

INPUTS AND OUTPUTS

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COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
6.RP.A	Understand ratio concepts and use ratio reasoning to solve problems.
6.RP.3	<p>Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations:</p> <ul style="list-style-type: none"> a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
6.NS.B	Compute fluently with multi-digit numbers and find common factors and multiples.
6.NS.3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
6.EE.A	Apply and extend previous understandings of arithmetic to algebraic expressions.
6.EE.2	<p>Write, read, and evaluate expressions in which letters stand for numbers:</p> <ul style="list-style-type: none"> a. Write expressions that record operations with numbers and with letters standing for numbers. b. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems.
6.EE.B	Reason about and solve one-variable equations and inequalities.
6.EE.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
6.EE.C	Represent and analyze quantitative relationships between dependent and independent variables.
6.EE.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. <i>For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.</i>
6.SP.A	Develop understanding of statistical variability.
6.SP.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.
6.SP.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

UNIT PLANNING

* Starred resources can be accessed under Unit Resources on the Teacher Portal.

Unit Pacing* Up to 14 class hours	7.0 Opening Problem: Pick a Dot (1 hour) 7.1 Visual Patterns (3 hours) 7.2 Comparing Prices (2-3 hours) 7.3 Rate Applications (2-3 hours) Review (3 hours) Assessment (1 hour)
Unit Resources* Up to 3 class hours	<ul style="list-style-type: none"> • Extra Problems • Essential Skills • Math Talks (Data, Picture, Number) • Nonroutine Problems • Technology Activities • Tasks (Soccer Fundraiser, Pineapple Party!) • Projects (A Savings Problem) • Parent Support Letters
Assessment Options* See Portal Unit 7 → Other Resources → Assessment, Follow-up and Feedback for more ideas	<ul style="list-style-type: none"> • On the Teacher Portal <ul style="list-style-type: none"> ✓ Unit Quizzes ✓ Cumulative Tests ✓ Tasks ✓ Projects ✓ The <i>MathLinks</i> Rubric (with rubric-worthy problems) • In the Student Packet <ul style="list-style-type: none"> ✓ Monitor Your Progress ✓ Unit Reflection • In the Teacher Edition <ul style="list-style-type: none"> ✓ References to Journals ✓ Suggested problems for the <i>MathLinks</i> Rubric
Materials	<ul style="list-style-type: none"> • Square tiles [7.1] • General supplies (e.g., colored pencils, rulers, tape, scissors, graph paper, calculators)
Slide Decks*	S7.0 Pick a Dot S7.1a What Comes Next? S7.1b Input-Output Rules S7.2 The Keychain Fundraiser S7.3 Raising Money for Music
Reproducibles*	R7-1 Match and Compare Sort Cards: Inputs and Outputs [Review] (1/student)
Prepare Ahead	<ul style="list-style-type: none"> • Organize the distribution of square tiles, cups, and counters (baggies are recommended). Use positive/negative counters as a substitute for tiles if they are not unavailable. • See Activity Routines in Program Information for directions for the <i>MathLinks</i> Rubric, Poster Problems, Match and Compare Sort, Why Doesn't it Belong?, and Computational Fluency Challenge [7.2, Review]
Other Resources on the Teacher Portal	<ul style="list-style-type: none"> • Getting Started Videos and Resources - General Resources • Skill Boosters - Teacher Access page (decimals, percent) • Puzzles / Games - Teacher Access page (SMASH Game 5, Pattern Grids Unlocked)

PLANNING FOR DIFFERENT USERS

Student Packet (SP)

Unit 7 component options for those who support students:

For teachers	<ul style="list-style-type: none"> • Teacher Edition (this document) • Teacher Portal (Unit Resources, General Resources) • Program Information
For substitutes	<ul style="list-style-type: none"> • SP (Practice 1 – 9 may be completed independently any time after instruction; Spiral Review; Vocabulary Review) • Unfinished work from previous SP's • Extra Problems • <i>MathLinks</i> Puzzles / Games
For parents	<ul style="list-style-type: none"> • Resource Guide • Parent Letter (English and Spanish)

Unit 7 component options to use with all students (all available on the Teacher Portal):

<ul style="list-style-type: none"> • SP (Word Bank, Opening Problem, Lessons, Activity Routines, Review, Student Resources, self-monitoring, journals, reflection prompts) • Student Packet Text file for Translation (EL) • Extra Problems (practice or assess by lesson) • Essential Skills (just-in-time review) 	<ul style="list-style-type: none"> • Math Talks (whole-class discourse) • Nonroutine Problems (enrichment) • Tasks (multi-part problems) • Projects (authentic multi-day experiences) • Technology Activities (variety) • <i>MathLinks</i> Puzzles / Games (fun challenges)
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Unit 7 component options for particular subgroups of students:

For English learners see pgs vii-viii for specific strategies	<ul style="list-style-type: none"> • SP Text File for Translation • SP features for language development (Word Bank, Vocabulary Review, consistent structure for reading and writing, and grouping opportunities for speaking and listening) • SP Activity Routines for language development (rubric-worthy problems with the <i>MathLinks</i> Rubric, Match and Compare Sort, Why Doesn't It Belong?, Poster Problems) • Math Talks for speaking and listening
For struggling learners see pgs vii-viii for specific strategies	<ul style="list-style-type: none"> • SP features for math confidence (Getting Started, Review including Spiral Review, Word Bank, Vocabulary Review, consistent structure, grouping options) • SP Activity Routines for math confidence (rubric-worthy problems with the <i>MathLinks</i> Rubric, Match and Compare Sort, Why Doesn't It Belong?, Poster Problems, Computational Fluency Challenge) • Essential Skills for just-in-time intervention • Extra Problems (by lesson) for practice, review, or assessment • Skill Boosters (Decimal and Percent Concepts)
For enrichment and advanced learners	<ul style="list-style-type: none"> • SP features for enrichment (see pg xi for specific options) • SP Activity Routines for enrichment (rubric-worthy problems with the <i>MathLinks</i> Rubric Match and Compare Sort, Why Doesn't It Belong?) • Nonroutine Problems (including problems from the Math Olympiad) • Technology Activities for variety • Projects for applications

MATH BACKGROUND

Independent and Dependent Variables

Independent variables are under our control, in the sense that we may specify their values. Once the values of the independent variables have been specified, the values of the dependent variables are completely determined. We have no control over them.

When two variables are in a proportional relationship, the values of either variable completely determine the values of the other. Either variable could be regarded as the independent variable, and the remaining variable would then be regarded as the dependent variable. Which of the two variables is the independent variable, and which is the dependent variable, depends on the context of the problem.

Consider this situation: Suppose text messages cost \$0.05 each.

Let n = the number of text messages sent

Let C = the cost of a text message bill (in dollars)

- If we know the number of text messages sent, then we can determine the cost of the text message bill as $C = 0.05n$. In this case, n is the natural independent variable and C is the dependent variable because the total cost depends on the number of text messages sent.
- If we are operating within a budget and we have a limit C on what we can spend for text messages, then we can determine the number of text messages we can send as $n = \frac{C}{0.05}$. In this case, C is the natural independent variable and n is the dependent variable because the number of text messages depends on the total cost.

TEACHING TIPS

Applying Standards for Mathematical Practice (SMP)

Here is an abbreviated version of the SMPs and some ways they are applied in this unit.

SMP1 Make sense of problems and persevere in solving them. <ul style="list-style-type: none"> • Understand a problem and look for entry points • Consider simpler or analogous problems • Monitor progress and alter solution course as needed • Make connections between multiple representations • Check answers with a different method 	[All Lessons] Multiple representations are used extensively throughout this unit to help students make connections between numbers in tables, graphs, double number lines, equations, and the contexts from which these are all derived.
SMP2 Reason abstractly and quantitatively. <ul style="list-style-type: none"> • Use numbers and quantities flexibly in computations • Attend to the meaning of quantities • Decontextualize a problem using symbols, manipulate them, and then interpret based on the context 	[All Lessons] Students decontextualize situations and manipulate algebraic representations for the purpose of problem solving.
SMP3 Construct viable arguments and critique the reasoning of others. <ul style="list-style-type: none"> • Use assumptions, definitions, established results, examples, and counter examples to analyze an argument and discuss its merits or flaws • Make and test conjectures based on evidence • Analyze situations by breaking them into cases • Understand and analyze the approaches of others 	[7.1] Students analyze diagrams made by two students. [Review] For Match and Compare Sort and Why Doesn't it Belong? , students must pay close attention to other's explanations or descriptions, offer critiques or advice, and revise work.
SMP4 Model with mathematics. <ul style="list-style-type: none"> • Attach meaningful mathematics to everyday problems and questions of interest • Make reasonable assumptions and approximations to simplify a situation • Identify quantities, use mathematical tools (such as multiple representations, formulas, equations) to analyze relationships • Interpret results and draw conclusions in the context of the situation 	[7.2] Students make a recommendation to a fundraising committee based upon pricing a survey. [7.2] In A Committee Decision , students use unit prices, data, and statistics to make a fundraising recommendation.

Applying Standards for Mathematical Practice (SMP) Continued		
SMP5	Use appropriate tools strategically. <ul style="list-style-type: none"> • Select and use tools strategically (and flexibly) to visualize, explore, and compare information • Use technological tools and resources to solve problems and deepen understanding 	[7.2] In A Committee Decision , students may choose to use a calculator for computations to focus more on the modeling/decision-making aspect of the problem.
SMP6	Attend to precision. <ul style="list-style-type: none"> • Calculate accurately and efficiently • Explain thinking using mathematical vocabulary • Use symbols appropriately • Specify units of measure 	<p>[7.0] Pick a Dot requires students to use precise, descriptive mathematical language. This activity demonstrates the value of mathematical notation and symbols.</p> <p>[7.1, 7.2, 7.3] Key mathematical vocabulary is underlined throughout the unit, precise definitions are provided, and students are expected to make sense of, and use, mathematical language.</p> <p>[7.3] The Raising Money for Music series of problems require attention to what is being asked to produce accurate calculations.</p> <p>[Review] Match and Compare Sort requires close attention to the meaning of new academic vocabulary.</p>
SMP7	Look for and make use of structure. <ul style="list-style-type: none"> • Recognize the structure of a symbolic representation and generalize it • See complicated objects as composed of chunks of simpler object 	[7.1, 7.2] Templates provided for Visual Patterns and Comparing Prices lessons help students identify structure within representations and generalize processes.
SMP8	Look for and make use of repeated reasoning. <ul style="list-style-type: none"> • Identify repeated calculations and patterns • Generalize procedures based on repeated patterns or calculations • Find shortcuts based on repeated patterns or calculations 	[All Lessons] Creating an input-output rule from given information is a shortcut for finding information quickly. For example, in the Poster Problems , students create input-output rules and then use them to find the number of tiles in step 100, or the step number when there are 120 tiles.

Strategies to Support Diverse Populations		
Classrooms typically include students with different learning styles and needs. Here are some specific ways that <i>MathLinks</i> supports special populations. Strategies essential to the academic success of English learners are noted with a star (*). See Universal Design for Learning in Program Information for more details.		
	General Examples	MathLinks Examples
Know your Learner	<ul style="list-style-type: none"> ✓ Understand student attributes that support or interfere with learning ✓ Determine preferred learning and interaction styles ✓ Assess student knowledge of prerequisite mathematics content ✓ Check for understanding continuously ✓ Provide differentiation opportunities for intervention to reach more learners ✓ Encourage students to write about their attitudes and feelings towards math ✓ Use contexts that link to students' cultures* 	<p>Built into the <i>MathLinks</i> Design:</p> <div style="border: 1px dashed black; padding: 5px;"> SP: Getting Started, Spiral Review, Monitor Your Progress, Unit Reflection TE: References to Journals UR: Extra Problems, Essential Skills, Projects OR: Skill Boosters, Assessment Options </div> <p>[7.0] Use the Pick a Dot opening problem to assess student prior knowledge of coordinate graphing.</p> <p>[Review] Group students by language proficiency to increase interaction on activities that require cooperative learning.*</p>
Increase Academic Language through Mathematics	<ul style="list-style-type: none"> ✓ Provide opportunities for students to read, write, speak, and listen ✓ Explain the academic vocabulary needed to access mathematical ideas, providing both examples and non-examples ✓ Use strategically organized groups that attend to language needs* ✓ Use rich mathematical contexts and sophisticated language to help ELs progress in their linguistic development* ✓ Use cognates and root words (when appropriate) to link new math terms to students' background knowledge* 	<p>Built into the <i>MathLinks</i> Design:</p> <div style="border: 1px dashed black; padding: 5px;"> SP: Word Bank, Vocabulary Review, Student Resources TE: Grouping suggestions, References to Journals, Suggested problems for The MathLinks Rubric UR: Math Talks OR: Critique student work on Slide Decks </div> <p>[All Lessons] Ask students to share translations of mathematical terms and write them next to key vocabulary in My Word Bank. Point out that many mathematical terms have Latin roots.*</p> <p>[7.0] Students with special needs may need a review of the definitions of horizontal and vertical.</p> <p>[Review] Match and Compare Sort requires students to engage in rich mathematical discussions with new terminology.*</p>

Components cited: Student Packet (SP), Teacher Edition (TE), Unit Resources (UR), Other Resource (OR)

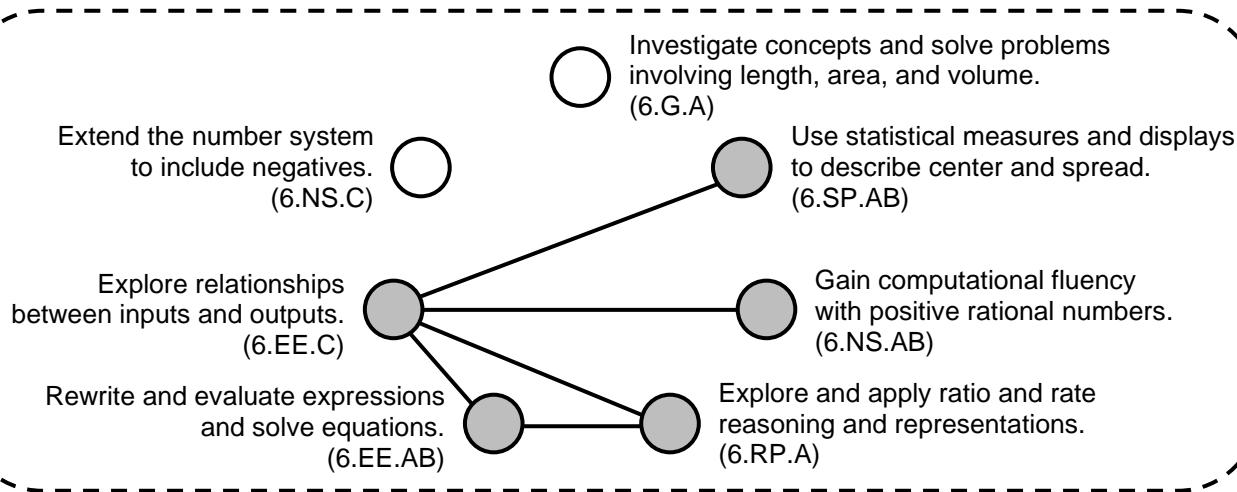
Strategies to Support Diverse Populations (Continued)		
	General Examples	MathLinks Examples
Increase Comprehensible Input	<ul style="list-style-type: none"> ✓ Link concepts to past learning ✓ Make concepts meaningful through hands-on activities, visuals, demonstrations, and color-coding ✓ Use a think-aloud strategy to model appropriate thinking processes and academic language use ✓ Use graphic organizers to help students record information and data, see patterns, and generalize them ✓ Use multiple representations (pictures, numbers, symbols, words, contexts) of math ideas to create meaning and make connections ✓ Strategically sequence and scaffold to make mathematics accessible ✓ Simplify written instructions, rephrase explanations, and use verbal and visual clues* 	<p>Built into the <i>MathLinks</i> Design:</p> <div style="border: 1px dashed black; padding: 5px;"> SP: Structured workspace TE: Slide Deck Alternatives, Reproducibles, Materials OR: Slide Decks </div> <p>[All Lessons] Use an explicit think-aloud process to model appropriate language to make connections between numbers in tables, ordered pairs, graphs, or equations.</p> <p>[7.1] Provide students with square tiles to build patterns.</p> <p>[Review] Match and Compare Sort includes a Venn diagram to help students see similarities and differences between two related words. Check in frequently with English learners during this language-intense activity.*</p>
Promote Student Interaction	<ul style="list-style-type: none"> ✓ Use flexible group configurations that support content objectives ✓ Use strategies and activities that promote teacher/student and student/student interactions (e.g., think-pair-share, Poster Problems) ✓ Encourage elaborate responses through questioning ✓ Allow processing time and appropriate wait time, recognizing the importance of the different requirements for speaking, reading, and writing in a new language* ✓ Allow alternative methods to express mathematical ideas (e.g., visuals, students' first language)* 	<p>Built into the <i>MathLinks</i> Design:</p> <div style="border: 1px dashed black; padding: 5px;"> SP: Lesson and Review activities TE: References for Journals, Suggested problems for The MathLinks Rubric UR: Math Talks, various games and puzzles OR: Slide Decks, Activity Routines </div> <p>[All Lessons] Use think-pair-share strategies to encourage communication in a safe environment for all learners, especially those who are learning English as a second language.*</p> <p>[7.0] For Pick a Dot, allow students to be creative in terms of how they want to express themselves (e.g., hand gestures) if language is a barrier.*</p> <p>[Review] Match and Compare Sort are specifically designed to encourage student interaction and deemphasize competition. Organize students into pairs or groups that support language and content needs.*</p>

Components cited: Student Packet (SP), Teacher Edition (TE), Unit Resources (UR), Other Resources (OR)

Big Ideas and Connections

The Center for Mathematics and Teaching is dedicated to igniting and nurturing passion for mathematics in middle school students. We see the classroom as a place of joy and wonder, collaboration and purpose, perseverance and empowerment. We want all students to succeed in mathematics, as they explore its beauty in patterns, concepts, connections, and applications.

MathLinks: Grade 6 is organized around seven big ideas. This graphic provides a snapshot of the ideas in Unit 7 and their connections to each other.



These ideas build on past work and prepare students for the future. Some of these include:

Prior Work	What's Ahead
<ul style="list-style-type: none"> • Generate and analyze patterns and relationships (4.OA.C, 5.OA.B) • Represent and interpret data (4.MD.B, 5.MD.B) • Fluently perform operations with positive numbers (4.OA.A, 4.NBT.B, 4.NF.B, 5.NBT.B, 5.NF.AB, 6.NS.B) • Graph points in the coordinate plane to solve real-world and mathematical problems (5.G.A) • Understand ratio concepts and use ratio reasoning to solve problems (6.RP.A) • Extend understanding of arithmetic to algebraic expressions (6.EE.A) • Summarize and describe distributions (6.SP.B) 	<ul style="list-style-type: none"> • Analyze proportional relationships and use them to solve problems (7.RP.A) • Use properties of operations to generate equivalent expressions (7.EE.A) • Solve problems involving numerical expressions and equations (7.EE.B) • Define, evaluate, and compare functions (8.F.A) • Use functions to model relationships between quantities (8.F.B) • Continue to explore the world of functions (HS)

Developing Language Skills through *MathLinks*

Language (reading, writing, speaking, and listening) helps students communicate math ideas and understand concepts. Here are examples for Unit 7.

Language Objectives

Students will:

(Lesson 1) Read the formal definitions input-output rule, independent variable, and dependent variable, and join in a group discussion of the meaning of each, as they are then assisted in writing something in My Word Bank that is meaningful to them. (Note that these terms don't come up again in the middle school Standards, though the concepts run through high school mathematics).

(Lesson 2) Talk about how the language and representations of proportional reasoning (e.g., unit price and double number line) connect to algebraic thinking (e.g., using expressions, equations, and graphs in the coordinate plane to describe changing quantities).

(Lesson 3) Use the language of proportional reasoning, orally and in writing, within the world of algebra. For example, as students make tables, graphs, and equations, they should use unit rate in their discussions as they point them out.

(Review) Collaboration is vital on all activities. Student participation must be high for all speaking and writing required to complete Poster Problems, Match and Compare Sort, and Why Doesn't it Belong?

Group Discussions to Promote Reading, Listening, and Speaking (2+)

Critique reasoning situations appear for discussion on slide decks. Examples: On S7.1a and pg 3, students must make a conjecture for the growing pattern and justify its reasonableness. On S7.2 students may make their own assumptions, wonder about details, and politely challenge the reasonableness of the assumptions of others. For A Committee Decision (pg 13 / 7), students might critique each other's recommendations based upon the data and preferences. Encourage pair/trio and/or whole class discussions.

While doing What Comes Next? (pg 3 / 1-2), after students have time to individually write their descriptions, stop and give time for pairs/trios to silently read each other's responses and orally critique each other's reasoning.

Input-Output Rules (pg 5 / 4-7) is based off of very new learning. Give individuals some time to write answers to the problems before reading them aloud to compare in pairs/trios.

In Review, Why Doesn't it Belong (pg 22 / 1) requires specificity to describe why an entry does not belong with the others. Allow discussion time for comparing and critiquing answers in pairs/trios before any whole-class sharing.

Journal Ideas to Promote Writing

(Explaining Concepts) See pg 12 for a specific content Journal prompt.

(New Language) Write a sentence or two to help you remember, in an input-output situation, which is the independent variable and which is the dependent variable. Example situation: if buying 4 apples is the input, the output is paying \$2.

(Language in the Real World) In the coordinate plane, describe what quadrants are. Where else do you see the root/prefix "quad" in mathematics? What about in the real world?

Students May Wonder...

Why is it important to identify the independent and dependent variables? For most problems at the 6th grade level it really doesn't matter. In fact, for any "invertible function", either variable could be the independent variable. But as students learn about different functions, the input (independent variable) becomes more natural, and general properties of function may only apply if the independent variable is defined properly. For example, quite often, we graph situations over time. In these cases, time is the independent variable.

Enrichment and Challenges for Advanced Learners

MathLinks: Grade 6 materials provide multiple opportunities for advanced students to investigate grade-level mathematics at a higher level of complexity without doing more work than their peers.

Every student does not need to do every problem in a Student Packet. Challenge those who are ready with these pages. Others might do unfinished work, Spiral Review, or more practice problems as needed.

- Practice 4: Extend Your Thinking (pg 8)
- Raising Money for Music with Practice 7 (pgs 15-16)
- Practice 9: Extend Your Thinking (pg 19)

Consider speeding up instruction, and skipping some Practice and Spiral Review. See also Planning for Different Users (TE, pg iii) and Enrichment for Advanced Learners and Those with Undiscovered Hidden Talents (Program Information → Universal Design for Learning) for more ideas.

The Algebra Progression in *MathLinks:* Grade 6

The Expressions and Equations standards are the focus of four units in *MathLinks:* Grade 6.

- In Unit 6, **Expressions**, students worked with both numerical and algebraic expressions. They learned to manipulate, simplify, and evaluate expressions using the distributive property and the conventions for order of operations. They also translated between words, numbers, and symbols.
- In Unit 7 (this unit), **Inputs and Outputs**, students use visuals and contexts to analyze and solve problems with multiple representations. Concepts related to proportional reasoning are reviewed and emphasized. Students' knowledge of expressions enables them to generate equations for relationships relating two variables, called "input-output rules."

Without explicitly defining "function" (this is done in grade 8), students begin to develop flexibility when working with variables, expressions, and equations. The problems introduced set the stage for solving a linear equation in one variable since these equations are of the form $x + p = q$ and $px = q$ (i.e., "one-step" equations") for cases in which p and q , and are nonnegative rational numbers.

- In Unit 8, **Solving Equations**, students will learn mental math and substitution strategies and formalize the equation-solving processes, using balance and tape diagrams to work toward traditional equation-solving procedures, as the need grows.
- In Unit 10, **The Number Lines and the Coordinate Plane**, students will graph solutions to simple inequalities.

The Proportional Reasoning Progression in *MathLinks: Grade 6*

The Ratio and Proportional Relationships standards are the focus of four units in *MathLinks: Grade 6*.

- In Unit 3, **Ratio Representations**, students began the transition from additive to multiplicative thinking by learning to create and interpret ratios, tape diagrams, tables, equivalent ratios, and double number lines in a variety of contexts.
- In Unit 4, **Division**, students explored the structure of rate problems and solved rate problems as they gained fluency with division.
- In Unit 5, **Percent**, students used sense-making strategies, computational procedures, and double number lines to make sense of percent and percent applications.
- In Unit 7 (this unit), **Inputs and Outputs**, students informally expand their notion of proportional relationships, specifically the use of unit rates, into the world of input-output rules (functions).

REPRODUCIBLES

R7-1 MATCH AND COMPARE SORT CARDS: INPUTS AND OUTPUTS

I UNIT RATE	△	I UNIT PRICE	○
II INPUT-OUTPUT RULE	△	II EQUATION	○
III INDEPENDENT VARIABLE	△	III DEPENDENT VARIABLE	○
IV DOUBLE NUMBER LINE	△	IV GRAPH IN COORDINATE PLANE	○
A <ul style="list-style-type: none">✓ a variable whose value may be specified✓ typically, the input	△	A <ul style="list-style-type: none">✓ on a horizontal and a vertical number line✓ ordered pairs in the form (x, y) may be graphed	○
B <ul style="list-style-type: none">✓ a diagram made of two parallel number lines✓ two quantities can be compared (like a ratio)	△	B <ul style="list-style-type: none">✓ a variable whose value is determined by the values of the independent variable✓ typically, the output	○
C <ul style="list-style-type: none">✓ the value of a ratio✓ example: 45 miles per hour	△	C <ul style="list-style-type: none">✓ a statement that asserts that two expressions are equal✓ example: $20 = 15 + 5$	○
D <ul style="list-style-type: none">✓ an equation that establishes a specific output value for each input value✓ example: $y = 2.5x$	△	D <ul style="list-style-type: none">✓ the price for one unit of measure✓ example: \$1.10 per orange	○

Name _____

Period _____

Date _____

UNIT 7
ANSWER KEY

Math Links

GRADE 6

INPUTS AND OUTPUTS

	Monitor Your Progress	Page
My Word Bank		0
7.0 Opening Problem: Pick a Dot		1
7.1 Visual Patterns <ul style="list-style-type: none"> Review graphing ordered pairs Describe sequences of numbers generated by visual patterns using verbal descriptions, tables of numbers, graphs, and input-output rules Understand the relationship between dependent and independent variables 	3 2 1 0 3 2 1 0 3 2 1 0	2
7.2 Comparing Prices <ul style="list-style-type: none"> Use tables of numbers, double number lines, graphs, equations, unit rates, and words to compare prices of similar items 	3 2 1 0	9
7.3 Rate Applications <ul style="list-style-type: none"> Use rates in problem solving contexts Identify unit rates in tables, graphs, and equations Deepen understanding of independent and dependent variables 	3 2 1 0 3 2 1 0 3 2 1 0	14
Review		20
Student Resources		27

Materials

Grouping

Reproducibles

Slide Deck

Journal Idea

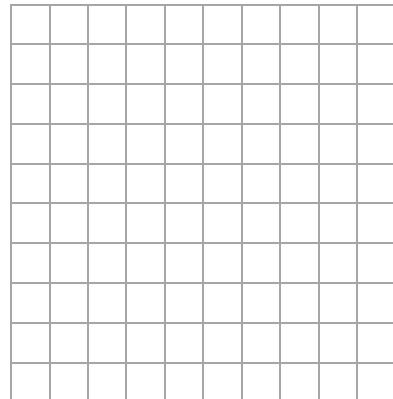
Parent (or Guardian) signature _____

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

coordinate plane / coordinate axes / origin

When a vocabulary word first comes up in context, take the time to support students in writing something that is meaningful to them, whether it's an explanation of the vocabulary in their own words, an example, and/or a picture.



dependent variable

independent variable

input-output rule

unit rate/unit price

OPENING PROBLEM: PICK A DOT

[SMP 6]

Follow your teacher's directions for (1) – (4). Descriptions will vary. Below is an example for one point.

(1) My dot is right in the middle of the slide.

(2) Go a little to the right of the dot on the x -axis, then pick the second dot going up vertically.

(3) From where the darker lines (rays) intersect (the origin), count 8 units to the right and then 7 units up.

(4) My dot is point O .

5. Record the meaning of coordinate plane in **My Word Bank**.

LESSON NOTES S7.0: PICK A DOT

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

The activity is intended to create the intellectual “need to know” for the coordinate plane. Graphing in one quadrant was introduced in a grade 5 (Common Core Standard 5.G.1). Graphing in four quadrants will be introduced in Unit 10.

- Slide 1: For (1), students pick a dot and keep their choice to themselves while writing the location description. Ask a few students to read their descriptions aloud to the class, while other selected classmates try to guess the point’s location. Or use this as a pair activity where students read each other’s descriptions and make guesses.

PICK A DOT

Without saying anything
pick any dot.

(1) Describe its location in writing as if you were trying to help someone in class find it.



1

 Math Links

- Slide 2: This slide reviews some important vocabulary. For (2), students write their new descriptions. Make sure students know that they must describe the **SAME** point as before.

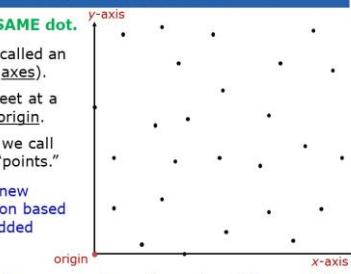
Why might these coordinate axes make describing the location of a point easier? Encourage the use of academic language so that students refer to specific axes or the origin as reference points.

DOES THIS HELP?

Find your SAME dot.

Each ray is called an axis (plural axes).
The axes meet at a point - the origin.
On a graph we call these dots “points.”

(2) Write a new description based on the added axes.



How did adding axes make writing a description easier?

2

 Math Links

LESSON NOTES S7.0: PICK A DOT

Continued

- Slide 3: For (3), students write a new description for the location of the SAME point.

How does the grid help to make your description even easier? Regardless of prior experiences with the coordinate plane, now students can count specific numbers of units from reference points like the origin or the axes. Use this as a formative assessment to see if students remember that graphing in the first quadrant involves starting at the origin and counting to the right and then up. Have students share as they did for the previous slides.

Informally discuss additional vocabulary associated with the coordinate plane as desired (e.g., ordered pairs, x -axis, y -axis, x -coordinate, y -coordinate).

- Slide 4: For (4), students write their final description, which can be as simple as stating the letter for the point. Collectively, these slides should highlight the utility of the coordinate graphing system.

HOW ABOUT THIS?

Find your SAME point.

(3) Write a new description based on the added grid.

How did adding a grid make writing a description easier?



3

MathLinks

THIS HELPS!

Find your SAME point.

(4) Write a new description based on the added letters to name each point.



How did adding "names" to each point make writing a description easier?

4

MathLinks

SLIDE DECK ALTERNATIVE S7.0: PICK A DOT

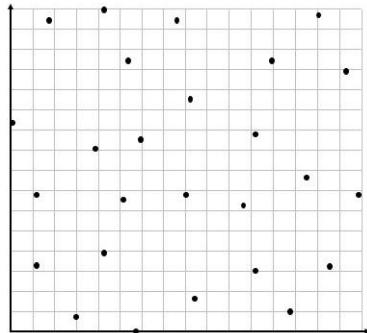
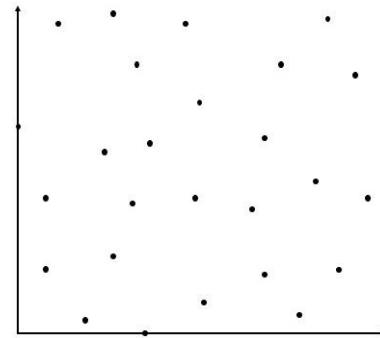
Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slides 1-4



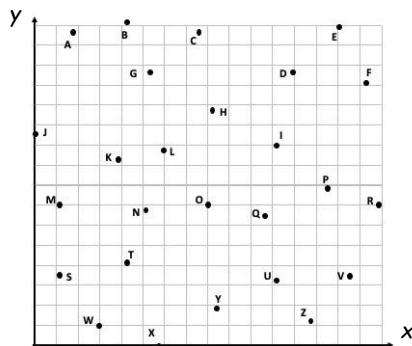
(1) Pick a dot. Describe its location in writing as if you were trying to help someone in the class find it.

(2) Write a new description based on the added axes.



(3) Write a new description based on the added grid.

(4) Write a new description based on the added letters to name each point.



VISUAL PATTERNS

We will review graphing ordered pairs. We will use words, tables of numbers, graphs, and algebraic input-output rules (equations) to describe visual patterns. We will identify what is typically the difference between independent variables and dependent variables.

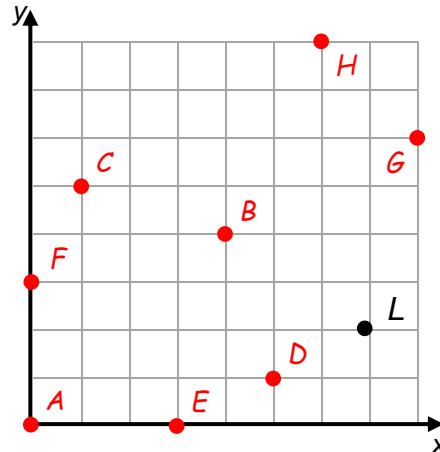
[6RP3a, 6EE2ab, 6EE6, 6EE9; SMP1, 2, 3, 6, 7, 8]

GETTING STARTED

Each small square on the grid represents 1 square unit.

- Graph and label the following ordered pairs. As an example, the ordered pair (7, 2) is graphed and labeled point *L*.

$A(0, 0)$ $B(4, 4)$ $C(1, 5)$ $D(5, 1)$
 $E(3, 0)$ $F(0, 3)$ $G(8, 6)$ $H(6, 8)$



- How can you remember that we count the x -coordinate in the direction along the x -axis first and the y -coordinate in the direction along the y -axis second when graphing ordered pairs?
Answers will vary. One way to remember is that since x comes before y in the alphabet, we count in the x direction first.

Use the word list below to fill in the blanks. Some words are used more than once. Use the coordinate plane below for reference or notes.

coordinate plane	horizontal	ordered pairs	origin	vertical
------------------	------------	---------------	--------	----------

- A coordinate plane is a plane with a horizontal axis and a vertical axis meeting at the point $(0, 0)$, called the origin.
- The horizontal axis is typically referred to as the x -axis.
- The vertical axis is typically referred to as the y -axis.
- Points in the coordinate plane are named by pairs of numbers called ordered pairs. They are written in the form (x, y) .
- From the origin to the point located at $(3, 5)$, move 3 units in the horizontal direction and 5 units in the vertical direction.

WHAT COMES NEXT?

[SMP1, 3]

Follow your teacher's directions.

(1) **Describe the way your pattern is growing.** Answers may vary.

Two possible answers:

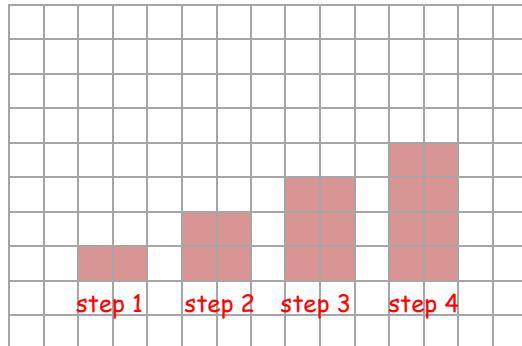
- Start with 2 tiles and add 2 more each time. (Mateo's illustration below)
- Start with 2 tiles and double the amount each time. (Anita's illustration below)

Watch for (2,4): Are students making precise drawings and graphs (with correct labels and scaling): to match?

(2) - (4) For both patterns: Draw steps 1-4 (build if desired); make tables; make graphs.

Mateo

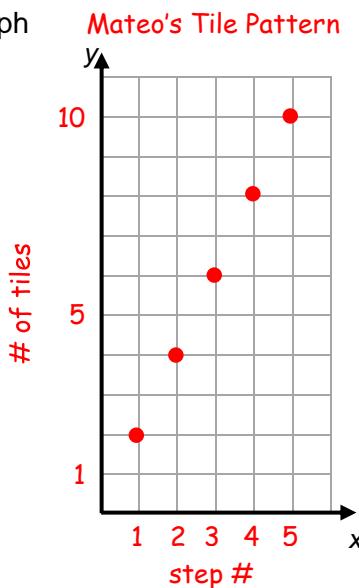
(2) Picture



(3) Table

Mateo's Tile Pattern	
step # (x)	# of tiles (y)
1	2
2	4
3	6
4	8
5	10

(4) Graph



Anita

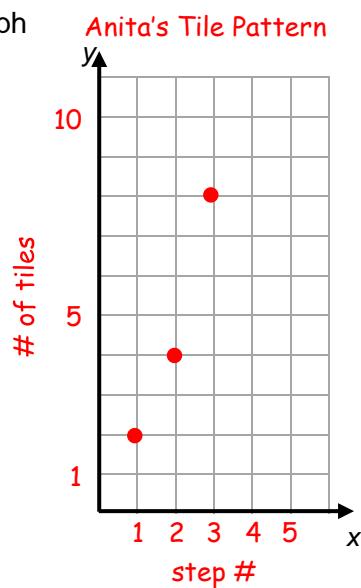
(2) Picture



(3) Table

Anita's Tile Pattern	
step # (x)	# of tiles (y)
1	2
2	4
3	8
4	16
5	32

(4) Graph



LESSON NOTES S7.1a: WHAT COMES NEXT?

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

We focus on growing geometric patterns for a review of multiple representations (i.e., recording numerical data in tables and graphs), a context that shows a relation between two variables, and a connection to rates. This work will continue in grade 7, and becomes formalized in grade 8 with a study of linear functions. Many students benefit from building each pattern with square tiles prior to drawing it.

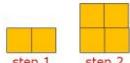
- Slide 1: Show the growing pattern. Ask students to build the first two steps, and then a third step that makes sense to them.

What is changing from step 1 to step 2? Two tiles are added, the number of tiles is doubled.

For (1), students write descriptions of the growing pattern. Ask students to describe their 3rd steps (one or more). Share as desired.

WHAT COMES NEXT?

Mateo and Anita are making multi-step patterns with square tiles. They both started with these two steps.



step 1 step 2

Mateo and Anita see this "growing" pattern differently and so they have different 3rd steps. They are both able to justify their 3rd step.

Build step 1 and step 2. Then build one or two 3rd steps you think make sense.

(1) Describe the way your pattern is growing, including how you built step 3.

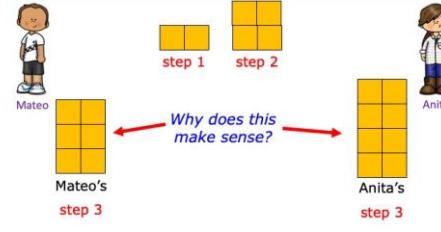
1 Math Links

- Slide 2: Reveal Mateo's and Anita's 3rd steps, even if students have already come up with these options.

Why can both of these be correct? Mateo sees the pattern as starting with 2 tiles and adding 2 more at each step. Anita sees it as starting with 2 tiles and doubling at each step.

For (2), ask students to draw steps 1 – 4 for both patterns.

ANALYZE STUDENT DIAGRAMS



step 1 step 2

Why does this make sense?

(2) Draw steps 1-4 for both Mateo's and Anita's patterns.

2 Math Links

- Slide 3: For (3), ask students to complete tables for both. Discuss appropriate titles and column headings as needed.

TABLE FOR MATEO'S PATTERN

Mateo's Tile Pattern	
step # (x)	# of tiles (y)
1	2
2	4
3	
4	
5	

step 1 step 2 step 3

Mateo Anita

Title for table?
Column Headings?

(3) Complete tables for both Mateo's and Anita's data.

3 Math Links

LESSON NOTES S7.1a: WHAT COMES NEXT?

Continued

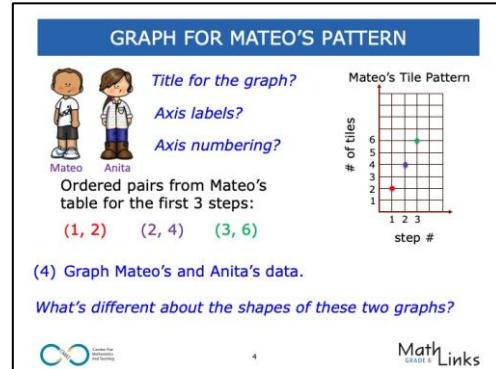
- Slide 4: For (4), ask students to complete graphs for both. If needed, discuss good title, axis labels, and axis numbering, and remind students that entry pairs from the table act as ordered pairs.

Do we need to plot all data points in the table? No. Some may not fit. We only need enough points to determine the shape of the graph.

Should we "connect the dots" for these graphs? This pattern context only makes sense for whole numbers of steps and tiles. It is permissible to draw a "trend line/curve" to show an overall shape.

What's different about the shapes of these graphs? Mateo's points appear on a straight line; data points increase at a constant rate. Anita's points appear on a curve; data points increase at increasing rate.

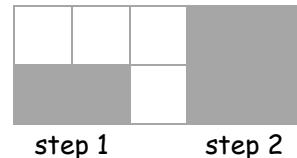
How is the change from one step to the next shown on the different representations for Mateo? On the graph and in the table, as each step number increases by 1, the number of tiles increases by 2. For each increase by 2 tiles upward, the step number increases by 1 to the right.



SLIDE DECK ALTERNATIVE 7.1a: WHAT COMES NEXT?

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Mateo and Anita are making multi-step patterns with square tiles. They both started with the two steps. They see the patterns growing differently so they have different 3rd steps.



Build steps 1 and 2. Then build one or two 3rd steps you think make sense.

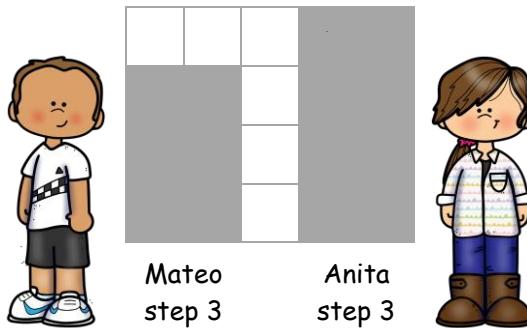
Slide 1

(1) Describe the way your pattern is growing.

Use your description to explain how you built step 3.

Slide 2

(2) Here is step 3 for Mateo's pattern and Anita's pattern. Draw steps 1-4 for both patterns.



Slide 3

(3) Complete tables for both Mateo's and Anita's data.

Slide 4

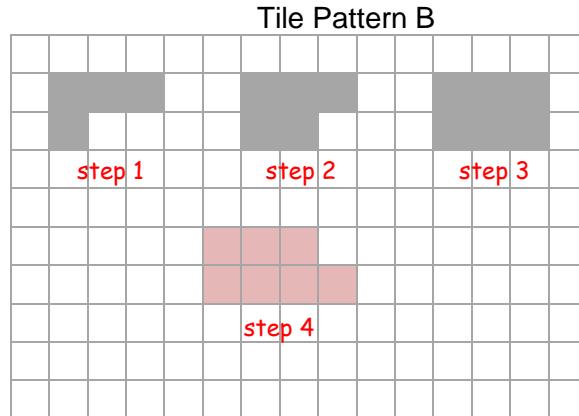
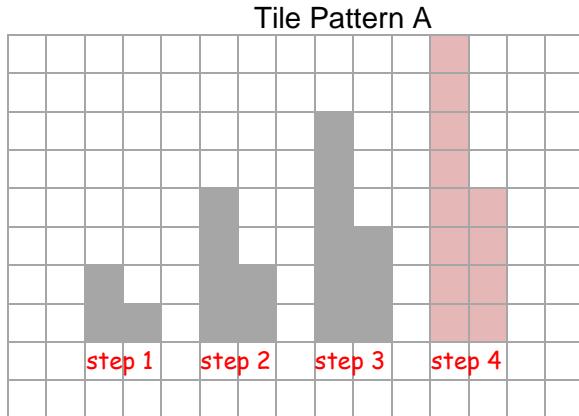
(4) Graph Mateo's and Anita's data.

What is different about the shapes of these two graphs?

PRACTICE 1

[SMP1]

1. Label steps 1 – 3 for tile patterns A and B, and draw step 4 for each pattern. Complete the tables and draw the graphs with titles and labels.

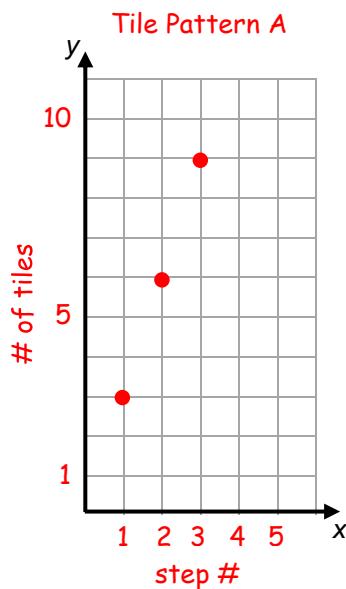


Tile Pattern A

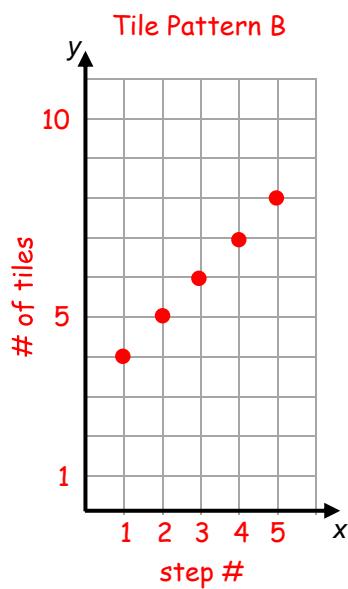
step # (x)	# of tiles (y)
1	3
2	6
3	9
4	12
5	15

Tile Pattern B

step # (x)	# of tiles (y)
1	4
2	5
3	6
4	7
5	8



trend lines may
be drawn to
stress the linear
relationships



2. Why is it appropriate to leave the points unconnected? Step numbers and numbers of tiles must be whole number amounts, so we do not want to include points that represent fractional step numbers or numbers of tiles.

INPUT-OUTPUT RULES

[SMP1, 2, 6]

Follow your teacher's directions for (1) – (2).

(1) *Input-output rule for Mateo in words.*
To get the number of tiles, multiply the step number by 2.

(2) *Input-output rule for Mateo in symbols.*
 $y = 2x$

3. Record the meaning of input-output rule in **My Word Bank**.4. Write an input-output rule for Pattern A on **Practice 1** in words and symbols.

To get the number of tiles, multiply the step number by 3.
 $y = 3x$

Watch for (4,5): Do students write full, descriptive input-output rules in words? For example, "multiply by 3" or "add 3" are incomplete.

5. Write an input output rule for Pattern B on **Practice 1** in words and symbols.

To get the number of tiles, add 3 to the step number.
 $y = x + 3$

The input values (x) are sometimes referred to as the “independent variable,” and output values (y) are sometimes referred to as the “dependent variable.”

Fill in the missing numbers and blanks based on the suggested numerical patterns. In the tables below, the x -value is considered the input value and the y -value is the output value.

6.

x	1	2	3	4	5	6
y	5	6	7	8	9	10

a. Rate of change: for every increase of x by 1, y increases by 1.

b. Input-output rule (words): add 4 to an x -value (independent variable) to get its corresponding y -value (dependent variable).

c. Input-output rule (equation): $y = x + \underline{4}$

7.

x	1	2	3	4	5	6
y	4	8	12	16	20	24

a. Rate of change: for every increase of x by 1, y increases by 4.

b. Input-output rule (words): Multiply an x -value (independent variable) by 4 to get its corresponding y -value (dependent variable).

c. Input-output rule (equation): $y = \underline{4} \cdot x$

LESSON NOTES S7.1b: INPUT-OUTPUT RULES

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

We revisit three geometric patterns and their corresponding tables of values to focus on the relationship between the "independent variables" and "dependent variables." Students also begin to write these relationships in words and with symbols (equations) as input-output rules.

- Slide 1: Revisit Mateo's pattern. Focus on the relationship between the input values and the output values, and then discuss input-output rules.

What can we do to each x-value to obtain the corresponding y-value? Multiply the x-value by 2 to get the y-value. If students reply to add 2 each time, they are recognizing a recursive rule that describes successive output values, which is valid, but not the focus here.

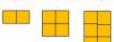
For (1) and (2), students write an input-output rule for Mateo's pattern in words and using symbols.

- Slide 2: Discuss terms that are associated with the input values (independent variable) and output values (dependent variable) as desired.

Why does the number of tiles depend on the step number? If we know the step number, we can multiply it by 2 to determine the number of tiles. That is why step number is the independent variable and the number of tiles is the dependent variable.

INPUT-OUTPUT RULES

step 1 step 2 step 3



What operation can be performed on each input value to obtain each corresponding output value?

An equation that expresses this relationship is called an input-output rule.

(1) Write the input-output rule for Mateo's pattern in words.

(2) Write the input-output rule for Mateo's pattern using symbols.

1 Math Links

INDEPENDENT AND DEPENDENT VARIABLES

We usually specify the input values. We typically refer to inputs as the independent variable.

The input values determine the output values. We typically refer to outputs as the dependent variable.

Does number of tiles seem to depend upon the step number?

What is the independent variable and the dependent variable in Mateo's tile pattern?

2 Math Links

SLIDE DECK ALTERNATIVE S7.1b: INPUT-OUTPUT RULES

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slide 1

step 1		step 2		step 3

Mateo's Tile Pattern	
step # (x)	# of tiles (y)
1	2
2	4
3	6
4	8
5	10

↑
input → output
values values

- (1) Write the input-output rule for Mateo's pattern in words.
- (2) Write the input-output rule for Mateo's pattern using symbols.

Slide 2

We usually specify the input values. We typically refer to the inputs as the independent variable. The output values are determined by the input values. We typically refer to the outputs as the dependent variable.

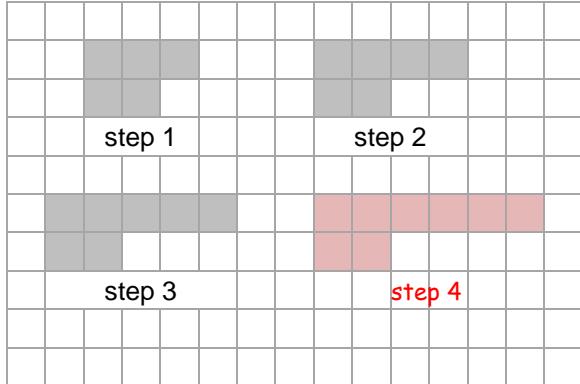
(3) For Mateo's tile pattern, specify the independent variable and the dependent variable.

PRACTICE 2

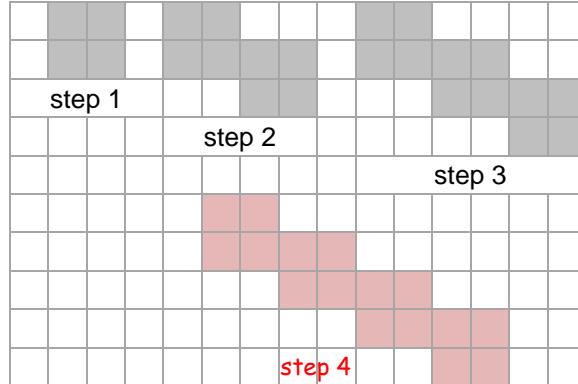
[SMP1, 2, 7]

1. Build steps 1–4 for tile patterns C and D if needed. Draw step 4 for each. Complete the tables and make graphs with titles and labels. Write an input-output rule (equation).

Tile Pattern C



Tile Pattern D



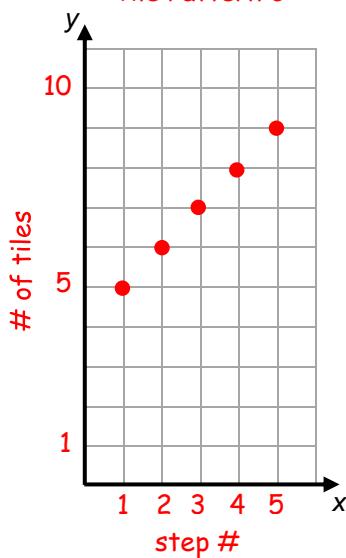
Tile Pattern C

step # (x)	# of tiles (y)
1	5
2	6
3	7
4	8
5	9

Tile Pattern D

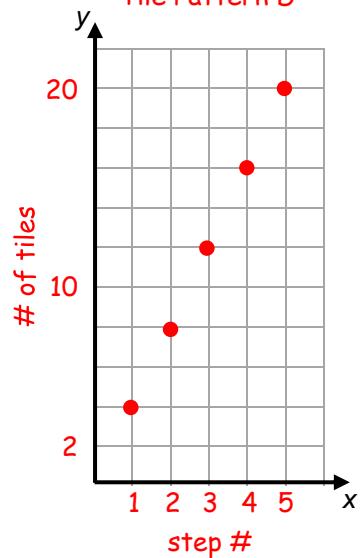
step # (x)	# of tiles (y)
1	4
2	8
3	12
4	16
5	20

Tile Pattern C



trend lines may
be drawn to
stress the linear
relationships

Tile Pattern D

Rule for C: $y = x + 4$ Rule for D: $y = 4x$

2. True or false: for these patterns, typically the step number is the independent variable.
true

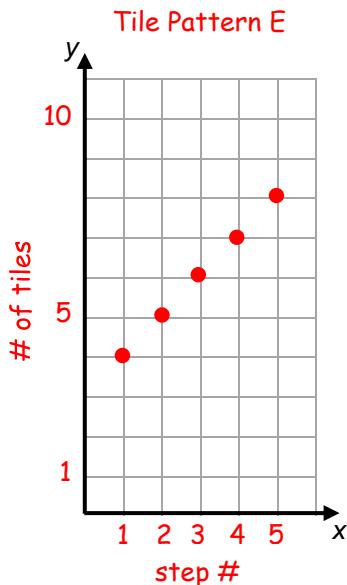
PRACTICE 3

[SMP1, 2, 7]

1. Pattern E is described with a table. Pattern F is described with a graph. Complete the other representations in any order.

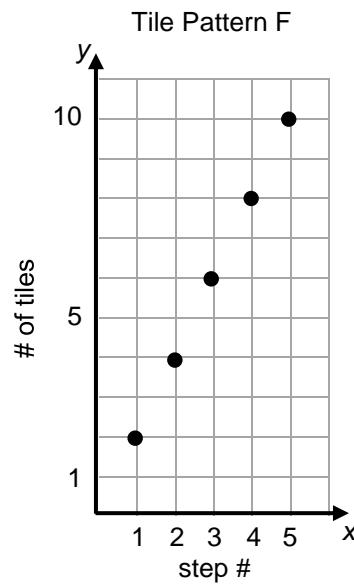
Tile Pattern E	
step # (x)	# of tiles (y)
1	3
2	4
3	5
4	6
5	7

Tile Pattern F	
step # (x)	# of tiles (y)
1	2
2	4
3	6
4	8
5	10

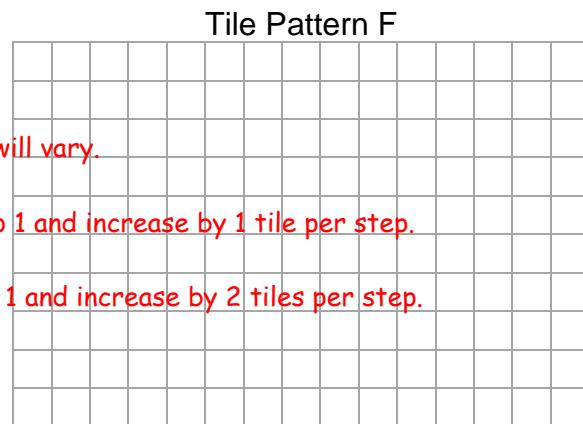
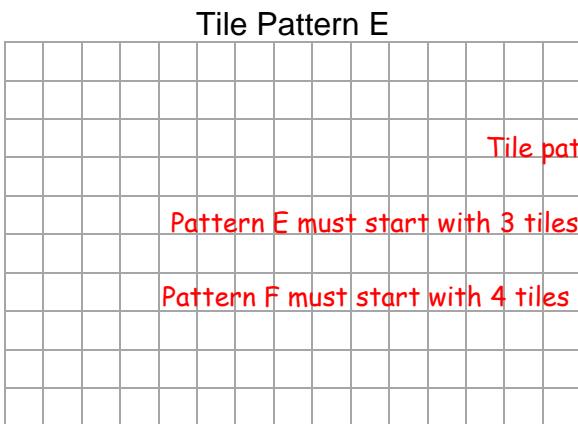


Rule for C: $y = x + 2$

trend lines may
be drawn to
stress the linear
relationships



Rule for D: $y = 2x$



Write the increase in number of tiles for each step for each pattern. E: 1 F: 2

2. True or false: For these patterns, typically the step number is the dependent variable. false

PRACTICE 4: EXTEND YOUR THINKING

[SMP1, 2, 8]

1. Fill in the chart based upon the work you did for tile patterns A – F.

- Column I: Copy each rule (make sure you have the correct rules before proceeding).
- Columns II-IV: Find the numbers of square tiles for the given step numbers.
- Column V: Find each step number when the number of square tiles is 60.

I	II	III	IV	V
Pattern	Step 10	Step 100	Step 1,000	Step Number for 60 tiles*
A \rightarrow $y = 3x$	30	300	3,000	20
B \rightarrow $y = x + 3$	13	103	1,003	57
C \rightarrow $y = x + 4$	14	104	1,004	56
D \rightarrow $y = 4x$	40	400	4,000	15
E \rightarrow $y = x + 2$	12	102	1,002	58
F \rightarrow $y = 2x$	20	200	2,000	30

* Students are expected to use any number-sense strategy that makes sense to them. More formal equation-solving strategies are introduced in Unit 8.

2. Complete the table and fill in the blanks.

a.	x	1	2	3	4	5	6	8	11	13
	y	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	4	$5\frac{1}{2}$	$6\frac{1}{2}$

b. Rate of change: for every increase of x by 1, y increases by $\frac{1}{2}$.

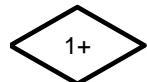
c. Input-output rule (words): Multiply an x -value by $\frac{1}{2}$ to get its corresponding y -value;

OR divide an x -value by 2 to get its corresponding y -value.

d. Input-output rule (equation): $y = \frac{1}{2} \cdot x$; OR $y = \frac{x}{2}$

e. If $x = 100$, then $y = 50$.

f. If $y = 100$, then $x = 200$.



COMPARING PRICES

We will use tables, double number lines, graphs, unit prices, equations, and words to compare prices.

[6RP3abc, 6NS3, 6EE2a, 6EE6, 6EE9, 6SP1, 6SP3; SMP1, 2, 4, 5, 7]

GETTING STARTED

Suppose you were shopping for groceries at Barter Jack's and have to make some choices. Explain which choices you'd make and why. *Answers may vary.*

Choice 1: energy bars	Choice 2: fruit
<p>Healthy Crunch → 2 bars for \$2.50</p> <p>OR</p> <p>Super Bar → 3 bars for \$2.50</p> <p><i>Answers will vary.</i></p> <p>Based upon quantity and price, you pay the same for Healthy Crunch and Super Bar (\$2.50), but get more Super Bars at that price, making them the better buy.</p> <p>However, taste or health considerations may factor into the decision.</p>	<p>bananas → 2 pounds for \$4.10</p> <p>OR</p> <p>apples → 2 pounds for \$4.95</p> <p><i>Answers will vary.</i></p> <p>Based upon quantity and price, you get the same number of pounds of apples as bananas (2 lb), but the bananas are cheaper, making them the better buy.</p> <p>However, taste or health considerations may factor into the decision.</p>

THE KEYCHAIN FUNDRAISER

[SMP1, 2, 7]

Follow your teacher's directions for (1) – (2).

(1a) – (1e) and (2a) – (2e) Copy pricing information, and complete the various representations.

(1a)

HI-TOPS
2 for \$3

(2a)

DONUTS
3 for \$4

(1b) Table

HI-TOPS	
# of keychains (x) (quantity)	Cost in \$ (y)
2	3
4	6
6	9
1	1.50
3	4.50

(2b) Table

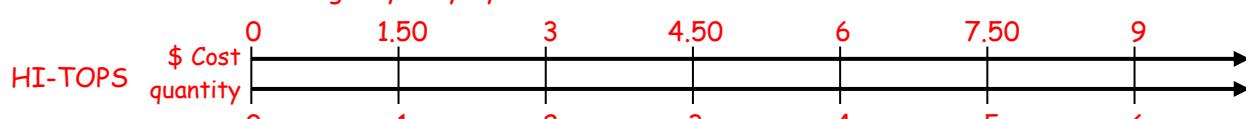
DONUTS	
# of keychains (x) (quantity)	Cost in \$ (y)
3	4
6	8
9	12
1	1.33
2	2.66 or 2.67

(1c) Rule: $y = 1.50x$

(2c) Rule: $y = 1.33x$

Double Number Lines Rounding may vary by one cent.

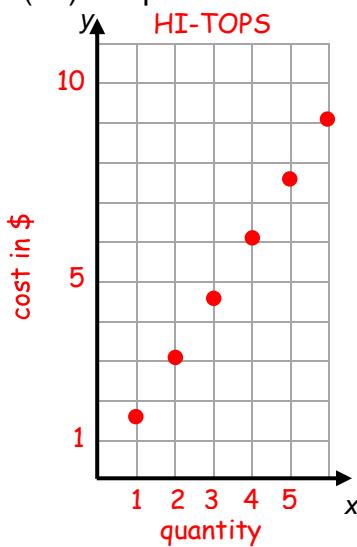
(1d)



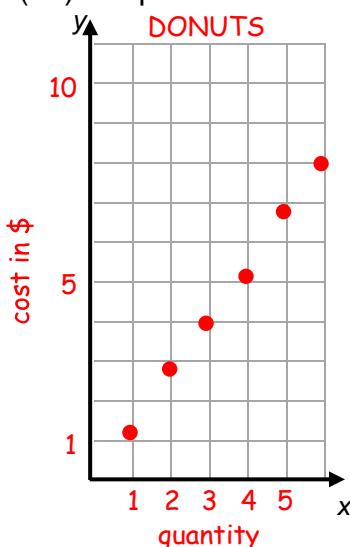
(2d)



(1e) Graph



(2e) Graph



Watch for (1bd,2bd): Do students recognize that the table values are not in order, but the double number line values must be put in order?

3. Record the meanings of unit rate and unit price in **My Word Bank**.

LESSON NOTES S7.2: THE KEYCHAIN FUNDRAISER

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

In this lesson, we analyze prices using multiple representations including tables, equations, double number lines, graphs, and unit prices. Students will compare prices and other features to decide which keychains might be preferable for a fundraiser. They may wonder about various details. For example, must packages of keychains be sold intact or can they be split up? Is a unit price analysis reasonable? Is the quality the same? Are different options equally desirable? Entertain questions as they arise, and discuss assumptions to keep the analysis reasonable.

- Slide 1: For (1a) – (1d), ask students to copy the pricing information for Hi-Tops and create the three representations. This is review. Discuss as needed.

Which appears to be the input value? Number of keychains (or quantity).

Are any entries in the table helpful for creating an input-output rule (equation)? Students may realize that the price for 1 keychain represents the coefficient of x .

- Slide 2: Connect a double number line to a coordinate graph. For (1e), students complete a graph.

How do double number lines relate to coordinate planes?

One of the number lines becomes the x -axis and the other the y -axis. Pairs of entries can be graphed. A graph in a coordinate plane shows how one variable appears to be changing in relation to the other at a quick glance.

THE KEYCHAIN FUNDRAISER

The Lincoln Middle School fundraising committee wants to sell keychains to raise money for the big dance. Keychains are packaged and sold in small quantities.

(1a) Copy the pricing information for the Hi-Tops keychains.

HI-TOPS
2 for \$3.00

(1b) Complete a table that relates the number of keychains (quantity) and cost.

(1c) Write an input-output rule that relates quantity and cost.

(1d) Complete a double number line.

1

Math Links
Grade 6

FROM NUMBER LINES TO GRAPHS

Cost in \$ quantity

How do double number lines relate to coordinate planes?

(1e) Complete a graph.

2

Math Links
Grade 6

- Slide 3: **What does the unit price mean in this context?** It is the price for one Hi-Top keychain (\$1.50). For this discussion it does not matter if they are sold separately.

Reveal the pricing information for Donuts. For (2a) – (2e) ask students to complete the representations.

Which keychain has a lower unit price? The DONUTS keychains are a better buy in terms of price per one keychain (unit rate or unit price), but this does not necessarily mean they are preferable for the fundraiser.

UNIT PRICE

HI-TOPS
2 for \$3.00

What is the unit price and what does it mean in this context?

(2a-2e) For the Donuts keychains:

DONUTS
3 for \$4.00

• Copy the pricing information.

• Complete a table, equation, double number line, and graph (round as needed).

Which keychain has the lower unit price?

3

Math Links
Grade 6

SLIDE DECK ALTERNATIVE S7.2: THE KEYCHAIN FUNDRAISER

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slides 1-2

The Lincoln Middle School fundraising committee wants to sell keychains to raise money for the big dance. Keychains are packaged and sold in small quantities.

Hi-Top Keychains are 2 for \$3.00.

(1a) Copy the pricing information for the Hi-Top keychains.

(1b) Complete a table that relates the number of keychains (quantity) and cost.



(1c) Write an input-output rule that relates quantity and cost.

(1d) Complete a double number line.

(1e) Complete a graph.

Slide 3

Donut Keychains are 3 for \$4.00.

(2a-e) For the Donuts keychains:

- Copy the pricing information.
- Complete a table, equation, double number line, and graph (round as needed).



Which keychain has the lower unit price?

PRACTICE 5

[SMP1, 2, 7]

Here are two more keychain packages. Complete the representations for both.
Rounding may vary by about one cent on this page.

GOOGLIES
5 for \$6



EMOJIS
6 for \$5



1a. Table

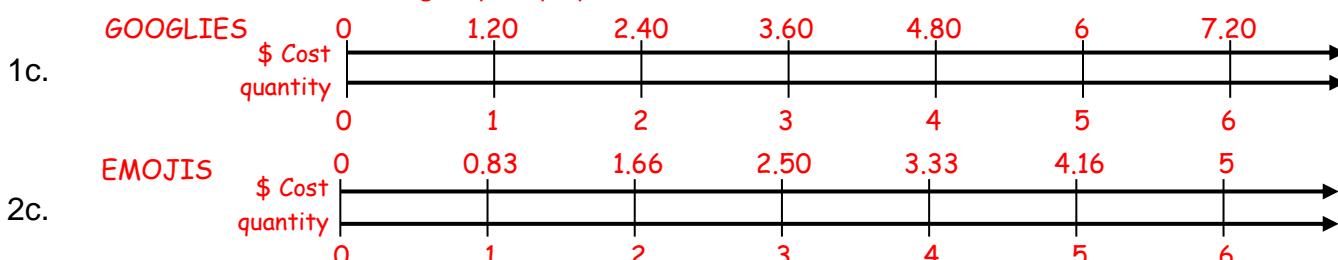
GOOGLIES	
quantity (x)	Cost in \$ (y)
5	6
10	12
1	1.20
2	2.40
3	3.60

2a. Table

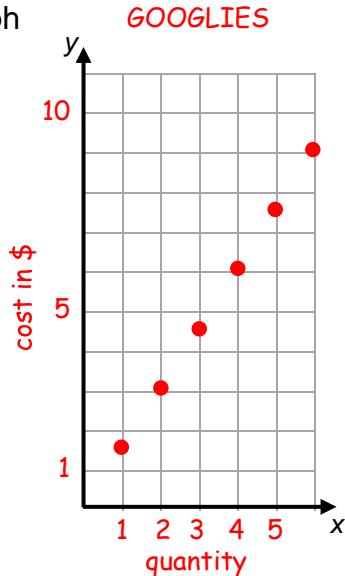
EMOJIS	
quantity (x)	Cost in \$ (y)
6	5
12	10
3	2.50
1	0.83
2	1.66

1b. Rule: $y = 1.20x$ 2b. Rule: $y = 0.83x$

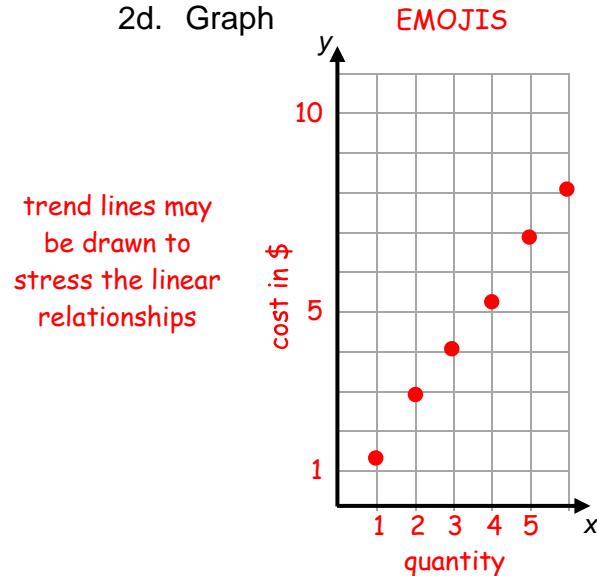
Double Number Lines Rounding may vary by one cent.



1d. Graph



2d. Graph

3. EMOJIS have the lower unit price because... $\$0.83 \text{ per keychain} < \$1.20 \text{ per keychain}$.

PRACTICE 6

[SMP1, 2, 7]

Here are two more keychain packages.

LOCKS
3 for \$6CUBES
2 for \$51. Complete each table below.

LOCKS	
quantity (x)	Cost in \$ (y)
3	6
6	12
1	2
2	4
5	10

CUBES	
quantity (x)	Cost in \$ (y)
2	5
4	10
6	15
1	2.50
3	7.50

2. Explain how you know which is the cheaper purchase based on unit price.

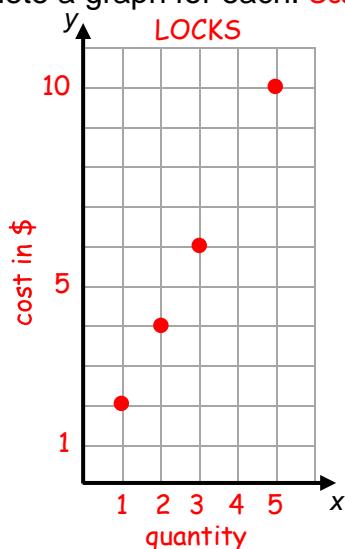
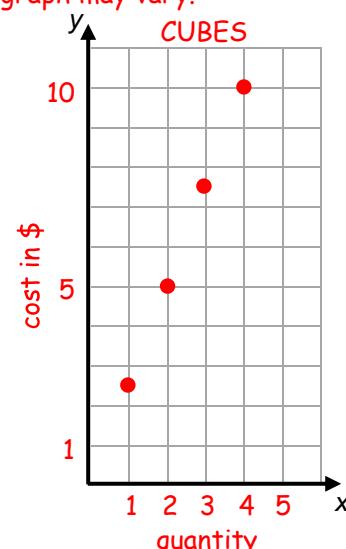
Locks are cheaper. For 1 keychain, \$2 per keychain < \$2.50 per keychain.

3. Explain how you know which is cheaper based on the entries with $x = 3$.

Locks are cheaper. For 3 keychains, \$6 < \$7.50.

4. Explain how you know which is cheaper based on the entries with $y = 10$.

Locks are cheaper. For \$10, 5 keychains > 4 keychains.

5. Write a rule for each. LOCKS: $y = 2.00x$ CUBES: $y = 2.50x$ 6. Complete a graph for each. Scaling and points chosen to graph may vary.trend lines may
be drawn to
stress the linear
relationships7. Which graph illustrates a greater cost increase per each additional keychain? CUBES
How can you see this when comparing the graphs? The graph for CUBES is "steeper," (increase of \$2.50 per additional keychain compared to an increase of \$2.00 per additional keychain).

A COMMITTEE DECISION

The MathLinks Rubric: See Activity Routines on the Teacher Portal for directions.

[SMP4, 5, 7]

Help the Lincoln Middle School fundraising committee decide which keychains to sell for the fundraiser. The six different keychains analyzed on the previous pages are listed below. In addition, a small survey was taken, the results of which are in the table below.

1. Complete the table. **Percent rounded to the nearest whole percent.**

Keychain	Price	Unit price (price per keychain)	Students polled who preferred this keychain:	
			Number	Percent
Hi-Tops	2 for \$3	\$1.50	18	30%
Donuts	3 for \$4	\$1.33	10	17%
Googlies	5 for \$6	\$1.20	20	33%
Emojis	6 for \$5	\$0.83	6	10%
Locks	3 for \$6	\$2.00	1	2%
Cubes	2 for \$5	\$2.50	5	8%
			Total: 60	Total: 100%

Consider unit prices from the table.

2. What is the range of prices? \$2.50 - \$0.83 = \$1.67	3. What is the median price? \$1.42	4. What is the mean price? \$1.56
---	--	--

Write one statistical question based on each. **Answers will vary. Examples:**

5. Unit prices What is the typical unit price of a keychain?	6. Students polled What keychains do students like best?
---	---

7. Recommend one or more keychains to the committee based on data from the table.

Answers will vary. Some possibilities:

- Recommend emoji's because they are the cheapest
- Recommend googlies because they are the most popular and 2nd cheapest

Watch for (7): Are students making recommendations based on data, as instructed?

RATE APPLICATIONS

We will deepen our understanding of independent and dependent variables. We will use rates in money contexts and in time-distance contexts. We will identify unit rate in tables, graphs, and equations.

[6RP3abc, 6NS3, 6EE2ab, 6EE6, 6EE9; SMP1, 2, 4, 5, 6, 7, 8]

GETTING STARTED

[SMP8]

Julie is saving \$10 each week from her allowance.

1. Fill in the table for the total amount saved at each week through week 6.

Week # (W)	1	2	3	4	5	6	10	25	37
Total \$ (T)	10	20	30	40	50	60	100	250	370

2. Rate of change: for every increase of one week, the total increases by \$10

3. Complete an input-output rule that relates week number to total dollars saved: $T = \underline{10W}$

4. For the last three entries in the table let $W = 10, 25$, and 37 . Use your rule to find the corresponding values for T .

5. For problem 2 above, typically, which quantity is the independent variable? **Week number**

6. After how many weeks of allowance will Julie have saved each amount?

a. \$400	b. \$500
40 weeks	50 weeks
c. \$450	d. \$425
45 weeks	43 weeks $10(42.5) = 425$, but allowances are not usually given for partial weeks

RAISING MONEY FOR MUSIC

[SMP1, 2, 6, 7, 8]

Follow your teacher's directions for (1) – (2).

The Springfield Education Foundation is trying to raise \$100,000 for its music programs. They have fundraisers throughout the year.

(1) The SEF sells holiday trees for its December fundraiser. Each tree sells for \$50.

Complete Tables I and II.

Table I	
# of trees sold (t)	Money earned in \$ (m)
1	50
2	100
5	250
10	500
50	2,500
100	5,000

Table II	
Money earned in \$ (m)	# of trees sold (t)
50	1
100	2
200	4
600	12
1,000	20
4,000	80

(2) *Write two different word statements that relate t and m .*

Table I: The number of money earned depends on the number of trees sold.

Table II: The number of trees sold depends on the amount of money earned.
(or the money needed to earn for a particular fundraising goal).

3. Write two different equations that relate t and m .

$$m = \underline{50t} \qquad t = \underline{\frac{m}{50}}$$

Use the equations from problem 3 to complete problems 4 – 5.

4. If they sell 1,000 trees, how much will they earn?

$$m = 50t = 50(1,000) = 50,000 \text{ dollars}$$

What percent of the way would they be to their goal?

$$\frac{50,000}{100,000} = 50\%$$

5. How many trees sold raises \$60,000?

$$t = \frac{m}{50} = \frac{60,000}{50} = 1,200 \text{ trees}$$

If they did this, how many more trees would they have to sell to reach their goal?

$$\text{This requires another } \$40,000 \rightarrow t = \frac{m}{50} = \frac{40,000}{50} = 800 \text{ trees.}$$

Watch for (all): Do students grasp the subtlety of dependent and independent variables? On this page, inverse situations are posed where the money earned depends on the number of trees sold, and then later the number of trees that are sold depends on the money that needs to be earned.

LESSON NOTES S7.3: RAISING MONEY FOR MUSIC

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

Here we analyze a fundraising effort using tables and equations. We also consider that the independent and dependent variables might change based upon our focus.

- Slide 1: Focus attention on the problem context and the tables started on this slide.

What's the same about each table? They both keep track of the number of trees sold and money earned in dollars.

What's different about each table? The variables are "switched." In Table I, number of trees sold is the input, and money earned in dollars is the output. Table II is the opposite.

For (1), students copy the cost of a tree and complete the tables.

- Slide 2: For (2), pose each sentence frame one at a time and discuss.

When does it make sense that number of trees is the input and money earned in dollars is the output? The amount of money we earn will depend upon how many trees we sell. In this case, the trees are the independent variable and amount of money is the dependent variable.

When does it make sense that money earned in dollars is the input and number of trees is the output? This may seem less obvious than the question above. We might think about a particular fundraising goal in such a way that the number of trees we want to sell depends upon the amount of money we want to earn. In this case, the amount of money is the independent variable and the number of trees is the dependent variable.

RAISING MONEY FOR MUSIC

What's the same about each table? What's different?

Table I		Table II	
# of trees sold (t)	Money earned in \$ (m)	Money earned in \$ (m)	# of trees sold (t)
1		50	
2		100	
5		200	
10		600	
50		1,000	
100		4,000	

(1) Each tree is \$50. Complete Tables I and II.

A CLOSER LOOK AT INPUTS AND OUTPUTS

(2) Write two different word statements that relate *t* and *m*:

Table I: The amount of money earned depends on...

Table II: The number of trees sold depends on...




SLIDE DECK ALTERNATIVE S7.3: RAISING MONEY FOR MUSIC

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slide 1

What is the same about each table? What is different?

- (1) Each tree is \$50. Complete Tables I and II.

Slide 2

- (2) Complete two different word statements that relate t and m :

Table I: The amount of money earned depends on...

Table II: The number of trees sold depends on...

PRACTICE 7

[SMP1, 2, 7, 8]

1. The Springfield Education Foundation hosts a pancake breakfast for its May fundraiser. They sell each breakfast for \$5.50. Complete Tables III and IV.

Table III

# of breakfasts sold (b)	Money earned in \$ (m)
1	5.50
2	11
4	22
12	66
40	220
100	550

Table IV

Money earned in \$ (m)	# of breakfasts sold (b)
5.50	1
11	2
55	10
110	20
440	80
660	120

2. Write two different equations that relate b and m .

a. $m = 5.50b$

b. $b = \frac{m}{5.50}$

3. In Table III, money earned in dollars depends on... **number of breakfasts sold**.

4. If they sell 1,000 breakfasts, how much will they earn? **\$5,500**

5. The Springfield Education Foundation sponsors a walk-a-thon for its August fundraiser. Each entry fee is \$25. Complete Tables V and VI.

Table V

# of entries sold (w)	Money earned in \$ (m)
1	25
2	50
3	75
5	125
10	250
100	2,500

Table VI

Money earned in \$ (m)	# of entries sold (w)
25	1
50	2
125	5
500	20
1,000	40
2,000	80

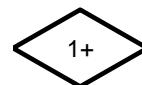
6. Write two different equations that relate m and w .

a. $m = 25w$

b. $w = \frac{m}{25}$

7. In Table VI, number of entries sold depends on **money earned in \$ (their goal)**.

8. How many entries sold accomplishes a \$6,500 goal? **260**



RUNNING

[SMP1, 2, 7, 8]

Martino is training for a marathon. Today he ran for 3 hours and 15 minutes at a constant rate of speed. A graph of his run is shown.

1. Write 3 hours and 15 minutes as a decimal.

3.25 hours

2. Write a title and axis labels. The input values are time (in hours). The output values are distance (in miles).

3. Why is it appropriate to connect the points on the graph?

Time and distance are continuous measures. In other words, fractional portions make sense in the context.

4. Explain what the point $(0,0)$ on the graph means in the context of this problem.

Martino ran 0 miles in 0 hours.

5. Complete the table below.

Time in hours (t)	1	2	3	0.5	0.25	1.5	3.25
Distance in miles (d)	4	8	12	2	1	6	13

6. Write an equation for distance in terms of time.

$d = 4t$

7. What is the meaning of the coefficient of t in the equation? Circle it in the table and the graph.

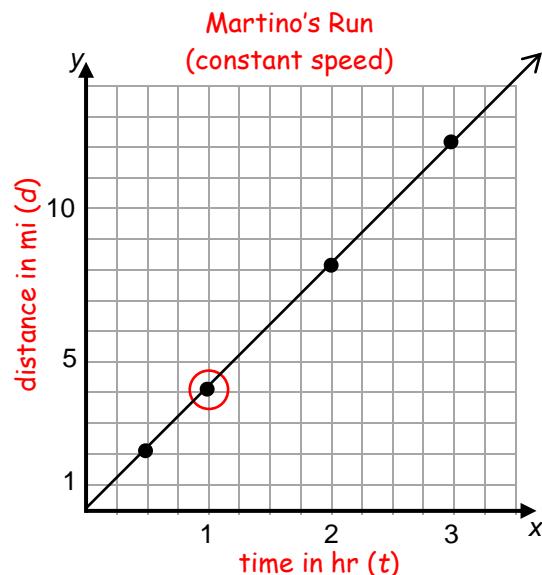
The 4 represents the number of miles run per one hour, or the unit rate.

8. At this rate, how far did Martino run in 2.5 hours?

10 miles

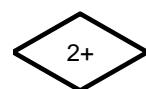
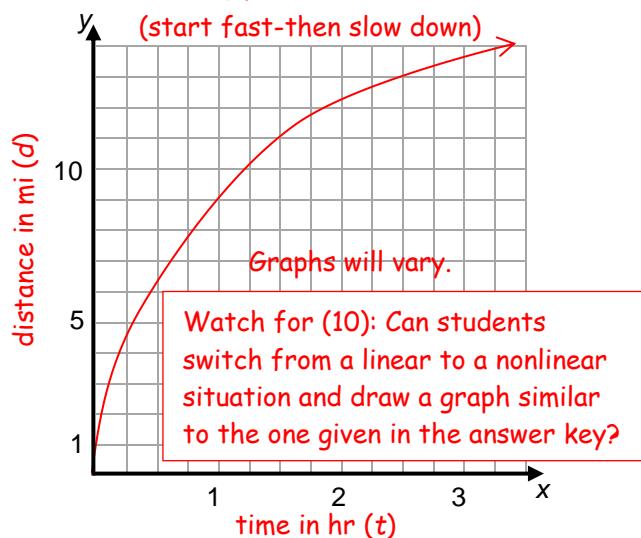
9. At this rate, how many hours would it take Martino to run 17 miles?

4.25 hours



10. Suppose Martino starts his run fast and then slows down. Make a properly labeled graph to illustrate this situation.

Martino's Run



PRACTICE 8

[SMP1, 2, 7, 8]

Darryl is doing a 52-mile bicycle ride that takes him 4 hours. He rides at a constant rate.

1. Complete the table below.

Time in hours (t)	4	2	1	0.5	0.25	0.75	1.5	2.75	3.25
Distance in miles (d)	52	26	13	6.5	3.25	9.75	19.5	35.75	42.25

2. Write an equation that relates distance and time. $d = 13t$

3. Draw a graph below. Include at least four points from the table. Be sure to label it completely. *Axis scaling and graphed points may vary. Placement of points may be estimated.*

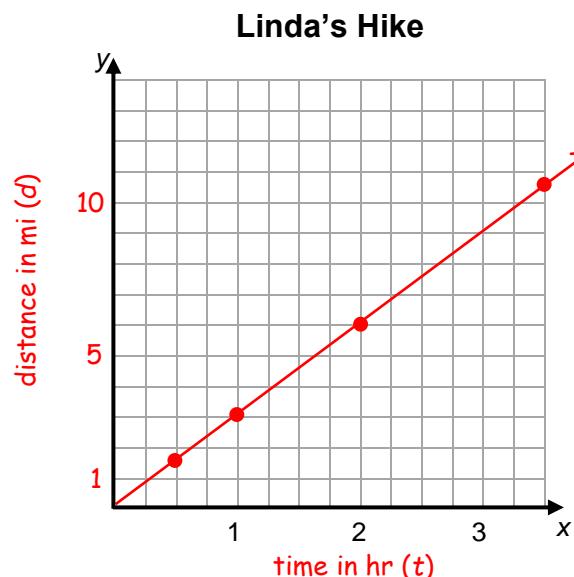
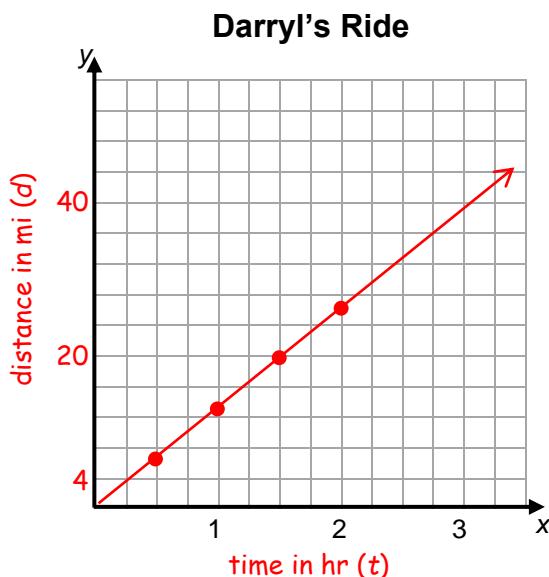
Linda is doing an 18-kilometer uphill hike that takes her 6 hours. She hikes at a constant rate.

4. Complete the table below.

Time in hours (t)	6	1	2	0.5	0.25	0.75	1.75	2.25	3.5
Distance in kilometers (d)	18	3	6	1.5	0.75	2.25	5.25	6.75	10.5

5. Write an equation that relates distance and time. $d = 3t$

6. Draw a graph below for Linda's hike. Include at least four points from the table. Be sure to label it completely. *Axis scaling and graphed points may vary. Placement of points may be estimated.*



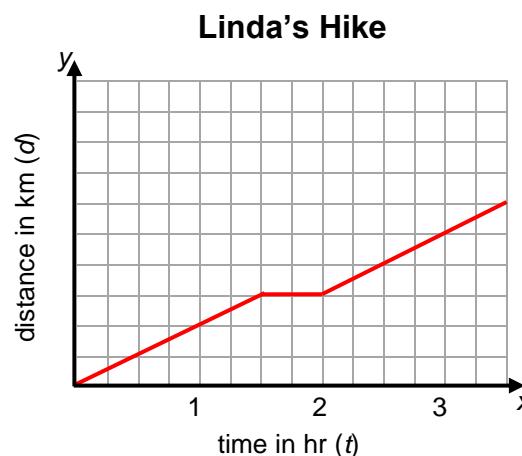
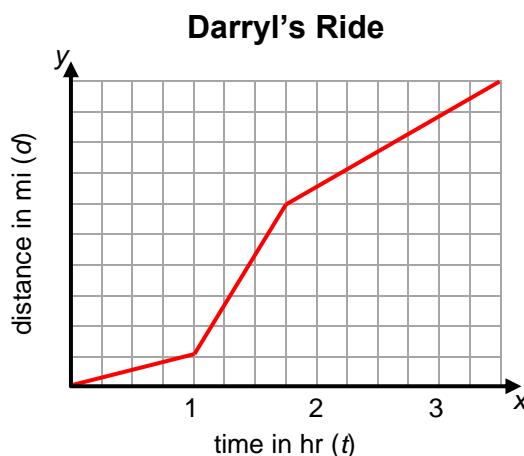
PRACTICE 9: EXTEND YOUR THINKING**The MathLinks Rubric:** See Activity Routines on the Teacher Portal for directions.

[SMP1, 2, 4, 7]

Refer to Darryl's ride and Linda's hike from the previous page.

Graphs will vary. Numbers for distance are purposely omitted, but may be included.

- Sketch a graph for Darryl if, instead of riding at a constant rate the entire race, he started off slowly uphill, then fast downhill, then finished at medium speed on flat land.
One possible answer is shown below.
- Sketch a graph for Linda if she hikes at some constant rate the entire time, except for a 30-minute break somewhere in the middle of the hike.
One possible answer is shown below.



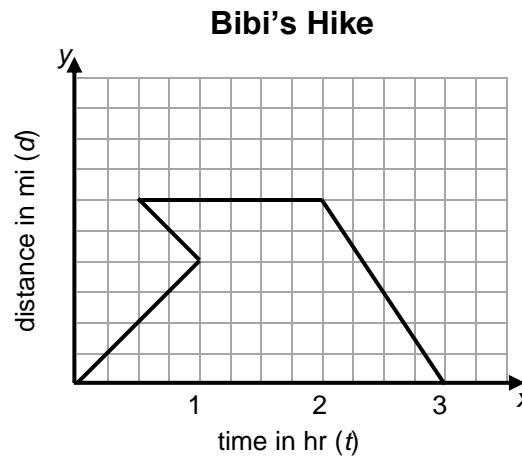
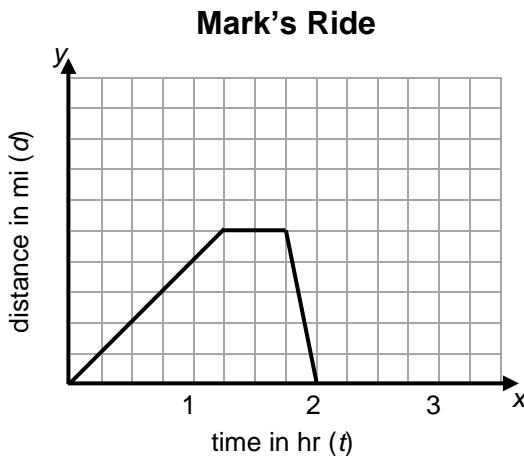
Notice that the graphs for Mark and Bibi below have distance on the vertical axes.

- Explain Mark's graph in writing.

Mark starts at $h = 0$ m ("sea level"), travels uphill at a constant rate for 1 hour and 15 minutes, stays at that height for 30 minutes, and then travels back down to his start height in 15 minutes.

- Explain the part of Bibi's hike that does not make sense.

There is a line segment that shows time going "backwards." In other words, as the height increases, time cannot decrease.

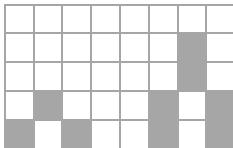
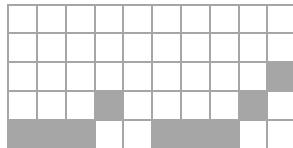
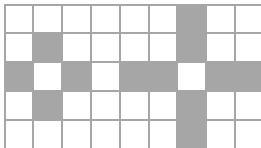
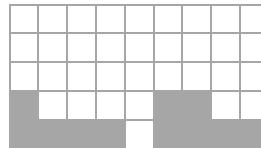


REVIEW**POSTER PROBLEMS: INPUTS AND OUTPUTS****See Activity Routines on the Teacher Portal for directions.****[SMP1, 2, 7, 8]**

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _____.
- Each group will have a different colored marker. Our group marker is _____.

Part 2: Do the problems on the posters by following your teacher's directions. Steps 1 and 2 of each pattern are given below.

Poster 1 (or 5)	Poster 2 (or 6)	Poster 3 (or 7)	Poster 4 (or 8)
 step 1 step 2	 step 1 step 2	 step 1 step 2	 step 1 step 2

- A. Copy steps 1 and 2 onto the poster and draw step 3. Explain your step 3 in words.
- B. Make a table, label it appropriately, and record values for steps 1 through 5.
- C. Make a graph and label it appropriately.
- D. Write an input-output rule that relates the total number of tiles to the step number.

Part 3: Return to your seats. Work with your group, and show all work.

Use your "start problem."

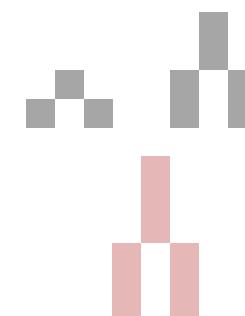
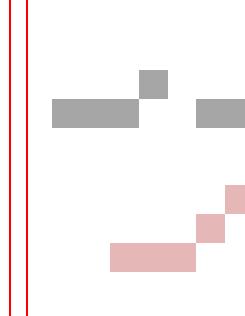
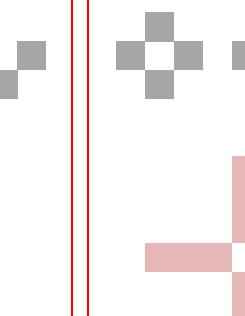
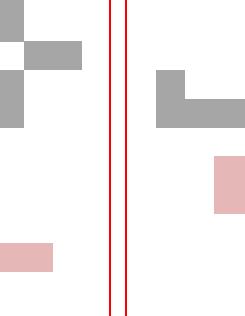
1. Find the number of tiles in step 100.

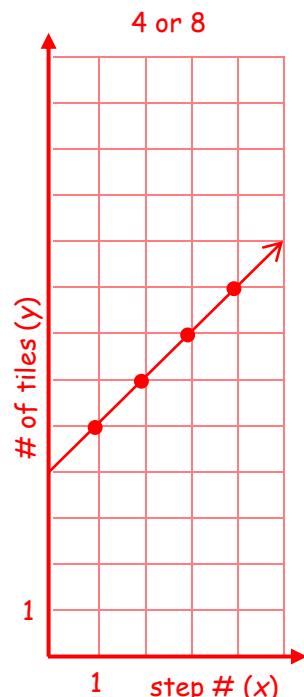
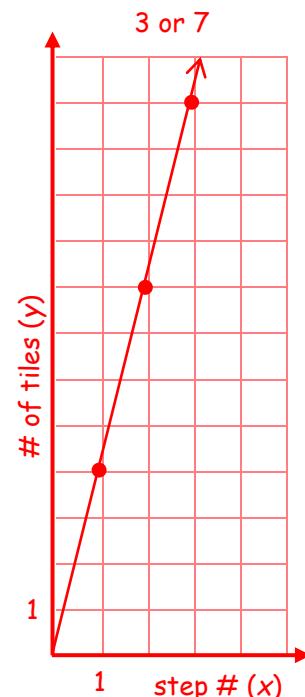
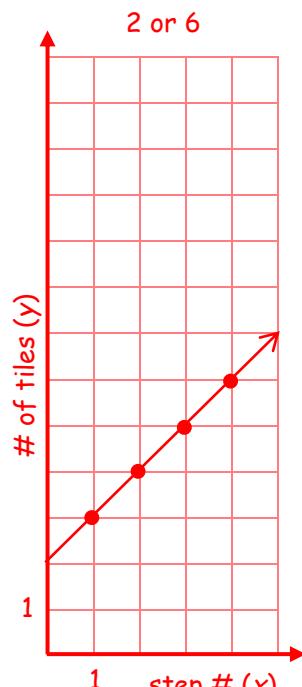
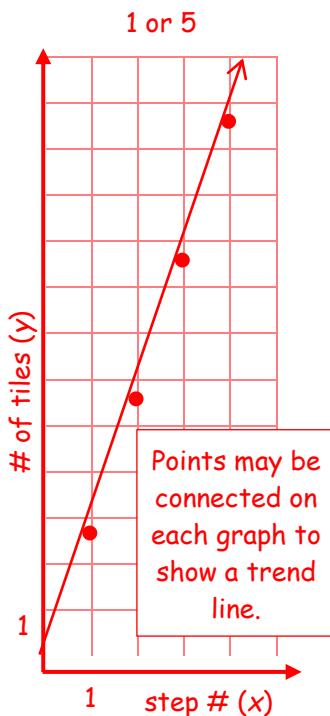
2. Find which step number has exactly 120 tiles.

POSTER PROBLEMS: INPUTS AND OUTPUTS

Answer Key

Likely patterns illustrated below, though they may vary if justified.

Poster 1 (or 5)		Poster 2 (or 6)		Poster 3 (or 7)		Poster 4 (or 8)	
							
Step # (x)	# of tiles (y)	Step # (x)	# of tiles (y)	Step # (x)	# of tiles (y)	Step # (x)	# of tiles (y)
1	3	1	4	1	4	1	5
2	6	2	5	2	8	2	6
3	9	3	6	3	12	3	7
4	12	4	7	4	16	4	8
5	15	5	8	5	20	5	9
$y = 3x$		$y = x + 3$		$y = 4x$		$y = x + 4$	
Step 100: 300 tiles		Step 100: 103 tiles		Step 100: 400 tiles		Step 100: 104 tiles	
120 tiles: step 40		120 tiles: step 117		120 tiles: step 30		120 tiles: step 116	



MATCH AND COMPARE SORT: INPUTS AND OUTPUTS

See Activity Routines on the Teacher Portal for directions.

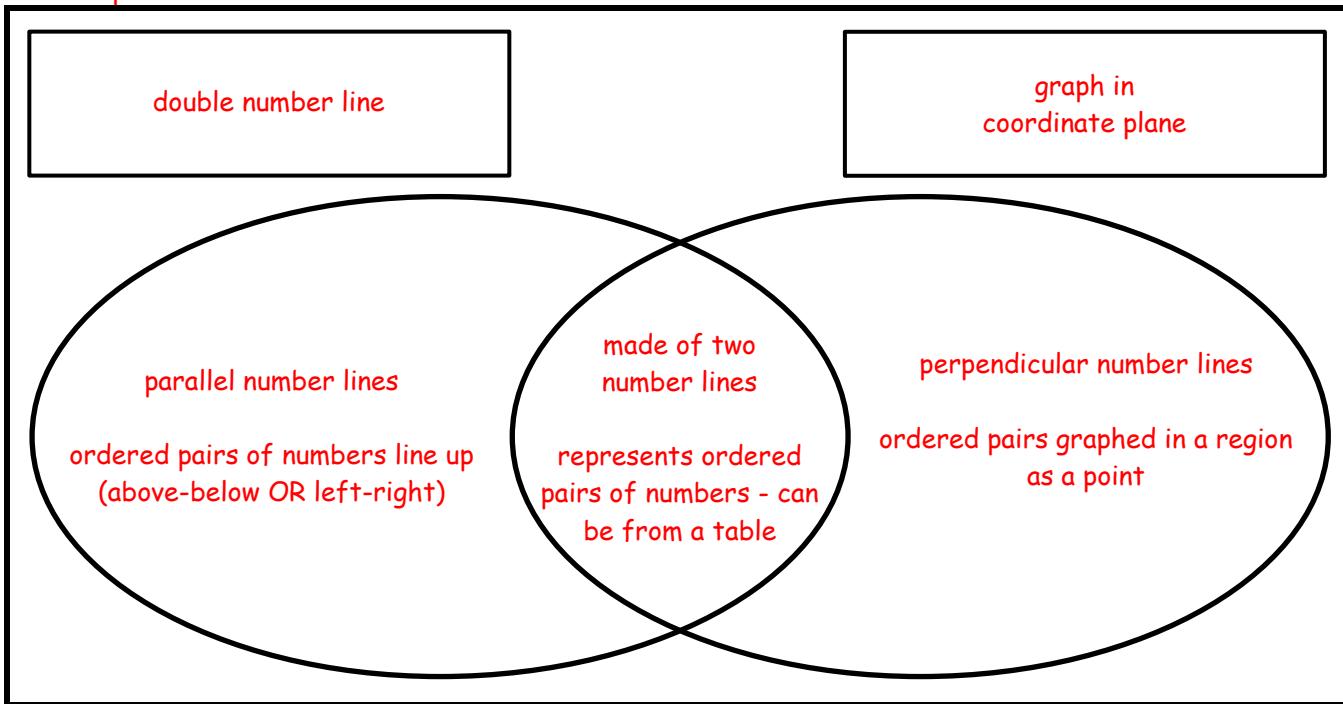
[SMP3, 6]

1. Individually, match words with descriptions. Record results.

Card set 			Card set 		
Card number	word	Card letter	Card number	word	Card letter
I	unit rate	C	I	unit price	D
II	input-output rule	D	II	equation	C
III	independent variable	A	III	dependent variable	B
IV	double number line	B	IV	graph in coordinate plane	A

2. Partners, choose a pair of numbered matched cards and record the attributes that are the same and those that are different. *Choice of vocabulary words to compare will vary.*

One possible answer:

3. Partners, choose another pair of numbered matched cards and discuss the attributes that are the same and those that are different.

WHY DOESN'T IT BELONG?: INPUTS AND OUTPUTS

See Activity Routines on the Teacher Portal for directions.

[SMP1, 3]

Four different stores near LaRonda's home sell yellowfin tuna by the pound. Though prices are listed differently, she can buy any number of pounds at each store at the given rate.

1. Choose one of these price rates and explain why it doesn't belong with the others. Then choose at least one more and explain why it doesn't belong.

A	B
\$3.50 for 1 pound	\$24.00 for 6 pounds
C	D
\$8.75 for 2.5 pounds	4 pounds for \$14.00

Possible responses:

A doesn't belong because it is the only one written as a unit price.

B doesn't belong because it is not \$3.50 per pound like the others.

C doesn't belong because it is not a whole number of pounds.

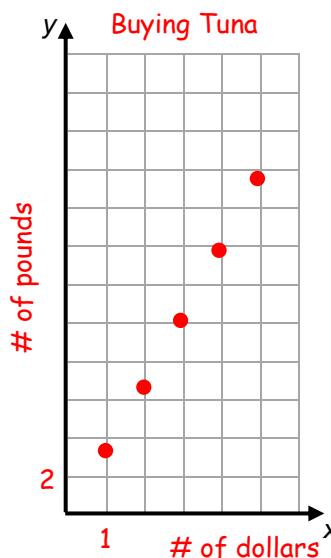
D doesn't belong because the ratio is written in the opposite order (pounds per dollars).

2. For the price that is common among three of the stores above, complete the following:

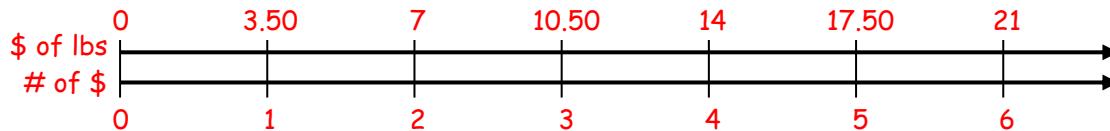
- a table
- a double number line
- a graph
- an input-output rule (equation)

Labeling and scaling may vary for the visual representations.
Points graphed are estimated locations. A trend line may be drawn.

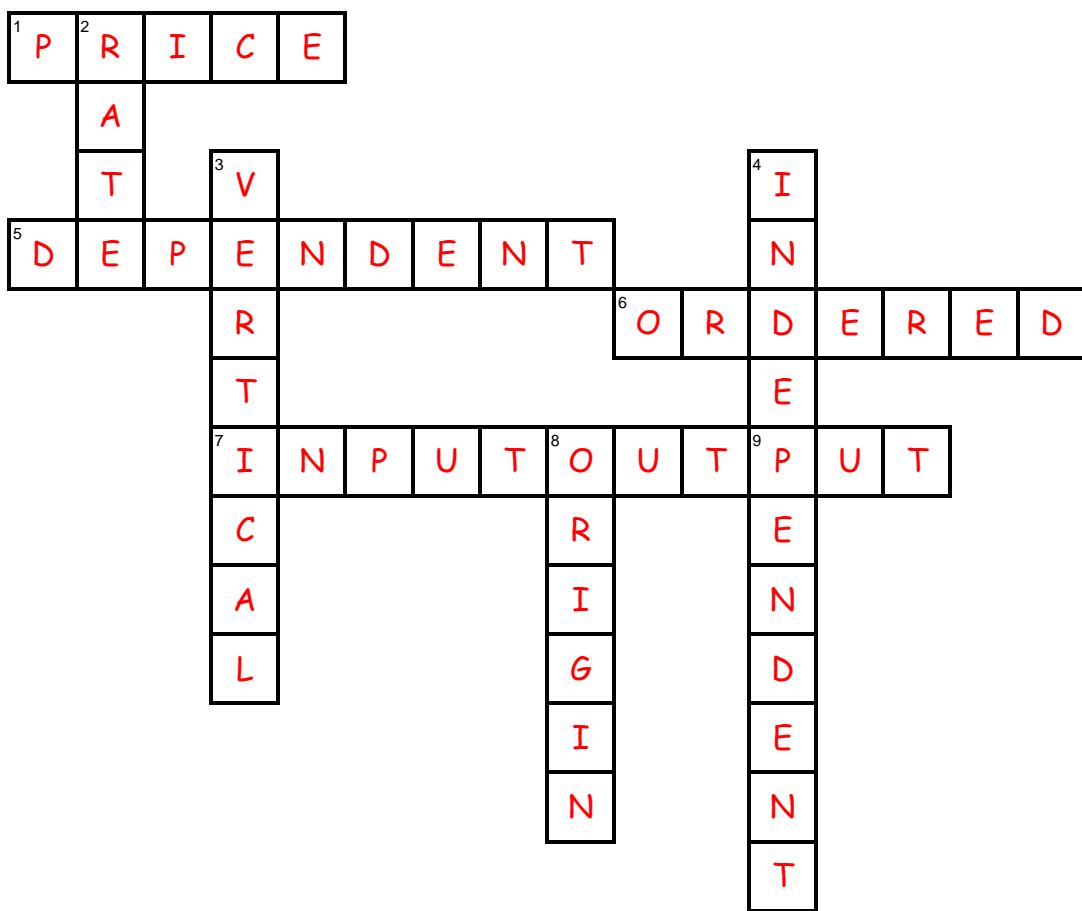
(x) # of pounds	(y) # of dollars
1	3.50
2	7.00
3	10.50
4	14.00
5	17.50



Equation:
 $y = 3.5x$



VOCABULARY REVIEW



Across

1 If 6 oranges cost \$4.50, the unit ___ per orange is \$0.75.

5 A variable that is typically thought of as the output

6 Pairs of numbers in the form (x, y) are called ___ pairs.

7 A rule where, given x , you can find y (2 words)

Down

2 If there are 12 pencils for a group of 4 students, the unit ___ is 3 pencils per each student.

3 Direction of the y -axis

4 A variable typically thought of as the input

6 Name of the point $(0, 0)$

Spiral Review

See Activity Routines on the Teacher Portal for directions.

1. **Computational Fluency Challenge.** This paper and pencil exercise will help you gain fluency with multiplication and division. Try to complete this challenge without any errors. No calculators!

- Start with 2.5. Multiply by 4. Divide the result by 5. Multiply the result by 20. Multiply the result by 40. Now you have a “big number”. My big number is 1600.
- Start with your big number. Divide it by 40. Divide the result by 10. Divide the result by 2. Multiply the result by 5. Divide the result by 4. What is the final result? 2.5

2. Evaluate each numerical expression below.

Evaluate each numerical expression below.		
a. $26 - 4^2$ 10	b. $2(6 + 4)$ 20	c. $2(6 - 4)^2$ 8
d. $2 + (6 + 4)^2$ 102	e. $2(6^2) + 2(4)^2$ 104	f. $6 + 4 \bullet 2$ 14

3. Alon needs to paint a rectangular picture that has an area of $12\frac{3}{8}$ square yards on a brick wall. The width of his painting is $2\frac{3}{4}$ yards. What does the height of the painting need to be?

$$12\frac{3}{8} \div 2\frac{3}{4} = 4\frac{1}{2} \text{ yd}$$

SPIRAL REVIEW**Continued**

4. Ryann bought 3 drinks that cost \$2 each and 3 hot dogs that cost \$2.50 each at the snack bar. Circle the expressions that could be used to compute the total cost.

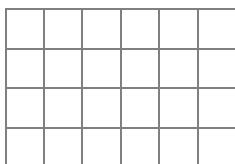
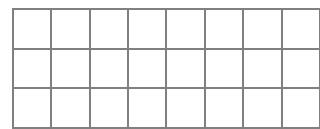
$$2 + 2 + 2(2.50)$$

$$3(2) + 3(2.50)$$

$$(2 + 2.5)(3)$$

$$3 + 2 + 3 + 2.50$$

5. Denali and Jena created plans for a quilted blanket in sewing class, using patches that are 6 inches on each side. Patches are represented by the small squares below.

Denali's Plan**Jena's Plan**

a. Whose blanket covers a greater area? Explain.
 $6 \times 4 = 24$ square patches
 $3 \times 8 = 24$ square patches

b. After they sew the patches together, they have to sew a ribbon around the edge of their blankets. Who will need to use a longer ribbon? Explain your answer in feet.
Parker needs 10 feet of ribbon. Levi needs 11 feet of ribbon. Levi needs more.

c. Leonardo wants to make a square blanket using 16 patches. Write two equivalent expressions using multiplication and exponents to represent its area.
 $4^2 = 4(4)$

6. Some nutritionists recommend that teens drink 8 cups of water per day.

a. Yesterday Helena drank 64 oz of water. Did Helena reach this goal? What percent of this goal did Helena drink? Yes, 100%

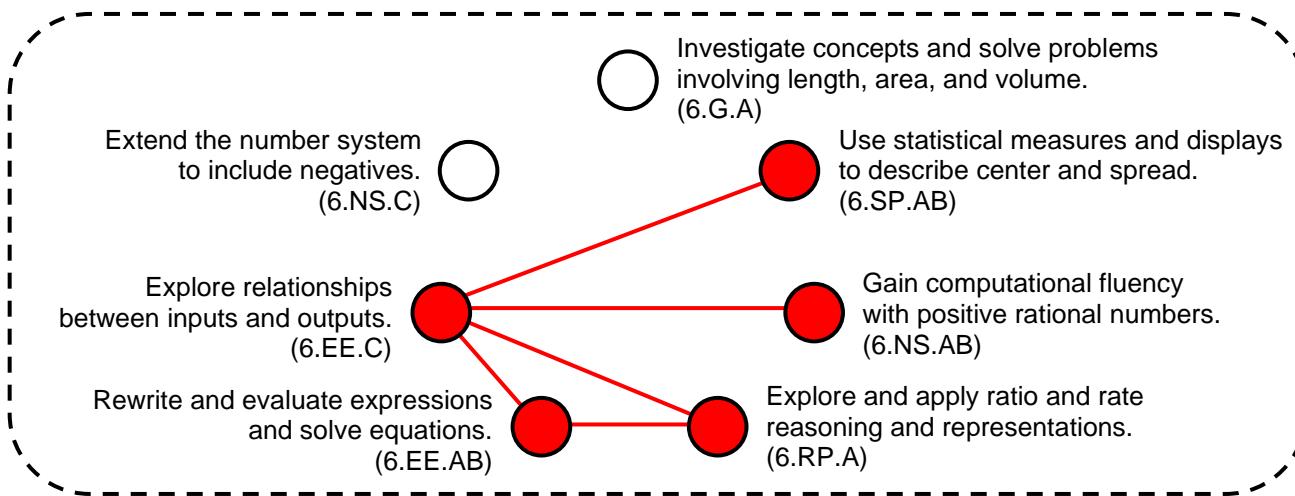
b. Yesterday Betina drank a quart of water. Did Betina reach this goal? What percent of this goal did Betina drink? No, 50%

c. Yesterday Caryn drank 6 cups of water. Did Caryn reach this goal? What percent of this goal did Caryn drink? No, 75%

REFLECTION

Answers will vary. Some possible answers:

1. **Big Ideas.** Shade all circles that describe big ideas in this unit. Draw lines to show connections that you noticed.



Give an example from this unit of one of the connections above.

- Making a recommendation about the emoji purchase for the fundraiser might be done by using the given data and proportional reasoning.
- Input-output relationships are seen in equations and graphs.

2. **Unit Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before or something you would still like to work on.

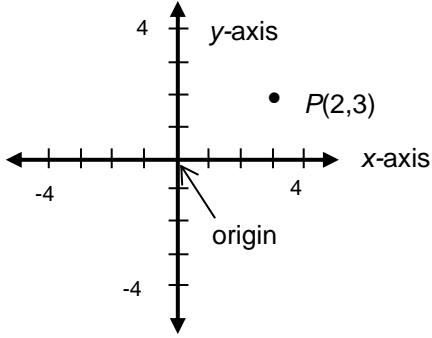
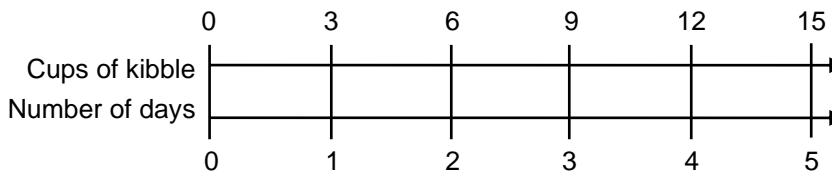
3. **Mathematical Practice.** Did you get stuck on any problems in this unit? What did you do to get unstuck [SMP1]? Then circle one more SMP on the back of this packet that you think was addressed in this unit and be prepared to share an example.

Perseverance when solving problems is a growth mindset behavior. Some strategies might be to break apart problems into simpler problems, work on the parts understood first, try different representations, ask for help from friend or teacher.

4. **Making Connections.** Give an example of how data along with your math skills helped you to make a decision or prediction.

- For keychain fundraiser, polling data and costs are analyzed to make recommendation to the committee.
- For music fundraiser, input-output data, tables, and equations are used to determine if a particular fundraiser would meet its fundraising goals.

STUDENT RESOURCES

Word or Phrase	Definition
coefficient	<p>A <u>coefficient</u> is a number or constant factor in a term of an algebraic expression.</p> <p>In the expression $3x + 5$, 3 is the coefficient of the term $3x$, and 5 is the constant term.</p>
coordinate plane	<p>A <u>coordinate plane</u> is a plane with two perpendicular number lines (<u>coordinate axes</u>) meeting at a point (<u>the origin</u>). Each point P of the coordinate plane corresponds to an ordered pair (a, b) of numbers, called the <u>coordinates</u> of P. The point P may be denoted $P(a, b)$.</p> <p>The coordinate axes are often referred to as the <u>x-axis</u> and the <u>y-axis</u> respectively. The origin has coordinates $(0, 0)$.</p> 
dependent variable	<p>A <u>dependent variable</u> is a variable whose value is determined by the values of the independent variables. See <u>independent variable</u>.</p>
double number line	<p>A <u>double number line</u> is a diagram made up of two parallel number lines that visually depict the relative sizes of two quantities. Double number lines are often used when the two quantities have different units, such as miles and hours.</p> <p>The proportional relationship "Wrigley eats 3 cups of kibble per day" can be represented in the following double number line diagram.</p> 
equation	<p>An <u>equation</u> is a mathematical statement that asserts the equality of two expressions.</p> <p>$18 = 8 + 10$ is an equation that involves only numbers. This is a numerical equation.</p> <p>$18 = x + 10$ is an equation that involves numbers and a variable and $y = x + 10$ is an equation that involves a number and two variables. These are both algebraic (variable) equations.</p>
expression	<p>A mathematical <u>expression</u> is a combination of numbers, variables, and operation symbols. When values are assigned to the variables, an expression represents a number.</p> <p>Some mathematical expressions are 19, $7x$, $a + b$, $\frac{8+x}{10}$, and $4v - w$.</p>

Word or Phrase	Definition														
independent variable	<p>An <u>independent variable</u> is a variable whose value may be specified. Once specified, the values of the independent variables determine the values of the dependent variables.</p> <p>For the equation $y = 3x$, y is the dependent variable and x is the independent variable. We may assign a value to x. The value assigned to x determines the value of y.</p>														
input-output rule	<p>An <u>input-output rule</u> for a sequence of values is a rule that establishes explicitly an output value for each given input value.</p> <table border="1" data-bbox="470 523 1388 593"> <tr> <td>input value (x)</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>x</td> </tr> <tr> <td>output value (y)</td> <td>1.5</td> <td>3</td> <td>4.5</td> <td>6</td> <td>7.5</td> <td>$1.5x$</td> </tr> </table> <p>In the table above, the input-output rule could be $y = 1.5x$. In other words, to get the output value, multiply the input value by 1.5. If $x = 100$, then $y = 1.5(100) = 150$.</p>	input value (x)	1	2	3	4	5	x	output value (y)	1.5	3	4.5	6	7.5	$1.5x$
input value (x)	1	2	3	4	5	x									
output value (y)	1.5	3	4.5	6	7.5	$1.5x$									
rate	See <u>unit rate</u> .														
unit price	<p>A <u>unit price</u> is a price for one unit of measure.</p> <p>If 4 apples cost \$1.00, then the unit price is $\frac{\\$1.00}{4} = \\0.25 for one apple, or 0.25 dollars per apple or 25 cents per apple.</p>														
unit rate	<p>The <u>unit rate</u> associated with a ratio $a : b$ of two quantities a and b, $b \neq 0$, is the number $\frac{a}{b}$, to which units may be attached. This is sometimes referred to as the <u>value of the ratio</u>.</p> <p>The ratio of 40 miles for every 5 hours has a unit rate of $\frac{40}{5} = 8$ miles per hour.</p>														
variable	<p>A <u>variable</u> is a quantity whose value has not been specified. Variables are used in many different ways. They may refer to quantities that vary in a relationship (as in a formula or an input-output rule). They may refer to unknown quantities in expressions, equations, or inequalities. Finally, they may be used to generalize rules of arithmetic.</p> <p>In the equation $d = rt$, the quantities d, r, and t are variables.</p> <p>In the equation $2x = 10$, the variable x may be referred to as the unknown.</p> <p>The equation $a + b = b + a$ generalizes the commutative property of addition for all numbers a and b.</p>														

The Coordinate Plane

A coordinate plane is determined by a horizontal number line (the x -axis) and a vertical number line (the y -axis) intersecting at the zero on each line. The point of intersection $(0, 0)$ of the two lines is called the origin. Points are located using ordered pairs (x, y) .

- The first number (x -coordinate) indicates how far the point is to the right of the y -axis.
- The second number (y -coordinate) indicates how far the point is above the x -axis.

Point, coordinates, and interpretation

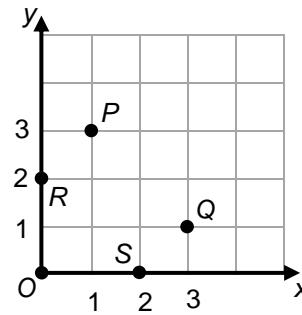
$O(0, 0)$ → at the intersection of the axes

$P(1, 3)$ → start at the origin, move 1 unit right, then 3 units up

$Q(3, 1)$ → start at the origin, move 3 units right, then 1 unit up

$R(0, 2)$ → start at the origin, move 0 units right, then 2 units up

$S(2, 0)$ → start at the origin, move 2 units right, then 0 units up



Multiple Representations: Tables, Graphs, and Equations

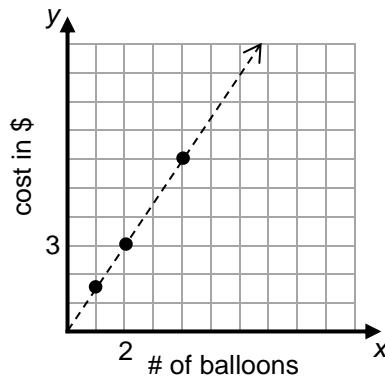
Suppose 4 balloons cost \$6.00 and each balloon is the same price. Here are some representations for this relationship.

Table

Number of Balloons	Cost in \$
4	6.00
2	3.00
1	1.50
8	12.00

Note that the unit price is \$1.50 per balloon

Graph



Numbers of balloons must be discrete values (specifically, whole numbers), however a trend line may be drawn to show a growth pattern.

Equation
(input-output rule)

Let y = cost in dollars and x = number of balloons.

We can see from the table that the unit price is 1.50 dollars per balloon.

It appears that multiplying any input value by 1.5 yields its corresponding output value.

Therefore, $y = 1.5x$.

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
6.RP.A	Understand ratio concepts and use ratio reasoning to solve problems.
6.RP.3	<p>Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations:</p> <ul style="list-style-type: none"> a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
6.NS.B	Compute fluently with multi-digit numbers and find common factors and multiples.
6.NS.3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
6.EE.A	Apply and extend previous understandings of arithmetic to algebraic expressions.
6.EE.2	<p>Write, read, and evaluate expressions in which letters stand for numbers:</p> <ul style="list-style-type: none"> a. Write expressions that record operations with numbers and with letters standing for numbers. b. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems.
6.EE.B	Reason about and solve one-variable equations and inequalities.
6.EE.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
6.EE.C	Represent and analyze quantitative relationships between dependent and independent variables.
6.EE.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. <i>For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.</i>
6.SP.A	Develop understanding of statistical variability.
6.SP.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.
6.SP.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

STANDARDS FOR MATHEMATICAL PRACTICE	
SMP1	Make sense of problems and persevere in solving them.
SMP2	Reason abstractly and quantitatively.
SMP3	Construct viable arguments and critique the reasoning of others.
SMP4	Model with mathematics.
SMP5	Use appropriate tools strategically.
SMP6	Attend to precision.
SMP7	Look for and make use of structure.
SMP8	Look for and express regularity in repeated reasoning.

Inputs and Outputs

*The English book asked the Math book why he was so sad.
You know what he said? “Because I have so many problems!”*

