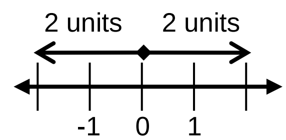
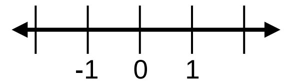
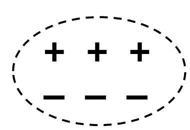
STUDENT RESOURCES

Word or Phrase	Definition
absolute value	The absolute value $\lvert x \rvert$ of a number x is the distance from x to 0 on the number line. $\lvert 2 \rvert = 2$ and $\lvert -2 \rvert = 2$, because both 2 and -2 are 2 units from 0 on the number line. 
addend	See sum.
additive identity property	The additive identity property states that $a + 0 = 0 + a = a$ for any number a. In other words, the sum of a number and 0 is the number. We say that 0 is an additive identity. The additive identity property is sometimes called the addition property of zero. $3 + 0 = 3$, $0 + 7 = 7$, $-5 + 0 = -5 = 0 + (-5)$
additive inverse	The additive inverse of a is the number b such that $a + b = b + a = 0$. The additive inverse of a is denoted by $-a$. -4 is the additive inverse of 4.
additive inverse property	The additive inverse property states that $a + (-a) = 0$ for any number a. In other words, the sum of a number and its opposite is 0. The number $-a$ is the additive inverse of a. $3 + (-3) = 0$, $-5 + 5 = 0$
difference	In a subtraction problem, the difference is the result of subtraction. The minuend is the number from which another number is being subtracted, and the subtrahend is the number that is being subtracted. 12 – 4 = 8 minuend subtrahend difference
integers	The integers are the whole numbers and their opposites. They are the numbers 0, 1, 2, 3, … and -1, -2, -3, … .
minuend	See difference.
negative numbers	Negative numbers are numbers that are less than zero, written $a < 0$. The negative numbers are the numbers to the left of 0 on a horizontal number line, or below zero on a vertical number line. The numbers -2, -4.76, and $-\frac{1}{4}$ are negative. The numbers 2 and 5.3, and 0 are NOT negative.

Word or Phrase	Definition
opposite of a number	The <u>opposite of a number</u> *n*, written $-n$, is its additive inverse. Algebraically, the sum of a number and its opposite is zero. Geometrically, the opposite of a number is the number on the other side of zero at the same distance from zero. The opposite of 1 is -1, because $1 + (-1) = -1 + 1 = 0$. The opposite of -1 is $-(-1) = 1$. Thus, the opposite of a number does not have to be negative. 
positive numbers	<u>Positive numbers</u> are numbers that are greater than zero, written $a > 0$. The positive numbers are the numbers to the right of 0 on a number line, or above zero on a vertical number line. The numbers 3, 2.6, and $\frac{3}{7}$ are positive. The numbers -3, -2.6, $-\frac{3}{7}$, and 0 are NOT positive.
rational numbers	<u>Rational number</u> are numbers expressible in the form $\frac{m}{n}$, where *m* and *n* are integers, and $n \neq 0$. $\frac{3}{5}$ is rational because it is a quotient of integers. $2\frac{1}{3}$ and 0.7 are rational numbers because they **can be** expressed as quotients of integers, namely $\frac{7}{3}$ and $\frac{7}{10}$, respectively. $\sqrt{2}$ and π are NOT rational numbers. They cannot be expressed as a quotient of integers.
subtrahend	See <u>difference</u>.
sum	A <u>sum</u> is the result of addition. In an addition problem, the numbers to be added are <u>addends</u>. 7 + 5 = 12 addend addend sum
whole numbers	The <u>whole numbers</u> are the natural numbers together with 0. They are the numbers 0, 1, 2, 3, … .
zero pair	In the counter model, a positive and a negative counter together form a <u>zero pair</u>. Let **+** represent a positive counter and let **–** represent a negative counter.  Then the figure to the right is an example of a collection of (three) zero pairs.

Mr. Mortimer's Magic Cubes

Mr. Mortimer discovered an amazing way to control the temperature of liquid. He invented magic hot and cold cubes to change the liquid's temperature. These magic cubes never melt or change in any way. For example, ice cubes melt, but magic cold cubes do not.

Hot Cubes (the basics):

- If you add 1 hot cube to a liquid, the liquid heats up by 1 degree.
- If you remove 1 hot cube from the liquid, the liquid cools down by 1 degree.

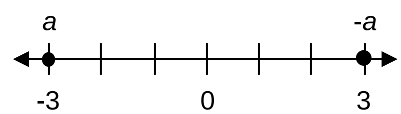
Cold Cubes (the basics):

- If you add 1 cold cube to the liquid, the liquid cools down by 1 degree.
- If you remove 1 cold cube from the liquid, the liquid heats up by 1 degree.

How this temperature change model works		**For 1 cube**
Hot Cubes Positive (+)	Put in Heat → Hotter	add (+1) → +(+1) = +1
	Remove Heat → Colder	subtract (+1) → -(+1) = -1
Cold Cubes Negative (–)	Put in Cold → Colder	add (-1) → +(-1) = -1
	Remove Cold → Hotter	subtract (-1) → -(-1) = +1

Here are a few examples to show temperature change using magic hot and cold cubes.

	Simplest ways:		**Other Ways:**	
+4 degrees	Put in 4 hot cubes +(+4) = 4	Remove 4 cold cubes -(-4) = 4	Put in 6 hot cubes and put in 2 cold cubes +(+6) + (-2) = 4	Remove 6 cold cubes and remove 2 hot cubes -(-6) – (+2) = 4
-2 degrees	Remove 2 hot cubes -(+2) = -2	Put in 2 cold cubes +(-2) = -2	Remove 3 hot cubes and remove 1 cold cube -(+3) – (-1) = -2	Put in 3 cold cubes and put in 1 hot cube +(-3) + (+1) = -2
0 degrees	Do nothing 0		Put in 4 hot cubes and put in 4 cold cubes +(+4) + (-4) = 0	Remove 3 hot cubes and remove 3 cold cubes -(+3) – (-3) = 0

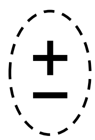
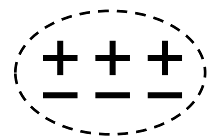
Representing the Additive Inverse	
The minus sign may be used to show additive inverses. The identity $a + (-a) = 0$ means that $-a$ is the additive inverse of a. It is what we add to a to get 0.	Example: If $a = -3$, then $-a = 3$ The statement, "If a is equal to minus 3, then minus a is equal to 3" can be read: • If a is equal to the opposite of 3, then the opposite of a is equal to 3. When we add -3 and 3, the result is 0. 

A Counter Model

This counter model is used to model integers.

Let + represent a positive counter with a value of positive 1
Let – represent a negative counter with a value of negative 1.

A <u>zero pair</u> is a pair with one positive counter and one negative counter.
Both representations below have a value of zero.

one zero pair:  three zero pairs: 

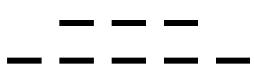
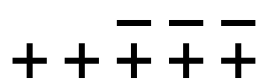
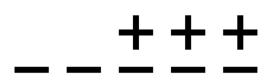
Below are some counter diagrams that represent the given integers:

	+4	-2	0
Simplest representation:	+ + + +	− −	(no counters)
Other representations:	+ + + + + −	− − + + − −	+ −
	+ + + + + + + − − −	− − − − − − + + + +	+ + + + − − − −

Counter Addition Sentence Frames

- Begin with a work space that has a value equal to 0.
- Build ___________ ______.
  positive/negative
- The plus (+) means to add.
- Add ______ ___________ counter(s).
  positive/negative
- The result is ______ ___________ counter(s).
  positive/negative

Integer Addition Using Counters

-3 + (-5) = -8	-3 + 5 = 2	3 + (-5) = -2
		
• Start with a work space equal to zero • Build negative 3 • The (+) means to add • Add 5 negative counters • The result is 8 negative counters	• Start with a work space equal to zero • Build negative 3 • The (+) means to add • Add 5 positive counters • The result is 2 positive counters	• Start with a work space equal to zero • Build positive 3 • The (+) means to add • Add 5 negative counters • The result is 2 negative counters

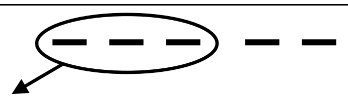
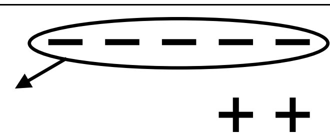
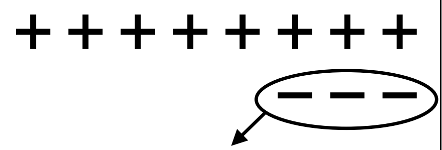
Rules for Addition of Integers

Rule 1: When the addends have the same sign, add the absolute values. Use the original sign in the answer.

Rule 2: When the addends have different signs, subtract the absolute values. Use the sign of the addend with the greatest absolute value in the answer.

Counter Subtraction Sentence Frames
• Begin with a work space that has a value equal to 0. • Build ___________ _____. positive/negative • The minus (–) means to subtract. • Subtract _____ ___________ counter(s). Introduce zero pairs if needed. positive/negative • The result is _____ ___________ counter(s). positive/negative

Integer Subtraction Using Counters

-5 – (-3) = -2	-3 – (-5) = 2	5 – (-3) = 8
		
• Start with a work space equal to zero. • Build negative 5. • The (–) means to subtract. • Subtract 3 negative counters. I do not need zero pairs. • The result is 2 negative counters.	• Start with a work space equal to zero. • Build negative 3. • The (–) means to subtract. • Subtract 5 negative counters. I need zero pairs to do this. • The result is 2 positive counters.	• Start with a work space equal to zero. • Build positive 5. • The (–) means to subtract. • Subtract 3 negative counters. I need zero pairs to do this. • The result is 8 positive counters.

Rule for Subtraction of Integers

Rule: In symbols, $a – b = a + (-b)$ and $a – (-b) = a + b$.

In words, the result is the same whether subtracting a quantity or adding its opposite.

Examples: 6 – 4 = 6 + (-4) = 2

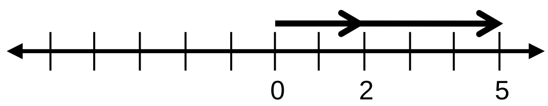
-3 – (-2) = -2 + 2 = -1

Addition and Subtraction on a Number Line

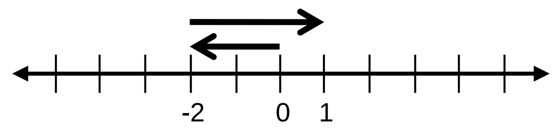
We can use arrows to represent addition and subtraction on a number line. For **adding** any two numbers:

- The absolute value of a number is represented by the arrow length.
- The first arrow begins at zero. If it's representing a positive number, the arrow points to the right. If it's representing a negative number, the arrow points to the left.
- If the second number is positive, the arrow points right. If the second number is negative, the arrow points left.
- The sum is represented by the end (tip) position of the second arrow.

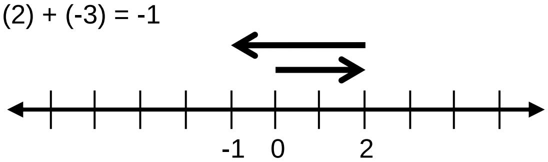
(2) + (3) = 5



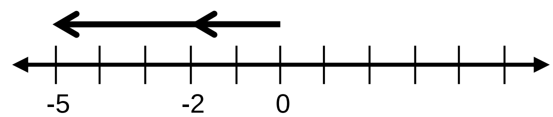
(-2) + (3) = 1



(2) + (-3) = -1



(-2) + (-3) = -5

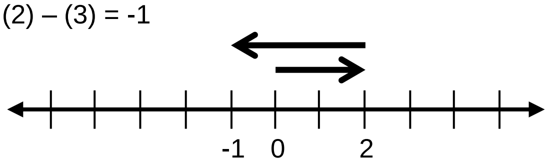


For **subtracting** any two numbers, remember that any minus sign signals doing the "opposite:"

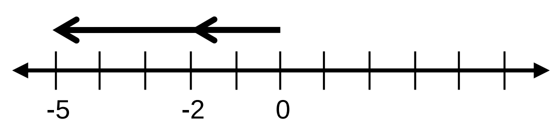
- The absolute value of a number is represented by the arrow length.
- The first arrow begins at zero. If it represents a positive number, the arrow points to the right. If it represents a negative number, the arrow points to the left.
- If the second number is positive, move the opposite of right (LEFT). If the second number is negative, move the opposite of left (RIGHT).
- The difference is represented by the end (tip) position of the second arrow.

Compare the subtraction problems below to the addition problems above. Notice that the first numbers and arrows are all identical to those above. Notice that the numbers subtracted are identical as well, and so the second arrows all point in the opposite direction.

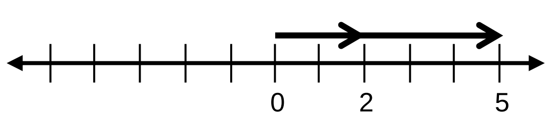
(2) – (3) = -1



(-2) – (3) = -5



(2) – (-3) = 5



(-2) – (-3) = 1