UNIT 1
TEACHER EDITION

MathLinks
GRADE 7

Annotated for new users

PROBABILITY

Unit Planning Information
(TE-UPI)
Numbered i, ii, iii, …

Annotated Answer Key,
including Student Packet
and Lesson Notes
(TE-AK)
Numbered 0, 1, 2, …

Each *MathLinks* Unit is organized this way

The start of TE-UPI

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
7.SP.C	**Investigate chance processes and develop, use, and evaluate probability models.**
7.SP.5	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
7.SP.6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
7.SP.7	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy:
a	Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.
b	Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?
7.SP.8	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation:
a	Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
b	Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.
c	Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*
7.NS.A	**Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.**
7.NS.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers:
d	Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Common Core State Standards for the Unit are listed here and also at the end of the Student Packet.

Individual sets of standards are listed at the beginning of each lesson.

Standards for the entire grade level are in Program Information.

UNIT PLANNING

* Starred resources can be accessed under Unit Resources on the Teacher Portal.

Unit Pacing* Up to 14 class hours Pacing detail (estimates)	1.0 Race to the Top (<1 hour) 1.1 Introduction to Probability (3 hours) 1.2 Flips, Rolls, and Sample Displays (3 hours) 1.3 Probability Experiments: Games and Puzzles (3 hours) Review (3 hours) Assessment (1 hour)
Unit Resources* Up to 3 class hours On the Teacher Portal	• Extra Problems • Essential Skills • Math Talks (Data, Dot) • Nonroutine Problems • Technology Activities • Tasks (Fair or Unfair, Strange Spinners) • Projects (Design a Game, Intransitive Spinners) • Parent Support Letters
Assessment Options* See Portal Unit 1 → Other Resources → Assessment, Follow-up, and Feedback for more Details in Program Information	• On the Teacher Portal ✓ Unit Quizzes ✓ Cumulative Tests ✓ Tasks ✓ Projects ✓ The *MathLinks* Rubric (with rubric-worthy problems) • In the Student Packet ✓ Monitor Your Progress ✓ Unit Reflection • In the Teacher Edition ✓ References to Journals ✓ Suggested problems for the *MathLinks* Rubric
Materials See Program Information for supplies for the year	• Number cubes [1.0, 1.2, 1.3] (2 per student) • Coin [1.1, 1.2] (1 per student) • Virtual number cubes online [1.1, 1.2, 1.3] (alternative option) • Paper clips [1.1] (1 per student) • General supplies (e.g., colored pencils, markers, rulers, tape, scissors, graph paper, calculators, chart paper)
Slide Decks* On the Teacher Portal	S1.0 Race to the Top S1.1a A Coin Flip Experiment S1.1b A Spinner Experiment S1.2a Investigating One-Third S1.2b Flip and Roll S1.3 Spinner Puzzles
Reproducibles* At the end of TE-UPI	R1-1 Will It Happen? Cards [1.1] (1/pair) R1-2 Spinner Clue Cards [1.3] (1/group) R1-3 Big Square Puzzle: Probability [Review] (1/pair or group) R1-4 Match and Compare Sort Cards: Probability [Review] (1/pair)
Prepare Ahead Use Activity Routines files in General Resources at the start of the year	• Cut up R1-1, R1-2, R1-3, R1-4 [1.1, 1.3, Review] vity Routines in Program Information for directions for the *MathLinks* ig Square Puzzle, Why Doesn't It Belong?, Match and Compare ster Problems, Math Path Fluency Challenge [1.1, 1.2, Review]
Other Resources on the Teacher Portal Watch Getting Started Videos in General Resources at the start of the year	• Getting Started Videos and Resources - General Resources • Skill Boosters - Teacher Access page (whole numbers, fractions) • Puzzles / Games - Teacher Access page (SMASH Game 1, EGAD)

PLANNING FOR DIFFERENT USERS

Student Packet (SP)

Unit 1 component options for those who support students:

For teachers	• Teacher Edition (this document) • Teacher Portal (Unit Resources, General Resources) • Program Information
For substitutes	• SP (Practice 1 – 6 may be completed independently any time after instruction; Spiral Review; Vocabulary Review) • Extra Problems • *MathLinks* Puzzles / Games
For parents	• Resource Guide • Parent Letter (English and Spanish)

Print copy and on the Portal

Created by Carole Greenes for *MathLinks* users; on the Portal; not linked to any course

Unit 1 component options to use with all students (all available on the Teacher Portal):

- SP (Word Bank, C
  Activity Routines,
  self-monitoring, jo
- Student Packet T
- Extra Problems (p
- Essential Skills (ju

In addition to unit-by-unit Parent Letters, the public website also has an introductory letter.

- Math Talks (whole-class discourse)
- Nonroutine Problems (enrichment)
- Tasks (multi-part problems)
- Projects (authentic multi-day experiences)
- Technology Activities (variety)
- *MathLinks* Puzzles / Games (fun challenges)

Unit 1 component options for particular subgroups of students:

For English learners see pgs ix-x for specific strategies	• SP Text File for Translation • SP features for language development (Word Bank, Vocabulary Review, consistent structure for reading and writing, grouping opportunities for speaking and listening) • SP Activity Routines for language development (rubric-worthy problems with the *MathLinks* Rubric, Big Square Puzzle, Why Doesn't It Belong?, Match and Compare Sort, Poster Problems) • Math Talks for speaking and listening
For struggling learners see pgs ix-x for specific strategies	• SP features for math confidence (Getting Started, Review including Spiral Review, Word Bank, Vocabulary Review, consistent structure, grouping options) • SP Activity Routines for math confidence (rubric-worthy p Rubric, Big Square Puzzle, Why Doesn't It Belong?, Mat Problems, Math Path Fluency Challenge) • Essential Skills for just-in-time intervention • Extra Problems (by lesson) for practice, review, or assessment • Skill Boosters (Whole Numbers, Fraction Concepts)
For enrichment and advanced learners	• SP enrichment (see page xii for specific options) • SP Activity Routines for enrichment (rubric-worthy problems with the *MathLinks* Rubric, Why Doesn't It Belong?, Match and Compare Sort) • Nonroutine Problems (including problems from the Math Olympiad) • Technology Activities for variety • Projects for applications

Use a translation app

A skills practice routine; on the Portal; not linked to any course

Activity Routines recur throughout a course. Use the introductory activities in General Resources first.

MATH BACKGROUND

Origins of Probability

An important day in the history of mathematics occurred in 1653 when Blaise Pascal, Antoine Gombaud, and Chevalier de Méré, found themselves sharing a carriage ride with a mutual friend. The Chevalier de Méré was not actually a nobleman – he was a writer who had invented the name "Chevalier de Méré" for a character who espoused his views in the dialogues he wrote. Apparently, his friends started calling him Chevalier de Méré, which he liked, and he eventually adopted the name for himself.

The Chevalier de Méré gambled, and he was very interested in the mathematics connected to gambling, presumably to assure that he had the advantage in any game of chance. As they bumped along in the carriage, he told Pascal about several problems that arise in gambling. This opened up new horizons for Pascal. The enthusiastic Pascal tackled the problems and made some headway. At the time, Pascal was in correspondence with Pierre de Fermat, one of the great mathematicians of the century, so it was natural for Pascal to write to Fermat about these problems. This correspondence between Pascal and Fermat, in the year 1654, laid the foundations of what would become modern probability theory.

Two specific problems that the Chevalier de Méré told Pascal about, and which were under some discussion at the time, had been treated by Cardano some hundred years earlier and subsequently by a number of mathematicians. The first problem ran along the following lines:

A dice thrower bets a sum of money—against anyone who matches the sum—that in 24 throws of a pair of dice he will roll a pair of sixes. De Méré thought that the dice thrower should have an advantage. The intuitive argument for the thrower to have an advantage was that the odds of getting a pair of sixes in one throw is 1 out of 36, so in 24 throws the odds of getting a pair of sixes should be about 24 out of 36, or 2 out of 3. It seemed like a safe bet, but de Méré was losing money. He thought he had discovered a contradiction in mathematics. (It turns out that to have an advantage, the dice thrower should bet that in 26 throws he will get a pair of sixes.)

The second problem concerned how the stakes of a game played in stages should be divided between two gamblers if the game must be terminated before it is over. This problem, called the "problem of points," was solved by Fermat in his correspondence with Pascal. The solution led to the first explicit reasoning about what today are known as expected values.

Math Background includes adult-level information and explanations written by our PhD mathematicians.

Dependent and Independent Events

A bag contains 2 green cubes and 1 brown cube. Two cubes are to be drawn from the bag.

Experiment #1: Suppose the cubes are removed from the bag **without** replacement. The events of “drawing a green cube on the first draw” and “drawing a brown cube on the second draw” are dependent. The likelihood of the second event (of drawing a brown cube on the second draw) is influenced by the occurrence of the first event (that is, by whether or not a green cube was drawn first).

Experiment #2: Now suppose one cube is removed from the bag, the color is noted, and it is replaced. After shaking the bag, another cube is removed from the bag and the color is noted. In this experiment of drawing **with** replacement, the event of drawing a green cube on the first draw is independent of the event of drawing a brown cube on the second draw. The likelihood of drawing a brown cube on the second draw is equal to the likelihood of drawing a brown cube on the first draw. It is not influenced by whether or not a green cube was drawn on the first draw.

What is Randomness?

Events that have no pattern or are unpredictable are referred to as random events. Rolling a 1 on a fair number cube is a random event. Rolling a 1 has probability $\frac{1}{6}$ of occurring on any given roll, but otherwise the occurrence of a 1 is unpredictable, and there is no pattern in the appearance of 1’s in a sequence of rolls.

Though the results of a probability experiment (such as rolling a number cube) may be unpredictable in the short term, certain statements can be made about the results of repeated trials in the long term. In the long term, the average number of 1’s appearing (the experimental probability of rolling 1) will get closer and closer to $\frac{1}{6}$ as the number of rolls increases.

In fact, it is a remarkable fact that if we repeatedly perform a probability experiment of this sort, the average of the outcomes will “fit” on a normal distribution with a specific mean and standard deviation. This signals the very special role that the normal distribution plays in mathematics and statistics.

If a player with a fair number cube rolls nine 1’s in a row, what is the probability that the player will get a 1 on the next roll? Though the answer may seem counterintuitive, the probability of rolling a 1 after having rolled nine consecutive 1’s is still $\frac{1}{6}$. The tenth roll is independent of the first nine rolls.

It is virtually impossible for people to discern whether a given sequence of digits, as the results of rolling a number cube, is random or not. However, if we roll a die enough times, seemingly impossible streaks such as 1,1,1,1,1,1 are almost certain to occur. In fact, there are statistical tests to determine whether a given sequence of digits is random, based on the occasional appearance of long strings of the same digit.

Truly random sequences, though, do not appear random to people. Sports fans regularly attribute NBA players’ streaks to hot hands, even when their performance is no better than chance. A Spotify developer reported that their random shuffle feature did not feel random to many of their listeners, and subsequently received many complaints. They decided to replace their truly random shuffle algorithm by an algorithm that fits the common intuition of a random selection.

A Clever Trick

Rational numbers have decimal expansions that repeat, either with a repeating pattern of nonzero digits, or with zeros from some point on (terminating decimals). This occurs because the decimal expansion of a quotient of integers obtained by long division eventually repeats, since there are only a finite number of possibilities for the remainder. For example, if the divisor is 7, then the only possibilities for the remainder are 0, 1, 2, 3, 4, 5, 6.

Conversely, every repeating decimal is the decimal expansion of a rational number. There is a “clever trick” for converting repeating decimals to fractions. We illustrate the clever trick by converting the decimal $x = 0.16666\ldots$ to a quotient of integers, following the scheme used in the Student Pages. Notice the unconventional order in writing down the steps. This is done to simplify the arithmetic for the students.

$$10x = 1.66666\ldots \qquad (2)$$

$$\textbf{Let } \boldsymbol{x = 0.16666\ldots} \qquad \textbf{(1)}$$

$$9x = 1.5 \qquad (3)$$

$$x = \frac{1.5}{9} = \frac{15}{90} = \frac{1}{6} \qquad (4)$$

Notice that step 2 is above step 1.

- The clever trick is to multiply both sides of the equation in step 1 by a power of 10 that will “line up” the repeating portion of the decimal.
- Then subtract the expressions in step 1 from step 2. This will make the repeating portion equal zero (step 3).
- Finally, solve for x and simplify your result into a quotient of integers (step 4).

The procedure for converting a decimal to a rational number gives a different rational number for each decimal except in the special case that the decimal is terminating or is repeating with all 9’s. For instance, it shows that 0.29999…. represents the same rational number as 0.3.

Students are often surprised that 0.9999… is a decimal representation of 1, as can be seen by using the clever trick to convert 0.9999… to a quotient of integers.

$$\begin{aligned} 10x &= 9.999\ldots \\ \underline{\text{Let } x} &\underline{{}= 0.999\ldots} \\ 9x &= 9 \\ x &= 1 \end{aligned}$$

TEACHING TIPS

Applying Standards for Mathematical Practice (SMP)

Here is an abbreviated version of the SMPs and some ways they are applied in this unit.

SMP1	**Make sense of problems and persevere in solving them.** • Understand a problem and look for entry points • Consider simpler or analogous problems • Monitor progress and alter solution course as needed • Make connections between multiple representations • Check answers with a different method	[1.0, 1.2] For **Race to the Top**, students perform an experiment and predict the results. Once they learn more about probability, they revisit the problem and revise their analysis. [1.3] For **Spinner Puzzles**, students make sense of clues in order to create a spinner, including one in which the clues are purposely flawed and must be corrected.
SMP2	**Reason abstractly and quantitatively.** • Use numbers and quantities flexibly in computations • Attend to the meaning of quantities • Decontextualize a problem using symbols, manipulate them, and then interpret based on the context	[1.1] On **When Will it Happen?**, students learn that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. [1.2, 1.3, Review] For **Flip and Roll**, **The Terminator**, and **Poster Problems**, students examine theoretical probabilities to determine if the game is fair.
SMP3	**Construct viable arguments and critique the reasoning of others.** • Use assumptions, definitions, established results, examples, and counter examples to analyze an argument and discuss its merits or flaws • Make and test conjectures based on evidence • Analyze situations by breaking them into cases • Understand and analyze the approaches of others	[1.1] Students critique Emmett’s justification of a computation of probability in **A Spinner Experiment**. [1.2] Students explain Javier’s error in constructing a sample space for the **Race to the Top Revisited** experiment.
SMP4	**Model with mathematics.** • Attach meaningful mathematics to everyday problems and questions of interest • Make reasonable assumptions and approximations to simplify a situation • Identify quantities, use mathematical tools (such as multiple representations, formulas, equations) to analyze relationships • Interpret results and draw conclusions in the context of the situation	[All Lessons] Starting with their predictions for **Race to the Top**, students explore and analyze probability experiments using various mathematical tools and representations. [1.1] Students predict how many tokens they will win based on theoretical probabilities.

Abbreviated descriptions of the Standards for Mathematical Practice appear in every unit.

Specific examples from the Student Packet for this unit.

Applying Standards for Mathematical Practice (SMP) Continued		
SMP5	**Use appropriate tools strategically.** • Select and use tools strategically (and flexibly) to visualize, explore, and compare information • Use technological tools and resources to solve problems and deepen understanding	[1.2] Students should choose to use a calculator to enhance pattern explorations for patterns in repeating decimals. [1.0, 1.2, 1.3] Students may want to find and use virtual coins to flip and dice to roll for probability experiments. This will allow for more repetitions, better generalizations, and deepen intuitive understanding of “the law of large numbers.”
SMP6	**Attend to precision.** • Calculate accurately and efficiently • Explain thinking using mathematical vocabulary • Use symbols appropriately • Specify units of measure	[All Lessons] Students record mathematics vocabulary as it is introduced in lessons. They use precise language in writing and exercises. Precise definitions are located in the **Student Resources** section at the end of the unit. [1.1, 1.2] Students encounter and use the new notation of probability and repeating decimals.
SMP7	**Look for and make use of structure.** • Recognize the structure of a symbolic representation and generalize it • See complicated objects as composed of chunks of simpler object	[1.2] For **Investigating One-Third**, students use and connect patterns in diagrams of $\frac{1}{3}$ to decimal and percent equivalents. [1.2] Students create organized lists for sample spaces, using patterns to be sure they have them all.
SMP8	**Look for and make use of repeated reasoning.** • Identify repeated calculations and patterns • Generalize procedures based on repeated patterns or calculations • Find shortcuts based on repeated patterns or calculations	[1.2] For **Investigating One-Third**, students write some rational numbers as repeating decimals by observing the repeating pattern in the division algorithm. [1.2, 1.3] Students begin to generalize the number of outcomes based on experiences with sample space visual representations. For example, the **Flip and Roll** experiment must produce $2 \bullet 6 = 12$ outcomes.

Strategies to Support Different Learners

Classrooms typically include students with different learning styles and needs. Here are some specific ways that *MathLinks* supports special populations. Strategies essential to the academic success of English learners are noted with a star (*). See Universal Design for Learning in Program Information for more details.

	General Examples	*MathLinks* Examples
Know your Learner	✓ Understand student attributes that support or interfere with learning ✓ Determine preferred learning and interaction styles ✓ Assess student knowledge of prerequisite mathematics content ✓ Check for understanding continuously ✓ Provide differentiation opportunities for intervention to reach more learners ✓ Encourage students to write about their attitudes and feelings towards math ✓ Use contexts that link to students' cultures*	Built into the *MathLinks* Design: SP: Getting Started, Spiral Review, Monitor Your Progress, Unit Reflection TE: References to Journals [Front of Unit] Enhance the **Monitor Your Progress** prompt at the end of each lesson with a journal prompt that focuses on attitudes. [Skill Boosters] Use the decimal-percent pre-assessment to determine student proficiency in this prerequisite skill. [1.2] For struggling learners, enlarge **Investigating One-Third** on colored paper. Have students cut out repeating decimals pieces, glue them on paper, and label the parts clearly. [All Lessons] Be sensitive to student cultural values where "gambling" may be an inappropriate application of probability. Consider whether referring to "games of chance" might be more appropriate or not.
Increase Academic Language through atics	✓ Provide opportunities for students to read, write, speak, and listen ocabulary needed ideas, providing -examples ✓ Use strategically organized groups that attend to language needs* ✓ Use rich mathematical contexts and help ELs development* ✓ Use cognates and root words (when appropriate) to link new math terms to students' background knowledge*	Built into the *MathLinks* Design: SP: Word Bank, Vocabulary Review, Student Resources TE: Grouping suggestions, References to Journals, Suggested problems for **The *MathLinks* Rubric** UR: Math Talks OR: Critique student work on Slide Decks [All Lessons] Probability requires the use of extensive academic language. Make a word wall for the vocabulary associa [1.1] Fo help students distingu robability experim and "experimental probability."* [1.1] Connect Aisha's description of probability $\frac{\text{it happened}}{\text{total}}$ to more formal definitions.* [Review] Students use academic language with partners in **Match and Compare Sort** and **Poster Problems**.*

Many strategies are consistently in components that are built into the MathLinks design

A "*" indicates a strategy for English learners.

These General Examples appear in every unit

Specific examples from the Student Packet for this unit

Components cited: Student Packet (SP), Teacher Edition (TE), Unit Resources (UR), Other Resource (OR)

Strategies to Support Different Learners (Continued)

	General Examples	***MathLinks* Examples**
Increase Comprehensible Input	✓ Link concepts to past learning ✓ Make concepts meaningful through hands-on activities, visuals, demonstrations, and color-coding ✓ Use a think-aloud strategy to model appropriate thinking processes and academic language use ✓ Use graphic organizers to help students record information and data, see patterns, and generalize them ✓ Use multiple representations (pictures, numbers, symbols, words, contexts) of math ideas to create meaning and make connections ✓ Strategically sequence and scaffold to make mathematics accessible ✓ Simplify written instructions, rephrase explanations, and use verbal and visual clues*	Built into the *MathLinks* Design: SP: Structured workspace TE: Slide Deck Alternatives, Reproducibles, Materials OR: Slide Decks [1.1] The terms likely, unlikely, certain, impossible, etc. introduce probability concepts using familiar language.* [All Lessons] Demonstrate hands-on experiments to be sure students understand the instructions.* [1.1, 1.2] Make connections and distinguish between experimental probability (in green on slide decks) and theoretical probability (in orange on slide decks). However, be aware that colorblind individuals may have difficulty with color-coding. [1.2] Language to communicate repeating decimals is awkward. Be sure the meaning of the language is understood by all learners. For example, 0.333… (“Zero-point-three-three-three-repeat” may also be stated “three-tenths plus three-hundredths plus three-thousandths, etc.”); $0.3\overline{3} = 0.\overline{33}$.*
Promote Student Interaction	✓ Use flexible group configurations that support content objectives ✓ Use strategies and activities that promote teacher/student and student/student interactions (e.g., think-pair-share, Poster Problems) ✓ Encourage elaborate responses through questioning ✓ Allow processing time and appropriate wait time, recognizing the importance of the different requirements for speaking, reading, and writing in a new language* ✓ Allow alternative methods to express mathematical ideas (e.g., visuals, students’ first language)*	Built into the *MathLinks* Design: SP: Lesson and Review activities TE: References for Journals, Suggested problems for **The *MathLinks* Rubric** UR: Math Talks, various games and puzzles OR: Slide Decks, Activity Routines [All Lessons] Use turn-and-talk strategies when possible to encourage communication in a safe environment for all learners, especially English learners.* [All Lessons] All games, experiments, and card sorts encourage student interaction. Make sure all learners get these opportunities.*

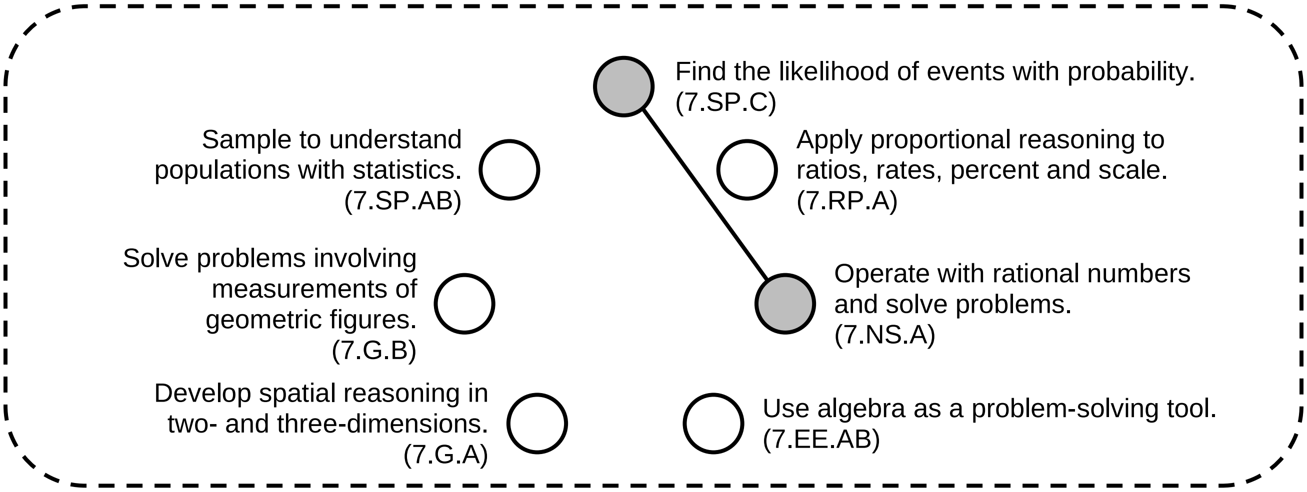
Components cited: Student Packet (SP), Teacher Edition (TE), Unit Resources (UR), Other Resources (OR)

Strategies to Support Different Learners combines the Universal Design for Learning (UDL) with other research-based strategies that are proven successful for a wide range of learners, especially those with special needs and English learners. See program Information for more details and a bibliography of cited references and resources for inspiration.

Big Ideas and Connections

The Center for Mathematics and Teaching is dedicated to igniting and nurturing passion for mathematics in middle school students. We see the classroom as a place of joy and wonder, collaboration and purpose, perseverance and empowerment. We want all students to succeed in mathematics, as they explore its beauty in patterns, concepts, connections, and applications.

MathLinks: Grade 7 is organized around seven big ideas. This graphic provides a snapshot of the ideas in Unit 1 and their connections to each other.



These ideas build on past work and prepare students for the future. Some of these include:

Prior Work	What's Ahead
• Analyze patterns and relationships (5.OA.B) • Understand the place value system (5.NBT.A) • Compute fluently with multi-digit numbers (5.NBT.B, 6.NS.B) • Understand ratio concepts and use ratio reasoning to solve problems. (6.RP.A) • Operations with fractions, decimals, percents (4.NF.B, 5.NBT.B, 5.NF.A 6.NS.AB)	• Use probability ideas to analyze samples of populations and draw inferences (7.SP.AC, HS) • Know that there are numbers that are not rational, and approximate them by rational numbers (8.NS.A, HS) • Explore and apply concepts of conditional probability and rules of probability (HS) make decisions (HS)

Connections made to:

- work from previous courses or from earlier in the grade (above)
- work later in grade or to future courses (to the right)

The "Big Idea wheel" helps teachers and students see connections among mathematical ideas within a unit. Big Ideas also ensure a cohesive and efficient design program. The Big Idea wheel also appears on the Reelection page of every Student Packet. The *MathLinks* version was inspired by the work of Randy Charles and Jo Boaler (see the References section of Program Information, Portal version only). It consists of Domains from the Common Core State Standards.

Appears in every TE-UPI

Students May Wonder…

- In the 1980 Winter Olympics, the USA hockey team beat the Soviet Union in the semifinals in a game that is called the "Miracle on Ice." In the previous four Olympics, the Soviet Union had won the gold medal. No one thought that the USA team could upset the heavily favored the Soviet team. In fact, an American victory was considered so unlikely that ABC did not even air the game live. After briefly explaining the context of the game, consider showing a short video clip that can be found online of the closing seconds of the game. Then connect this example to the language underlined in **1.1 Getting Started**: likely, impossible, improbable, etc.

  Which of these words best describes "miracles"? Answers may vary, but miracles could be described as outcomes that were so unlikely and improbable that they were considered impossible. You can extend this conversation to help students understand the implications of depending on probabilities to make programming decisions.

  In general, TV networks want to show programming that people will probably want to watch so that the networks can charge more money to advertisers and subscribers. How does a better understanding of probability help networks make more money? Networks need to predict shows that will probably be watched by lots of people and avoid shows that people probably won't watch.

- Many people like to gamble in casinos like those that are found in Las Vegas, Nevada.

  Why do you think that casinos make so much money? The answer is simple. The "house" always has a better probability of winning—at every game of chance—than the gambler. In other words, the game is not fair. Over long periods of time, with many gamblers, the casinos do a lot more winning than losing, and it adds up. Some gamblers think they can "beat the odds," and it's true that some do, but overwhelmingly gamblers lose and casinos win.

With all probability work, please be sensitive to community values. For example, if gambling is an inappropriate topic in your community, please do not use this prompt.

Appears in every TE-UPI

Enrichment and Challenges for Advanced Learners

MathLinks: Grade 7 materials provide multiple opportunities for advanced students to investigate grade-level mathematics at a higher level of complexity without doing more work than their peers.

Within this Student Packet, here are some problems that have accessible entry points and opportunities for extensions (low floors / high ceilings).

- Race to the Top (pgs 1, 15, 16)
- Cereal Box Simulation (pg 22)

Every student does not need to do every problem in a Student Packet. Challenge those who are ready with these pages and problems. Others might do unfinished work, Spiral Review, or more practice problems as needed.

- An extension for The Terminator: Theoretical Probability (pgs 18-19): ***Suppose you use two 12-sided dice for The Terminator. Is this a fair game? What about for 20-sided dice? What strategies make finding this solution more efficient?***

Consider speeding up instruction and skipping some Practice and Spiral Review. Consider some EGAD Puzzles from the Puzzles and Games tab. See also Planning for Different Users (TE, pg iii) and Enrichment for Advanced Learners and Those with Undiscovered Hidden Talents (Program Information → Universal Design for Learning) for more ideas.

Appears in every TE-UPI

Developing Language Skills through *MathLinks*

Language (reading, writing, speaking, and listening) helps students communicate math ideas and understand concepts. Here are some language examples in Unit 1.

Language Objectives

The students will:

(Lesson 1) Distinguish between the meaning of <u>experimental probability</u>, <u>theoretical probability</u>, and probability experiment.

(Lesson 2) Explain the difference between a <u>terminating decimal</u> and a <u>repeating decimal.</u>

(Lesson 3) Read and interpret spinner puzzle clues (R1-2) for Spinner Puzzles (Lesson 3, page 20) with peers.

(Review) Contribute ideas to the solutions of the Poster Problems (page 25).

Group Discussions to Promote Reading, Listening, and Speaking (2+)

Critique reasoning situations appear on Slide Decks S1.1a and S1.1b and Practice pages 3, 5 and 16. Use a partner strategy such as “turn and talk” to encourage students to critique the reasoning of other students respectfully.

Group students to promote meaningful interaction. For Race to the Top (Opening Problem, page 1; Revisited, pages 15-16), the Coin Flip Experiment (Lesson 1, page 4), the Spinner Experiment (Lesson 1, page 5), Flip and Roll (Lesson 2, page 12), and The Terminator (Lesson 3, page 18), ask groups of 3-4 to compile individual data and then report to the class. Circulate around the room and encourage elaborate explanations.

Spinner Puzzles (Lesson 3, page 20) are designed for groups of four. Consider pairs/trios to encourage students to help each other read and interpret clues.

Review activities are designed for group interaction and discussion. For example, Match and Compare Sort (page 24) is a pairs activity. Listen closely as students compare related mathematical terms.

Journal Ideas to Promote Writing

(Explaining Concepts) Journal ideas appear on pages 11, 19, and 25.

(New Language) Choose at least one entry from the Word Bank that has a conversational English meaning and a mathematical meaning. Describe whether these two meanings have similarities or not.

(Language in the Real World) Explain a situation in the real world where someone might use the probability studied in this unit. Explain why you think it might be important.

Virtual Manipulatives

Consider using virtual coins and number cubes for probability experiments. They can be found online with a search for “virtual manipulatives.”

Teaching Tips later in the TE-UPI are about specific instructional strategies. Most units will have more than just this one at the end.

REPRODUCIBLES

R1-1 WILL IT HAPPEN? CARDS

Reproducibles are typically for games, card sorts, templates, and manipulatives. They are listed on the TE-UPI Unit Planning page (always page ii) with details of where they are used and how many copies are needed. There is also a reminder on the TE-AK page where they are needed, though sometimes they are listed as optional.

A. You toda	vill
B. It will get dark tonight.	H. A monkey will read the Declaration of Independence in your social studies class today.
C. Everyone in 2nd period has the same favorite movie.	I. You will breathe today.
D. We will have a fire drill tomorrow.	J. You win a game of “rock-paper-scissors” against your friend.
E. From a full deck of shuffled cards, you will pick five cards and they will all be aces.	K. You will roll a number cube and get a number greater than 1.
F. You will roll two number cubes and get a sum less than 10.	M. You will flip a coin 20 times and it will land on heads every time.

R1-2 SPINNER CLUE CARDS

Pass Spinner Puzzle B out first.

SPINNER PUZZLE B
The spinner has four colors. You are equally as likely to get blue as you are to get red.

SPINNER PUZZLE B
You are twice as likely to get green as you are to get red.

SPINNER PUZZLE B
If you spun 800 times, you would land on blue about 100 times.

SPINNER PUZZLE B
You are four times as likely to get a yellow as you are to get a blue.

Pass Spinner Puzzle C out after Puzzle B is completed.

SPINNER PUZZLE C
On this spinner you can win a homework free pass, a day of math games, or be principal for the day.

SPINNER PUZZLE C
The chances of winning a day of math games is three times as likely as a homework free pass.

SPINNER PUZZLE C
The chances of winning a homework free pass is 6 out of 24 spins.

SPINNER PUZZLE C
P(Principal for the day) is equal to P(no homework)

R1-3 BIG SQUARE PUZZLE: PROBABILITY

$\frac{2}{5}$ $0.22\overline{2}$	0.01 $\frac{3}{6}$ 0.4	0.04 $\frac{22}{100}$ $\frac{1}{100}$	$\frac{2}{3}$ $\frac{1}{25}$
$\frac{2}{9}$ $\frac{1}{20}$ $\frac{3}{8}$	0.500 1 $0.444...$ 0.05	0.22 0.9 0.125 $\frac{10}{10}$	$0.\overline{66}$ 0.75 $\frac{9}{10}$
0.375 $\frac{14}{25}$ $0.8\overline{3}$	$\frac{4}{9}$ $\frac{7}{8}$ 0.5 0.56	$\frac{1}{8}$ $0.5\overline{5}$ $\frac{1}{6}$ 0.875	$\frac{3}{4}$ $\frac{1}{9}$ $\frac{5}{9}$
$\frac{5}{9}$ $\frac{90}{100}$	$\frac{1}{2}$ $\frac{2}{6}$ 0.9	$0.1\overline{6}$ $\frac{3}{20}$ $0.33\frac{1}{3}$	$0.11\overline{1}$ 0.15

R1-4 MATCH AND COMPARE SORT CARDS: PROBABILITY

I △ EXPERIMENTAL PROBABILITY	I ○ THEORETICAL PROBABILITY
II △ OUTCOME	II ○ SAMPLE SPACE
III △ TERMINATING DECIMAL	III ○ REPEATING DECIMAL
IV △ FAIR GAME	IV ○ RANDOM SAMPLE
A △ ✓ the result of a probability experiment ✓ example: Flip a coin and roll a number cube. The result is H-6	A ○ ✓ a measure of the likelihood of the event occurring ✓ example: We will get tails about 50% of the time when flipping a coin
B △ ✓ a decimal whose digits are 0 from some point on ✓ Example: $\frac{1}{4}$ = 0.25 = 0.25000…)	B ○ ✓ a selection in a probability experiment done by chance ✓ when flipping a fair coin, heads or tails are equally likely
C △ ✓ in a game of chance, all players have an equal chance of winning; or if a single player, the chance of winning and losing are the same	C ○ ✓ the set of all possible outcomes ✓ sometimes shown as an outcome grid or tree diagram
D △ ✓ the number of times the event occurs divided by the number of trials ✓ example: Flip a coin 4 times, and get 3 heads ($\frac{3}{4}$ of the time)	D ○ ✓ a decimal that ends in repetitions of the same block of digits ✓ Example: $\frac{1}{3}$ = 0.333… = $0.\overline{3}$

Name ______________________________ Period ____________ Date ________________

UNIT 1
ANSWER KEY

The start of TE-AK

MathLinks
GRADE 7

PROBABILITY

For student self- assessment after each lesson →

		Monitor Your Progress	Page
	My Word Bank		0
1.0	**Opening Problem: Race to the Top**		1
1.1	**Introduction to Probability**		2
	• Compare estimates from probability experiments to theoretical probabilities.	3 2 1 0	
	• Understand that probability is a number from 0 to 1.	3 2 1 0	
	• Understand vocabulary related to probability.	3 2 1 0	
	• Represent probabilities as fractions, decimals, and percents.	3 2 1 0	
1.2	**Flips, Rolls, and Sample Space Displays**		9
	• Explore patterns in repeating decimals.	3 2 1 0	
	• Use outcome grids, tree diagrams, and lists to represent the sample space of a probability experiment.	3 2 1 0	
	• Analyze outcomes from a probability experiment.	3 2 1 0	
1.3	**Probability Experiments: Games and Puzzles**		17
	• Play and analyze a probability game that involves terminating and repeating decimals.	3 2 1 0	
	• Use theoretical probability to determine the fairness of the game.	3 2 1 0	
	• Create and analyze probability spinner puzzles.	3 2 1 0	
	Review		23
	Student Resources		30

Materials Grouping Reproducibles Slide Deck Journal Idea

Look for these icons at the bottom of many TE-AK pages for instructional suggestions.

Parent (or Guardian) signature ____________

Parent Support letters are available on the Teacher Portal AND on the public website for those who help your students at home.

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

event	experimental probability
outcome	repeating decimal
sample space	terminating decimal
theoretical probability (probability)	trial

When a vocabulary word first comes up in context, take the time to support students in writing something that is meaningful to them, whether it's an explanation of the vocabulary in their own words, an example, and/or a picture.

This is the first of many annotations you'll see in red comic sans. This one is meant as a timely teaching note.

OPENING PROBLEM: RACE TO THE TOP

[SMP 1, 4, 5]

Answers will vary based upon numbers rolled, which may lead to misconceptions. This activity is revisited at the end of Lesson 2 for students to apply new learning of theoretical probability.

Follow your teacher's directions.

(1) I predict I will roll a sum of ________ most often because…

(2) **Record X's below.** Results will vary.

Every unit begins with an opening problem to motivate learning and pique interest. Some are wrapped up within a class hour. Many require more concepts and skills than students have at the beginning of a unit, so are revisited later. When to revisit is always stated directly in Lesson Notes or on a student page. This one is referenced on pages 15-16.

						8	9	10	11	12

"Follow your teacher's directions" is a signal that some direct instruction is required. Look for the accompanying Slide Deck on the Teacher Portal, Lesson Notes on the following page(s), and an alternative page to the deck if some other lesson delivery method is desired.

(3) **Record class data below.** Results will vary.

sum	2	3	4	5	6	7	8	9	10	11	12
number of occurrences											

(4) **Summary of the class discussion.**
Some possible discussion points: common winning sums are usually 6 – 9, because there are more number combinations for these, and least common winning sums are usually 2, 3, 11, and 12, because there are fewer number combinations for these. For new rounds, individual and class results will probably be different, but students will find with experience in probability that over many trials, general results are consistent.

Detailed answers support teacher instruction and student learning. Sometimes the answers provided surpass what may be expected from students.

(5) **Predict the sum that will occur the most.**
One possible prediction: 7 because it came up

(6) **Predict the sum that will occur the least.**
One possible prediction: 2 because it rarely came up in our rolls.

(7) **Predict which is more likely, an even sum or a sum of 13.**
An even sum, becaus

(8) **Predict which is mo**
One possible predic

Here are some helpful icons. You'll need a few simple materials. Slide Deck 1.0 is available to use (or its alternative page), with corresponding Lesson Notes. During instruction, it's suggested that students be grouped (2 or more).

2 number cubes per student or pair

S1.0

2+

LESSON NOTES S1.0: RACE TO THE TOP

On slide decks, blue italic text suggests disc onses.

Review Slide Decks (or the alternative page(s)) and Lesson Notes prior to instruction. These notes are not intended as a script.

This intuitive context sets the stage for ex r cubes (perform an experiment and predict results) on 7.2 to build a connection between experimental and theoretical probability. Students may want to find and use virtual number cubes for this experiment.

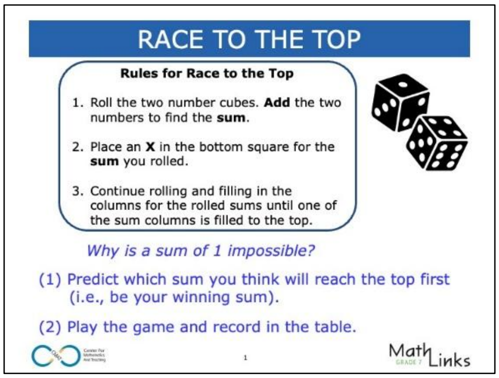
We think the ***bold italic*** questions are good to ask. Modify or add to them them to suit your students' needs.

- Slide 1: Introduce t understanding.

  Which sums are possible with two number cubes? 2-12.

  Why is a sum of 1 not possible? Since the smallest value on each cube is 1, the smallest sum is 1 + 1 = 2.

  Students write a prediction for (1). Have some students share their predictions and reasoning. For (2), students play the game and record results.



RACE TO THE TOP

Rules for Race to the Top

1. Roll the two number cubes. **Add** the two numbers to find the **sum**.
2. Place an **X** in the bottom square for the **sum** you rolled.
3. Continue rolling and filling in the columns for the rolled sums until one of the sum columns is filled to the top.

Why is a sum of 1 impossible?

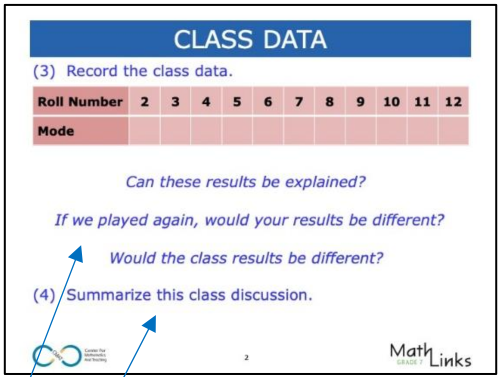
(1) Predict which sum you think will reach the top first (i.e., be your winning sum).

(2) Play the game and record in the table.

- Slide 2: For (3), record the total class results for each sum.

  Which sums were the most common and least common winners for the class? Can we explain why? Answers may vary. If students aren't ready for this discussion, table it until they have more experience. More than likely, the winning sums were from 6 to 9 because there are more number combinations for these sums. The least common sums were likely 2 ,3, 11, and 12 because there are fewer number combinations for these sums.

  Students summarize the class discussion for (4).



CLASS DATA

(3) Record the class data.

Roll Number	2	3	4	5	6	7	8	9	10	11	12
Mode											

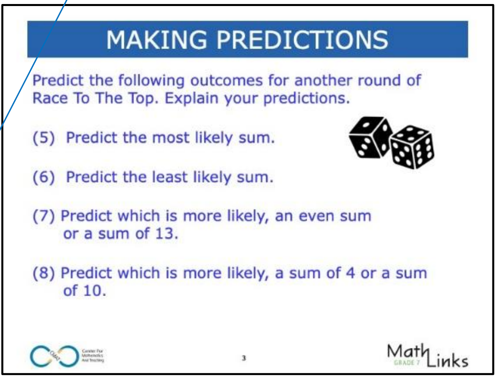
Can these results be explained?

If we played again, would your results be different?

Would the class results be different?

(4) Summarize this class discussion.

- Slide 3: For (5) – (8), students predict what might happen if playing another round of **Race to The Top**. Discuss as a class. If there is time, play another round and compare results to predictions.



MAKING PREDICTIONS

Predict the following outcomes for another round of Race To The Top. Explain your predictions.

(5) Predict the most likely sum.

(6) Predict the least likely sum.

(7) Predict which is more likely, an even sum or a sum of 13.

(8) Predict which is more likely, a sum of 4 or a sum of 10.

Discussion questions are always in italics in this blue color.

The same blue is used for numbered problems that require student writing.

SLIDE DECK ALTERNATIVE S1.0: RACE TO THE TOP

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slide 1

Rules for Race to the Top

- Roll the two number cubes. **Add** the two numbers to find the **sum**.
- Place an **X** in the bottom square for the **sum** you rolled.
- Continue rolling and filling in the columns for the rolled sums until one of the sum columns is filled to the top.

Why is a sum of 1 impossible?

(1) I predict I will roll a sum of ____ most often because…

(2) Play the game and record your results in the table.

As an alternative to using the slides, some teachers project this page on a white board or use a document camera.

Note that all of the important parts of the slide decks are included on the alternative page.

Slides 2 – 3

(3) Record the class data.

Can these results be explained?

If we played again, would your results be different?
Would the class results be different?

(4) Summarize the class discussion.

Predict the following outcomes for another round of **Race to the Top**. Make sure to explain your predictions.

(5) Predict the most likely sum.	(6) Predict the least likely sum.
(7) Predict which is more likely, an even sum or a sum of 13.	(8) Predict which is more likely, a sum of 4 or a sum of 10.

INTRODUCTION TO PROBABILITY

We will perform probability experiments and use the data to compute experimental probabilities. We will compare experimental probabilities to theoretical probabilities. We will use vocabulary related to probability, and write the probabilities of events using fractions, decimals, and percents.

[7.SP.5, 7.SP.6, 7.SP.7ab; SMP2, 3, 4]

GETTING STARTED

Each lesson begins with this gray box containing a lesson summary and Common Core content and practice standards.

Getting Started problems follow and are intended to review or preview important content for the lesson.

[SMP3]

Kyla plays trombone ticket to support the band's fundraiser. S of winning the raffle?"

1. Explain what Ky . One possible answer: She may want to k other tickets were purchased. The more "competition" there is, the less chance she probably has of winning.

We sometimes use percent values to describe the likelihood of an event occurring. You may have seen this on a weather app that is reporting the chance of rain. For each phrase below, write a percent value that corresponds to the **bold** word.

2. "It is **certain** that it will rain."
100%

3. "It is **unlikely** that it will rain."
Percent may vary. One possible answer: 10%

4. "It is **impossible** for it to rain."
0%

5. "It is **probable** that it will rain."
Percent may vary. One possible answer: 80%

6. "It is **possible** that may rain."
Percent may vary. One possible answer: 60%

7. "It is **improbable** that it will rain."
Percent may vary. One possible answer: 5%

8. "It is **likely** that it may rain."
Percent may vary. One possible answer: 75%

9. "It is **equally likely** that it will rain or not rain."
50%

Change each fraction to a decimal and percent.

10. $\frac{3}{5}$ $\frac{3}{5}\left(\frac{2}{2}\right) = \frac{6}{10} = 0.6 = 60\%$	11. $\frac{18}{25}$ $\frac{18}{25}\left(\frac{4}{4}\right) = \frac{72}{100} = 0.72 = 72\%$	12. $\frac{1}{20}$ $\frac{1}{20}\left(\frac{5}{5}\right) = \frac{5}{100} = 0.05 = 5\%$

WILL IT HAPPEN?

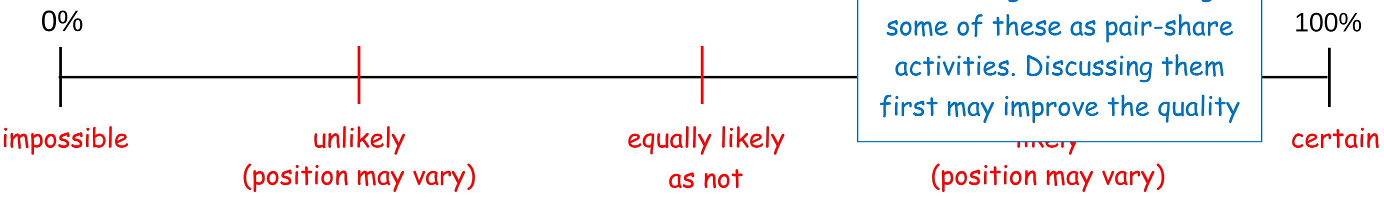
[SMP2]

Instructional idea:

For problems that require students to explain their reasoning, consider doing some of these as pair-share activities. Discussing them first may improve the quality

1. Place the following words/phrases in a reasonable posit rtain

   likely unlikely equally likely as not rtain



2. Label each column below with the words/phrases from problem 1 in the order you placed them from left to right on the number line. Your teacher will give you a set of **Will It Happen?** Cards. Place the corresponding letter for each card in the appropriate column.

   Some answers may vary. One possible sort is below.

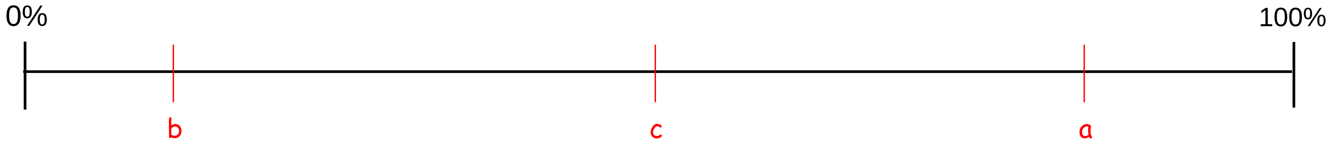
Impossible	Unlikely	Equally Likely as Not	Likely	Certain
E, H	C, D, G, M	J	A, F, K	B, I

3. Choose an activity that you participate in and know well. Write three events below that might happen during the activity. Determine a potential likelihood of each event and explain why. Represent each event on the probability line below.

   Answers will vary. One possible answer:

   My activity is: Soccer

Event	Chance of Event Happening (and Why)
a. I will score a goal.	Likely, since I score in almost every game.
b. Our goalie will save every attempted shot.	Unlikely, since we do not usually hold opponents to 0 goals.
c. Our team will win the game.	Equally likely as not, since we win about half of our games.



Fill in each blank with a reasonable number. Answers will vary.

4. Tai says to Fatima, “I think it’s **impossible** for you to beat LeBron James in a one-on-one basketball game. You have a 0 % chance.” Fatima replies, “Maybe, but I’m 100 % **certain** that I can beat you.

A COIN FLIP EXPERIMENT

[SMP2, 3, 4, 5, 6]

Follow your teacher's directions for (1) – (8).
Students flip a coin 20 times and record results. Answers will vary.

	(1) PREDICTION			(2) MY DATA			(7) CLASS DATA		
	Number	**Fraction**	**Percent**	**Tally**	**Fraction**	**Percent**	**Total**	**Fraction**	**Percent**
Heads									
Tails									
Total	20	$\frac{\quad}{20}$			20				

Notice the uncluttered, structured work space. Most *MathLinks* teachers do not require students to take traditional notes, since they are doing and discussing mathematics constantly, and all of the information students need is contained within the Student Packet.

(3) ***When you flip a coin, what are all the possible <u>outcomes</u> in the <u>sample space</u>?***
There are two possible outcomes in the sample space. They are heads or tails (H, T).

(4) ***Is the <u>event</u> of landing on heads (a subset of the sample space) a random event?***
An event is a subset of the sample space. Landing on heads (H) is a random event.

(5) ***How many <u>trials</u> did you perform of the experiment?***
20 trials

A reminder that vocabulary should be recorded on page 0. New vocabulary is always underlined the first time it appears.

(6) ***Find the <u>theoretical probability</u> (probability) of each of these events.***
P(tails) = $\frac{1}{2}$ P(heads or tails) = 1 P(heads and tails) = 0

(8) Record the meanings of <u>experimental probability</u>, <u>theoretical probability</u>, <u>event</u>, <u>outcome</u>, <u>sample space</u>, and <u>trial</u> in **My Word Bank**.

9. If you flip a coin 500 times, about how many times would you expect it to land on heads?
About 250 times, since the probability of the coin landing heads up is $\frac{1}{2}$ or 50%.

10. Suppose you flipped a coin 20 times and it landed on tails each time. What is the probability of the next flip landing on tails?
Still $\frac{1}{2}$ or 50%. All trials are random and independent of one another (the result of one flip does not impact the result of the next flip). However, the chance of actually getting 20 tails in a row in the first place is highly unlikely.

Coins S1.1a 2+

LESSON NOTES S1.1a: A COIN FLIP EXPERIMENT

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

Students review fraction, decimal, and percent equivalences as they explore experimental and theoretical probability in a very intuitive context, flipping a coin. They learn that probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring, and grapple with and make sense of the new notation as they analyze the experiment. Virtual coins may be used if desired.

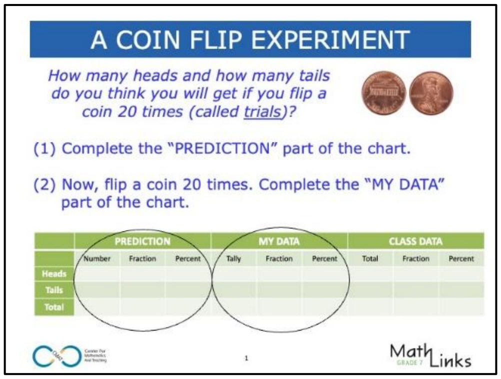
- Slide 1: Begin with the intuitive question about flipping tails. Introduce probability concepts using appropriate language.

  When you flip a coin, what are the possible <u>outcomes</u>? Heads or tails.

  Is the result of flipping it a random <u>event</u>? Yes, we cannot know what the exact outcome will be when flipping a coin once or more.

  Explain the experiment and pose (1), giving time for students to make their predictions and record their results.

  For (2), students individually flip a coin and record results.



A COIN FLIP EXPERIMENT

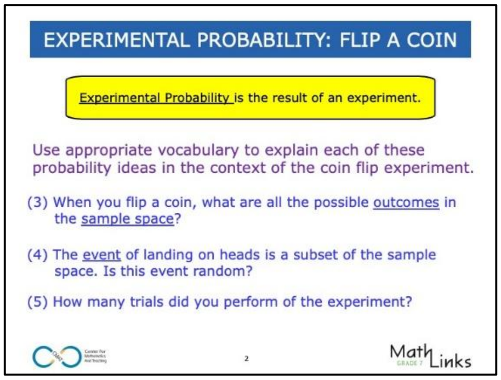
How many heads and how many tails do you think you will get if you flip a coin 20 times (called <u>trials</u>)?

(1) Complete the "PREDICTION" part of the chart.

(2) Now, flip a coin 20 times. Complete the "MY DATA" part of the chart.

	PREDICTION			MY DATA			CLASS DATA		
	Number	Fraction	Percent	Tally	Fraction	Percent	Total	Fraction	Percent
Heads									
Tails									
Total									

- Slide 2: Continue to discuss the meaning of the vocabulary and concepts associated with experimental probability, which is calculated from an experiment. Encourage students to refer to **Student Resources** for definitions and to use vocabulary correctly to complete (3) – (5).



EXPERIMENTAL PROBABILITY: FLIP A COIN

<u>Experimental Probability</u> is the result of an experiment.

Use appropriate vocabulary to explain each of these probability ideas in the context of the coin flip experiment.

(3) When you flip a coin, what are all the possible <u>outcomes</u> in the <u>sample space</u>?

(4) The <u>event</u> of landing on heads is a subset of the sample space. Is this event random?

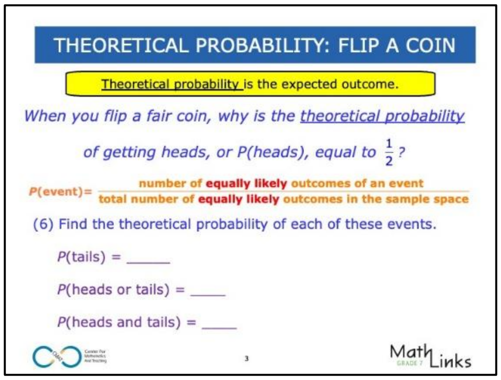
(5) How many trials did you perform of the experiment?

- Slide 3: Compare theoretical probability to experimental probability using the coin flipping example.

  Ask students to complete (6) and discuss.

  Can the coin land either heads OR tails at each flip? Yes, it is certain that one of these will happen.

  Can the coin land both heads AND tails at each flip? No, it is impossible for this to happen.



THEORETICAL PROBABILITY: FLIP A COIN

<u>Theoretical probability</u> is the expected outcome.

When you flip a fair coin, why is the <u>theoretical probability</u> of getting heads, or P(heads), equal to $\frac{1}{2}$?

$$P(\text{event}) = \frac{\text{number of equally likely outcomes of an event}}{\text{total number of equally likely outcomes in the sample space}}$$

(6) Find the theoretical probability of each of these events.

P(tails) = ______

P(heads or tails) = _____

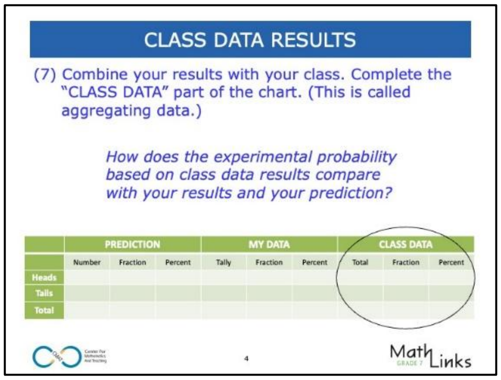
P(heads and tails) = _____

LESSON NOTES S1.1a: A COIN FLIP EXPERIMENT

Continued

- Slide 4: For (7), students aggregate class data from the coin flip experiment. Discuss the results.

  Are class results a better estimate of theoretical probability than individual results? Probably. ***Why do you think this happened?*** Individual results will have more variability. Intuitively, the more trials that are done on a fair experiment, the more likely the experimental probability will approximate theoretical probability.



CLASS DATA RESULTS

(7) Combine your results with your class. Complete the "CLASS DATA" part of the chart. (This is called aggregating data.)

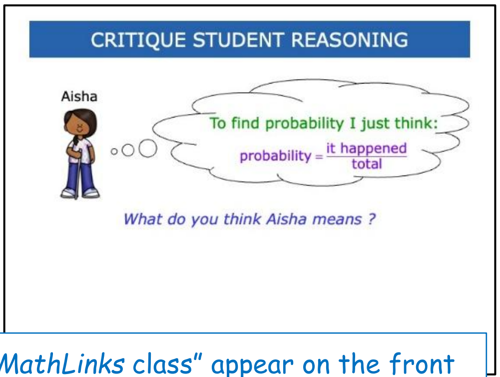
How does the experimental probability based on class data results compare with your results and your prediction?

	PREDICTION			MY DATA			CLASS DATA		
	Number	Fraction	Percent	Tally	Fraction	Percent	Total	Fraction	Percent
Heads									
Tails									
Total									

- Slide 5: Critiquing student ideas or student work are recurring features of the program. Aisha expresses her early thinking about probability using a few simple words. Give students time to interpret Aisha's shortcut (imprecise) expression $\frac{\text{it happened}}{\text{total}}$. Students' understanding will grow as these lessons progress.

  What does "it happened" mean? For experimental probability, it is the number of times an an experiment. For theoretical probabili outcomes of an event in the sample spac

  What does "total" mean? For experime the total number of trials of the experi probability, it is the total number of equ the sample space.

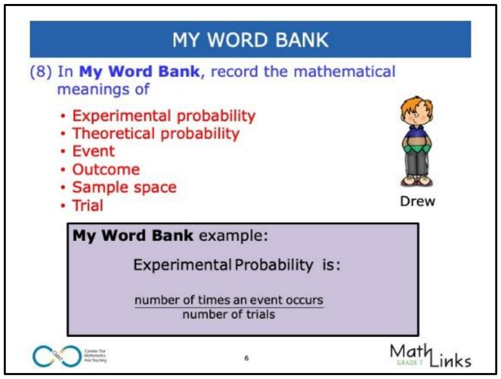


CRITIQUE STUDENT REASONING

What do you think Aisha means ?

The students in our "*MathLinks* class" appear on the front of every Student Packet in grades 6 – 8. They portray a wide range of cultures and qualities. These students often share their thinking, their work, or their errors on slides, which is intended to stimulate class discussions. They typically do not appear in Student Packets to minimize clutter and maximize work space on student pages.

- Slide 6: For (8), introduce students to a vocabulary routine in the program. Periodically, students will be prompted to record explanations and examples of vocabulary in **My Word Bank** at the front of the unit. Give students time to do this, using knowledge from the activity and **Student Resources** in the back of the unit to help them. An example is provided.



MY WORD BANK

(8) In **My Word Bank**, record the mathematical meanings of

- Experimental probability
- Theoretical probability
- Event
- Outcome
- Sample space
- Trial

My Word Bank example:

Experimental Probability is:

$\frac{\text{number of times an event occurs}}{\text{number of trials}}$

Numbered problems not in parentheses (such as 9 – 10 in this activity) are not referenced in the slide deck, and are intended for practice individually or in groups.

SLIDE DECK ALTERNATIVE S1.1a: A COIN FLIP EXPERIMENT

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slide 1

How many heads and how many tails do you think you will get if you flip a coin 20 times (called trials)?

(1) Complete the "PREDICTION" part of the chart.

(2) Now flip a coin 20 times. Complete the "MY DATA" part of the chart.

Slide 2

Experimental Probability is the result of an experiment.

Use appropriate vocabulary to explain each of these probability ideas in the context of the coin flip experiment:

(3) When you flip a coin, what are all the possible outcomes in the sample space?

(4) The event of landing on heads is a subset of the sample space. Is this event random?

(5) How many trials did you perform of the experiment?

Slide 3

Theoretical probability is the expected outcome.

When you flip a fair coin, why is the theoretical probability of getting heads equal to $\frac{1}{2}$?

$$P(\text{event}) = \frac{\text{number of equally likely outcomes in an event}}{\text{total number of equally likely outcomes in the sample space}}$$

(6) Find the probability of each of these events.

P(tails) = _____ P(heads or tails) = _____ P(heads and tails) = _____

SLIDE DECK ALTERNATIVE S1.1a: A COIN FLIP EXPERIMENT

Continued

Slide 4

(7) Combine your results with your group or class. Complete the "CLASS DATA" section of the chart.

How does the experimental probability based on class results compare with your results and your prediction?

Slide 5

Aisha says:

To find probability, I just think: probability = $\frac{\text{it happened}}{\text{total}}$

What do you think Aisha means?

Aisha

Slide 6

(8) In **My Word Bank**, record the mathematical meanings of

- Experimental probability (or estimate of a probability)
- Theoretical probability (or probability)
- Event
- Outcome
- Sample space
- Trial

My Word Bank example:

Experimental Probability is

$$\frac{\text{number of times an event occurs}}{\text{number of trials}}$$

Drew

PRACTICE 1

A bag of marbles contains 10 blue, 5 yellow, 4 green, and 1 red. You pick a marble without looking into the bag. Determine the probability of each event occurring on your first pick. Write each probability as a fraction, decimal, and percent.

Watch for (1-6): Do students use effective strategies to change from fractions to decimals to percents?

Practice follows the introduction of a concept.

1. P(a blue marble)

$\frac{10}{20}$ = 0.5 = 50%

een marble)

$\frac{4}{20}$ = 0.2 = 20%

3. P(a red marble)

$\frac{1}{20}$ = 0.05 = 5%

4. P(a yellow marble)

$\frac{5}{20}$ = 0.25 = 25%

5. P(a marble that is not blue)

$\frac{10}{20}$ = 0.50 = 50%

6. P(blue or yellow)

$\frac{15}{20}$ = 0.75 = 75%

7. Ryder conducted an experiment with the same bag of marbles. She drew a marble out of the bag 10 times (without looking, and replacing it after each turn). Check the box that contains the best description of likelihood of each event.

Event	**Impossible**	**Unlikely**	**Equally Likely as Not**	**Likely**	**Certain**
***P*(red)**		x			
***P*(not red)**				x	
***P*(pink)**	x				
***P*(blue)**			x		
***P*(a marble)**					x

8. Carmen wants to play a game with the marbles above. She will choose a marble out of the bag 20 times (and replace it after each turn). If Carmen chooses a blue or yellow marble, she gets a point. If she chooses a red or green marble, you get a point. Is this game fair? Explain using words, diagrams, and/or numbers.

It is not fair. Carmen has a greater probability of winning, since P(blue or yellow) = 75% and P(red or green) = 25%. Any game where each player has an equal chance of winning is fair. This game is not fair because 75% is three times 25%.

Appropriate for independent practice or for partners.

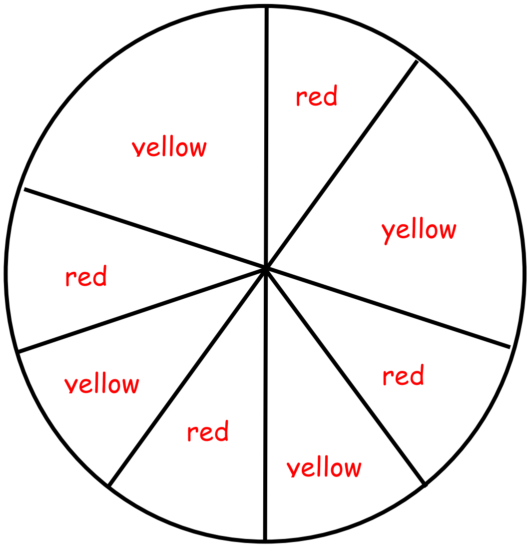
1+

A SPINNER EXPERIMENT

[SMP2, 3, 4]

Follow your teacher's directions for (1) – (4).

(1)



(2) ***What are the possible outcomes in the sample space?*** Land on yellow (yellow); land on red (red)

(3) If I perform __25__ trials of the spinner experiment, I predict: Answers will vary.
I will land on yellow _____ times (_____%).
I will land on red _____ times (_____%).

Students spin their spinner 25 times. They record individual and class results. Answers will vary.

(4)

	MY DATA				**CLASS DATA**			
	Tally	**Fraction**	**Decimal**	**Percent**	**Total**	**Fraction**	**Decimal**	**Percent**
yellow								
red								
Total	25	$\frac{\quad}{25}$						

Find the theoretical probabilities for each event as a fraction, decimal, and percent.

5. P(yellow) = $\frac{6}{10} = 0.6 = 60\%$

6. P(red) = $\frac{4}{10} = 0.4 = 40\%$

7. P(yellow or red) = $\frac{10}{10} = 1 = 100\%$

8. P(yellow and red) = $\frac{0}{10} = 0 = 0\%$

9. Which problems above focused on experimental probability? 2, 3, 4

10. Which problems above focused on theoretical probability? 2, 5, 6, 7, 8

11. How did the experimental probability results compare to theoretical probabilities?
Aggregated class results represented as fractions or percents were likely to be closer to theoretical probabilities than individual results. The more trials that are done on a fair experiment, the more likely the experimental probability will approximate theoretical probability.

Paperclips S1.1b 2+

LESSON NOTES S1.1b: A SPINNER EXPERIMENT

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

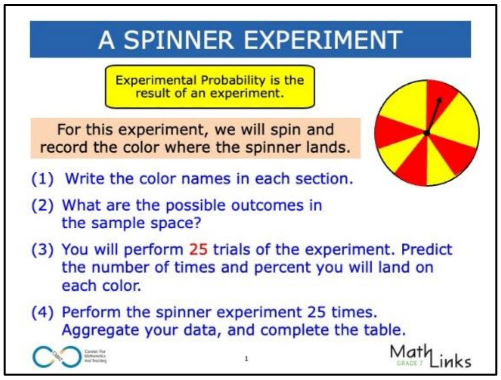
Students deepen their understanding of experimental and theoretical probability by finding probabilities of events that are not equally likely. They write outcomes using fractions, decimals, and percents as they analyze the experiment.

- Slide 1: Review vocabulary associated with probability as it occurs in context. Remind students that experimental probability is calculated using data gathered from an experiment. Show students how to use a paperclip as a spinner, which will be used for the experiment.

  Reveal problems (1) – (4) one at a time, giving students time to gather data, complete the table, and discuss.

  What are all the possible outcomes for the spinner experiment? Landing on yellow (or "yellow"), landing on red (or "red"). The class may want to agree that landing between colors does not count as a spin.

  Review writing fractions as decimals and percents if needed. ***Suppose we landed on red 8 times out of 25. What are the decimal and percent values for this?***
  $\frac{8}{25}\left(\frac{4}{4}\right) = \frac{32}{100} = 0.32 = 32\%$



A SPINNER EXPERIMENT

Experimental Probability is the result of an experiment.

For this experiment, we will spin and record the color where the spinner lands.

(1) Write the color names in each section.

(2) What are the possible outcomes in the sample space?

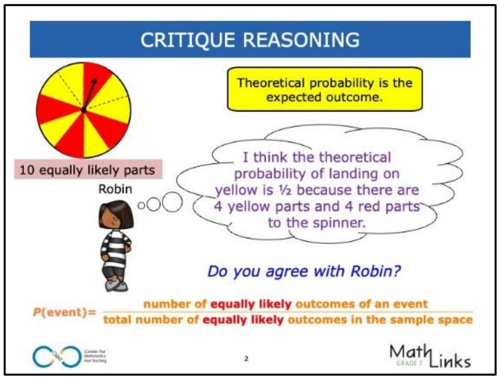
(3) You will perform 25 trials of the experiment. Predict the number of times and percent you will land on each color.

(4) Perform the spinner experiment 25 times. Aggregate your data, and complete the table.

- Slide 2: Remind students that theoretical probability (or probability) is a measure of the likelihood of an event. For example, *P*(red) is the fraction of the area of the circle that is red.

  Is Emmett correct? Explain. Emmett is not correct. He is counting the number of parts of the circle that are yellow, but each section is not the same size (does not have the same area) so they are not **equally likely**.

  Since two of the yellows sectors are each twice the size of the others, there are actually 10 equally likely portions of the circle, and therefore *P*(yellow) = $\frac{6}{10}$. The formula at the bottom of the slide is a reminder about the importance of considering equally likely outcomes when computing the probability of an event.



CRITIQUE REASONING

Theoretical probability is the expected outcome.

10 equally likely parts

Robin

I think the theoretical probability of landing on yellow is ½ because there are 4 yellow parts and 4 red parts to the spinner.

Do you agree with Robin?

$P(\text{event}) = \frac{\text{number of equally likely outcomes of an event}}{\text{total number of equally likely outcomes in the sample space}}$

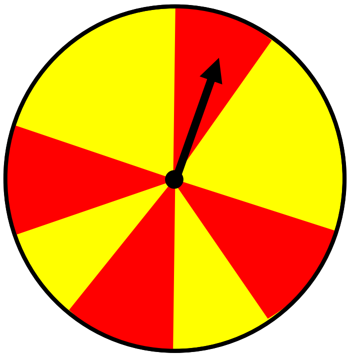
SLIDE DECK ALTERNATIVE S1.1b: A SPINNER EXPERIMENT

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slide 1

Experimental Probability is the result of an experiment.

This spinner has sectors that are yellow (Y) and red (R). For this experiment, we will spin and record the color where the spinner lands.



(1) Write the color names in each section.

(2) What are the possible outcomes in the sample space?

(3) You will perform 25 trials of the experiment. Predict the number of times and percent of each outcome.

(4) Perform the spinner experiment 25 times and record your results. Then aggregate your data with your class.

Slide 2

Theoretical probability is the expected outcome.

Emmett says:
I think the probability of landing on yellow is $\frac{1}{2}$ because there are 4 yellow parts and 4 red parts on the spinner.

Critique Emmett's reasoning.

A *MathLinks* class error analysis opportunity. (SMP3)

Emmett

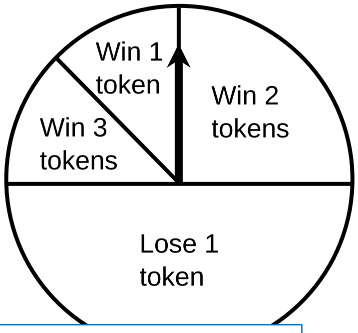
$$P(\text{event}) = \frac{\text{number of EQUALLY LIKELY outcomes of an event}}{\text{total number of EQUALLY LIKELY outcomes in the sample space}}$$

PRACTICE 2

The *MathLinks* Rubric: See Activity Routines on the Teacher Portal for directions. [SMP4]

Students predict how many tokens they will win based on theoretical probabilities.

Use the spinner to the right. All problems are based on the idea that the location where the pointer stops is random. Find the probability that the spinner will stop on each of the areas below—for any single spin—as a fraction, decimal, and percent.



1. P(lose 1 token) $\frac{1}{2}$ = 0.5 = 50%	2. P(win 1 token) $\frac{1}{8}$ = 0.125 = 12.5%
3. P(win 2 tokens) $\frac{1}{4}$ = 0.25 =25%	4. P(win 3 tokens) $\frac{1}{8}$ = 0.125 = 12.5%
5. P(win at least 2 tokens) $\frac{3}{8}$ = 0.375 = 37.5%	6. P(lose 2 tokens) 0 = 0%

See Program Information and the Teacher Portal (General Resources → Activity Routines) for more information and to prepare for using The *MathLinks* Rubric.

7. For 100 trials, about how many times will you expect to win exactly 2 tokens?
   About 25 times.

8. For 100 trials, about how many times will you expect to win?
   About 50 times.

9. For 100 trials, explain if you expect to win or lose more tokens, and by how much.
   Answers will vary. One possible analysis is given.
   You can expect to win about 50% of the time. From the top half of the spinner, you can expect to win an average of 2 tokens per winning spin. From the bottom half of the spinner, you can expect to lose 1 token per losing spin. Therefore, in 100 spins, since you can expect to win 2(50) = 100 tokens and lose 1(50) = 50, and 100 – 50 = 50, you can expect to win 50 tokens.

10. If you spin 20 times and get 1 token each time, what is probability of getting one token on the next spin? Explain. It is still $\frac{1}{8}$. Each spin is random and independent of the other.

PRACTICE 3

Suppose you roll a six-sided cube numbered 1 – 6.

Find the probability of each event.

1. *P* (3) $\frac{1}{6}$	2. *P* (2 or 5) $\frac{1}{3}$	3. *P* (odd number) $\frac{1}{2}$
4. *P* (number less than 7) 1	5. *P* (10) 0	6. *P* (prime number) $\frac{1}{2}$

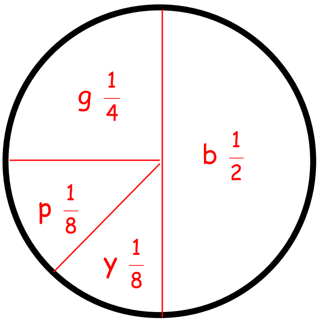
Write a number cube rolling event different than above that would have each of the following probabilities. Explain your reasoning in words or numbers. Answers will vary. Some possible answers:

7. $P(E) = 0$ Rolling a 7	8. $P(E) = 1$ Rolling any number from 1 to 6
9. $P(E) = \frac{1}{6}$ R	10. $P(E) = \frac{1}{2}$ n number

A "watch for" is a timely formative assessment heads-up where many students traditionally have misconceptions, make mistakes, or struggle with concepts.

11. Draw a spinner with a blue, a green, and a pink space such that *P* (blue) is twice *P* (green), *P* (pink) is half *P* (green), and the rest is yellow. Clearly write the fractional amount with the color in the circle.

Let the first letter of each color word represent that color.



Watch for (11): Do students make reasonable divisions on the spinner?

At the end of each lesson, this prompt reminds teachers to ask students to self-assess their progress on the lesson goals on the front of the Student Packet.

12. In t you think it means for a trial of an experiment to be ra ituation as an example.

When rolling a number cube, for example, the outcome is unpredictable. We can expect a 3 on $\frac{1}{6}$ of our rolls, but we never know when a 3 will happen or if it will happen the expected number of times.

1+

Monitor Your Progress

FLIPS, ROLLS, AND SAMPLE SPACE DISPLAYS

We will convert fractions into terminating and repeating decimals. We will continue to explore probability concepts and learn techniques for organizing data. We will more analyze experiments by comparing experimental probabilities to the theoretical probabilities.

[7.NS.2d, 7.SP.6, 7.SP.7ab, 7.SP.8ab; SMP1, 2, 3, 4, 5, 6, 7, 8]

Language used in directions and problems is precise and consistent from unit to unit.

GETTING STARTED

1. Write the product as a mixed number: $\frac{1}{3}(10)$. $3\frac{1}{3}$

Fill in the blanks using the distributive property.

2. $3\frac{1}{3}(10) = (\underline{\ 3\ })10 + (\underline{\ \frac{1}{3}\ })10$ $= \underline{\ 30\ } + \underline{\ 3\frac{1}{3}\ }$ $= \underline{\ 33\frac{1}{3}\ }$	3. $33\frac{1}{3}(10) = (\underline{\ 33\ })10 + (\underline{\ \frac{1}{3}\ })10$ $= \underline{\ 330\ } + \underline{\ 3\frac{1}{3}\ }$ $= \underline{\ 333\frac{1}{3}\ }$

Change each of the following fractions to a decimal and a percent using any method.

4. $\frac{3}{10}$ 0.3 or 0.30 30%	5. $\frac{3}{20}$ 0.15 15%	6. $\frac{3}{50}$ 0.06 6%
7. $\frac{3}{4}$ 0.75 75%	8. $\frac{3}{8}$ 0.375 37.5 %	9. $\frac{3}{9}$ 0.333… (by calculator). Exact decimal and percent values are not expected yet. It is between $\frac{3}{8}$ and $\frac{3}{10}$.

Answers will vary. Some possible answers:

10. How is the number in problem 9 the same as those in problems 4 – 8?

They all have 3 as the numerator. Each is a fraction that can be converted to a decimal and a percent.

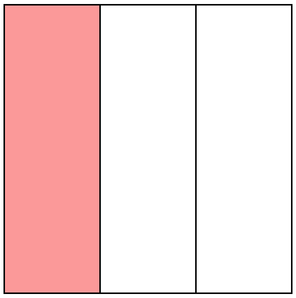
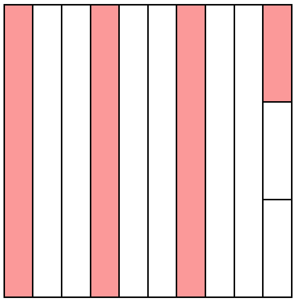
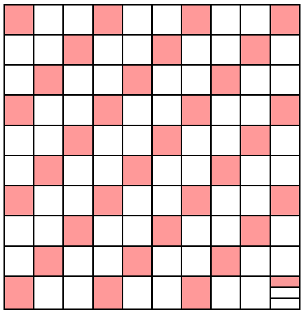
How is it different?

It is not equivalent to any of them. While the number of parts are all the same (equal numerators), all of the sizes of the parts are different (unequal denominators). If using long division to find the decimal value for $\frac{3}{9}$, the dividing never seems to "end." Also, it is the only fraction that is not in simplest form (numerator and denominator have no common factors).

INVESTIGATING ONE-THIRD

[SMP5, 6, 7, 8]

Follow your teacher's directions for (1) – (7).

(1) One-third is equal to…  $\frac{1}{3}$ of a whole = $\frac{\frac{1}{3}}{1}$ OR $\frac{1}{3}$	(2) One-third is equal to…  3 tenths + $\frac{1}{3}$ of a tenth = $3\frac{1}{3}$ tenths OR $\frac{3\frac{1}{3}}{10}$	(3) One-third is equal to…  3 tenths + 3 hundredths + $\frac{1}{3}$ of a hundredth = $33\frac{1}{3}$ hundredths OR $\frac{33\frac{1}{3}}{100}$
(4) ***Write the shaded fractional part that comes next in the pattern.*** $\frac{333\frac{1}{3}}{1000}$	(5) ***Divide.*** $3\overline{)1.000}$	(6) ***Write $\frac{1}{3}$ as an exact percent in two ways.*** $33\frac{1}{3}\%$ or $33.\overline{3}\%$ (Something like $33.\overline{33}\%$ is acceptable as well.)
(7) ***Write $\frac{2}{3}$ as an exact percent in two ways.*** $66\frac{2}{3}\%$ or $66.\overline{6}\%$ (Something like $66.\overline{66}\%$ is acceptable as well.)	8. Round the decimals in problems 6 and 7 to the nearest hundredths. Then write these as percents. 0.33 = 33% 0.67 = 67%	9. Round the decimals in problems 6 and 7 to the nearest thousandths. Then write these as percents. 0.333 = 33.3% 0.667= 66.7%

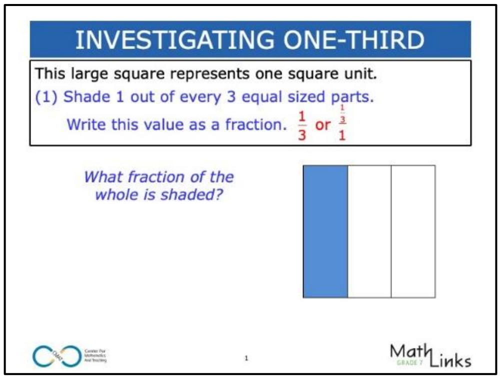
10. Record the meanings of repeating decimal and terminating decimal in **My Word Bank**.

LESSON NOTES S1.2a: INVESTIGATING ONE-THIRD

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

Splitting a whole square into successively smaller parts illustrates the nature of a repeating decimal. This visual investigation makes plausible to students that a fraction’s conversion to a decimal might have an infinite number of steps. Connections to the decimal algorithm and percents are made as well.

- Slide 1: Define the whole and allow students work time for (1). Share and discuss as desired. Emphasize the fraction forms $\frac{1}{3}$ (one-third) = $\frac{\frac{1}{3}}{1}$ (one-third of a whole), as this representation will be important in the exploration.



INVESTIGATING ONE-THIRD

This large square represents one square unit.

(1) Shade 1 out of every 3 equal sized parts.

Write this value as a fraction. $\frac{1}{3}$ or $\frac{\frac{1}{3}}{1}$

What fraction of the whole is shaded?

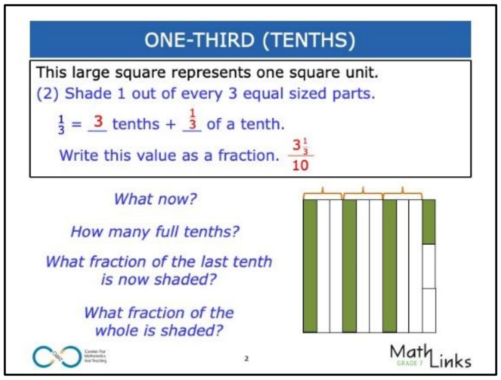
- Slide 2: Proceed as before for (2).

  After 3 of the first 9 tenths are shaded, how can we shade $\frac{1}{3}$ of the last tenth? Divide it into three equal parts and shade 1 of them.

  How many full tenths are shaded? 3

  What fraction of the last tenth is shaded? $\frac{1}{3}$

  What fraction of the whole is shaded? Students write this value as the fraction $\frac{3\frac{1}{3}}{10}$. Stress that this is still $\frac{1}{3}$ of the whole.



ONE-THIRD (TENTHS)

This large square represents one square unit.

(2) Shade 1 out of every 3 equal sized parts.

$\frac{1}{3}$ = _3_ tenths + _$\frac{1}{3}$_ of a tenth.

Write this value as a fraction. $\frac{3\frac{1}{3}}{10}$

What now?

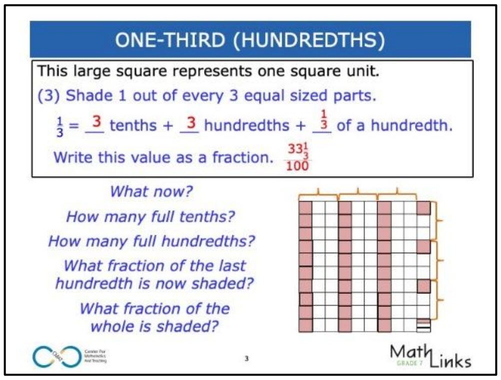
How many full tenths?

What fraction of the last tenth is now shaded?

What fraction of the whole is shaded?

LESSON NOTES S1.2a: INVESTIGATING ONE-THIRD

Continued



ONE-THIRD (HUNDREDTHS)

This large square represents one square unit.

(3) Shade 1 out of every 3 equal sized parts.

$\frac{1}{3}$ = 3 tenths + 3 hundredths + $\frac{1}{3}$ of a hundredth.

Write this value as a fraction. $\frac{33\frac{1}{3}}{100}$

What now?

How many full tenths?

How many full hundredths?

What fraction of the last hundredth is now shaded?

What fraction of the whole is shaded?

- Slide 3: Proceed as before for (3).

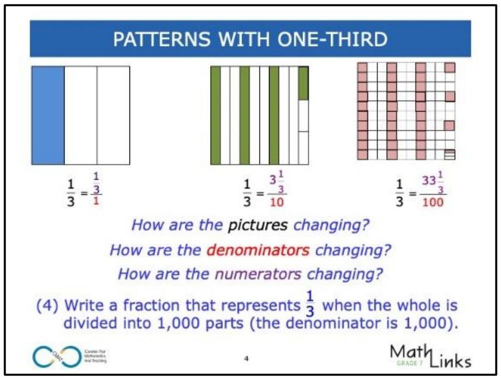
  After 33 of the first 99 hundredths are shaded, how can we shade $\frac{1}{3}$ of the last hundredth? Divide it into three equal parts and shade 1 of them.

  How many full tenths are shaded? 3

  How many full hundredths are shaded? 3

  What fraction of the last hundredth is shaded? $\frac{1}{3}$

  What fraction of the whole is shaded? Students write this value as the fraction $\frac{33\frac{1}{3}}{100}$. Stress that this is still $\frac{1}{3}$ of the whole.



PATTERNS WITH ONE-THIRD

$\frac{1}{3} = \frac{\frac{1}{3}}{1}$ $\frac{1}{3} = \frac{3\frac{1}{3}}{10}$ $\frac{1}{3} = \frac{33\frac{1}{3}}{100}$

How are the pictures changing?

How are the denominators changing?

How are the numerators changing?

(4) Write a fraction that represents $\frac{1}{3}$ when the whole is divided into 1,000 parts (the denominator is 1,000).

- Slide 4: Ask students to study the three pictures and their numerical representations all side-by-side.

  How are the pictures changing? The number of parts is increasing by a factor of 10, but the size of the parts is decreasing by a factor of 10.

  How are the denominators changing? Multiply by 10 to get to the next. 1(10) = 10; 10(10) = 100

  How are the numerators changing? Same relationship. $\left(\frac{1}{3}\right) \bullet (10) = 3\frac{1}{3}$; $\left(3\frac{1}{3}\right) \bullet (10) = 33\frac{1}{3}$.

  If we continue this pattern to another square, into how many parts would we divide the whole? 10(100) = 1,000.

  How many parts would be shaded? $\left(33\frac{1}{3}\right)(10) = 333\frac{1}{3}$.

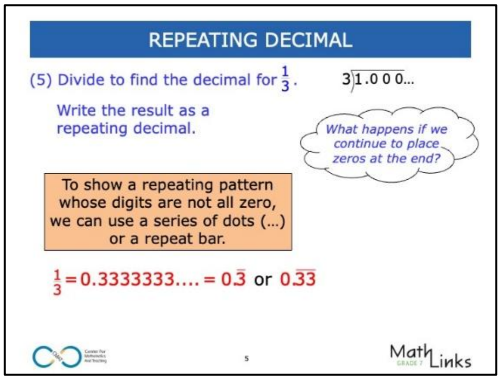
  How many times can we continue to divide the whole square into smaller and smaller parts? Infinitely many times. Then pose (4) and give students time to summarize these ideas.

LESSON NOTES S1.2a: INVESTIGATING ONE-THIRD

Continued

- Slide 5: Students use long division for (5) to find the decimal equivalent for $\frac{1}{3}$. Continue to place zeros until it's clear that, just like the visual, the decimal repeats infinitely. Demonstrate the notation for <u>repeating decimals</u> using a series of dots and also a repeat bar.

  How is this notation different from fraction notation for one-half? We do not need a repeat bar for one-half because it is 0.5. The digit 5 does not repeat in the decimal value. We call a decimal such as 0.5 a <u>terminating decimal</u>.



REPEATING DECIMAL

(5) Divide to find the decimal for $\frac{1}{3}$.

$3\overline{)1.000}$...

Write the result as a repeating decimal.

What happens if we continue to place zeros at the end?

To show a repeating pattern whose digits are not all zero, we can use a series of dots (…) or a repeat bar.

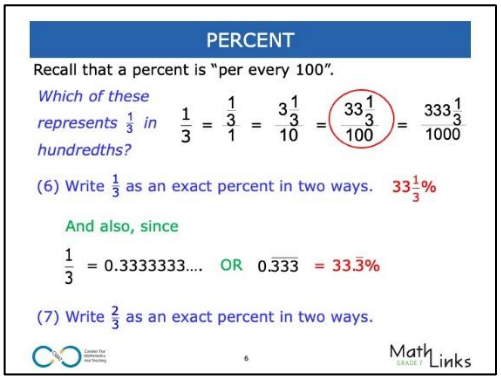
$\frac{1}{3} = 0.3333333.... = 0.\overline{3}$ or $0.\overline{33}$

- Slide 6: Review the literal translation of "percent." Reveal the different fraction values for one-third.

  Which of these represents $\frac{1}{3}$ in hundredths? $\frac{33\frac{1}{3}}{100}$. Since percent is "for every hundred", the percent equivalent is $33\frac{1}{3}$%.

  If one-third is equivalent to $0.\overline{333}$, how do we write this decimal as an exact percent? Multiply the amount by 100, which would give us $33.\overline{3}\%$. Students fill in both percent values for (6).

  For (7), students write another percent for two-thirds in two ways.



PERCENT

Recall that a percent is "per every 100".

Which of these represents $\frac{1}{3}$ in hundredths?

$$\frac{1}{3} = \frac{\frac{1}{3}}{1} = \frac{3\frac{1}{3}}{10} = \boxed{\frac{33\frac{1}{3}}{100}} = \frac{333\frac{1}{3}}{1000}$$

(6) Write $\frac{1}{3}$ as an exact percent in two ways. $33\frac{1}{3}\%$

And also, since

$\frac{1}{3} = 0.3333333....$ OR $0.\overline{333}$ $= 33.\overline{3}\%$

(7) Write $\frac{2}{3}$ as an exact percent in two ways.

SLIDE DECK ALTERNATIVE S1.2a: INVESTIGATING ONE-THIRD

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

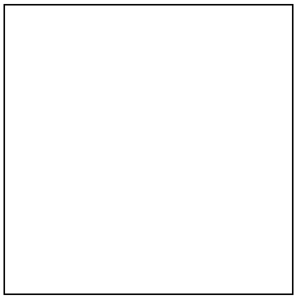
Slides 1 – 4

Each "large square" below represents one square unit.

(1) Shade 1 out of every 3 equal sized parts.

What fraction is shaded?

Write this value as a fraction.

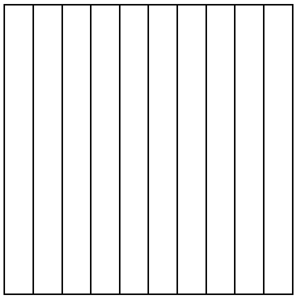


(2) Shade 1 out of every 3 equal sized parts.

How do we shade one-third of the last tenth?

$\frac{1}{3}$ = _____ tenths + _____ of a tenth.

Write this value as a fraction.

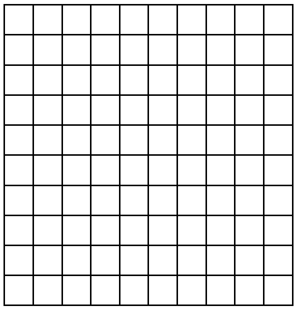


(3) Shade 1 out of every 3 equal sized parts.

How do we shade one-third of the last hundredth?

$\frac{1}{3}$ = _____ tenths + _____ hundredths + _____ of a hundredth.

Write this value as a fraction.



Look at the 3 pictures you just created.
How are the pictures changing?
How are the denominators changing?
How are the numerators changing?

(4) Write a fraction that represents $\frac{1}{3}$ when the whole is divided into 1,000 parts (the denominator is equal to 1,000).

SLIDE DECK ALTERNATIVE S1.2a: INVESTIGATING ONE-THIRD

Continued

Slide 5

(5) Divide to find the decimal for $\frac{1}{3}$.

What happens if we continue to place zeros at the end?

To show a repeating pattern whose digits are not all zero, we can use a series of dots (…) or a repeat bar.

$\frac{1}{3} =$ 0.333… or $0.\overline{3}$

$$3\overline{)1.000}$$

Slide 6

Recall that percent is "per every hundred".
Which of the following represents $\frac{1}{3}$ in hundredths?

$$\frac{1}{3} = \frac{\frac{1}{3}}{1} = \frac{3\frac{1}{3}}{10} = \frac{33\frac{1}{3}}{100} = \frac{333\frac{1}{3}}{1000}$$

(6) Write $\frac{1}{3}$ as an exact percent in two ways.

- With a fraction within the percent:
- With a repeating decimal within the percent:

(7) Write $\frac{2}{3}$ as an exact percent in two ways.

- With a fraction within the percent:
- With a repeating decimal within the percent:

PRACTICE 4

Rounded and exact answers given [SMP3, 6, 8]

Change each fraction to a decimal and a percent, using a repeat bar when necessary.

1. $\enclose{circle}{\frac{1}{6}}$ $0.1\overline{6}$; $16.\overline{6}\%$	2. $\enclose{circle}{\frac{2}{6}}$ $0.\overline{3}$; $33.\overline{3}\%$	3. $\boxed{\frac{3}{6}}$ 0.5; 50%	4. $\enclose{circle}{\frac{4}{6}}$ $0.\overline{6}$, $66.\overline{6}\%$
5. $\enclose{circle}{\frac{5}{6}}$ $0.8\overline{3}$; $83.\overline{3}\%$	6. $\boxed{\frac{6}{6}}$ 1 or 1.00; 100%	7. $\enclose{circle}{\frac{1}{12}}$ $0.08\overline{3}$; $8.\overline{3}\%$	8. $\boxed{\frac{3}{12}}$ 0.25; 25%
9. $\enclose{circle}{\frac{5}{12}}$ $0.41\overline{6}$; $41.\overline{6}\%$	10. $\enclose{circle}{\frac{7}{12}}$ $0.58\overline{3}$; $58.\overline{3}\%$	11. $\boxed{\frac{9}{12}}$ 0.75; 75%	12. $\enclose{circle}{\frac{11}{12}}$ $0.91\overline{6}$; $91.\overline{6}\%$

13. For each fraction above, circle those that are equivalent to repeating decimals, and box those that are equivalent to terminating decimals.

14. Channing thinks that the decimal equivalent for $\frac{1}{6}$ is $0.1\overline{6}$. Carter thinks it's $0.1\overline{6}$. Explain who is correct.
Bryce is correct. Only the digit 6 repeats in the decimal value (0.166666…, NOT 0.161616…).

Problem 14 is suggested as possible journal idea.

1+

Journal Problem 14

FLIP AND ROLL

[SMP2, 4, 5, 6, 7, 8]

Follow your teacher's directions.

(1) ***Restate the rules in your own words.***

You win: the coin lands heads and the cube shows a 2, 4, or 6 OR the coin lands tails and the cube shows 3 or 6

You lose: all other outcomes

(2) ***Is this a fair game? Explain.***

Answers will vary. All explanations may be considered for now, but it will only be fair if all players have equal probabilities of winning.

(3) ***Record your data for 10 trials.***

Trial #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Flip Result										
Roll Result										
Win or Lose?										

(4) ***Record your results and aggregate class data.***

	EXPERIMENTAL PROBABILITY							
	My Data				**Class Data**			
	My Results	**Fraction**	**Decimal**	**Percent**	**Class Results**	**Fraction**	**Decimal**	**Percent**
Wins								
Losses								
Totals								

(5) ***Record all of the outcomes in the sample space in an outcome grid.***

		Outcome Grid					
		Number Rolled					
		1	2	3	4	5	6
Coin Flip	Heads (H)	H-1	H-2	H-3	H-4	H-5	H-6
	Tails (T)	T-1	T-2	T-3	T-4	T-5	T-6

Coins, number cubes, calculators (optional)

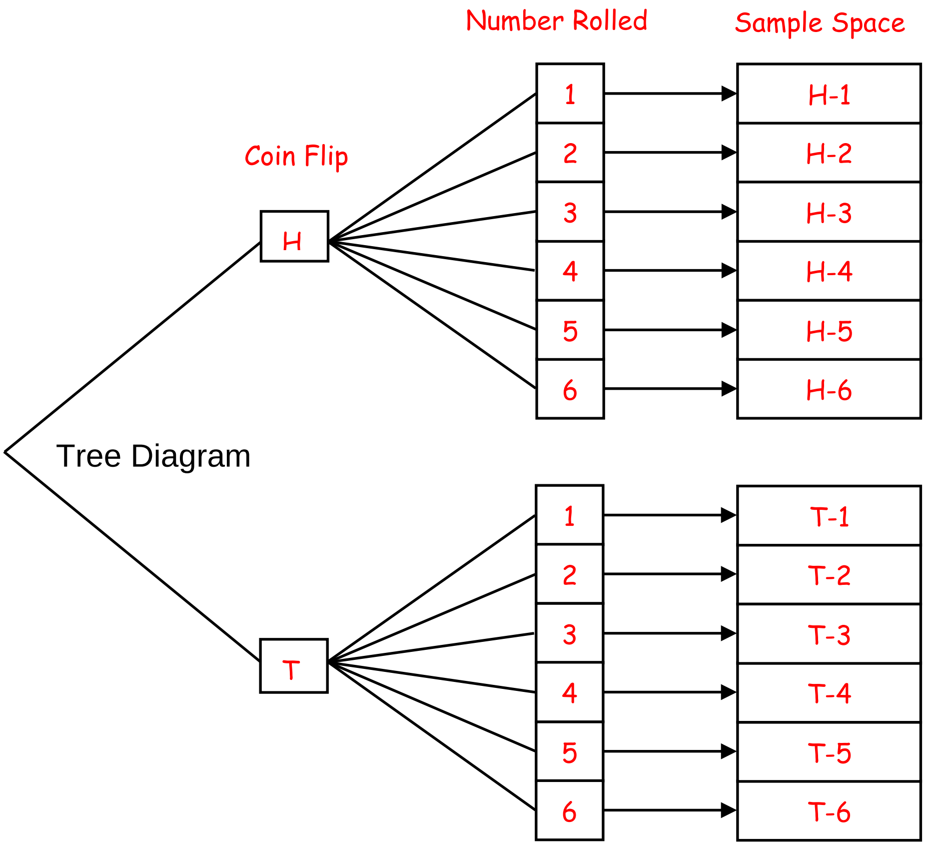
S1.2b

2+

FLIP AND ROLL

Continued

(6) – (7) ***Record all of the outcomes in the sample space in a tree diagram and an organized list.***



Organized List

H-1
H-2
H-3
H-4
H-5
H-6
T-1
T-2
T-3
T-4
T-5
T-6

(8) ***Determine theoretical probabilities for the events of winning and losing the game.***

	THEORETICAL PROBABILITY		
	Fraction	**Decimal**	**Percent**
***P*(Win)**	$\frac{5}{12}$	$0.41\overline{6}$	$41.\overline{6}\%$
***P*(Lose)**	$\frac{7}{12}$	$0.58\overline{3}$	$58.\overline{3}\%$
Total	$\frac{12}{12}$	1.00 or 1	100%

(9) ***Revisit your prediction. Is Flip and Roll a fair game? Explain.***
No. There is not an equal probability of winning and losing. The probability of winning is less than the probability of losing ($\frac{5}{12} \approx$ 42% compared to $\frac{7}{12} \approx$ 58%).

Coins, number cubes, calculators (optional)

S1.2b

2+

LESSON NOTES S1.2b: FLIP AND ROLL

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

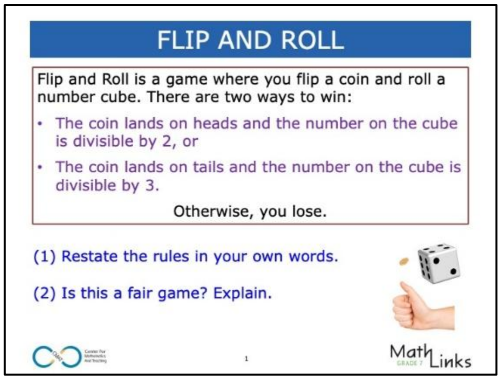
Students calculate experimental probability from an experiment and find theoretical probabilities for a compound event (flip a coin and roll a number cube) using outcome grids, tree diagrams, and organized lists. They compare experimental probabilities to the theoretical probabilities and determine if a game is fair.

- Slide 1: Ask students to read the rules of the game and check for understanding.

  If you flip a head and roll a 4, will you win? Yes. ***Why?*** 4 is an even number.

  If you flip a tail and get a 4, will you win? No. ***Why?*** 4 is not a multiple of 3.

  For (1), students restate the rules in their own words, and for (2) they indicate whether they think it is fair and why. Encourage students to pair-share their predictions.



FLIP AND ROLL

Flip and Roll is a game where you flip a coin and roll a number cube. There are two ways to win:

- The coin lands on heads and the number on the cube is divisible by 2, or
- The coin lands on tails and the number on the cube is divisible by 3.

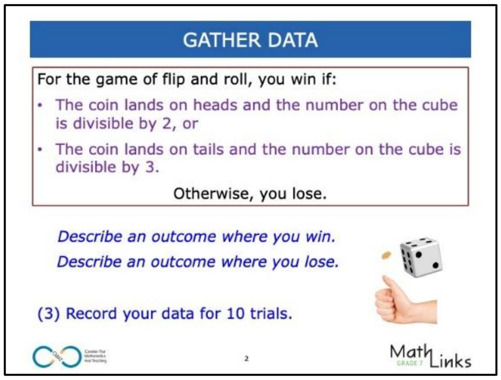
Otherwise, you lose.

(1) Restate the rules in your own words.

(2) Is this a fair game? Explain.

- Slide 2: Remind students that a trial consists of one coin flip combined with one number cube roll. For (3), students complete 10 trials and record their data in the table.

  What shorthand notation might we use to record? Use H for heads, T for tails, W for win, and L for lose.



GATHER DATA

For the game of flip and roll, you win if:

- The coin lands on heads and the number on the cube is divisible by 2, or
- The coin lands on tails and the number on the cube is divisible by 3.

Otherwise, you lose.

Describe an outcome where you win.

Describe an outcome where you lose.

(3) Record your data for 10 trials.

Questioning suggestions support students with problem comprehension, which is especially important for struggling learners and English learners.

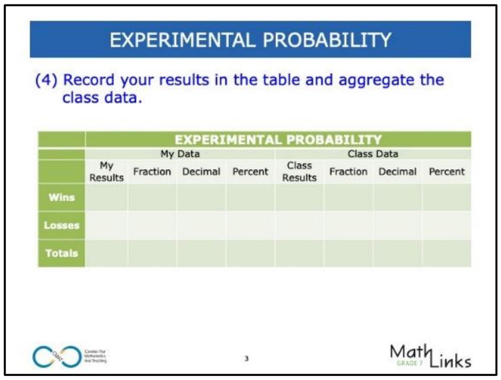
LESSON NOTES S1.2b: FLIP AND ROLL

Continued

- Slide 3: For (4), students record individual data and aggregated class data in the table as whole numbers, fractions, decimals, and percents.

  What should be the totals in the "My Data" section? 10 trials, $\frac{10}{10}$, 1.00, and 100%

  Which results do you think are best estimates of the probability of winning this game? Why? Answers may vary, but probably the class data, because as a sample size grows, the experimental probabilities get closer to the theoretical probabilities. This illustrates the law of large numbers, an important idea in probability and statistics.



EXPERIMENTAL PROBABILITY

(4) Record your results in the table and aggregate the class data.

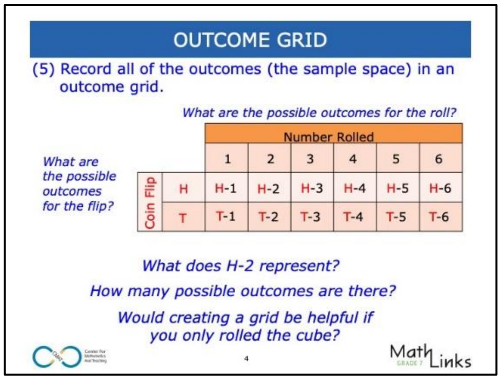
EXPERIMENTAL PROBABILITY	My Data				Class Data			
	My Results	Fraction	Decimal	Percent	Class Results	Fraction	Decimal	Percent
Wins								
Losses								
Totals								

- Slide 4: For (5), use the slide as needed to guide the creation of an outcome grid for this compound event.

  What are the possible outcomes for the coin flip? Heads or tails. ***For rolling the number cube?*** 1, 2, 3, 4, 5, 6

  How many equally likely outcomes are there when rolling a number cube and flipping a coin? 12

  Would this display be helpful if we only rolled a cube? Probably not. The only outcomes are the numbers 1 – 6.



OUTCOME GRID

(5) Record all of the outcomes (the sample space) in an outcome grid.

What are the possible outcomes for the roll?

What are the possible outcomes for the flip?

		Number Rolled					
		1	2	3	4	5	6
Coin Flip	H	H-1	H-2	H-3	H-4	H-5	H-6
	T	T-1	T-2	T-3	T-4	T-5	T-6

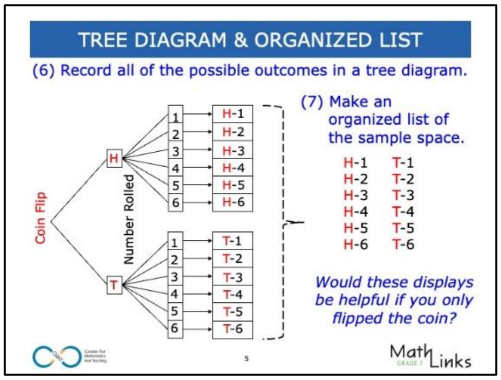
What does H-2 represent?

How many possible outcomes are there?

Would creating a grid be helpful if you only rolled the cube?

LESSON NOTES S1.2b: FLIP AND ROLL

Continued



TREE DIAGRAM & ORGANIZED LIST

(6) Record all of the possible outcomes in a tree diagram.

(7) Make an organized list of the sample space.

H-1	T-1
H-2	T-2
H-3	T-3
H-4	T-4
H-5	T-5
H-6	T-6

Would these displays be helpful if you only flipped the coin?

- Slide 5: For (6), use the slide to guide students as they label and create a tree diagram.

  Where should we record possible outcomes from the coin flip? In the first branch of the tree.

  From rolling a number cube? In the second branch.

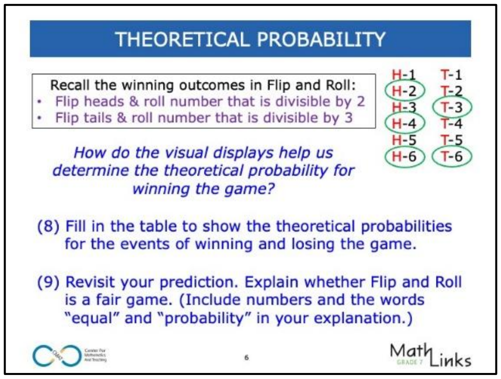
  For the outcomes of the flip and roll experiment? After the arrows.

  Why do we need to record outcomes for rolling the number cube twice? Each of the 6 outcomes can occur with either heads or tails.

  What would the tree look like if we recorded outcomes from rolling the number cube first? The six outcomes for rolling the number cube would be first. Each would have two branches for the two outcomes from flipping a coin. There would still be 12 outcomes in the sample space.

  For (7), students make an organized list of the sample space.

  How do we know we have them all and didn't miss any? Emphasize the importance of being systematic.



THEORETICAL PROBABILITY

Recall the winning outcomes in Flip and Roll:
- Flip heads & roll number that is divisible by 2
- Flip tails & roll number that is divisible by 3

How do the visual displays help us determine the theoretical probability for winning the game?

(8) Fill in the table to show the theoretical probabilities for the events of winning and losing the game.

(9) Revisit your prediction. Explain whether Flip and Roll is a fair game. (Include numbers and the words "equal" and "probability" in your explanation.)

- Slide 6: Compare the outcome grid, tree diagram, and organized list for **Flip and Roll.**

  Where is the sample space located on each display? Allow time for students to discuss and locate the sample spaces.

  How do the visual displays help us determine the theoretical probability for winning? We can see the total outcomes easily and locate the winning outcomes by circling them on each data display.

  For (8), students find theoretical probabilities for the events of winning or losing the game, and for (9) they revisit their prediction about the fairness of the game and justify their answer. Discuss as needed.

SLIDE DECK ALTERNATIVE S1.2b: FLIP AND ROLL

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slides 1 – 3

Flip and Roll is a game where you flip a coin and roll a number cube.

You win if:

- The coin lands on heads and the number on the cube is divisible by 2, or
- The coin lands on tails and the number on the cube is divisible by 3.

Otherwise, you lose.

(1) Restate the rules of the game in your own words.

(2) Do you think this is a fair game? Explain.

Describe one outcome where you win and one where you lose.

(3) Record your data for 10 trials.

(4) Record your results in the table and aggregate the class data.

Do you think this is a fair game?

Slides 4 – 6

(5) Record all of the outcomes (the sample space) in an outcome grid.

How many possible outcomes are there?

(6) Record all of the possible outcomes in a tree diagram.

(7) Make an organized list of the sample space.

How do the visual displays help us determine the theoretical probability for winning?

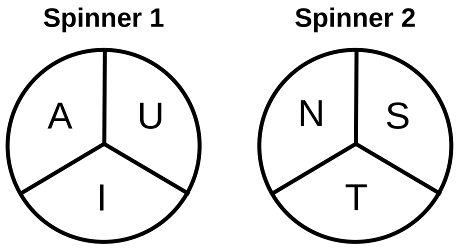
(8) Fill in the table for the probabilities for the events of winning and losing the game.

(9) Revisit your prediction. Is Flip and Roll a fair game? Explain.

PRACTICE 5

[SMP8]

For the two spinners to the right, one trial is spinning each spinner once, at the same time.

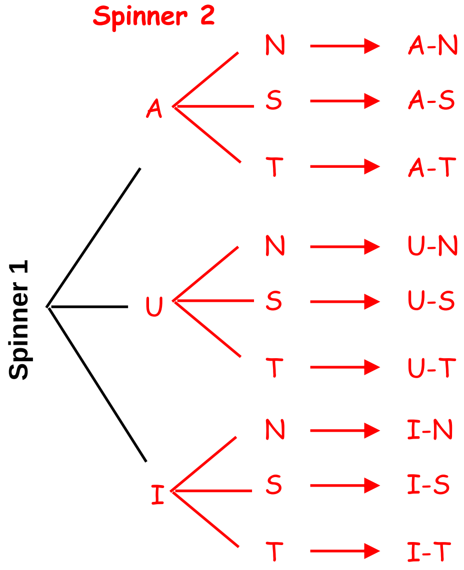


1. Make an outcome grid to display the sample space and a tree diagram to display the sample space.
   Order may vary. One possible answer for each:

Outcome Grid

		Spinner 2		
		N	S	T
Spinner 1	A	A-N	A-S	A-T
	U	U-N	U-S	U-T
	I	I-N	I-S	I-T

Tree Diagram



2. How many equally likely outcomes are there? 9

You spin each spinner once. Determine the theoretical probability of each event as a fraction, a decimal, and a percent. Use a repeat bar when necessary.

3. P(spinning an A) $\frac{3}{9}$; $0.\overline{3}$; $33.\overline{3}\%$

4. P(spinning the word “IT”) $\frac{1}{9}$; $0.\overline{1}$; $11.\overline{1}\%$

5. P(spinning any real word) $\frac{7}{9}$; $0.\overline{7}$; $77.\overline{7}\%$

6. P(spinning a nonsense word) $\frac{2}{9}$; $0.\overline{2}$; $22.\overline{2}\%$

7. Predict the number of times you are likely to spin the word “AT” in 300 spins.
   $P(AT) = \frac{1}{9}$. For 300 spins, $\frac{1}{9}(300) = 33.\overline{3}$, so about 33 times.

8. Change one letter in one of the spinners so that P(spinning a real English word) = 100%.
   Answers may vary. One possible answer: Change U to O.

RACE TO THE TOP REVISITED

The *MathLinks* Rubric: See Activity Routines on the Teacher Portal for directions. [SMP1, 3, 4, 8]
Students revisit the problem and revise their analysis as needed.

1. In the **Race to the Top** experiment, you rolled two number cubes and recorded the sums. Create a sample space (outcome grid, tree diagram, or list) for this experiment.

   One possible answer: Outcome Grid

		Number Cube #2					
		1	**2**	**3**	**4**	**5**	**6**
Number Cube #1	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

Revisiting contexts is common when continuing concept development.

2. Write the sum that has the greatest probability of occurring, and write the probability as a fraction.

   $P(\text{rolling a } 7) = \frac{6}{36} = \frac{1}{6}$

3. Write the sum that has the least probability of occurring, and write the probability as a fraction.

   $P(\text{rolling a } 2) = P(\text{rolling a } 12) = \frac{1}{36}$

RACE TO THE TOP REVISITED

Continued

4. Use your data from problems 2 – 3 in the **Race to the Top** experiment to complete this table. Answers will vary.

	2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**Total Trials**
Your results												
Fraction												
Class results												
Fraction												

5. Compare the theoretical probabilities to the experimental probabilities.

Answers will vary. Typically, the aggregated class data is closer to the theoretical probabilities than all of the individual student probabilities.

6. Javier made this sample space for the sum of two number cubes. Then he used the sample space to claim that P (sum = 7) = $\frac{3}{21} = \frac{1}{7}$. What is wrong with his work?

He didn't list all of the equally likely outcomes. For example, rolling a 5 on cube #1 and 2 on cube #2 is a different outcome than rolling a 2 on cube #1 and a 5 on cube #2, even though the result is the same.

1-1	1-2	1-3	1-4	1-5	1-6
2-2	2-3	2-4	2-5	2-6	
3-3	3-4	3-5	3-6		
4-4	4-5	4-6			
5-5	5-6				
6-6					

PROBABILITY EXPERIMENTS: GAMES AND PUZZLES

We will play a probability game that involves converting fractions into decimals that terminate or repeat, and determine the fairness of the game. We will create spinners from clues and analyze these probability puzzles.

[7.NS.2d, 7.SP.6, 7.SP.7ab, 7.SP.8abc; SMP1, 2, 4, 5, 8]

GETTING STARTED

Watch for (all): Do students correctly place the decimal point?

Find the decimal equivalent for each fraction below. Use a repeat bar when necessary.

1. $\frac{1}{1}$ 1 or 1.0	2. $\frac{1}{2}$ 0.5	3. $\boxed{\frac{1}{3}}$ $0.\overline{3}$	4. $\frac{1}{4}$ 0.25
5. $\frac{1}{5}$ 0.2	6. $\boxed{\frac{1}{6}}$ $0.1\overline{6}$	7. $\boxed{\frac{1}{7}}$ $0.\overline{142857}$	8. $\frac{1}{8}$ 0.125
9. $\boxed{\frac{1}{9}}$ $0.\overline{1}$	10. $\frac{1}{10}$ 0.1	11. $\boxed{\frac{1}{11}}$ $0.\overline{09}$	12. $\boxed{\frac{1}{12}}$ $0.08\overline{3}$

13. Box all the fractions that are equivalent to repeating decimals. Circle all the fractions that are equivalent to terminating decimals.

THE TERMINATOR: EXPERIMENTAL PROBABILITY

[SMP2,4,5,8]

The Game: Roll two number cubes labeled 1 – 6 and create a fraction less than or equal to 1, using the values you rolled. For example, if you roll a 4 and 3, the fraction will be $\frac{3}{4}$.

- You win if the fraction results in a repeating decimal.
- You lose if the fraction results in terminating decimal.

1. Roll the cubes 20 times and record the results in the table. Circle each trial # you win.

Trial #	Numbers Rolled	Fraction	Decimal
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

Trial #	Numbers Rolled	Fraction	Decimal
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

	My Game Data			Class Game Data		
	Fraction	Decimal	Percent	Fraction	Decimal	Percent
***P*(losing)**						
***P*(winning)**						

2. Based on “My Game Data” results, which represent your experimental probability, do you think this is a fair game? Explain.
Answers may vary. Students will probably make a preliminary determination based upon what they personally observe while playing, and revisit this question after determining the theoretical probability.

Number cubes, calculators (optional)

2+

THE TERMINATOR: THEORETICAL PROBABILITY

1. Make an outcome grid to determine the theoretical probabilities of winning and losing the Terminator game. Using two different colored cubes helps to keep track of outcomes.

Number Cube (__________)

Number Cube (__________)

	1	**2**	**3**	**4**	**5**	**6**
1	$\frac{1}{1} \to$ T	$\frac{1}{2} \to$ T	$\frac{1}{3} \to$ R	$\frac{1}{4} \to$ T	$\frac{1}{5} \to$ T	$\frac{1}{6} \to$ R
2	$\frac{1}{2} \to$ T	$\frac{2}{2} \to$ T	$\frac{2}{3} \to$ R	$\frac{2}{4} \to$ T	$\frac{2}{5} \to$ T	$\frac{2}{6} \to$ R
3	$\frac{1}{3} \to$ R	$\frac{2}{3} \to$ R	$\frac{3}{3} \to$ T	$\frac{3}{4} \to$ T	$\frac{3}{5} \to$ T	$\frac{3}{6} \to$ T
4	$\frac{1}{4} \to$ T	$\frac{2}{4} \to$ T	$\frac{3}{4} \to$ T	$\frac{4}{4} \to$ T	$\frac{4}{5} \to$ T	$\frac{4}{6} \to$ R
5	$\frac{1}{5} \to$ T	$\frac{2}{5} \to$ T	$\frac{3}{5} \to$ T	$\frac{4}{5} \to$ T	$\frac{5}{5} \to$ T	$\frac{5}{6} \to$ R
6	$\frac{1}{6} \to$ R	$\frac{2}{6} \to$ R	$\frac{3}{6} \to$ T	$\frac{4}{6} \to$ R	$\frac{5}{6} \to$ R	$\frac{6}{6} \to$ T

2. Determine the theoretical probabilities of winning and losing as a fraction, decimal and percent.

$P(\text{losing}) = \frac{24}{36} = \frac{2}{3} = 0.\overline{6} = 66.\overline{6}\%$ $P(\text{winning}) = \frac{12}{36} = \frac{1}{3} = 0.\overline{3} = 33.\overline{3}\%$

3. Based on the theoretical probabilities, out of 3,000 rolls, about how many times is winning expected? About 1,000 times.

4. Go back to "My Game Data" and "Class Game Data" on the previous page.

   a. Explain how **your** experimental probability compares to the theoretical probability.
   Answers will vary.

   b. Explain how **the class's** experimental probability compares to the theoretical probability.
   Answers will vary.

5. Based on the theoretical probabilities, explain why this is not a fair game.
The ratio of outcomes is 2 losses for every 1 win. Losing is twice as likely as winning.

6. Explain one way to make this a fair game.
Answers may vary. One way is to change the rules to say, "You win if the fraction results in a repeating decimal or a fraction that is equal to 1".

1+

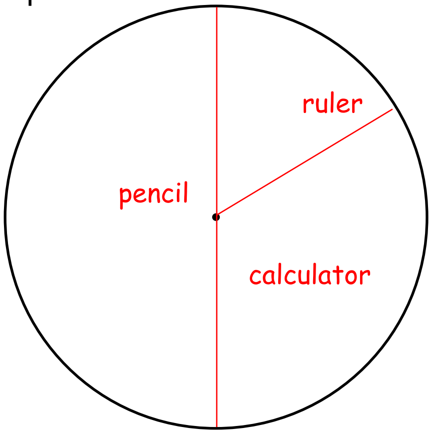
Journal
Problem 5, 6

SPINNER PUZZLES

[SMP1,4]

Follow your teacher's directions. Order of items in the chart may vary. One possibility for each:

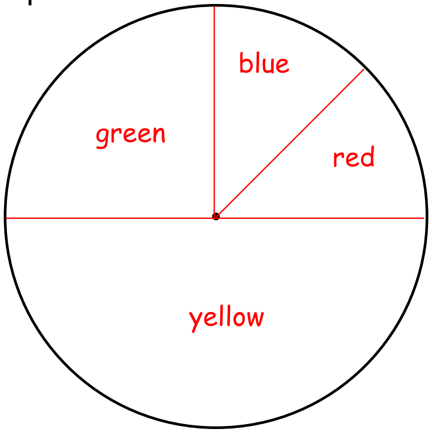
(1) Spinner A



	Fraction	Decimal	Percent
***P*(pencil)**	$\frac{1}{2}$	0.5	50%
***P*(ruler)**	$\frac{5}{30}=\frac{1}{6}$	$0.1\overline{6}$	$16.\overline{6}\%$
***P*(calculator)**	$\frac{2}{6}=\frac{1}{3}$	$0.\overline{3}$	$33.\overline{3}\%$
TOTAL	$\frac{6}{6}=1$	$0.\overline{99}$ or 1*	$99.\overline{9}\%$ or 100%*

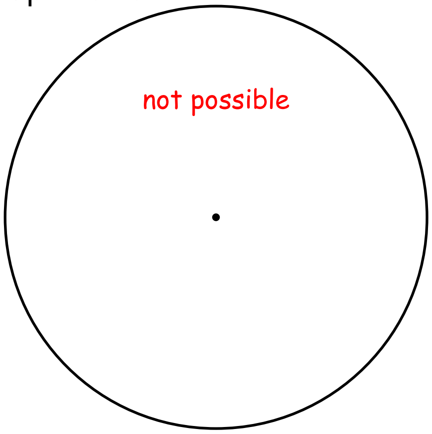
*See Math Background: "A Clever Trick" for more information.

(2) Spinner



	Fraction	Decimal	Percent
***P*(blue)**	$\frac{1}{8}$	0.125	12.5%
***P*(yellow)**	$\frac{4}{8}=\frac{1}{2}$	0.5	50%
***P*(green)**	$\frac{2}{8}=\frac{1}{4}$	0.25	25%
***P*(red)**	$\frac{1}{8}$	0.125	12.5%
TOTAL	$\frac{8}{8}=1$	1.00	100%

(3) Spinner C



	Fraction	Decimal	Percent
***P*(HW free)**	$\frac{6}{24}=\frac{1}{4}$	0.25	25%
***P*(games)**	$\frac{3}{4}$	0.75	75%
***P*(principal)**	$\frac{1}{4}$	0.25	25%
TOTAL	$1\frac{1}{4}$	1.25	125%

(4) **Change one clue to make Spinner C work.** Answers will vary. One possible answer: The probability of spinning a day of math games is twice the chance of spinning a homework free pass.

LESSON NOTES S1.3: SPINNER PUZZLES

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

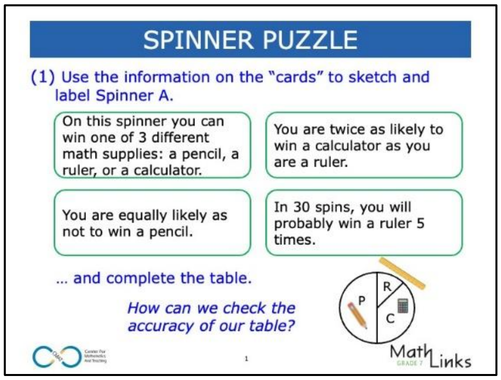
Students work in groups to create a circular spinner from a set of clues. The clues describe the probabilities of each spinner section in words and numbers. A table accompanies each spinner as a more complete recording mechanism. Note that one set of clues is purposely flawed and must be corrected.

- Slide 1: Explain the activity to the students. Reveal the clue one at a time, pausing for a class discussion after each. For (1), students work together to create a spinner and fill in the table. Ask questions after each card is revealed:

    - ***What does this card tell us?***
    - ***What can we record in our table?***
    - ***If nothing can be recorded in the table yet, what relevant notes can we make?***

  How can we check the accuracy of our table? Check to see the totals are correct.

  What should the totals be for each column? 1, 1, and 100%, respectively. This might be students' first experience with $0.\overline{99}$ = 1 and $99.\overline{9}\%$ = 100%. (See Math Background: "A Clever Trick", for more information.)



SPINNER PUZZLE

(1) Use the information on the "cards" to sketch and label Spinner A.

On this spinner you can win one of 3 different math supplies: a pencil, a ruler, or a calculator.

You are twice as likely to win a calculator as you are a ruler.

You are equally likely as not to win a pencil.

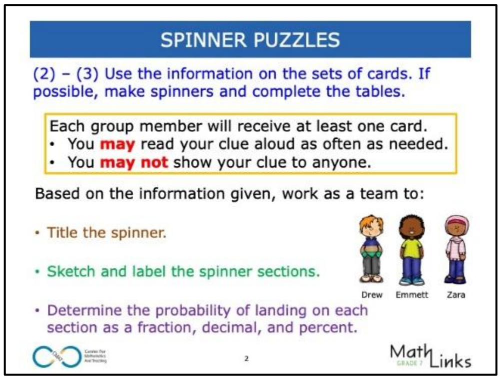
In 30 spins, you will probably win a ruler 5 times.

… and complete the table.

How can we check the accuracy of our table?

- Slide 2: For (2) and (3), discuss the directions for the activity. Be sure students notice the "if possible" portion of the directions. Distribute **R1-2 Spinner Clue Cards**, first the B cards and then the C cards.

  Remind students to take notes as their partners read aloud their clues. Encourage them to work collaboratively.



SPINNER PUZZLES

(2) – (3) Use the information on the sets of cards. If possible, make spinners and complete the tables.

Each group member will receive at least one card.
- You **may** read your clue aloud as often as needed.
- You **may not** show your clue to anyone.

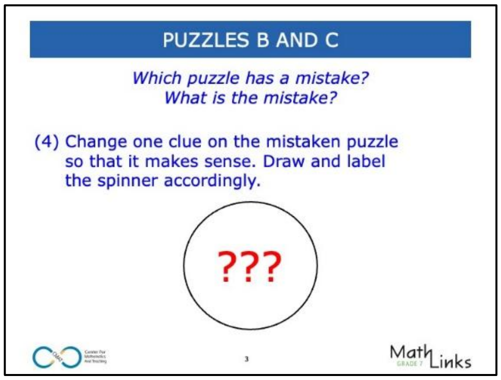
Based on the information given, work as a team to:

- Title the spinner.
- Sketch and label the spinner sections.
- Determine the probability of landing on each section as a fraction, decimal, and percent.

- Slide 3: Discuss.

  Which puzzle has a mistake? Puzzle C
  What is the mistake? The total is more than 1 whole.

  For (4), students change one of the clues on Spinner Puzzle C so that the puzzle makes sense. They draw and label the spinner to show the edits. Ask students to share.



PUZZLES B AND C

Which puzzle has a mistake?
What is the mistake?

(4) Change one clue on the mistaken puzzle so that it makes sense. Draw and label the spinner accordingly.

SLIDE DECK ALTERNATIVE S1.3: SPINNER PUZZLES

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction.
The Slide Deck Alternative offers a modified option.

Slide 1

(1) Use the information on the "cards" below to sketch and label Spinner A and fill in the table.

On this spinner you can win one of 3 different math supplies: a pencil, a ruler, or a calculator.	You are twice as likely to win a calculator as you are a ruler.
You are equally likely as not to win a pencil.	In 30 spins, you will probably win a ruler 5 times.

How can we check the accuracy of our table?

Slide 2

(2) – (3) Use the information on the sets of cards. If possible, make spinners and complete the tables.

Each group member will receive at least one card.
- You **may** read your clue aloud as often as needed.
- You **may not** show your clue to anyone.

Based on the information given, work as a team to:
- Title the spinner.
- Sketch and label the spinner sections.
- Determine the probability of landing on each section as a fraction, decimal, and percent.

Slide 3

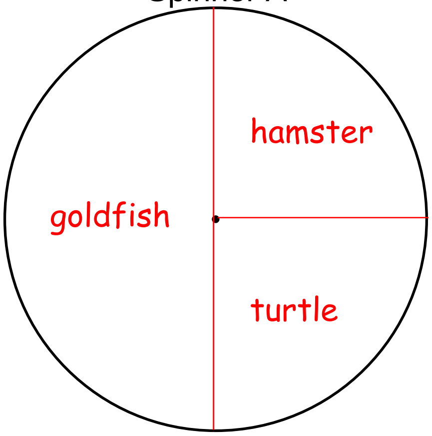
Which puzzle has a mistake? What is the mistake?

(4) Change one clue on the flawed puzzle so that it makes sense. Draw and label the spinner accordingly. Share and discuss.

PRACTICE 6

1. Draw Sinner A to match the clues below. Express each probability as a fraction, decimal, and percent in the table.

Spinner A



Clue #1	**Clue #2**
The Pet Palace is giving away turtles, hamsters, and goldfish as prizes. You are certain to get exactly one prize.	The probability you will get a hamster is $\frac{1}{4}$.
Clue #3	**Clue #4**
The probability you will get a goldfish is twice the probability you will get a turtle.	It is more likely you will get a goldfish than a turtle.

2. In the table below, let *P*(hamster) refer to the probability of getting a hamster by spinning, and so on.

	Fraction	**Decimal**	**Percent**
***P*(turtle)**	$\frac{1}{4}$	0.25	25%
***P*(hamster)**	$\frac{1}{4}$	0.25	25%
***P*(goldfish)**	$\frac{2}{4}=\frac{1}{2}$	0.5	50%
TOTAL	$\frac{4}{4}=1$	1.0	100%

3. What do you notice about the total in each column?
They all are equivalent to 1.

4. What is the probability of getting a dog? 0% An animal? 100%

5. If you spin the spinner 400 times, about how many hamsters will you expect to get?
You would expect the spinner to land on hamster about 100 times.

6. What is the greatest value the probability of an event can have? 1 or 100%

The least? 0 or 0%

7. Were there any clues you did not need? Explain.
Clue #4 is unnecessary because Clue #3 already stated that *P*(goldfish) > *P*(turtle).

THE CEREAL BOX SIMULATION

Part 1: There are four different animal prizes in Krispi Krunchy Cereal, and you want to collect all four. You have an equally likely chance of getting any of the prizes when buying a box. How many boxes do you think you need to buy to get all four? Design an experiment. Carry it out.

1. First make a prediction. What is your "gut feeling?" _______________

   Answers will vary.

2. What tools or materials will you use to generate a simulation for collecting 4 different objects? How many times will you perform the experiment?

   Answers will vary. Good tools include a number cube (use 1-4 only), a spinner with 4 equal-sized areas, 4 slips of paper, four different colors of cubes, etc. Randomness is important. If students work in groups, they may choose to perform their experiment more times and combine results.

3. Perform your experiment. Collect and organize your data.

4. Compare your data to your prediction and write a conclusion.

Part 2: Now, suppose there are four different animal prizes, AND each of them comes in two different colors.

5. Repeat problems 1-4 with this new information.

6. How is the simulation in Part 2 different than the one described in Part 1?

   Part 1 suggests a uniform probability model that assigns equal probabilities to all outcomes. Part 2 suggests a compound probability model, although students may design an experiment with 8 different equally likely outcomes (uniform probability).

REVIEW

BIG SQUARE PUZZLE: PROBABILITY

See Activity Routines on the Teacher Portal for directions.

Your teacher will give you a **Big Square Puzzle** to complete with partners. After finishing, do the following.

1. In the puzzle, $\frac{1}{6}$ = __$0.16\overline{6}$__ . Circle all numbers below that are also equal to $\frac{1}{6}$.

[✓ $\frac{2}{12}$] [✓ $\frac{0.5}{3}$] 0.16 [✓ 0.1666…] [✓ $0.1\overline{66}$] [✓ $0.1\overline{6}$]

2. Write $\frac{1}{6}$ as:
   a. an exact percent value __$16.\overline{6}$ %__
   b. a percent rounded to the nearest whole percent __17%__
   c. a percent rounded to the nearest tenth of a percent __16.7

Why Doesn't it Belong? And Big Square Puzzles are Activity Routines that appear mostly in the Review section in several units in grades 6 – 8.

Why Doesn't it Belong? promotes creative and divergent thinking, and encourages mathematical reasoning and justification.

Big Squares are motivating collaborative practice.

WHY DOESN'T IT BELONG?: PROB

See Activity Routines on the Teacher Portal for directions.

Four different weather apps' reports about the probability it will ra

1. Choose one of these news stations' predictions and explain w e others. Then choose at least one more and explain why it do

A There is a $83.3\overline{3}$% chance of rain.	**B** There is a $\frac{5}{6}$ chance of rain.
C There is a $0.8\overline{3}$ chance of rain.	**D** There is a 0.83 chance of rain.

Some possible answers:

A doesn't belong because it is the only value as a percent.

B doesn't belong since it is the only value as a fraction.

C doesn't belong because it is the only value as a decimal that repeats (in non-zero digits).

D doesn't belong because it is the only value as a decimal that repeats in zeroes (terminates) and is not equivalent to $\frac{5}{6} = 83.3\overline{3}\% = 0.8\overline{3}$.

MATCH AND COMPARE SORT: PROBABILITY

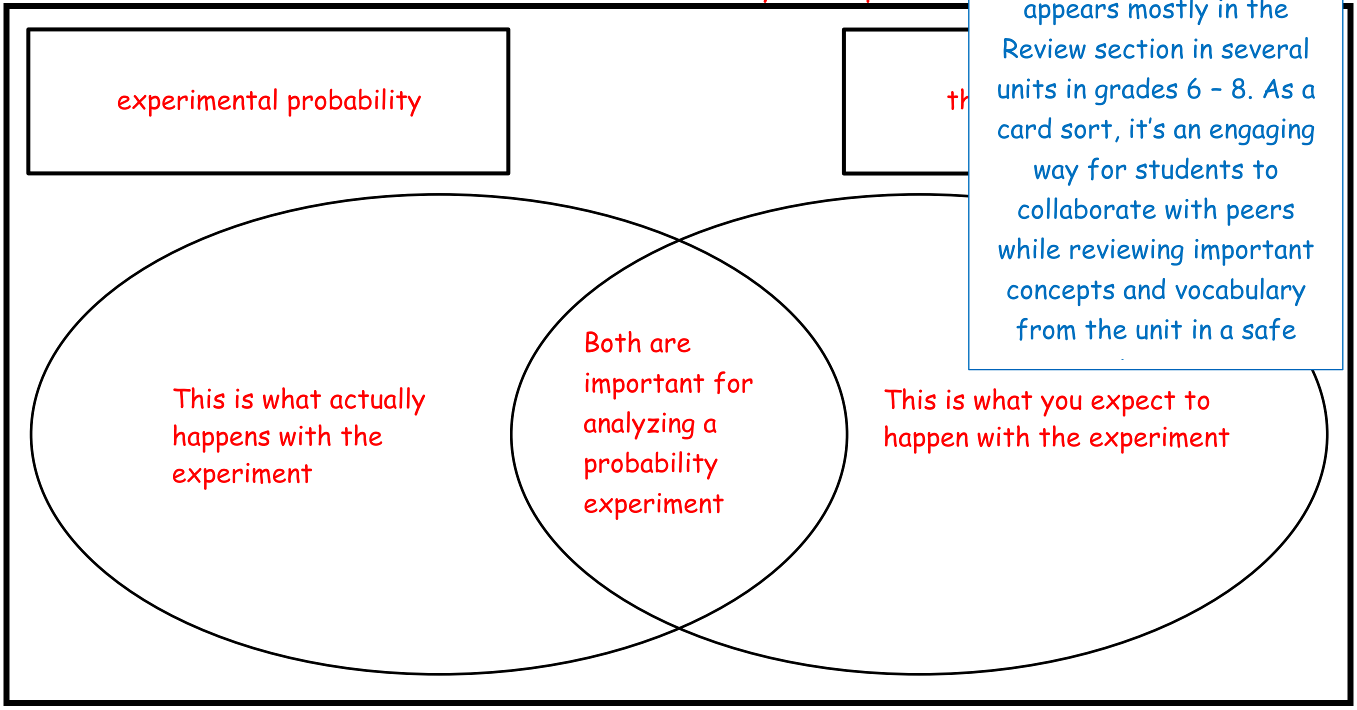
See Activity Routines on the Teacher Portal for directions. [SMP6]

1. Individually, match your word cards to your description cards, discuss with your partner(s), and record all of your results in the table.

Card set △			Card set ○		
Card number	**word**	**Card letter**	**Card number**	**word**	**Card letter**
I	experimental probability	D	**I**	theoretical probability	A
II	outcome	A	**II**	sample space	C
III	terminating decimal	B	**III**	repeating decimal	D
IV	fair game	C	**IV**	random sample	B

2. Partners, choose a pair of numbered matched cards and record t same and those that are different. Answers will vary. One possible

Match and Compare Sort is an Activity Routine that appears mostly in the Review section in several units in grades 6 – 8. As a card sort, it's an engaging way for students to collaborate with peers while reviewing important concepts and vocabulary from the unit in a safe



3. Partners, choose another pair of numbered matched cards and discuss the attributes that are the same and those that are different.

POSTER PROBLEMS: PROBABILITY

See Activity Routines on the Teacher Portal for directions. [SMP2,3,4]

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start post
- Each group will have a different colored marker. Our group marl

Part 2: Do the problems on the posters by following your teacher's d as needed.

Poster 1 (or 5)	Poster 2 (or 6)	Poster 3 (or 7)
You roll two number cubes and find the product. You win if the product is odd.	You flip a coin and spin a fair spinner with the colors red, yellow, and blue. You win if you flip heads and land on red or blue.	You roll a fair 6-side number cube (numbered 1 – 6) and pull a marble o of a bag with 3 red and 2 blue marbles. You win if you roll a number less than 3 and choose a red marble.

Poster Problems is an Activity Routine that appears in the Review section in every unit in grades 6 – 8. It is an engaging way for students to solve problems and analyze work together. When teachers watch and listen, they are able to gauge student understanding and identify areas for review or reteaching. By establishing classroom norms, all students will find a safe place to contribute ideas and collaborate with peers.

A. On your poster:
   - Summarize the probability experiment. Abbreviate as needed.
   - Summarize the event that describes how you win. Abbreviate as needed.

B. Create a sample space display for the two events. Make sure to label the display.

C. Find the probability for winning the game as a fraction, decimal and percent.

D. Edit the event that describes winning the game to make it fair. Write the new winning event on the poster.

Part 3: Go to your start poster and copy the abbreviated descriptions of the probability experiment and the **new** event describing how you win. At your seat, use words and numbers to justify whether it is now a fair game.

Chart paper or board space, markers

4

Journal Part 3

POSTER PROBLEMS: PROBABILITY

Answer Key

Poster 1 (or 5):

B. TWO CUBES

	1	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	13
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

C. $P(\text{win}) = \frac{9}{36} = \frac{1}{4}$; 0.25 = 25%

D. *Answers will vary. One possibility:*
You win if the product is less than 10
or the product is 36.

Poster 2 (or 6):

B. COIN AND SPINNER GAME

	Red	**Yellow**	**Blue**
H	H-R	H-Y	H-B
T	T-R	T-Y	T-B

C. $P(\text{win}) = \frac{2}{6} = \frac{1}{3}$; $0.\overline{3} = 33.\overline{3}\%$

D. *Answers will vary. One possibility:*
You win if you spin red (regardless of coin)
or if you flip heads and spin yellow.

Poster 3 (or 7):

B. CUBE AND MARBLES

	1	**2**	**3**	**4**	**5**	**6**
R	R1	R2	R3	R4	R5	R6
R	R1	R2	R3	R4	R5	R6
R	R1	R2	R3	R4	R5	R6
B	B1	B2	B3	B4	B5	B6
B	B1	B2	B3	B4	B5	B6

C. $P(\text{win}) = \frac{6}{30} = \frac{1}{5}$ = 0.20 = 20%

D. *Answers will vary. One possibility:*
You win if you roll a number less than 4 and
pull red marble OR number greater than 3
and pull a blue marble.

Poster 4 (or 8):

B. COIN AND NUMBER CUBE

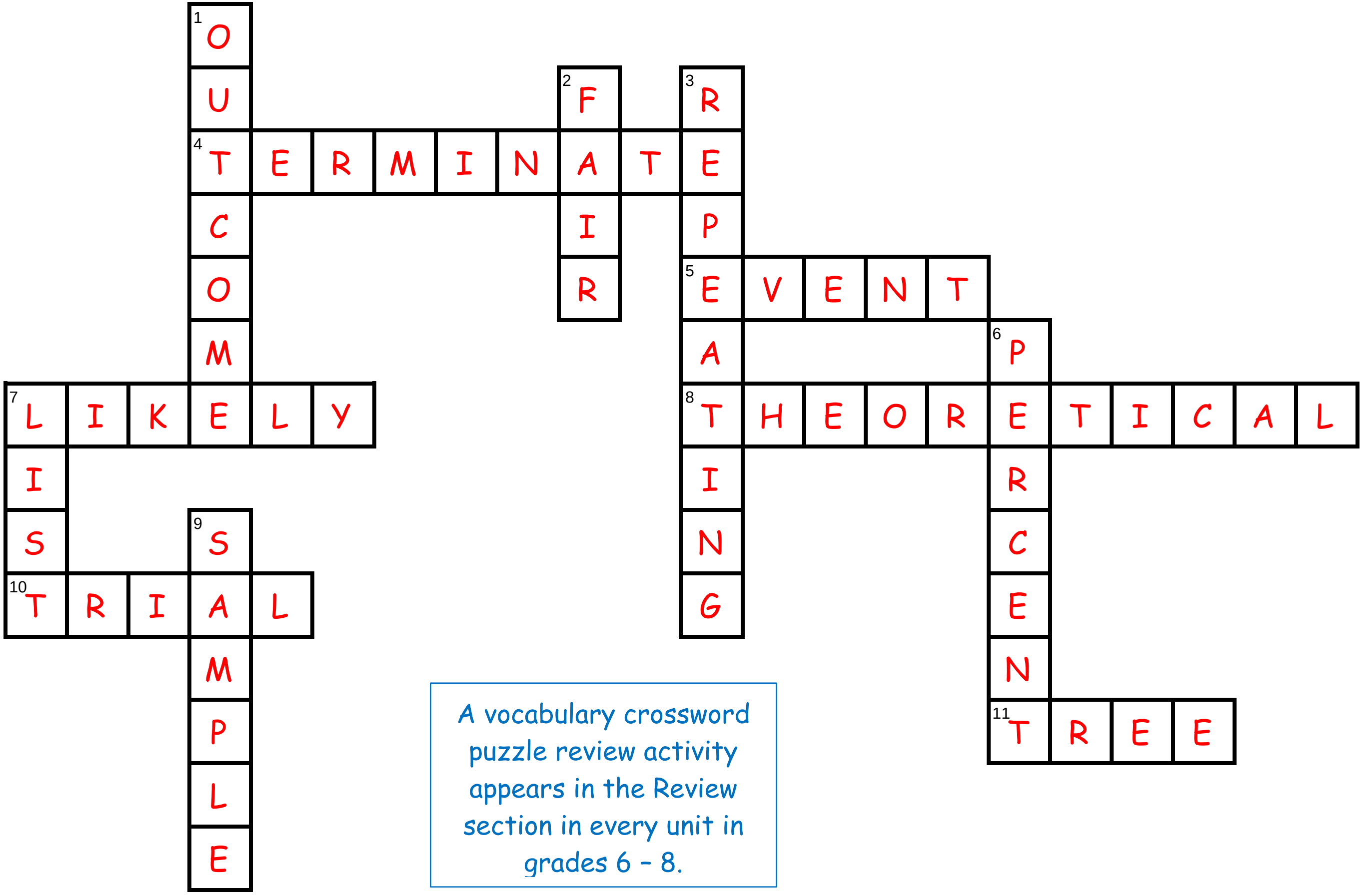
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
H	H1	H2	H3	H4	H5	H6	H7	H8
T	T1	T2	T3	T4	T5	T6	T7	T8

C. $P(\text{win}) = \frac{4}{16}$ = 0.25 = 25%

D. *Answers will vary. One possibility:*
You win if you roll heads and a number less
than 5 or you roll tails and a prime number.

VOCABULARY REVIEW

[SMP6]



A vocabulary crossword puzzle review activity appears in the Review section in every unit in grades 6 – 8.

Across

4 When converted, $\frac{3}{5}$ is an example of a decimal that will ____.

5 Subset of the sample space

7 If P(event) > 70%, it is ____ it will happen

8 When flipping a fair coin, the ___ probability of getting heads is 50%.

10 One repetition of a probability experiment

11 One way to display probability outcomes: _____ diagram

Down

1 The result of a probability experiment

2 A game where each player has the same chance of winning.

3 A number that ends in a repetition of a block of digits, like 0.23232323… is an example of a ___ decimal

6 Per hundred

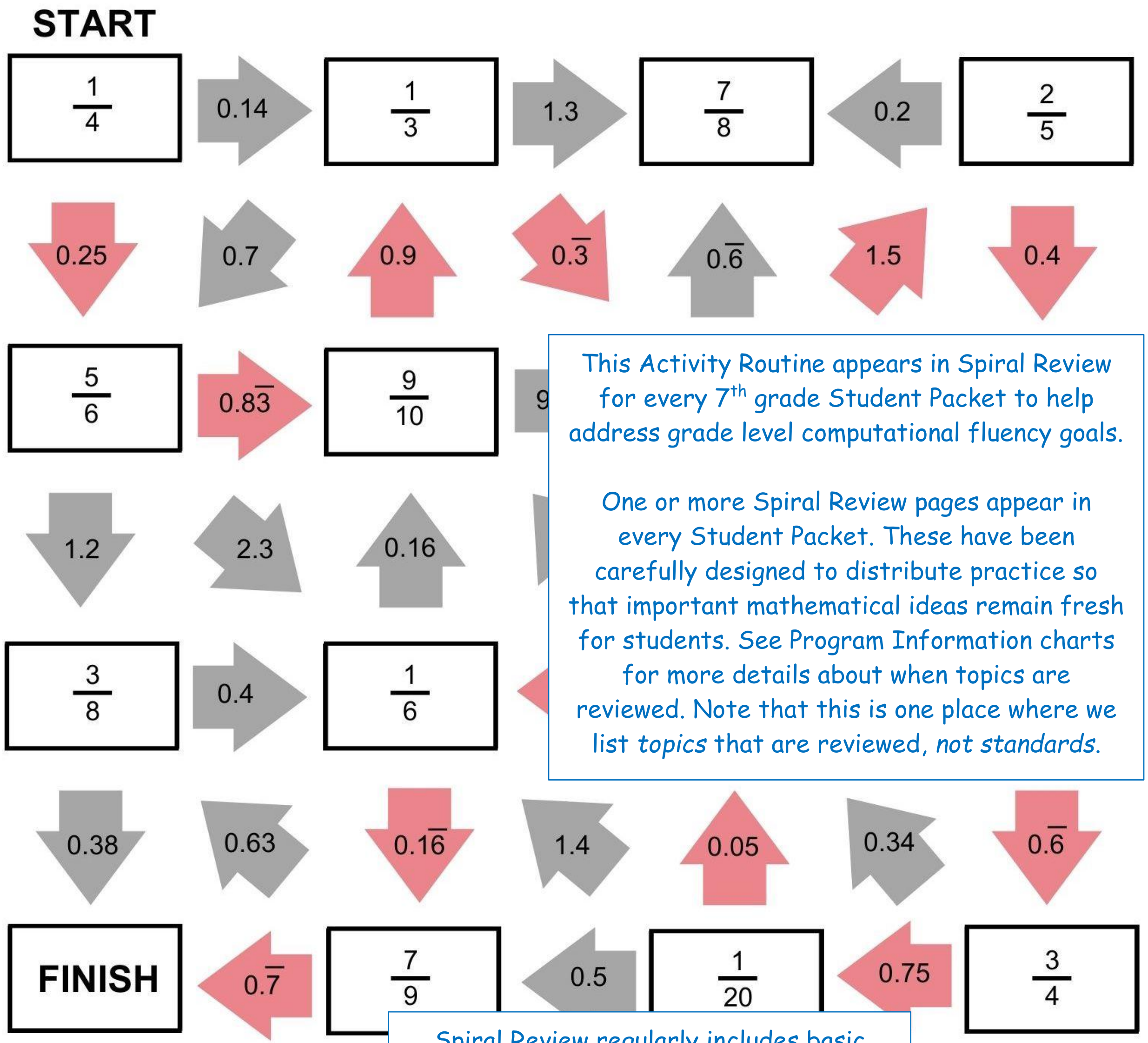
7 An organized way to display probability outcomes

9 All possible outcomes in a probability experiment are called the _____ space.

SPIRAL REVIEW

See Activity Routines on the Teacher Portal for directions.

1. **Math Path Fluency Challenge**: Use what you know about fraction-decimal equivalencies to find the correct path from Start to Finish. Correct arrows are indicated in red.



This Activity Routine appears in Spiral Review for every 7th grade Student Packet to help address grade level computational fluency goals.

One or more Spiral Review pages appear in every Student Packet. These have been carefully designed to distribute practice so that important mathematical ideas remain fresh for students. See Program Information charts for more details about when topics are reviewed. Note that this is one place where we list *topics* that are reviewed, *not standards*.

Spiral Review regularly includes basic conversions and mental math problems.

2. Complete the table:

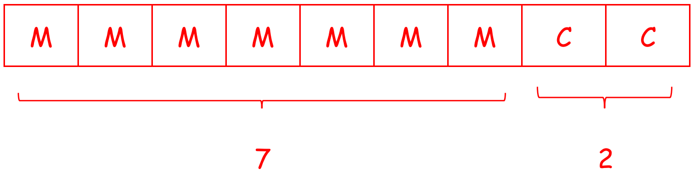
	10%	20%	5%	15%	1%
$100	$10	$20	$5	$15	$1
$36	$3.60	$7.20	$1.80	$5.40	$0.36
$55	$5.50	$11	$2.75	$8.25	$0.55

SPIRAL REVIEW

Continued

3. Rocco likes to make his own chocolate milk. He mixes 7 ounces of milk with 2 ounces of chocolate syrup each day.

   a. Make a tape diagram to represent this relationship.



   b. Rocco is going to the grocery store to buy chocolate milk supplies for 5 days. How many ounces of milk and chocolate syrup should he buy?
   35 oz of milk and 10 oz syrup

   c. Rocco is having friends over, and he makes a giant batch of chocolate milk. When he finishes mixing, he has 54 ounces of chocolate milk. How many ounces of milk did he use?
   42 oz of milk

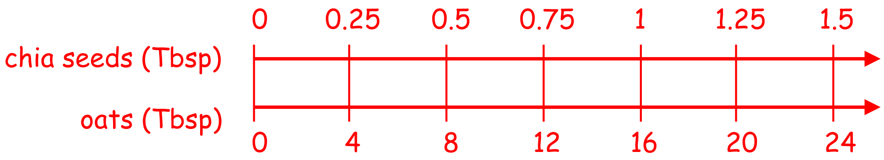
4. An overnight oats recipe calls for $\frac{1}{2}$ Tbsp chia seeds and 8 Tbsp oats for each person.

   Entries in table and on double number line may vary.

   a. Make a table that displays different equivalent ratios for the statement.

chia seeds (Tbsp)	0.5	1	0.25	1.5	2
oats (Tbsp)	8	16	4	24	32

   b. Make a double number line or graph to display different equivalent ratios. Include some of the data from your table.



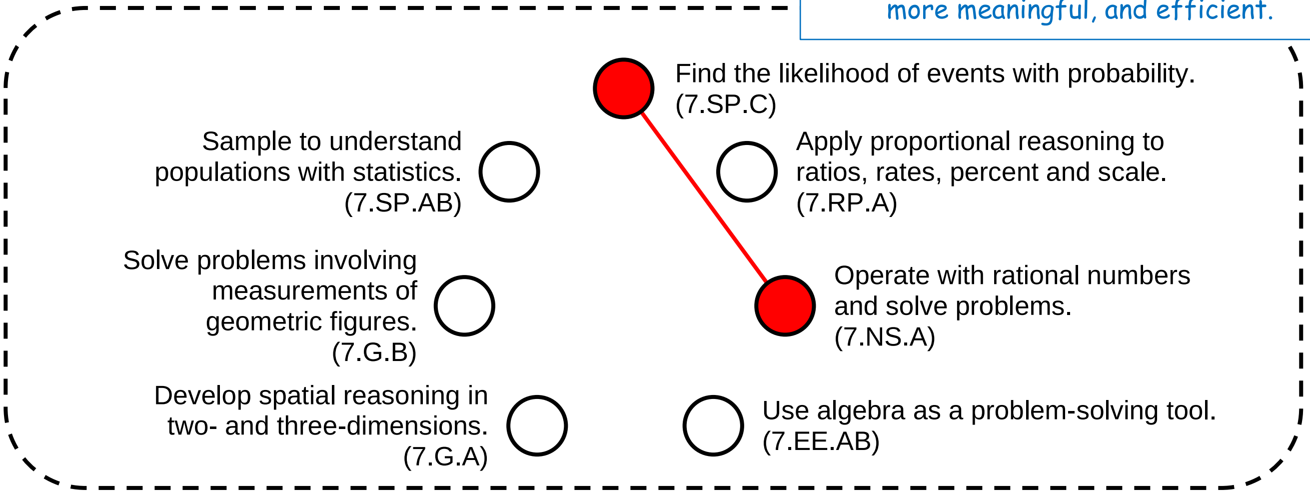
Spiral Review for every unit has been carefully designed to distribute practice so that important topics remain fresh. See the chart in Program Information for more details about when topics are reviewed.

REFLECTION

Identifying Big Ideas and their connections helps students view mathematics as a cohesive and connected body of knowledge. It also makes learning more meaningful, and efficient.

Answers will vary. Some possible answers:

1. **Big Ideas**. Shade all circles that describe big ideas in this unit and draw connections between any you noticed.



- ● Find the likelihood of events with probability. (7.SP.C)
- ○ Sample to understand populations with statistics. (7.SP.AB)
- ○ Apply proportional reasoning to ratios, rates, percent and scale. (7.RP.A)
- ○ Solve problems involving measurements of geometric figures. (7.G.B)
- ● Operate with rational numbers and solve problems. (7.NS.A)
- ○ Develop spatial reasoning in two- and three-dimensions. (7.G.A)
- ○ Use algebra as a problem-solving tool. (7.EE.AB)

Give an example from this unit of one of the connections above.

- Probabilities as fractions, decimals, and percents
- Probabilities are like rates because they represent a part to whole relationship

2. **Unit Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before, OR something you would still like to work on.

3. **Mathematical Practice.** Explain a situation where you used theoretical probabilities (i.e., a probability model) to determine if a game was fair [SMP 2, 3, 4]. Then circle one more SMP on the back of this packet that you think was addressed in this unit and be prepared to share an example.

- For Flip and Roll, an outcome grid and tree diagram showed the game was not fair
- For the Terminator, the ratio of losses to wins is 2:1, a new rule is needed to make the game fair

4. **Making Connections.** How might use probability in your life?

When playing dice games, it is understandable which sums will probably come up more frequently

This section is available on the public website for parents and others who support students.

STUDENT RESOURCES

Word or Phrase	Definition
dependent events	Two events are dependent if the occurrence (or nonoccurrence) of one event affects the likelihood of the other. See independent events.
event	An event is a subset of the sample space. See sample space. In the probability experiment of rolling a number cube, "rolling an even number" is an event, because getting a 2, 4, or 6 is a subset (part) of the sample space of {1, 2, 3, 4, 5, 6}.
experimental probability	In a repeated probability experiment, the experimental probability of an event is the number of times the event occurs divided by the number of trials. This is also called empirical probability. If, in 25 rolls of a number cube, we obtain an even number 11 times, we say that the experimental probability of rolling an even number is $\frac{11}{25} = 0.44 = 44\%$.
fair game	A game of chance is a fair game if all players have equal probabilities of winning. A two-person game of chance is a fair game if each player has probability $\frac{1}{2}$ of winning, that is, if each player has the same probability of winning as of losing.
independent events	Two events are independent if the occurrence (or nonoccurrence) of one event does not affect the likelihood of the other. See dependent. In the probability experiment of rolling a number cube and flipping a coin, the event of rolling a 1 is independent of the event of getting heads on the coin flip. The probability of rolling the 1 is $\frac{1}{6}$, no matter what the outcome of the coin flip is. In other words, the cube roll does not depend at all on the coin flip.
outcome	An outcome is a result of a probability experiment. If we roll a number cube, there are six possible outcomes: 1, 2, 3, 4, 5, 6.
percent	A percent is a number expressed in terms of the unit $1\% = \frac{1}{100}$.

This first part of Student Resources includes precise definitions. When they first arise in a unit, teachers help their students unpack these definitions and record them in My Word Bank using their own words, examples, and/or diagrams.

Word or Phrase	Definition
probability	The probability of an event is a measure of the likelihood of that event occurring. The probability $P(E)$ of the event E occurring satisfies $0 \le P(E) \le 1$. If the event, E, is certain to occur, then $P(E) = 1$. If the event E is impossible, then $P(E) = 0$. When flipping a fair coin, the probability that it will land on heads is $\frac{1}{2} = 0.5 = 50\%$.
probability experiment	A probability experiment is an experiment in which the results are subject to chance. Rolling a number cube can be considered a probability experiment.
repeating decimal	A repeating decimal is a decimal that ends in repetitions of the same block of digits. The repeating decimal 52.19343434… ends in repetitions of the block “34.” An abbreviated notation for the decimal is $52.19\overline{34}$, where the bar over 34 indicates that the block is repeated. The terminating decimal 4.62 is regarded as a repeating decimal. Its value is 4.620000…
sample space	The sample space for a probability experiment is the set of all possible outcomes of the experiment. In the probability experiment of rolling a number cube, the sample space can be represented as the set {1, 2, 3, 4, 5, 6}.
simulation	Simulation is the imitation of one process by means of another process. We may simulate rolling a number cube by drawing a card blind from a group of six identical cards labeled one through six. We may simulate the weather by means of computer models.
terminating decimal	A terminating decimal is a decimal whose digits are 0 from some point on. Terminating decimals are regarded as repeating decimals, though the final 0’s in the expression for a terminating decimal are usually omitted. See repeating decimal. 4.62 = 4.62000000… is a terminating decimal with value $4 + \frac{6}{10} + \frac{2}{100}$.
theoretical probability	The theoretical probability of an event is a measure of the likelihood of the event occurring. In the probability experiment of rolling a (fair) number cube, there are six equally likely outcomes, each with probability $\frac{1}{6}$. Since the event of rolling an even number corresponds to 3 of the outcomes, the theoretical probability of rolling an even number is 3 out of 6, or $3 \bullet \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$.
trial	Each performance or repetition of a probability experiment is called a trial. Flipping a coin 25 times can be viewed as 25 trials of the probability experiment of flipping a coin once.

Phrases That Describe Probabilities

In their assessment reports on climate change, climate scientists attach the following probabilities to common expressions of likelihood:

Virtually certain:	> 99% probability
Extremely likely:	> 95% probability
Very likely:	> 90% probability
Likely:	> 66% probability
More likely than not:	> 50% probability
About as likely as not:	33 to 66% probability
Unlikely:	< 33% probability
Very unlikely:	< 10% probability
Extremely unlikely:	< 5% probability
Exceptionally unlikely:	< 1% probability

This next part of Student Resources includes examples and explanations for use in class or at home.

Estimating Probabilities from an Experiment With Equally Likely Outcomes

To estimate the probability of an event E, repeat the experiment a number of times and observe how many times the event occurs. The estimate for the probability of the event E occurring is then given by the fraction:

$$\text{estimate} = \frac{\text{number of times an event } E \text{ occurs}}{\text{number of trials}} = \frac{\text{numerator}}{\text{denominator}}$$

In a probability experiment of rolling a number cube with six equally likely outcomes, each has probability $\frac{1}{6}$.

The event of rolling an odd number corresponds to three outcomes: 1, 3, or 5. Below is data from an experiment where a cube is rolled 10 times.

Trial #	1	2	3	4	5	6	7	8	9	10
Outcome	4	5	6	3	5	2	1	6	4	2

In this experiment, an odd number occurred 4 times.

$$\text{estimate(odd)} = \frac{4}{10} = \frac{2}{5} = 40\%$$

Since the estimate is based on an experiment, different experiments may lead to different estimates.

Finding Theoretical Probabilities

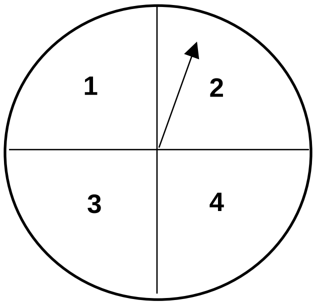
In a probability experiment of rolling a number cube with six equally likely outcomes, each has probability $\frac{1}{6}$.

The event of rolling an odd number corresponds to three outcomes: 1, 3, or 5. Thus the theoretical probability of rolling an odd number is given by the fraction:

$$P(E) = \frac{\text{number of outcomes in an event } E}{\text{total number of outcomes}} = \frac{3}{6} = \frac{1}{2} = 50\%$$

Sample Space Displays

Suppose our experiment is to flip a coin and then spin the spinner.

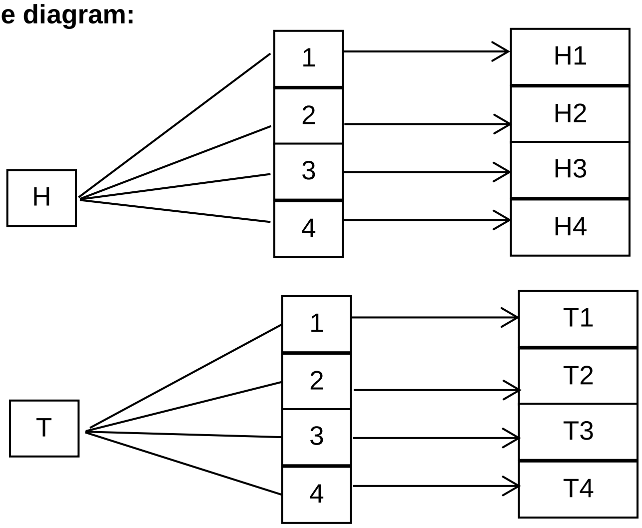


Below are three ways to show all the outcomes (or the sample space) of the experiment.

1. **Outcome grid:**

		Spinner			
		1	**2**	**3**	**4**
Coin Flip	**Heads (H)**	H1	H2	H3	H4
	Tails (T)	T1	T2	T3	T4

2. **Tree diagram:**



3. **List**

H1 H2 H3 H4
T1 T2 T3 T4

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
7.SP.C	**Investigate chance processes and develop, use, and evaluate probability models.**
7.SP.5	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
7.SP.6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
7.SP.7	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy:
a	Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.
b	Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?
7.SP.8	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation:
a	Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
b	Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.
c	Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood*
7.NS.A	**Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.**
7.NS.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers:
d	Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

STANDARDS FOR MATHEMATICAL PRACTICE	
SMP1	Make sense of problems and persevere in solving them.
SMP2	Reason abstractly and quantitatively.
SMP3	Construct viable arguments and critique the reasoning of
SMP4	Model with mathematics.
SMP5	Use appropriate tools strategically.
SMP6	Attend to precision.
SMP7	Look for and make use of structure.
SMP8	Look for and express regularity in repeated reasoning.

The final page of a Student Packet lists all of the Common Core content and practice standards addressed in the unit. Note that some standards develop over time and are included in multiple units.

Which tool is best for math? The multi-pliers.

Probability

Do you know why the two 4s didn't go to the cafeteria for lunch? They already 8!

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