FOCUS, COHERENCE, AND RIGOR

A group of educators at Rutgers University led by Dr. Roberta Shorr (Young, 2009) conducted a seven-year study on problem-solving with minority and low-income students in low-performing schools. They found that giving conceptually challenging problems to students caused frustration, but at the same time, students were engaged and motivated. Working through the frustration in an emotionally safe environment, students gained "satisfaction, pride, and a willingness to work harder next time." This work also resulted in higher standardized test scores (e.g., average scores for Newark fourth graders rose from 45 to 79 percent).

At the Center for Mathematics and Teaching, we know that all students have the potential to achieve in mathematics, we believe that the development of mathematics should reflect the connectedness of Big Ideas into a coherent whole, and we make mathematics inviting and inclusive to more students. One of the ways we put these beliefs and intentions into action is by attending to focus, coherence, and rigor in program development.

FOCUS

According to Achieve the Core (2014), "Not all content in a given grade is emphasized equally in the Standards. Some clusters require greater emphasis than others based on the depth of ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness."

Achieve the Core organizes mathematics clusters into three categories: major clusters, supporting clusters, and additional clusters. This table identifies the clusters for 7th grade and their alignment with *MathLinks* lessons.

GRADE 7: FOCUS IN *MATHLINKS* BASED ON MAJOR, MINOR, AND SUPPORTING CLUSTERS

CLUSTER	Standards	Lesson Focus	Additional Lessons (L) Spiral Review (SR)
Major Clusters			
• 7.RP.A Analyze proportional relationships and use them to solve real-world and mathematical problems.	7.RP.1,2,3	2.1, 2.2, 2.3, 3.1, 3.2, 3.3,	L: 6.1, 6.2, 6.4, 7.2, 9.1, 9.2, 9.3, 10.3 SR: 3, 4, 5, 6, 7, 8, 9, 10
• 7.NS.A Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	7.NS.1,2,3	1.2, 1.3, 4.1, 4.2, 4.3, 5.1, 5.2, 5.3,	L: 2.1, 2.2, 2.3, 3.1, 3.2, 3.3, 6.4, 7.2, 7.3, 7.4, 9.1, 9.2, 9.3 SR: 1, 2, 3, 4, 5, 6, 7, 8, 10
• 7.EE.A Use properties of operations to generate equivalent expressions.	7.EE.1,2	6.1, 6.3, 6.4, 7.2, 7.3, 7.4,	L: 2.1 SR: (7, 8, 9)
• 7.EE.B Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	7.EE.3,4	7.1, 7.2, 7.3, 7.4	L: 8.1 SR: (8, 9, 10)
Supporting Clusters			
• 7.SP.A Use random sampling to draw inferences about a population.	7.SP.1,2	10.1, 10.3, 10.3	
• 7.SP.C Investigate chance processes and develop, use, and evaluate probability models	7.SP.5,6,7,8	1.1, 1.2, 1.3, 10.1	L: 4.2 SR: 6
Additional Clusters			
• 7.G.A Draw, construct and describe geometrical figures and describe the relationship between them.	7.G.1,2,3	2.3, 3.1, 8.2, 8.3	SR: 9, 10
• 7.G. B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	7.G.4,5,6	9.1, 9.2, 8.3	SR: 10
• 7.SP.B Draw informal comparative inferences about two populations	7.SP.3,4	10.1, 10.2, 10.3	SR: 8

Ratio and Proportional Relationships Number Sense Expressions and Equations Statistics and Probability Geometry

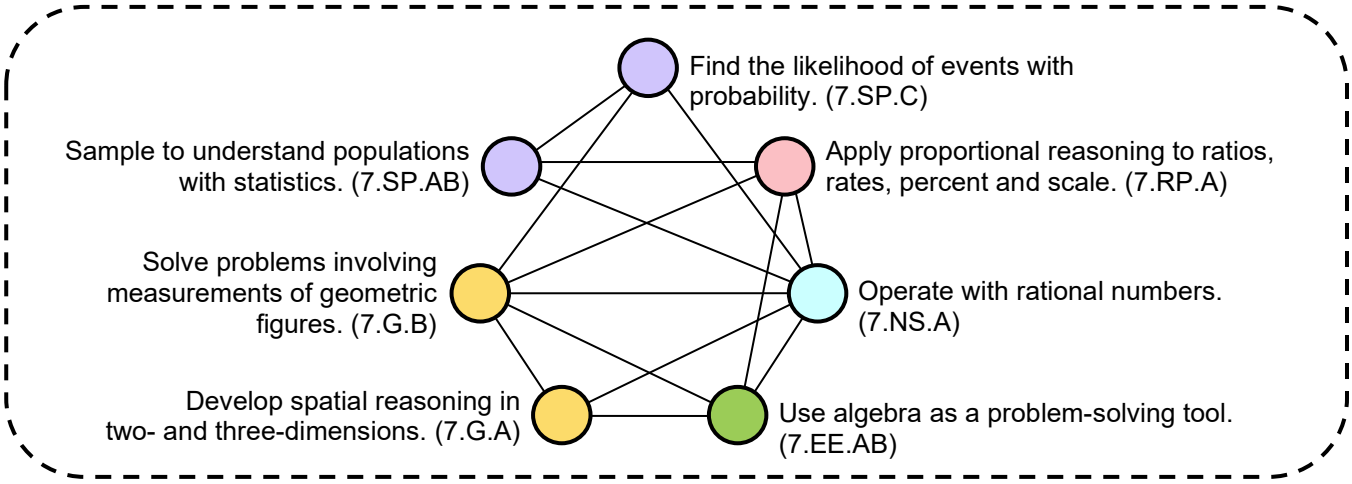
MathLinks intentionally avoids the trap of a "mile wide and inch deep" curriculum that uses standards as a checklist. Rather, we focus students on understanding grade-level mathematics deeply, connecting Big Ideas (see next page), and offering students problems, tasks, and investigations worthy of engagement.

COHERENCE

According to the 2023 California Mathematics Framework, “instructional materials should primarily involve tasks that invite students to make sense of big ideas, elicit wondering in authentic contexts, and necessitate mathematics. Big ideas in math are central to the learning of mathematics, link numerous mathematical understandings into a coherent whole, and provide focal points for students’ investigations.”

Inspired by the work of Jo Boaler (2018) and based on the Content Domains, the *MathLinks* team synthesized the work of the 7th grade into seven Big Ideas. These ideas and their connections within the entire program are shown here.

Grade 7: Big Ideas and Connections



The work in *MathLinks* is organized into 10 units. An expanded dive into the specific Big Ideas, connections, and progressions across grades are included in the front section of each unit of the Teacher Edition. Toward the end of each Student Packet, students identify the Big Ideas within the unit and reflect upon how they are connected to each other.

RIGOR

Achieve the Core (2015) states, “a rigorous math program will pursue with equal intensity conceptual understanding, procedural skill and fluency, and applications.” We now examine the *MathLinks*: Grade 7 program through this lens.

Conceptual Understanding in *MathLinks*

Conceptual understanding, the bedrock of a *MathLinks* course, frequently drives the other two components of rigor. It is a *MathLinks* philosophy to make sure all students have the opportunity to make meaning for every concept presented; we focus on the conceptual development of Big Ideas in depth and make them plausible through investigations, activities, and practice. This is commonly done throughout lessons in all units, oftentimes with the help of teacher Lesson Notes and Slide Decks (or Slide Deck Alternatives).

The table below identifies some examples of concept development in *MathLinks:* Core Grade 7 (unit number and lesson number in parentheses). Typically, these lessons include guided instruction with a slide deck, lesson notes, class discussions, or group work. Opportunities for independent work within a Student Packet appear as Practice pages within lessons, in the Review section as activities, and as Spiral Reviews in subsequent units. Unit Resources on the Teacher Portal also contain Extra Problems, Tasks, and Projects to support conceptual development.

GRADE7: EXAMPLES OF CONCEPT DEVELOPMENT IN *MATHLINKS*	
CLUSTER DESCRIPTION	**(Lesson Number) CONCEPT DEVELOPMENT ACTIVITY**
Major Clusters	
• 7.RP.A (RP 1-3) Analyze proportional relationships and use them to solve real-world and mathematical problems.	(3.1) Proportional Relationships (3.3) Double Number Lines and Equations
• 7.NS.A (NS 1-3) Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	(1.2) Investigating One-Third (4.2) Subtracting Integers with Counters 1 (4.2) Subtracting Integers with Counters 2
• 7.EE.A (EE 1-2) Use properties of operations to generate equivalent expressions.	(6.1) How Many on the Border? (6.3) Introduction to Cups and Counters (6.3) The Upside-Down Cup
• 7.EE.B (EE 3-4) Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	(7.2) Solving Equations with Balance (7.2) Solving Equations Algebraically
Supporting and Additional Clusters	
• 7.SP.A (SP 1-2) Use random sampling to draw inferences about a population.	(10.1) Populations and Sampling
• 7.SP.B (SP 3-4) Draw informal comparative inferences about two populations	(10.2) Math Score Samples
• 7.SP.C (SP 5-8) Investigate chance processes and develop, use, and evaluate probability models	(1.2) Flip and Roll
• 7.G.A (G 1-3) Draw, construct and describe geometrical figures and describe the relationship between them	(8.0) Tear it Up (8.2) Sketching Figures
• 7.G. B (G 4-6) Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	(9.1) Circles (9.2) An Area Investigation

Ratio and Proportional Relationships Number Sense Expressions and Equations Statistics and Probability Geometry

Applications in *MathLinks*

Problem-solving is an important driver of instruction within *MathLinks* courses. In *MathLinks*, we include engaging mathematical problems and applications with accessible entry points for all students, multiple approaches or solutions, and extensions to challenge and enrich. All units begin with an Opening Problem, which introduces a concept or establishes a need to know. In many cases, students require more instruction throughout the unit before they are fully prepared to bring the problem to its conclusion. Substantial problems exist throughout the units as well.

The table below identifies some examples of mathematical problems and applications in *MathLinks:* Grade 7 (unit number and lesson number in parentheses). Some of these problems include guided instruction with a slide deck and lesson notes. Others are appropriate for independent work. Unit Resources on the Teacher Portal (e.g., Tasks, Projects) also contain mathematical problems and applications.

GRADE 7: EXAMPLES OF PROBLEM-SOLVING IN *MATHLINKS*	
CLUSTER DESCRIPTION	**(Unit Number.Lesson Number) MATHEMATICAL PROBLEMS AND APPLICATIONS**
Major Clusters	
• 7.RP.A (RP 1-3) Analyze proportional relationships and use them to solve real-world and mathematical problems.	(2.0, 2.1) Using Coupons, Using Coupons Revisited (3.1) Twinkie the Dog, Practice 2 (3.2) Capt'n Sherman's Shrimp Shop
• 7.NS.A (NS 1-3) Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	(3.3) Jenna's Cornbread, Practice 6 (4.2) A Zero-Sum Game
• 7.EE.A (EE 1-2) Use properties of operations to generate equivalent expressions.	(6.0, 6.1) Crossing the Lake, Crossing the Lake Revisited (6.1) Practice 1, Practice 2
• 7.EE.B (EE 3-4) Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	(6.3) Trousers for Sale (7.1) The Hundred Chart Puzzle (7.2) Joan's Phones
Supporting and Additional Clusters	
• 7.SP.A (SP 1-2) Use random sampling to draw inferences about a population.	(10.3) Estimating Fish Populations
• 7.SP.B (SP 3-4) Draw informal comparative inferences about two populations	(10.2) Practice 3, Practice 4
• 7.SP.C (SP 5-8) Investigate chance processes and develop, use, and evaluate probability models	(1.0, 1.2) Race to the Top, Race to the Top Revisited (1.2) Flip and Roll (1.3) Cereal Box Simulation
• 7.G.A (G 1-3) Draw, construct and describe geometrical figures and describe the relationship between them	(2.3) The Birdhouse (8.2) A Polygon Investigation
• 7.G.B (G 4-6) Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	(9.0, 9.2) Felix the Sheep, Practice 4 (9.2) Penny Drop Probabilities

Ratio and Proportional Relationships **Number Sense** **Expressions and Equations** **Statistics and Probability** **Geometry**

Procedural Skill and Fluency in *MathLinks*

Procedural skill and fluency require more than just computational speed and accuracy. In a 2014 position paper, NCTM described procedural fluency as "the ability to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another."

In *MathLinks*, we thoughtfully develop new procedural skills and provide opportunities for students to gain fluency throughout the year. Skills practice in each unit is found in the Student Packets in the following ways:

- Practice pages – These pages support concept development.
- Review activities – These pages often include skills practice.
- Spiral Review – These pages have distributed practice of prior skills.

- Math Path Fluency Challenges – This Activity Routine, in the Spiral Review section, utilizes mental math skills and supports fluency development in a puzzle format.

While CCSS-M does not identify any specific standards for fluency in Grade 7, Achieve the Core's Instructional Materials Evaluation Tool (2021) suggests attention be given to three Standards: 7.NS.1, 7.EE.1, and 7.EE.4a. The table below shows some examples in 7th grade Student Packets where these procedural skills are developed and students have opportunities to gain fluency through independent practice.

GRADE 7: EXAMPLES OF FLUENCY WORK IN *MATHLINKS*		
Standard / Description	**Lesson**	**Opportunities to gain fluency**
7.NS.A Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers.	(4.1) Counters and Adding Integers (4.2) Counters and Subtracting Integers (4.3) Rational Number Addition and Subtraction (5.1) Multiplying and Dividing Integers (5.2) Multiplying and Dividing Rational Numbers (5.3) Order of Operations	(4.1, 4.2, 4.3) Practice 1 – Practice 9, Review (5.1, 5.2, 5.3) Practice 1 – Practice 9, Review Spiral Review in Packets 2 – 10
7.EE.1 Apply properties of operations as strategies to add, subtract, and expand linear expressions with rational coefficients.	(6.3) Expressions with Cups and Counters (6.4) Fluency with Expressions	(6.3, 6.4) Practices 9, 10, Review (7.1) Hundred Chart Puzzle (7.1, 7.2) Practice 1 – Practice 4 (8.1) Using Algebra to Find Angle Measures Spiral Review in Packet 9
7.EE.4a Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.	(7.1) Solving Equations Using Substitution (7.2) Solving Equations Using Algebra	(7.1) Hundred Chart Puzzle (7.1, 7.2) Practice 1 – Practice 5 (8.1) Using Algebra to Find Angle Measures (8.1) Practice 3 Spiral Review in Packets 9, 10

Number Sense **Expressions and Equations**

Grade-level skills practice is in each unit, as well as practice to fill in gaps. Both can be found on the Teacher Portal in Other Resources in the following ways:

- Essential Skills – This entire section reviews skills and concepts important for success in a given unit. Activity Routines such as Big Square Puzzles, Open Middle Problems, and Four-in-a-Row games are also in these sections for some units. They provide a practice alternative to "drill and kill."

- Extra Problems – Skills practice by lesson is available for all units.

- Nonroutine Problems – In addition to skills practice that is embedded in nonroutine problems, Big Square Puzzles, Open Middle Problems, and Four-in-a-Row games are located in this section for some units.

Finally, for students who need to fill large gaps in skills, consider Skill Boosters, which are on the Teacher Portal. This resource helps students catch up on below-grade-level work without losing extensive instructional time on grade-level work. A Skill Booster routine is intended to take about 5 – 10 minutes per day. Skill Boosters do not directly connect to any course or unit within a course.

Connecting the Three Aspects of Rigor

Interpretations of rigor are evolving. Achieve the Core described "rigor" as the pursuit of conceptual understanding, procedural skill and fluency, and application with equal intensity. The 2023 California Mathematics Framework added the element of integration, interpreting rigor as "an integrated way in which conceptual understanding, strategies for problem-solving and computation, and applications are learned so that each supports the other." A closer look at two major domains in 7th grade illustrates how this is done in *MathLinks*.

GRADE 7: THE ALGEBRA PROGRESSIONS IN MATHEMATICS

Algebra topics primarily appear in the CCSS-M Expressions and Equations and Ratios and Proportional Relationships domains. These areas are the focus of four units in *MathLinks*: Grade 7, and they extend work introduced in 6th grade.

- In Unit 2, **Percent and Scale**, students analyze and solve problems involving numerical and algebraic expressions and involving percent.
- In Unit 3, **Proportional Relationships**, students connect different representations (i.e., visual contexts, tables, graphs, equations, word descriptions) as they solve problems involving proportional relationships. Special attention is paid to whether two quantities are in a proportional relationship by analyzing tables, graphs, and equations. Students continue to develop flexibility when working with variables, expressions, and equations. Double number lines facilitate the learning of how to solve proportions (i.e., equations in the form $\frac{x}{a} = \frac{b}{c}$).
- In Unit 6, **Expressions**, students use a visual context to write numerical and algebraic expressions, paving the way to greater flexibility when working with variables and expressions. Equations of the form $y = mx + b$ are explored without formally addressing function, slope, and vertical intercept, which is done in 8th grade.

  The counter manipulative used for developing integer operations in Units 4 and 5 (**Rational Number Addition and Subtraction** and **Rational Number Multiplication and Division**) is extended to include cups to represent an unknown in an equation. This model gives students a tool for exploring and rewriting more difficult expressions.
- In Unit 7, **Solving Equations and Inequalities,** students extend the use of substitution to solve equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Cups and counters help to facilitate the learning of equation solving procedures. Students also learn to solve inequalities with negative coefficients and open/closed boundary points.

Additionally, in Units 8, 9, and 10 (**Plane and Solid Figures**; **Length, Area, and Volume**; **and Sampling**), students apply their knowledge of proportional relationships and algebra to solve problems in other domains.

CONCLUSION

The authors of *MathLinks* carefully researched and thoughtfully operationalized focus, coherence, and rigor in designing the program. We integrated mathematical concepts, skills, and applications through contexts and problems in a natural way. The result is an efficient core program that contains just 10 units with 32 lessons that can be completed in about 100 class hours. This leaves ample time for review, intervention, enrichment, assessment, and choice for both teachers and students, using what's available in Student Packets and Other Resources on the Teacher Portal.