

### 7-3 NONROUTINE PROBLEMS RUNNING RATES

Lilia, Chris, Braxton, and Trinity run on the cross-country team.

1. Select all tables that represent a proportional relationship between the runners' times and distances run.

a. Lilia

<b>Time (min)</b>	0	7.5	15	18.75
<b>Distance (mi)</b>	0	1	2	2.5

b. Chris

<b>Time (min)</b>	0	14.6	36.25	50
<b>Distance (mi)</b>	0	2	5	7

c. Braxton

<b>Time (min)</b>	0	3	12	19.5
<b>Distance (mi)</b>	0	$\frac{1}{2}$	$2\frac{1}{2}$	$3\frac{1}{4}$

d. Trinity

<b>Time (min)</b>	0	1.75	5.25	10.5
<b>Distance (mi)</b>	0	$\frac{1}{4}$	$\frac{3}{4}$	$1\frac{1}{2}$

Jason just joined the team and runs 3 miles in 18.45 minutes.

2. How long would it take Jason to run 1 mile at this rate? Write this value in minutes and also in minutes and seconds.

6.15 min; 6 min 9 sec.

3. Circle all statements below that are proportional to the relationship between these two values that describe the rate at which Jason runs.

a. Jason runs half a mile in 3.075 minutes.

b. Jason runs 4.5 miles in 19.95 minutes.

c. Jason runs about 1.63 miles in 10 minutes. (if we accept slight rounding)

d. Jason runs about 6 miles in 21.45 minutes.

# 7-3 NONROUTINE PROBLEMS RUNNING RATES Continued

Andrea runs a 3-mile race in 24.15 minutes.

4. How long would it take Andrea to run 1 mile at this rate? Write this value in minutes and also in minutes and seconds.

8.05 min; 8 min 3 sec.

5. Assuming Andrea runs at a constant rate of speed, circle ALL statements below that are true.

a. Andrea runs at a rate of 8.05 miles/minute.

☒ b. Andrea runs at a rate of 8.05 minutes/mile.

c. The equation  $y = 8.05x$  relates the time in minutes ( $x$ ) and distance in miles ( $y$ ) of Andrea's run.

☒ d. The equation  $y = 8.05x$  relates the distance in miles ( $x$ ) and time in minutes ( $y$ ) of Andrea's runs.

6. Levon ran  $\frac{5}{6}$  of a mile in 7.5 minutes.

- a. If he continues at the same rate of speed, how long will it take Levon to run 1 mile?

9 minutes

- b. Draw a double number line or tape diagram to support your answer to part a.

Representations will vary. One possibility for a tape:

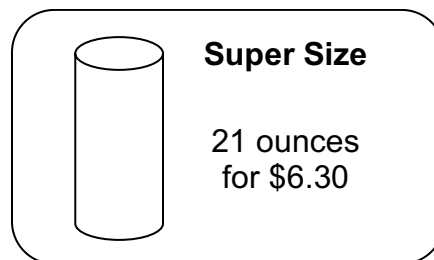
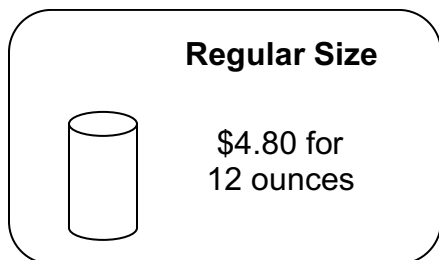
6(1.5) = 9 miles					
$\frac{1}{6}$ mi	$\frac{1}{6}$ mi	$\frac{1}{6}$ mi	$\frac{1}{6}$ mi	$\frac{1}{6}$ mi	$\frac{1}{6}$ mi
1.5 min	1.5 min	1.5 min	1.5 min	1.5 min	1.5 min
7.5 min					
7.5 min + 1.5 min = 9 min					

7. Ming runs a 3-mile race and averages 7 min/mile. In the same race Drayton runs each mile in exactly 7 minutes? Explain how Ming's and Drayton's rates are the same, but potentially different too. Answers may vary. One possible answer:

They both ran at the same average speed. However, we don't know that Ming ran each mile at the exact same pace the way Drayton did.

### 7-3 NONROUTINE PROBLEMS PROTEIN DRINKS

Suppose you are shopping for your favorite protein drink. You find there are two different size drinks available.



1. Which drink is the better buy? Justify using a graph AND one other representation (choose from a table, equation, double number line, tape diagram, or a picture). Use appropriate labels, scales, etc.

*Representations will vary. Check for proper labeling and scaling.*

*Graphs will be lines through the origin. The Regular line will be steeper than the Super line.*

*Another possible method is to calculate the unit rate per ounce.*

*Regular Size is \$0.40 per ounce.*

*Super Size is \$0.30 per ounce.*

2. Find the points (1,  $y$ ) on the two lines graphed for problem 1. Explain what each of these coordinate pairs means in the context of the problem.

*This relates to the unit rate, which is the change in cost per 1 ounce.*

<p>Regular: (1, <u>0.4</u> )</p> <p>The Regular Size is 1 ounce for 40 cents.</p>	<p>Super: (1, <u>0.3</u> )</p> <p>The Super Size is 1 ounce for 30 cents.</p>
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3. Other than the representations and calculations used for this problem, explain anything else you might take into consideration when choosing between the protein drinks.

*Answers will vary. If 21 ounces of protein drink is too much, the Super Size, even though a better buy, would probably be undesirable.*

### 7-3 NONROUTINE PROBLEMS

#### STRATEGIES FOR SIMPLIFYING COMPLEX FRACTIONS

A complex fraction is a fraction whose numerator or denominator is a fraction.

1. Here are two mathematical strategies for simplifying complex fractions. Fill in the missing parts of the fractions.

Equivalent fractions, such as  $\frac{2}{3}$ , are acceptable results as well.

<p><b>Strategy 1:</b> Write the complex fraction as a division problem.</p> $\frac{\frac{1}{2}}{\frac{3}{4}} = \frac{1}{2} \div \frac{\boxed{3}}{\boxed{4}} = \frac{\boxed{1}}{\boxed{2}} \cdot \frac{\boxed{4}}{\boxed{3}} = \frac{\boxed{4}}{\boxed{6}}$ <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <span>step 1</span> <span>step 2</span> <span>step 3</span> </div>	<p><b>Strategy 2:</b> Multiply by a form of the “big one” to create a denominator equal to one.</p> $\frac{\frac{1}{2}}{\frac{3}{4}} \cdot \frac{\frac{4}{3}}{\frac{4}{3}} = \frac{\frac{1}{2} \cdot \frac{\boxed{4}}{\boxed{3}}}{\frac{3}{4} \cdot \frac{\boxed{4}}{\boxed{3}}} = \frac{\frac{\boxed{4}}{\boxed{6}}}{\frac{\boxed{1}}{\boxed{6}}} = \frac{\boxed{4}}{\boxed{6}}$ <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <span>step 1</span> <span>step 2</span> <span>step 3</span> <span>step 4</span> </div>
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Circle all “step numbers” that apply. Note that a procedure may occur in one strategy or both, and may occur at one step or over two steps.

	<b>Strategy 1 Step #</b>	<b>Strategy 2 Step #</b>
2. The complex fraction is rewritten using the “÷” symbol for division.	1 2 3	1 2 3 4
3. A form of the “big one” (multiplicative identity) is used to create a denominator equal to one.	1 2 3	1 2 3 4
4. The “multiply by the reciprocal” method for fraction division is applied.	1 2 3	1 2 3 4
5. The “multiply across” method for fraction multiplication is applied.	1 2 3	1 2 3 4
6. Shows that it is clear that Strategy 1 and Strategy 2 will give the same result.	1 2 3	1 2 3 4

### 7-3 NONROUTINE PROBLEMS STRATEGIES FOR SIMPLIFYING COMPLEX FRACTIONS

Continued

Simplify each fraction by using Strategy 1.

<p>7. <math>\frac{\frac{3}{5}}{\frac{1}{5}}</math>      <math>\frac{3}{5} \div \frac{1}{5} = \frac{3}{5} \cdot \frac{5}{1} = 3</math></p>	<p>8. <math>\frac{\frac{2}{3}}{\frac{5}{6}}</math>      <math>\frac{2}{3} \div \frac{5}{6} = \frac{2}{3} \cdot \frac{6}{5} = \frac{4}{5}</math></p>
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Simplify each fraction by using Strategy 2.

<p>9. <math>\frac{\frac{1}{4}}{\frac{5}{8}}</math>      <math>\frac{\frac{1}{4} \cdot \frac{8}{5}}{\frac{5}{8} \cdot \frac{8}{1}} = \frac{\frac{2}{5}}{5} = \frac{2}{25}</math></p>	<p>10. <math>\frac{\frac{5}{9}}{\frac{3}{6}}</math>      <math>\frac{\frac{5}{9} \cdot \frac{6}{3}}{\frac{3}{6} \cdot \frac{6}{3}} = \frac{\frac{10}{9}}{1} = \frac{10}{9}</math> or <math>1\frac{1}{9}</math></p>
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11. Which strategy do you prefer? *Answers will vary.*

Simplify each complex fraction. Use a strategy of your choice.

<p>12. <math>\frac{\frac{5}{3}}{\frac{1}{9}}</math>      <b>15</b></p>	<p>13. <math>\frac{\frac{2}{15}}{\frac{2}{5}}</math>      <b><math>\frac{1}{3}</math></b></p>
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14. Blakely said, "I think I know a shortcut for simplifying complex fractions," and then she drew the markings below. Is her work correct? Explain what she did.

$$\frac{\frac{1}{2}}{\frac{2}{3}} = \frac{4}{6} = \frac{2}{3}$$

*Yes. Her method had the effect of multiplying the first fraction by the reciprocal of the second.*

### 7-3 NONROUTINE PROBLEMS MIXED PROBLEMS

1. A newspaper recently published a story that was titled: "Should People with Bigger Feet Pay More for Shoes?" *Answers will vary. One possible answer for each:*
  - a. Based on what you've learned so far about ratios and proportional relationships, what evidence or arguments might this article be trying to make about why people with bigger feet should pay more for shoes?  
*It takes more material to make bigger shoes, so it should cost more.*
  - b. What might be a counterargument for why people with bigger feet should NOT pay more for shoes?  
*It would be difficult to price every size differently. The company could just find the average price to make all of the shoes of a certain style (no matter the size) and use that for pricing. That way, all of the shoes for that style could cost the same amount. There are many more factors that go into pricing shoes than just cost of materials. Some may be rent for building space, cost of wages for workers, insurance, marketing, and sales costs. These, and more, may actually contribute a larger portion to the cost of the shoes than many people think.*
2. A plumber is hired to install a bathroom. The plumber claims that the relationship between the number of hours worked and the total work fee is proportional. The fee for 6 hours of work is \$174. Select ALL combinations of values for the plumber's work hours and total work fee that match this rate.
  - a. \$30 for 1 hour of work
  - b. 8.75 hours work for \$250
  - ☒ c. \$116 for 4 hours work
  - ☒ d. 10.5 hours work for \$304.50
3. Select all tables that represent a proportional relationship between  $x$  and  $y$ .

☒ a.

$x$	0	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$
$y$	0	$\frac{1}{3}$	$\frac{2}{3}$	1

b.

$x$	0	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$
$y$	0	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$

c.

$x$	0	1	2	3
$y$	0	1	4	9

☒ d.

$x$	0	1	6	9
$y$	0	2.5	15	22.5

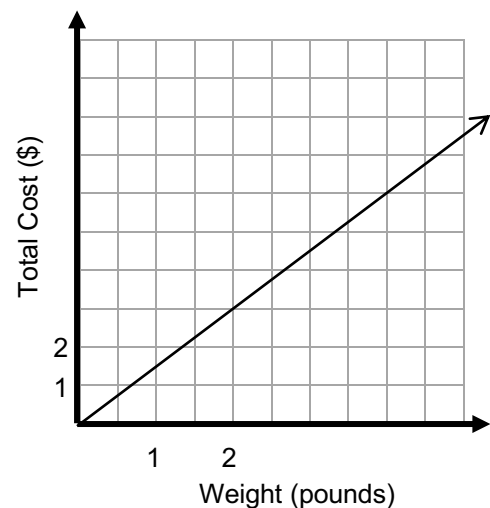
### 7-3 NONROUTINE PROBLEMS MIXED PROBLEMS

Continued

4. Lee wants to buy pies for her party for about 30 to 40 guests. One pie will serve 4 to 5 people. Each pie costs \$8.75.
- How many pies should Lee buy so that all guests get pie? Justify your answer.  
*Answers may vary. One possible answer: The maximum number of people is 40 guests. Assuming one pie serves 4 people (the lesser amount to be safe), Lee will need to buy 10 pies.*
  - What will the total cost be?  
*If Lee buys 10 pies, the total cost is \$87.50.*
  - If Lee has a budget of \$100 for the party, how much money is left over, if any?  
*If Lee buys 10 pies, \$12.50 is left*
5. When playing a video game, Mason wins 6 out of every 8 games she plays. Select all of the statements that describe Mason's situation.
- The ratio of Mason's wins to losses is 3:4.
  - ☒ The ratio of Mason's wins to games played is 3:4.
  - The equation  $8x = 6y$  shows the relationship between the number of games Mason wins ( $x$ ) and the number of games she loses ( $y$ ).
  - ☒ The equation  $8x = 6y$  shows the relationship between the number of games Mason wins ( $x$ ) and the total number of games she plays ( $y$ ).
6. This graph shows the relationship between the pounds of apples bought at the market and the total cost, in dollars, for the apples.

Select ALL true statements about this graph.

- ☒ The point (0, 0) shows the cost of 0 pounds of apples is \$0.00.
- The point (1, 1.5) shows the cost is \$1.00 for 1.5 pounds of apples.
- The point (1, 1.5) shows the cost is \$1.50 for 1 apple.
- ☒ The point (1, 1.5) shows the cost is \$1.50 for 1 pound of apples.
- ☒ The point (2, 3) shows the cost of 2 pounds of apples is \$3.00.



### 7-3 NONROUTINE PROBLEMS FROM THE MATH OLYMPIAD

1. Four chefs require 10 minutes to prepare 20 desserts. At this rate, how many chefs are needed to prepare 75 desserts in 15 minutes?

"Chef-minutes" are the total time required by all the chefs to do a job.

To prepare 20 desserts requires  $(4)(10) = 40$  chef-minutes.

So each dessert requires  $\frac{40}{20} = 2$  chef-minutes.

To prepare 75 desserts would then require  $(75)(2) = 150$  chef-minutes.

Therefore in 15 minutes,  $\frac{150}{15} = 10$  chefs are required.

2. For 8 weeks of work, Melanie will receive \$600 and a new computer. After only 6 weeks of work, she would be entitled to a new computer but only an additional \$150. What is the value of the computer in dollars?

With 2 weeks less work, Melanie earns less by  $\$600 - \$150 = \$450$  (she already has the computer).

This is \$225 per week. At this rate in eight weeks, she would have earned a total of \$1800.

The value of the computer is  $\$1800 - \$600 = \$1200$ .

3. Avi used 11 toothpicks to form a row of five attached triangles, as shown. Suppose he continued this pattern, using 89 toothpicks in all. What is the total number of triangles formed?



The first three toothpicks form one triangle. After that every two additional toothpicks form another triangle. After the first three toothpicks are used, 86 are left. These 43 pairs of toothpicks form 43 more triangles. Including the first triangle, 44 triangles are formed.

If using algebra, this pattern can be represented by the equation  $y = 2x + 1$ , where  $x$  is the number of triangles and  $y$  is the number of toothpicks. Solving this equation for  $y = 89$  yields  $x = 44$ .

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