STUDENT RESOURCES

Word or Phrase	Definition
boundary point of a solution set	A <u>boundary point of a solution set</u> is a point for which any interval surrounding the point on the number line contains both solutions and non-solutions. If the solution set is an interval, the boundary points of the solution set are the endpoints of the interval. The boundary point for BOTH $x < -1$ AND x £ -1 is $x = -1$. In the first case the boundary point IS NOT part of the solution set (open circle). In the second case it IS (closed circle).  
equation	An <u>equation</u> is a mathematical statement that asserts the equality of two expressions.
expression	A mathematical <u>expression</u> is a combination of numbers, variables, and operation symbols. When values are assigned to the variables, an expression represents a number. Some mathematical expressions are $7x$, $a + b$, $4v – w$, $\frac{8 + x}{10}$, and 19.
inequality	An <u>inequality</u> is a mathematical statement that asserts the relative size or order of two objects. When the expressions involve variables, a <u>solution to the inequality</u> consists of values for the variables which, when substituted, make the inequality true. $5 > 3$ is an inequality. $x + 3 > 4$ is an inequality. All values for x that are greater than 1 are solutions to this inequality.
solution to an equation	A <u>solution to an equation</u> involving variables consists of values for the variables which, when substituted, make the equation true. The value $x = 8$ is a solution to the equation $10 + x = 18$. If we substitute 8 for x in the equation, the equation becomes true: $10 + 8 = 18$.
solve an equation	To <u>solve an equation</u> refers to finding all values for the variables in the equation that, when substituted, make the equation true. Values that make an equation true are called solutions to the equation. To solve the equation $2x = 6$, one might think “two times what number is equal to 6?” Since $2(3) = 6$, the only value for x that satisfies this condition is 3. Therefore 3 is the solution.
substitution	<u>Substitution</u> refers to replacing a value or quantity with an equivalent value or quantity. If $x + y = 10$, and $y = 8$, then we may substitute this value for y in the equation to get $x + 8 = 10$.

Properties of Arithmetic

Properties of arithmetic govern the manipulation of expressions (mathematical phrases).
For any three numbers *a*, *b*, and *c*:

- ✓ Associative property of addition
  $a + (b + c) = (a + b) + c$
- ✓ Commutative property of addition
  $a + b = b + a$
- ✓ Additive identity property
  (addition property of 0)
  $a + 0 = 0 + a = a$
- ✓ Additive inverse property
  $a + (-a) = -a + a = 0$
- ✓ Associative property of multiplication
  $a \bullet (b \bullet c) = (a \bullet b) \bullet c$
- ✓ Commutative property of multiplication
  $a \bullet b = b \bullet a$
- ✓ Multiplicative identity property
  (multiplication property of 1)
  $a \bullet 1 = 1 \bullet a = a$
- ✓ Multiplicative inverse property
  $a \bullet \frac{1}{a} = \frac{1}{a} \bullet a = 1$
- ✓ Distributive property relating addition and multiplication
  $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for any three numbers *a*, *b*, and *c*.

Properties of Equality

Properties of equality govern the manipulation of equations (mathematical sentences).

For any three numbers *a*, *b*, and *c*:

- ✓ Addition property of equality
  (Subtraction property of equality)
  If $a = b$ and $c = d$, then $a + c = b + d$.
- ✓ Multiplication property of equality
  (Division property of equality)
  If $a = b$ and $c = d$, then $ac = bd$
- ✓ Reflexive property of equality: $a = a$
- ✓ Symmetric property of equality:
  If $a = b$, then $b = a$
- ✓ Transitive property of equality:
  If $a = b$, and $b = c$, then $a = c$

Solving Equations Using a Substitution Strategy

Method 1: To solve an equation using substitution, apply your knowledge of arithmetic facts to find values that make the equation true.

Example 1: Solve $-3x = 15$.

- Think: ***What number times -3 is 15?***
- Since $-3(-5) = 15$, $x = -5$.

Example 2: Solve $12 = 20 - k$.

- Think: ***20 minus what equals 12?***
- Since $20 - 8 = 12$, $k = 8$.

Method 2: Use the "cover-up" method and proceed as above.

Example 3: Solve $\frac{n + 20}{3} = 8$

- Cover up $n + 20$ →  $\frac{\square}{3} = 8$
- Think: ***What divided by 3 equals 8?***
- Since $\frac{24}{3} = 8$, you are covering up 24
- Think: ***What plus 20 equals 24?***
- Since $4 + 20 = 24$, $n = 4$

Example 4: Solve $5(m - 2) = -20$.

Cover up $m - 2$ → $5(\square) = -20$ 

- Think: ***5 times what equals -20?***
- Since $5(-4) = -20$, you are covering up -4
- Think: ***What minus 2 equals -4?***
- Since $-2 - 2 = -4$, $m = -2$

Similar Phrases with Different Meanings

Sometimes it is useful to "translate" a string of words into symbols.

String of Words	Example	Symbols	Classification
is less than	4 is less than 10	$4 < 10$	inequality
less than	4 less than 10	$10 - 4$	expression
is greater than	7 is greater than 2 + 3	$7 > 2 + 3$	inequality
greater than	7 greater than 2 + 3	$(2 + 3) + 7$	expression

"Is greater than" includes the word "is." Therefore, it behaves like a mathematical verb. This string of words is used to make a mathematical sentence (an inequality in this case).

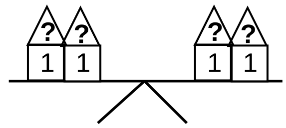
"Greater than" is a string of words without a verb. It translates into an expression. In English, we connect phrases with verbs to make sentences. The same is true in mathematics.

Balance Scales and Laws of Equality

Balance scales are physical representations of equations because both sides of a balanced scale must have the same weight, and both sides of an equation must have the same value.

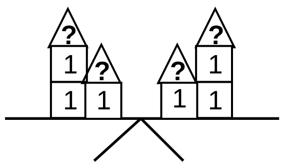
We imagine that each  represents one unit of weight and each  represents an unknown weight (not equal to zero). To represent unknowns, a popular variable is *x*.

Start with a balanced scale like the one to the right, which represents the equation $2x + 2 = 2x + 2$.



Example 1: Add the same thing to both sides, like 1.

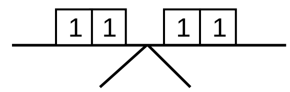
New scale:
(still balanced)



New equation: $2x + 2 + 1 = 2x + 2 + 1$
$2x + 3 = 2x + 3$

Example 2: Subtract the same thing from both sides, like $2x$.

New scale:
(still balanced)

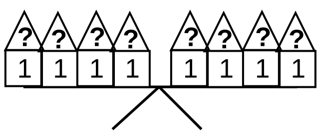


New equation: $2x + 2 - 2x = 2x + 2 - 2x$
$2 = 2$

In examples 1 and 2 the <u>addition property of equality</u> is applied (see Properties of Equality). Note that this property extends to subtraction as well. (See example 2 above).

Example 3: Multiply both sides by the same thing, like 2.

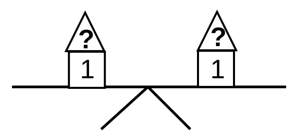
New scale:
(still balanced)



New equation: $2(2x + 2) = 2(2x + 2)$
$4x + 4 = 4x + 4$

Example 4: Divide both sides by the same thing, like 2.

New scale:
(still balanced)



New equation: $\frac{2x+2}{2} = \frac{2x+2}{2}$
$x + 1 = x + 1$

In examples 3 and 4 the <u>multiplication property of equality</u> is applied (see Properties of Equality). Note that this property extends to division as well.

Solving Equations Using a Model

Let **+** represent 1 Let **V** represent the unknown (like x)

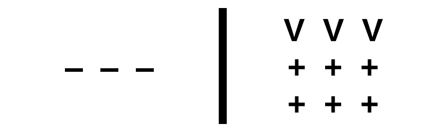
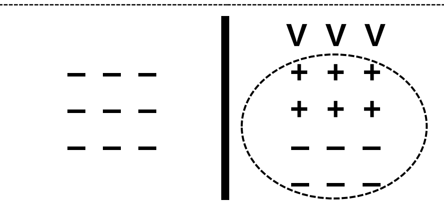
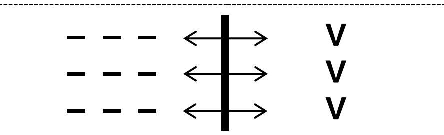
Let **–** represent -1 Let **Λ** represent the opposite of the unknown (like $-x$)

As you solve equations, think:

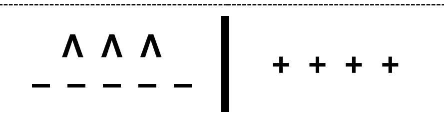
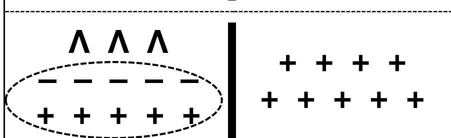
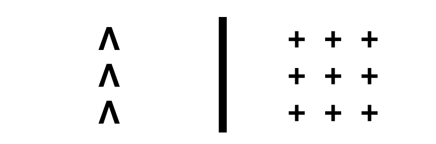
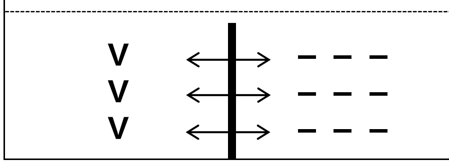
- Can I simplify one or both sides? That is, focus on what can be done to each expression alone.
- What can I do to both sides? That is, focus on what can be done to the equation.

The following examples illustrate one solution path. Others paths are possible to arrive at the same solutions.

Example 1: Solve $-3 = 3(x + 2)$.
Check (after solving): $3(-3) + 6 \rightarrow -9 + 6 \rightarrow -3 = -3$

Picture	Equation	Comments
	$-3 = 3x + 6$	build the equation (think: 3 groups of $x + 2$) and rewrite
	$-3 = 3x + 6$ $+(-6) = +(-6)$ $-9 = 3x$	add -6 to both sides and remove zero pairs
	$\frac{-9}{3} = \frac{3x}{3}$ $-3 = x$	divide both sides by 3 to put counters equally into cups

Example 2: Solve: $-2x - 5 - x = 4$
Check (after solving): $-2(-3) - 5 - (-3) \rightarrow 6 - 5 + 3 \rightarrow 1 + 3 = -3 + 7 \rightarrow 4 = 4$

Picture	Equation	Comments
	$-3x - 5 = 4$	build the equation and rewrite (collect like terms)
	$-3x - 5 = 4$ $+5 = +5$ $-3x = 9$	add 5 to both sides and remove zero pairs
	$(-1)(-3x) = (-1)(9) \rightarrow 3x = -9$ $\frac{3x}{3} = \frac{-9}{3}$ $x = -3$	take the opposite of both sides (accomplished by multiplying both sides by -1), and then divide both sides by 3
	$\frac{-3x}{-3} = \frac{9}{-3}$ $x = -3$	The above actions have the same effect as diving by -3 in a single step

Using Algebraic Techniques to Solve Equations

To solve equations using algebra:

- Use the properties of arithmetic to simplify each side of the equation (e.g., associative properties, commutative properties, inverse properties, distributive property).
- Use the properties of equality to isolate the variable (e.g., addition property of equality, multiplication property of equality).

- Solve: $-2 - 3 = 5x - 2x + 7$ for x

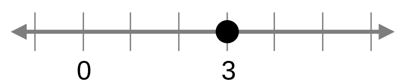
Equation	Comments
$-5 = 3x + 7$	• Collect like terms ($-2 - 3 = -5$; $5x - 2x = 3x$). Note that this is an application of the distributive property because $(5 - 2)x = 3(x)$.
$-5 = 3x + 7$ $\underline{-7} \quad \underline{-7}$ $-12 = 3x$	• Addition property of equality (subtract 7 from both sides) • Additive inverse property ($7 + (-7) = 0$)
$\frac{-12}{3} = \frac{3x}{3}$ $-4 = x$	• Multiplication property of equality (divide both sides by 3 or multiply both sides by $\frac{1}{3}$) • Multiplicative identity property ($1x = x$)

Graphing Inequalities

When graphing solutions to inequalities on the number line, we will use arrows to represent sets of solutions that extend indefinitely in one direction or the other. These arrows should not be confused with the arrows used to denote distance and direction above number ines in Units 3 and 4.

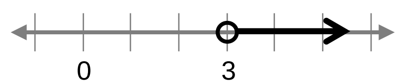
- Integers are graphed as closed circles.

  Example: $x = 3$



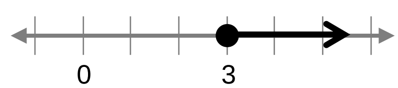
- Solutions to inequalities that involve the statements "is less than" (<) and "is greater than" (>) are graphed with an open circle, indicating that the boundary number *is not* included in the solution set.

  Example: $x > 3$



- Solutions to inequalities that involve the statements "is less than or equal to" (≤) and "is greater than or equal to" (≥) are graphed with a closed circle, indicating that the boundary number *is* included in the solution set.

  Example: $x \geq 3$



Reversing the Direction of an Inequality

When multiplying or dividing both sides of an inequality by a negative number, the direction of the inequality reverses.

Original inequality	Do to both sides	Resulting inequality	Direction reverses?
10 > -4	Add 2	12 > -2	No
	Subtract 2	8 > -6	No
	Multiply by 2	20 > -8	No
	Divide by 2	5 > -2	No
-10 < 4	Add -2	-12 < 2	No
	Subtract -2	-8 < 6	No
	Multiply by -2	20 > -8	**Yes**
	Divide by -2	5 > -2	**Yes**

The direction of an inequality reverses **ONLY** when multiplying or dividing both sides of an inequality by a negative number.

Note that it does not matter if there are negative numbers in the original inequality or not.

Solving Inequalities in One Variable

When solving a linear inequality, treat the inequality as if it were an equation. When multiplying or dividing both sides of the inequality by a negative number, reverse the direction of the inequality.

Example 1	Comments
$-4x + 1 \le 13$ $-1 \quad -1$	(Subtraction) Do not reverse the inequality.
$-4x \le 12$ $\frac{-4x}{-4} \ge \frac{12}{-4}$	(Division by a negative number) **Reverse the inequality symbol.**
$x \ge -3$	Solutions

Example 2	Comments
$2x - 9 \le -13$ $+9 \quad +9$	(Addition) Do not reverse the inequality.
$2x \le -4$ $\frac{2x}{2} \le \frac{-4}{2}$	(Division by a positive number) **Do not reverse the inequality symbol.**
$x \le -2$	Solutions

Error alert. The inequality does not always reverse when solving an inequality that includes negatives. Example 2 illustrates this.