STUDENT RESOURCES

Word or Phrase	Definition
coefficient	A coefficient is a number or constant factor in a term of an algebraic expression. In the expression $3x + 5$, 3 is the coefficient of the linear term $3x$, and 5 is the constant coefficient.
constant term	A constant term in an algebraic expression is a term that has a fixed numerical value. In the expression $5 + 2x + 3$, the terms 5 and 3 are constant terms. If this expression is rewritten as $2x + 8$, the term 8 is the constant term of the new expression.
distributive property	The distributive property states that $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for any three numbers a, b, and c. 3(4 + 5) = 3(4) + 3(5); (4 + 5)8 = 4(8) + 5(8); 6(8 – 1) = 6(8) – 6(1)
equation	An equation is a mathematical statement that asserts the equality of two expressions. 18 = 8 + 10 is an equation that involves only numbers. This is a numerical equation. $18 = x + 10$ is an equation that involves numbers and a variable and $y = x + 10$ is an equation that involves a number and two variables. These are both algebraic (variable) equations.
equivalent expressions	Two mathematical expressions are equivalent if, for any possible substitution of values for the variables, the two resulting numbers are equal. In particular, two numerical expressions are equivalent if they represent the same number. See expression. The numerical expressions 3 + 2 and 6 – 1 are equivalent, since both are equal to 5. The algebraic expressions $3(x + 2)$ and $3x + 6$ are equivalent. For any value of the variable x, the expressions represent the same number.
evaluate	Evaluate refers to finding a numerical value. To evaluate an expression, replace each variable in the expression with a value and then calculate the value of the expression. To evaluate the numerical expression 3 + 4(5), we calculate 3 + 4(5) = 3 + 20 = 23. To evaluate the variable expression $2x + 5$ when $x = 10$, we calculate $2x + 5 = 2(10) + 5 = 20 + 5 = 25$.
exponential notation	The exponential notation b^n (read as " b to the power n ") is used to express n factors of b. The number b is the base, and the number n is the exponent. $2^3 = 2 \bullet 2 \bullet 2 = 8$; The base is 2 and the exponent is 3. $3^2 \bullet 5^3 = 3 \bullet 3 \bullet 5 \bullet 5 \bullet 5 = 1{,}125$; The bases are 3 and 5. The exponents are 2 and 3.

Word or Phrase	Definition
expression	A mathematical expression is a combination of numbers, variables, and operation symbols. When values are assigned to the variables, an expression represents a number. Some mathematical expressions are 19, $7x$, $a + b$, $\frac{8 + x}{10}$, and $4v - w$.
greatest common factor	The greatest common factor (GCF) of two numbers is the greatest factor that divides the two numbers. The factors of 12 are 1, 2, 3, 4, 6, and 12. The factors of 18 are 1, 2, 3, 6, 9, and 18. Therefore the GCF of 12 and 18 is 6.
like terms	Terms of a mathematical expression that have the same variable part are referred to as like terms. See term. In the mathematical expression $2x + 6 + 3x + 5$, the terms $2x$ and $3x$ are like terms, and the terms 6 and 5 are like terms.
simplify	Simplify refers to converting a numerical or variable expression to a simpler form. A variable expression might be simplified by combining like terms. A fraction might be simplified by dividing numerator and denominator by a common divisor. $2x + 6 + 5x + 3 = 7x + 9$ $\frac{8}{12} = \frac{2}{3}$
square number	A square number, or perfect square, is a number that is a square of a natural number. The area of a square with side-lengths that are natural numbers is a square number. The square numbers are $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, $16 = 4^2$, $25 = 5^2$, … .
terms	The terms in a mathematical expression involving addition (or subtraction) are the quantities being added (or subtracted). Terms that have the same variable part are referred to as like terms. The expression $2x + 6 + 3x + 5$ has four terms: $2x$, 6, $3x$, and 5. The terms $2x$ and $3x$ are like terms, since each is a constant multiple of x. The terms 6 and 5 are like terms, since each is a constant.
variable	A variable is a quantity whose value has not been specified. Variables are used in many different ways. They may refer to quantities that vary in a relationship (as in a formula or an input-output rule). They may refer to unknown quantities in expressions, equations or inequalities. Finally, they may be used to generalize rules of arithmetic. In the equation $d = rt$, the quantities d, r, and t are variables. In the equation $2x = 10$, the variable x may be referred to as the unknown. The equation $a + b = b + a$ generalizes the commutative property of addition for all numbers a and b.

The Distributive Property

The distributive property relates the operations of multiplication and addition. The term “distributive” arises because the property is used to distribute the factor outside the parentheses over the terms inside the parentheses.

Suppose you earn $9.00 per hour. If you work 3 hours on Saturday and 4 hours on Sunday, one way to compute your earnings is to compute your wages for each day and then add them. Another way is to multiply the hourly wage by the total number of hours. This example illustrates the distributive property.

$$(9 \times 3) + (9 \times 4) = 9(3 + 4)$$
$$27 + 36 = 9(7)$$

Order of Operations

There are many mathematical conventions that enable us to interpret mathematical notation and to communicate efficiently. The agreed-upon rules for interpreting mathematical notation, important for simplifying arithmetic and algebraic expressions, are called the standard order of operations.

Step 1: Do the operations in grouping symbols first (e.g., use rules 2-4 inside parentheses).
Step 2: Calculate all the expressions with exponents.
Step 3: Multiply and divide in order from left to right.
Step 4: Add and subtract in order from left to right.

Example: $$\frac{3^2 + (6 \bullet 2 - 1)}{5} = \frac{3^2 + (12 - 1)}{5} = \frac{3^2 + (11)}{5} = \frac{9 + (11)}{5} = \frac{20}{5} = 4$$

There are many times when these rules make complete sense and are quite natural. Take this case, for example:

You purchase 2 bottles of water for $1.50 each and 3 bags of peanuts for $1.25 each. Write an expression for this situation, and simplify the expression to find the total cost.

Expression: $$\underbrace{2 \bullet (1.50)}_{3.00} + \underbrace{3 \bullet (1.25)}_{3.75} = 6.75$$

In this problem, it is natural to find the cost of the 2 bottles of water and then the cost of the 3 bags of peanuts prior to adding these amounts together. In other words, we perform the multiplication operations before the addition operation.

However, if we were to perform the operations in order from left to right (as we read the English language from left to right), we would obtain a different result:

$$2(1.50) = 3 \longrightarrow 3 + 3 = 6 \longrightarrow 6(1.25) = 7.50\text{, wrong answer!}$$

Using Order of Operations to Simplify Expressions		
Order of Operations	**Expression**	**Comments**
	$2^3 \div 2(5-2)$ $4 + 2 \bullet 10$	
1. Simplify expressions within grouping symbols.	$2^3 \div 2(3)$ $4 + 2 \bullet 10$	Parentheses are grouping symbols: Therefore, 5 – 2 = 3. The fraction bar is also a grouping symbol, so the first step here is to simplify the numerator and denominator.
2. Calculate powers.	$\frac{8 \div 2(3)}{4 + 2 \bullet 10}$	$2^3 = 2 \bullet 2 \bullet 2 = 8$
3. Perform multiplication and division from left to right.	$\frac{12}{4 + 20}$	In the numerator: Divide 8 by 2, then multiply by 3. In the denominator: Multiply 2 by 10.
4. Perform addition and subtraction from left to right.	$\frac{12}{24} = \frac{1}{2}$	Perform the addition: 4 + 20 = 24. Now the groupings in both the numerator and denominator have been simplified, so the final division can be performed.

Writing Expressions

The notation used for algebra is sometimes different from the notation used for arithmetic. For example:

- 54 means the sum of five tens and four ones, that is, 5(10) + 4.
- $5\frac{1}{2}$ means the sum of five and one-half. that is, $5 + \frac{1}{2}$.
- $5x$ means the product of 5 and x, which can also be written $5(x)$ or $5 \bullet x$. We typically do not write $5 \times x$ because the multiplication symbol '$\times$' is easily confused with the variable x.

Evaluate or Simplify?
We use the word "evaluate" when we want to calculate the value of an expression. Example: To evaluate $16 - 4(2)$, follow the rules for order of operations and compute: $16 - 4(2) = 16 - 8 = 8$. To evaluate $6 + 3x$ when $x = 2$, substitute 2 for x and calculate: $6 + 3(2) = 6 + 6 = 12$. We use the word "simplify" when rewriting a number or an expression in a form more easily readable or understandable. Example: To simplify $2x + 3 + 5x$, combine like terms: $2x + 3 + 5x = 7x + 3$. Sometimes it may not be clear what the simplest form of an expression is. For instance, by the distributive property, $4(x + 2) = 4x + 8$. For some applications, $4(x + 2)$ may be considered simpler than $4x + 8$, but for other applications, $4x + 8$ may be considered simpler than $4(x + 2)$.