TEACHER EDITION

MathLinks GRADE 6

COMPILED MATH BACKGROUND

MATH BACKGROUND

"The Quartile" or "In the Quartile"

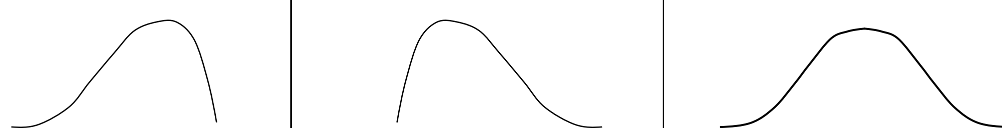
The word "quartile" is used in statistics in two different ways. Most often, it is used to denote numbers that separate an ordered data set into four equal parts. In the sibling data set {2, 2, 2, 2, 3, 3, 4, 5, 5, 7, 11}, Q_3 (or the third quartile) is 5.

The word "quartile" can also refer to a set of values, namely, one of those four equal parts. In the data set above, the fourth quartile is the set {5, 7, 11}.

Thus, "the first quartile is 2," but "the value 2 lies both in the first quartile and in the second quartile." This ambiguous use of terms occurs often in mathematics. For instance, the word "circle" usually refers to the boundary of a disk, but it can also refer to the entire disk.

The Tale is in the Tail

A data set that is not symmetric is said to be skewed. One way a data set can be skewed is to have a tail, that is, a higher concentration of data at one end of the distribution than the other, as indicated in the figures below. We say the data is skewed to the right if the tail of the distribution goes to the right. And we say the data is skewed to the left if the tail of the distribution goes to the left. The tail may provide valuable insight into the data set when interpreted in context.



tail (skewed to the left) | tail (skewed to the right) | symmetric

Using Data to Understand Our World

A key to understanding the world we live in is to collect data: temperature, wind velocity, rainfall, water levels, animal populations, cases of malaria, and so on. The data have a story to tell. It is the job of mathematics and statistics to read the story. We need mathematics and statistics to tell us what the trends are and with what certainty. Once we've read the story, it is our job to take steps to make the world a better place to live in.

MATH BACKGROUND

Why Is 1 neither Prime nor Composite?

Euclid (about 300 BCE) included "1" in the definition of a prime number. However, the number 1 had to be treated as a special case in so many theorems that by the time of Gauss (about 1800 AD), the definition of prime number had been changed, and 1 was excluded from the family of prime numbers. (See box below.)

There are many definitions in mathematics that have metamorphosed over time. Originally, the definition of "rectangles" did not include "squares," but it has become standard to include squares as a subset of the rectangle family because it makes many properties easier to explain.

To complicate the picture even more, some mathematical terms are defined differently in different textbooks and in different parts of the world. For example, some books define a trapezoid as a quadrilateral with exactly one pair of parallel sides, while others define it as a quadrilateral with at least one pair of parallel sides.

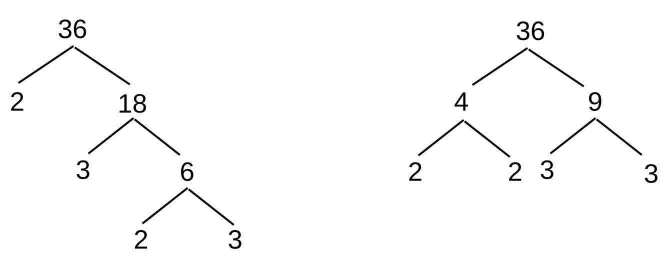
Something similar occurs in other sciences. For instance, Pluto was classified as a planet for almost a century, before it was demoted by astronomers to asteroid status. It is now a "dwarf planet."

The Fundamental Theorem of Arithmetic

The <u>Fundamental Theorem of Arithmetic</u> has two parts. First, any number $N \geq 2$ can be factored as a product of primes. Second, this prime factorization is unique up to the order of the factors.

A factor tree is a useful tool for organizing and recording the factors of a number. There may be different ways to make a factor tree for a given number, but the Fundamental Theorem of Arithmetic tells us that the end result (prime factorization) will always be the same.

Example 1: Create two different factor trees to illustrate that the prime factorization of 36 is $2^2 \bullet 3^2$.



Example 2: How do you know, without dividing, that 7 is not a factor of 36?
From the factor trees above we see that the prime factorization of $36 = 2^2 \bullet 3^2$. By the Fundamental Theorem of Arithmetic, we know that the factorization is unique, up to the order of factors. Since the prime number 7 does not appear, it cannot be a factor of 36. In fact, the only factors of 36 are 1, 2, 3, $2 \bullet 3 = 6$, $2 \bullet 3 \bullet 3 = 18$, $2 \bullet 2 = 4$, $2 \bullet 2 \bullet 3 = 12$, and $2 \bullet 2 \bullet 3 \bullet 3 = 36$.

Note, if 1 were included as a prime number, there would actually be infinitely many "prime factorizations" (including $2^2 \bullet 3^2 \bullet 1$, $2^2 \bullet 3^2 \bullet 1^2$, $2^2 \bullet 3^2 \bullet 1^3$ …). One reason to exclude 1 from the definition of a prime number is that we do not wish to count $2^2 \bullet 3^2 \bullet 1$ or $2^2 \bullet 3^2 \bullet 1^2$ as distinct prime factorizations of 36.

MATH BACKGROUND

Ratios Are Everywhere

Under every rug there is a ratio.

In mathematics:

- the ratio of the circumference of a circle to its diameter
- the ratio of lengths of corresponding sides of similar triangles
- the ratios of side lengths of right triangles (trigonometric ratios)
- the ratio of the "increase in the y-variable" to the "increase in the x-variable"

In science:

- laws of physics, such as the ratio of momentum to velocity of falling objects
- conversion rates, such as feet to meters or minutes to hours
- comparisons, such as nineteen out of twenty glaciers are receding

In daily activities:

- two cups water for every cup oatmeal (recipe)
- a dozen almonds per serving
- thirty miles per hour (a speed limit)
- twenty-seven miles per gallon (fuel consumption)

In pricing:

- cheese at $5 per pound
- farmland at $8,000 per acre

In sports and exercise:

- odds of Boston winning the baseball championship
- calories burned in fifteen minutes jogging

Whenever we refer to percentages, we are using ratio reasoning. The battery life of our electronic device, the sales tax on our pizza, and the discount on sale items are given as a percentage.

Ratios and Rates

Many definitions in mathematics have evolved over time. Originally, the definition of “rectangles” did not include “squares,” but it has become standard to include squares as a subset of the rectangle family because it makes many properties easier to explain. To complicate the picture even more, some mathematical terms are defined differently in different textbooks and in different parts of the world. For example, some books define a trapezoid as a quadrilateral with **exactly one pair** of parallel sides, while others define it as a quadrilateral with **at least one pair** of parallel sides.

In the Progressions to the Common Core document that focuses on proportional reasoning, the authors comment on the need for standardized definitions.

> “Because many different authors have used ratio and rate terminology in widely differing ways, there is a need to standardize the terminology for use with the Standards and to have a common framework for curriculum developers, professional development providers, and other education professionals to discuss the concepts.”

Here we state the Common Core definitions for some relevant vocabulary in this unit, with comments. This is not necessarily how the concepts should be presented to students in Grades 6 and 7.

A ratio is a pair of positive numbers in a specific order. The ratio of a to b is denoted by $a : b$ (read “a to b,” or “a for every b”).

Example: If there are 3 coins and 2 paperclips in your pocket, then the ratio of coins to paperclips may be denoted 3 to 2 or 3 : 2.

Comment: According to this definition, a ratio is not a fraction. Also, a ratio must be composed of positive numbers.

The value of a ratio $a : b$ is the number $\frac{a}{b}$, $b \neq 0$.

Example: The value of the ratio of 3 : 2 is $\frac{3}{2} = 1.5$.

Comment: According to this definition, the value of the ratio is a fraction.

The unit rate associated to a ratio $a : b$, where a and b have units attached, is the number $\frac{a}{b}$, with the units “a-units per b-unit” attached.

Example: The ratio of 400 miles for every 8 hours corresponds to the unit rate 50 miles per hour.

Comment: There is no formal definition of “rate.” It is treated as a word in common language. Such phrases as “at that rate” or “at the same rate” are used.

MATH BACKGROUND

Definition of Division

Division is the mathematical operation that is inverse to multiplication. For $b \neq 0$, division by b is multiplication by the multiplicative inverse $\frac{1}{b}$ of b,

$$a \div b = a \bullet \frac{1}{b}.$$

In this division problem, the number a to be divided is the dividend, the number b by which a is divided is the divisor, and the result $a \div b$ of the division is the quotient:

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient} \qquad \text{divisor}\overline{)\text{dividend}}^{\text{quotient}}$$

Some other notations for $a \div b$ are $\frac{a}{b}$ and a/b.

Division with remainder is a division problem for natural numbers n and d in which n is expressed a $n = qd + r$, where q and r are whole numbers, and $0 \leq r < d$. We say that the quotient of n divided by d is q with remainder r. This may be written as:

$$d\overline{)n}^{\,q\ \text{R}r} \qquad 3\overline{)14}^{\,4\ \text{R}2}$$

Rules for Division of Fractions

Division by a number b is defined to be multiplication by the multiplicative inverse of b. Thus, any statement about division can be restated in terms of multiplication and multiplicative inverses. Mathematically speaking, the operation of division is completely subservient to the operation of multiplication. Here are several rules on multiplicative inverses and division that follow this principle.

Multiplicative inverse of a fraction: The multiplicative inverse of $\frac{a}{b}$ is $\frac{b}{a}$. This can be justified easily by using the multiply-across rule, the commutative property, and the "Big one" identity:

$$\frac{a}{b} \bullet \frac{b}{a} = \frac{a \bullet b}{b \bullet a} = \frac{a \bullet b}{a \bullet b} = 1.$$

Another way to express this, using the notation ˎ for division, is that the multiplicative inverse of $a \div b$ is $b \div a$ that is,

$$\frac{1}{a \div b} = b \div a\ .$$

Multiply-by-the-reciprocal rule: The multiply-by-the-reciprocal rule for fraction division is that

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c}\text{, for } b, c, d \neq 0.$$

This rule follows immediately from the definition of division (division by $\frac{c}{d}$ is defined to be multiplication by the multiplicative inverse of $\frac{c}{d}$), and the fact noted above that the multiplicative inverse of $\frac{c}{d}$ is $\frac{d}{c}$.

Divide-across rule: The divide-across rule is that

$$\frac{a}{b} \div \frac{c}{d} = \frac{a \div c}{b \div d} \text{ for } b, c, d \neq 0.$$

The divide-across rule is completely analogous to the multiply-across rule. It can be derived with the aid of the multiply-by-the-reciprocal rule, together with definitions and other facts previously established, through the following sequence of identities:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c} = \frac{a \bullet d}{b \bullet c} = \frac{a \bullet d}{c \bullet b} = \frac{a}{c} \bullet \frac{d}{b} = \frac{a}{c} \div \frac{b}{d} = \frac{\frac{a}{c}}{\frac{b}{d}} = \frac{a \div c}{b \div d}$$

MATH BACKGROUND

Percent AS a Number and Percent OF a Number

We may regard a "percent" as a number, or we may talk about "a percent of a quantity." These two concepts are related, but they are different, and the difference may lead to confusion. A parallel situation occurs for fractions. We may regard $\frac{3}{4}$ as a number, or we may talk about $\frac{3}{4}$ of something.

Percent as a number

If P is a nonnegative number, then <u>P percent</u> (denoted $P\%$) is P hundredths:

$$P\% = \frac{P}{100}$$

We may think of a percent as a number expressed in terms of the unit of measure % where $1\% = \frac{1}{100}$.

To express p percent as a number, write p hundredths as a fraction, or write its decimal equivalent by dividing p by 100.

Example: Fifteen percent = $15\% = \frac{15}{100} = 0.1$

To convert a positive number to a percent, multiply by 1 in the form of 100% or $\frac{100}{100}$.

Example: $4 = 4 \times 100\% = 400\%$

Percent of a number

Three fourths of a number is the product of $\frac{3}{4}$ and the number. Proceeding in analogy, we say that <u>P percent of a number</u> is the product of $P\%$ and the number:

$$P\% \text{ of } n = P\% \bullet n = \frac{P}{100} \bullet n$$

Example: 15% of 300 is $15\% \bullet 300 = \frac{15}{100} \bullet 300 = 45$

MATH BACKGROUND

Order of Operations

There are many mathematical conventions that enable us to communicate better about common situations. For example, when using the coordinate plane, an agreed-upon convention is that we generally call the horizontal axis the x-axis, and the vertical axis the y-axis. This allows for easier, common communication.

There are agreed-upon rules as well for interpreting and evaluating arithmetic and algebraic expressions. We call these rules the standard order of operations. In a nutshell, the standard convention for order of operations is to simplify first the expressions within grouping symbols and exponents, then to perform multiplications and divisions, and finally additions and subtractions. More specifically:

1. Do the operations inside grouping symbols first, that is, use rules 2-4 inside parentheses, braces, brackets, and above and below fraction bars.
2. Calculate all the exponential expressions.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

Example:

$$\frac{3^2 + (6 \bullet 2 - 1)}{5} = \frac{3^2 + (12 - 1)}{5} = \frac{3^2 + (11)}{5} = \frac{9 + (11)}{5} = \frac{20}{5} = 4$$

Usually these rules make complete sense and are quite natural. Take this case, for example:

You purchase 2 bottles of water for $1.50 each and 3 bags of peanuts for $1.25 each. Write and evaluate an expression for the total cost.

Expression: $\underbrace{2 \bullet 1.50} + \underbrace{3 \bullet 1.25}$

Simplification: $3.00 + 3.75 = \$6.75$

In this problem, it is natural to find the cost of the 2 bottles of water and then the cost of the 3 bags of peanuts prior to adding these amounts together. In other words, we perform the multiplication operations before the addition operation.

Note, however, that if we were to perform the operations in order from left to right (as we read the English language from left to right) we would obtain a different result:

$2 \bullet 1.50 + 3 \bullet 1.25 = ((2 \bullet 1.50) + 3) \bullet 1.25 = (3 + 3) \bullet 1.25 = 6 \bullet 1.25 = \$7.50$, wrong answer!

Variables in Algebra	
Loosely speaking, variables are quantities that may vary. They have many different uses in mathematics. Variables are represented by letters or symbols. The introduction of literal notation together with the rules of arithmetic makes algebra a powerful tool. Three important ways that variables appear in algebra are the following:	
An *unknown quantity* in an equation or inequality: In this case, the equation is valid only for specific value(s) of the unknown, and we are challenged to find those value(s).	$x + 4 = 9$ $5x = 20$ $x^2 + 2x < 1$
Quantities that vary in a relationship: In this case, there is always more than one variable in the equation. The equation might be a formula, it might be a rule describing a function, or it might be a more complicated relation.	Formula: $P = 2l + 2w$, $A = s^2$ Function: $y = 5x$, $y = x + 3$ More complicated relation: $y^2 = x^3 + 4x + 1$
Quantities that generalize rules of arithmetic, quantities in identities, or inequalities that flow from rules of arithmetic: In this case, there may be one or more variables. The identity or inequality is true, except possibly for certain specific values as noted.	Commutative property of addition: $x + y = y + x$ Distributive property: $x(y + z) = xy + xz$ Multiplicative identity: $x\left(\frac{1}{x}\right) = 1$ for $x \neq 0$ Identities: $5(x + 2) = 5x + 10$ Inequalities: $x < x + 1$

MATH BACKGROUND

Independent and Dependent Variables

Independent variables are under our control, in the sense that we may specify their values. Once the values of the independent variables have been specified, the values of the dependent variables are completely determined. We have no control over them.

When two variables are in a proportional relationship, the values of either variable completely determine the values of the other. Either variable could be regarded as the independent variable, and the remaining variable would then be regarded as the dependent variable. Which of the two variables is the independent variable, and which is the dependent variable, depends on the context of the problem.

Consider this situation: Suppose text messages cost \$0.05 each.

Let n = the number of text messages sent
Let C = the cost of a text message bill (in dollars)

- If we know the number of text messages sent, then we can determine the cost of the text message bill as $C = 0.05n$. In this case, n is the natural independent variable and C is the dependent variable because the total cost depends on the number of text messages sent.
- If we are operating within a budget and we have a limit C on what we can spend for text messages, then we can determine the number of text messages we can send as $n = \frac{C}{0.05}$. In this case, C is the natural independent variable and n is the dependent variable because the number of text messages depends on the total cost.

MATH BACKGROUND

Different Definitions for Quadrilaterals

Is a rectangle a trapezoid? Is a square a rhombus? To a certain extent, the answers to these questions are a matter of convention and historical whim. According to the definitions we have given, a parallelogram is a trapezoid, and a square is a rhombus. However, there is room for disagreement. Some textbook writers do not allow parallelograms to be trapezoids, and some textbook writers do not allow squares to be rhombi.

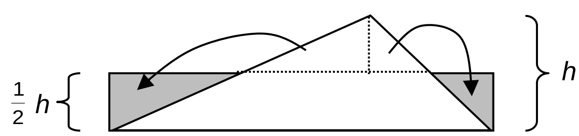
To complicate the issue further, definitions sometimes vary from one country to another. In England a "trapezium" is a trapezoid, while in the United States a "trapezium" is a quadrilateral in which no two sides are parallel.

Alternate Methods for Determining Area Formulas by Composing and Decomposing Figures

In Lesson 9.1, **Area of Polygons**, students are led to discover the formulas for area of a parallelogram, triangle, and trapezoid through compositions and decompositions of figures. We will call these processes "cut-up" strategies. Here are a few alternate methods to those posed in the lesson.

Triangle alternate #1 (Figure A): Orient the triangle with the longest side as the base. Cut the triangle along a line parallel to the base and at half the height. Cut the top triangular piece along the perpendicular to the base from the top vertex. The two resulting pieces from the top can be positioned over the shaded pieces of the rectangle with the same base as the triangle and half the height, so that the three pieces from the triangle cover the rectangle exactly once.

Figure A



Trapezoid alternate #1 (Figure B): Cut the trapezoid along a line parallel to a base and at half the height. Reassemble the two trapezoids as indicated to form a parallelogram. The height of the parallelogram will be half the height of the trapezoid, and the base of the parallelogram will be equal to the sum of the bases of the trapezoid. The new parallelogram will have the same area as the original trapezoid.

Trapezoid alternate #2 (Figure C): Cut the trapezoid along one of its diagonals, forming two triangles. The height of the trapezoid will be equal to the height of each triangle, and each of the two bases of the trapezoid will be equal to a base of one of the two triangles.

Figure B

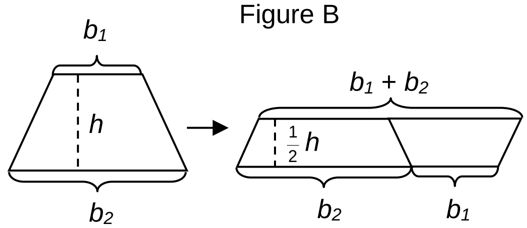
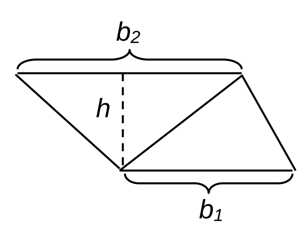


Figure C



Polygons

A polygon is a simple closed curve in a plane formed by laying straight line segments end-to-end. The line segments are the sides (or edges) of the polygon, and their endpoints are the vertices of the polygon. By "closed," we mean that the final endpoint of the final segment laid down is the initial endpoint of the first segment laid down. By "simple" we mean that each edge meets only two other edges, namely, the preceding edge and the succeeding edge at the vertices where they are joined.

polygon | not closed, not a polygon | not simple, not a polygon | not simple, not a polygon

A polygon divides the plane into a bounded region (the inside of the polygon) and an unbounded region (the outside of the polygon). The inside of the polygon is referred to as a polygonal figure. We use the word "polygon" also to refer to this polygonal figure, and the meaning of "polygon" can sometimes only be inferred from context. The area of a polygon refers to the area of the polygonal figure enclosed by the polygon.

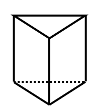
Polyhedra

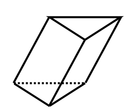
A polyhedron (plural: polyhedra) is a closed figure in space consisting of a finite number of polygonal figures that are joined at edges. Each polygonal figure is a face of the polyhedron, each side of a face is an edge of the polyhedron, and each vertex of a face is a vertex of the polyhedron.

The word "polyhedron" is used occasionally to refer to the solid figure comprised of the polyhedron and the region it encloses. Thus, the volume of a polyhedron refers to the volume of the solid figure enclosed by the polyhedron.

A prism is a polyhedron that has two congruent and parallel polygons as bases, and lateral faces that are parallelograms. Prisms in which the bases are perpendicular to the lateral faces are called right prisms. The lateral faces of right prisms are rectangles.

right triangular prism

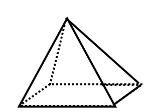




oblique triangular prism

A pyramid is a polyhedron with a base that can be any polygon, one further vertex called the apex, and lateral faces that are triangles with a vertex at the apex and opposite sides of the base.

Prisms and pyramids are named for the shape of their base(s). For example, a prism with a triangular base is called a triangular prism. A pyramid with a square base is called a square pyramid. A right prism with a rectangular base is called a right rectangular prism.



Many polyhedra (oriented in space) have a "bottom" and a "top," and these are sometimes both referred to as the bases. The faces of a polyhedron other than its bases are referred to as its "sides." The area of the sides of a polyhedral figure is referred to as its lateral area. A rectangular prism sitting on a table has six faces, including its top, its bottom, and its four sides. A pyramid sitting on a table has a base (the bottom) and several sides meeting at the apex. The lateral area of a pyramid is the area of the sides meeting at the apex.

There is an analogous nomenclature for cylinders. The bases of a cylinder are the two disks at either end, and the lateral area of the cylinder is the area of the curved tubular surface connecting these disks.

Base, Altitude, and Height

When a polygon is represented visually, it is often pictured as sitting on one of its sides. It is natural to call that side the "base" of the polygon. If we rotate the polygon, it may be sitting on another side, and we could refer to that other side as its base. Mathematically, the specification of the base of a polygon is arbitrary. We specify beforehand which side is the base, irrespective of how the polygon might be represented visually, and we develop definitions and formulas for a polygon with a predesignated base.

In the case of a rectangle, we may designate any side as the base of the rectangle. In the formula

$$\text{Area} = (\text{length}) \times (\text{width}),$$

the "length" then refers to the length of the base, while the "width" refers to the length of the sides perpendicular to the base.

An <u>altitude of a parallelogram</u> on a given base is a line segment perpendicular to the base and connecting a point on the base (extended if necessary) to a point on the opposite side. The <u>height</u> of the parallelogram is the length of the altitude. The area of a parallelogram is given by

$$\text{Area} = (\text{length of base}) \times (\text{height}).$$

Though the factors in this product depend on which side of the parallelogram is selected to be the base, the area will be the same no matter which side we designate as the base. In the case of a triangle, there are three choices for base, three different altitudes, and possibly three different heights. The formulas for area all yield the same result.

Something similar happens for rectangular prisms. We may designate any face of a rectangular prism as the base of the prism. The "height" then refers to the length of the edges perpendicular to the base, and the formula

$$\text{Volume of prism} = (\text{area of base}) \times (\text{height})$$

is valid, no matter which face we designate as the base.

MATH BACKGROUND

What is Sea Level?

Elevations are measured with respect to mean sea level (MSL), usually in feet or meters. The highest point in the contiguous United States is Mount Whitney, with an elevation of 14,505 feet *above* sea level. The lowest point is Badwater Basin in Death Valley, with an elevation of 282 feet *below* sea level. What do these MSL figures mean?

Mean sea level is an average of sea levels, averaged over wave motion, tides, lunar cycles, and so on. In past centuries the MSL figures were determined by Earth-based measurements, often based on some sort of average of sea levels at a particular location. The sea level standard varied from country to country, so that for instance the sea level standard in France was different from the sea level standard in England.

Determining a sea level standard around the globe is quite complicated. The Earth is not round, but has an irregular shape that is pushed in a bit at the poles. The sea is in constant motion, affected by many factors such as tides, wind, atmospheric pressure, temperature, and salinity. For instance, the sea level off the coast of California and Oregon rises during El Niño cycles and falls during La Niña cycles.

The branch of science called geodesy, which is a combination of applied mathematics and geology, has the goal of understanding the geometric shape of Earth and its gravity field, as well as changes of these in time. Scientists have adopted a precise model for theoretical sea level, known as the Geoid. The Geoid models the level to which water would settle under the gravitational pull of Earth. By definition, the Geoid is the true zero surface for determining elevations. It is a smooth surface that takes into account the local variations in the gravitational field of Earth, such as the increased gravitational attraction of the mass of the Rocky Mountains.

The measurement of heights with respect to sea level was revolutionized in 1992 when the first satellite to monitor the shape of Earth through GPS (Global Positioning System) was launched. This system adopts a certain ellipsoid as a rough approximation to mean sea level, and it measures vertical distances to the ellipsoid. These measurements vary in places by as much as 100 meters from MSL as determined by the Geoid, and the measurements are modified accordingly.

Sea levels are known to have varied greatly over geological time scales. Careful measurement of variations in sea levels produces information about the effect of the melting of glaciers and the warming of oceans on sea levels. Analyzing these measurements requires sophisticated probabilistic techniques. Estimates of sea level rise are of particular importance to coastal communities for establishing policies related to development, such as building codes, and also to insurers. Recent research puts average sea level rise at 1.2 mm per year in the pre-GPS period between 1900 and 1992, and at 3.0 mm per year in the GPS era from 1992 to the present. In other words, the rate of rise in sea levels is currently a little over one inch per decade and increasing.

See more about connections to the environment in General Resources on the Teacher Portal.

Interpreting the Minus Sign

Here are three ways to interpret the minus sign, along with some examples.

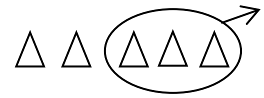
Operation Interpretation

When the minus sign is between two expressions, it means "subtract the second expression from the first."

Example: 5 – 3

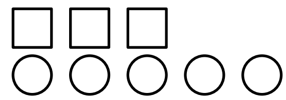
The phrase "5 minus 3" can be read:

- 5 take away 3, or subtract 3 from 5



Subtract or take away.
There are 2 triangles remaining.

- The difference between 5 and 3



Difference between two sets.
There are 2 more circles than squares.

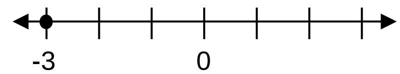
Geometric Interpretation

In front of a number or variable, a minus sign means "opposite." Geometrically, minus can be thought of as a reflection or mirror image. In this case, we are reflecting the number line through zero.

Example: -3

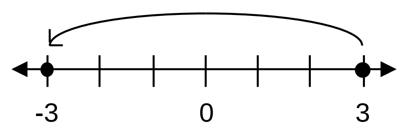
The phrase "minus 3" can be read:

- Negative 3



Pictorially, this is a location on the number line that is 3 units left of zero.

- Opposite of 3



This is the value you get by first locating 3 on the number line, and then locating that same distance on the opposite side of zero. Geometrically, minus can be thought of as a reflection or mirror image. In this case, the reflection of 3 through zero is -3.

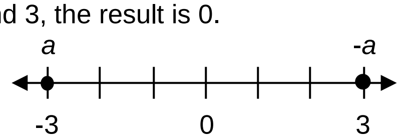
Algebraic Interpretation

The minus sign is used to show additive inverses. The identity $a + (-a) = 0$ means that $-a$ is the additive inverse of a. It is what we add to a to get 0.

Example: If $a = -3$, then $-a = 3$

The statement, "If a is equal to minus 3, then minus a is equal to 3" can be read:

- If a is equal to the opposite of 3, then the opposite of a is equal to 3. When we add -3 and 3, the result is 0.



Be careful not to read the variable $-a$ as "negative a" because the variable may not represent a number less than zero. However, reading $-a$ as "the opposite of a" is appropriate. For example, $-(-3) = 3$ may be read "the opposite of the opposite of 3 is 3.

An Algebraic Definition of Absolute Value

The absolute value $|x|$ of a number x is defined as the distance from x to 0 on the number line. As with distance, absolute value is always a nonnegative number.

The definition of the absolute value of x can be expressed algebraically:

$$\text{If } x \geq 0\text{, then } |x| = x.$$
$$\text{If } x < 0\text{, then } |x| = -x.$$

Examples	$|3| = 3$	Since $3 \geq 0$, we use the first line of the definition above.
	$|-3| = -(-3) = 3$	Since $-3 < 0$, we use the second line of the definition above.