathLinks*

Procedural skill and fluency require more than just computational speed and accuracy. In a 2014 position paper, NCTM described procedural fluency as "the ability to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another."

In *MathLinks*, we thoughtfully develop new procedural skills and provide opportunities for students to gain fluency throughout the year. Skills practice in each unit is found in the Student Packets in the following ways:

- Practice pages – These pages support concept development.
- Review activities – These pages often include skills practice.
- Spiral Review – These pages have distributed practice of prior skills.
- Math Path Fluency Challenges – This Activity Routine, in the Spiral Review section, utilizes mental math skills and supports fluency development in a puzzle format.

Achieve the Core (2015) explicitly identifies two CCSS-M standards (6.NS.2 and 6.NS.3) where fluency is expected in 8th grade. The EdReports Evidence Guide (2021) also includes 6.EE.1 as a fluency expectation. The table below shows some examples in 8th grade Student Packets where these procedural skills are developed and students have opportunities to gain fluency through independent practice.

GRADE 8: EXAMPLES OF COMPUTATIONAL FLUENCY WORK IN *MATHLINKS*		
Standard / Description	**Lesson**	**Opportunities to gain fluency**
8.EE.7 Solve linear equations in one variable.	(7.2) Solving Equations Using Cups and Counters (7.3) Solving Equations Algebraically (8.1) Solving Equations Involving Rational Numbers	(8.3) Algebra Applications Spiral Review in Packets 1 – 10
8.EE.8b Solve systems of two linear equations in two variables, and estimate solutions by graphing the equations. Solve simple cases by inspection.	(7.1) Solving Systems by Graphing (8.2) Solving Systems Using Algebra	Spiral Review in Packets: 1, 3, 5, 7, 9, 10

Expressions and Equations

Grade-level skills practice is in each unit, as well as practice to fill in gaps. Both can be found on the Teacher Portal in Other Resources in the following ways:

- Essential Skills – This entire section reviews skills and concepts important for success in a given unit. Activity Routines such as Big Square Puzzles, Open Middle Problems, and Four-in-a-Row games are also in these sections for some units. They provide a practice alternative to "drill and kill."
- Extra Problems – Skills practice by lesson is available for all units.
- Nonroutine Problems – In addition to skills practice that is embedded in nonroutine problems, Big Square Puzzles, Open Middle Problems, and Four-in-a-Row games are located in this section for some units.

Finally, for students who need to fill large gaps in skills, consider Skill Boosters, which are on the Teacher Portal. This resource helps students catch up on below-grade-level work without losing extensive instructional time on grade-level work. A Skill Booster routine is intended to take about 5 – 10 minutes per day. Skill Boosters do not directly connect to any course or unit within a course.

Connecting the Three Aspects of Rigor

Interpretations of rigor are evolving. Achieve the Core described "rigor" as the pursuit of conceptual understanding, procedural skill and fluency, and application with equal intensity. The 2023 California Mathematics Framework added the element of integration, interpreting rigor as "an integrated way in which conceptual understanding, strategies for problem-solving and computation, and applications are learned so that each supports the other." A closer look at two major domains in 8th grade illustrates how this is done in *MathLinks*.

The Algebra Progression in *MathLinks*: Grade 8

Algebra topics primarily appear in the CCSS-M Expressions and Equations and Functions domains. They are also in the Statistics and Probability domain. These areas are the focus of six units in *MathLinks*: Grade 8.

- In Units 1 and 2, **Plane and Solid Figures** and **Real Numbers and the Pythagorean Theorem**, students apply 7th grade algebra to new 8th grade topics.
- In Unit 3, **The Algebra of Exponents and Roots**, students observe patterns in numerical expressions with exponents and generalize them to obtain the product, power, and quotient rules for exponents. They also use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number.
- In Unit 4, **Introduction to Functions**, students continue the work started in 6th and 7th grades with multiple representations by connecting real-life or visual contexts to tables, graphs, and algebraic equations. Functions are formally defined and students represent them in tables, graphs, and mapping diagrams. They interpret different characteristics of functions (e.g., increasing or decreasing, linear or nonlinear).
- In Unit 5, **Linear Functions**, slope is formally defined as students find slopes of lines by counting on a grid, and by using the slope formula, which then leads into using the slope-intercept form of a line, and applying this knowledge to solving various problems.
- In Unit 6, **Bivariate Data**, students graph bivariate numerical data as scatter plots, describe patterns of association (if they exist), estimate lines of best fit to points showing linear associations, write equations of these estimated lines from which predictions are made, and draw conclusions from the data.
- In Unit 7, **Linear Equations and Systems 1**, students solve linear systems of equations by graphing, noting that these systems have exactly one solution, infinitely many solutions, or no solutions. Estimating solutions creates the need for a more precise solution method. The process of substitution is introduced as a way to take a system of two equations in slope-intercept form and creating one equation in one variable. This creates a need to learn to solve equations with variables on both sides.

  Students then revisit the "cups and counters" model (introduced in Grade 7) for solving equations. The model aids the transition to formal procedures. Students see a parallel to the work done with systems, as linear equations in one variable may have one, infinitely many, or no solutions.
- In Unit 8, **Linear Equations and Systems 2**, students revisit the use of procedures to solve equations in one variable, though made harder by the introduction of non-integer values. With newly acquired equation-solving skills, students use the substitution method to solve systems algebraically. The elimination method is introduced as an alternative algebraic method, but not rigorously pursued, being left to high school mathematics. The unit culminates with applications that utilize skills learned in Units 7 and 8.
- The concept of a function is revisited in Units 9 and 10, **Congruence** and **Similarity**, as mapping diagrams are introduced, and transformations are defined as functions that map points in the plane to points in the plane. Algebraic rules are used to describe translations on coordinate planes

In Conclusion

The authors of *MathLinks* carefully researched and thoughtfully operationalized focus, coherence, and rigor in designing the program. We integrate mathematical concepts, skills, and applications through contexts and problems in a natural way. The result is an efficient core program that contains just 10 Student Units with 30 lessons that can be completed in about 100 class hours. This leaves ample time for review, intervention, enrichment, and choice for both teachers and students, using the resources available in Student Packets and on the Teacher Portal.