FOCUS, COHERENCE, AND RIGOR

A group of educators at Rutgers University led by Dr. Roberta Shorr (Young, 2009) conducted a seven-year study on problem-solving with minority and low-income students in low-performing schools. They found that giving conceptually challenging problems to students caused frustration, but at the same time, students were engaged and motivated. Working through the frustration in an emotionally safe environment, students gained "satisfaction, pride, and a willingness to work harder next time." This work also resulted in higher standardized test scores (e.g., average scores for Newark fourth graders rose from 45 to 79 percent).

At the Center for Mathematics and Teaching, we know that all students have the potential to achieve in mathematics, we believe that the development of mathematics should reflect the connectedness of Big Ideas into a coherent whole, and we make mathematics inviting and inclusive to more students. One of the ways we put these beliefs and intentions into action is by attending to focus, coherence, and rigor in program development.

FOCUS

According to Achieve the Core (2014), "Not all content in a given grade is emphasized equally in the Standards. Some clusters require greater emphasis than others based on the depth of ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness."

Achieve the Core organizes mathematics clusters into three categories: major clusters, supporting clusters, and additional clusters. This table identifies the clusters for 6th grade and their alignment with *MathLinks* lessons.

GRADE 6: FOCUS IN *MATHLINKS* BASED ON MAJOR, MINOR, AND SUPPORTING CLUSTERS

CLUSTER	Standards	Lesson Focus	Additional Lessons (L) Spiral Review (SR)
Major Clusters			
• 6.RP.A Understand ratio concepts and use ratio reasoning to solve problems.	6.RP.1, 2, 3abcd	3.1, 3.2, 3.3, 3.4, 5.1, 5.2, 5.3	L: 4.2, 4.3, 7.2, 7.3, 9.3 SR:4,5,6,7,8,9,10
• 6.NS.A Apply and extend previous understandings of multiplication and division to divide fractions by fractions	6.NS.1	4.3, 4.4	SR:5,7,9,10
• 6.NS.C Apply and extend previous understandings of numbers to the system of rational numbers	6.NS.5, 6abc, 7abcd, 8	10.1, 10.2, 10.3, 10.4	<end of course, no spiral review>
• 6.EE.A Apply and extend previous understandings of arithmetic to algebraic operations.	6.EE.1, 2abc, 3, 4	6.1, 6.2, 6.3	L: 7.1 7.2, 7.3, 9.1, 9.3 SR:7,8, 9, 10
• 6.E.B Reason about and solve one-variable equations and inequalities	6.EE.5, 6, 7, 8	6.2, 6.3, 8.1, 8.2, 8.3	L: 7.1, 7.2, 7.3, 9.1, 9.3, 10.1, 10.2 SR:9, 10
• 6.EE.C Represent and analyze quantitative relationships between dependent and independent variables	6.EE.9	7.1, 7.2, 7.3	L: 8.3, 9.1, 9.2, 9.3
Supporting Clusters			
• 6.G.A Solve real-world and mathematical problems involving area, surface area, and volume	6.G.1, 2, 3, 4	9.1, 9.2, 9.3, 10.4	<end of course, no spiral review>
Additional Clusters			
• 6.NS.B Compute fluently with multidigit numbers and find common factors and multiples	6.NS.2, 3, 4	2.1, 2.2, 2.3 4.1, 4.2, 6.1	SR:(all)
• 6.SP.A Develop understanding of statistical variability	6.SP.1, 2, 3	1.1, 1.2, 1.3	L: 5.3
• 6.SP.B Summarize and describe distributions	6.SP.4, 5abcd	1.1, 1.2, 1.3	L: 5.3, 7.2 SR:9

Ratio and Proportional Relationships Number Sense Expressions and Equations Statistics and Probability Geometry

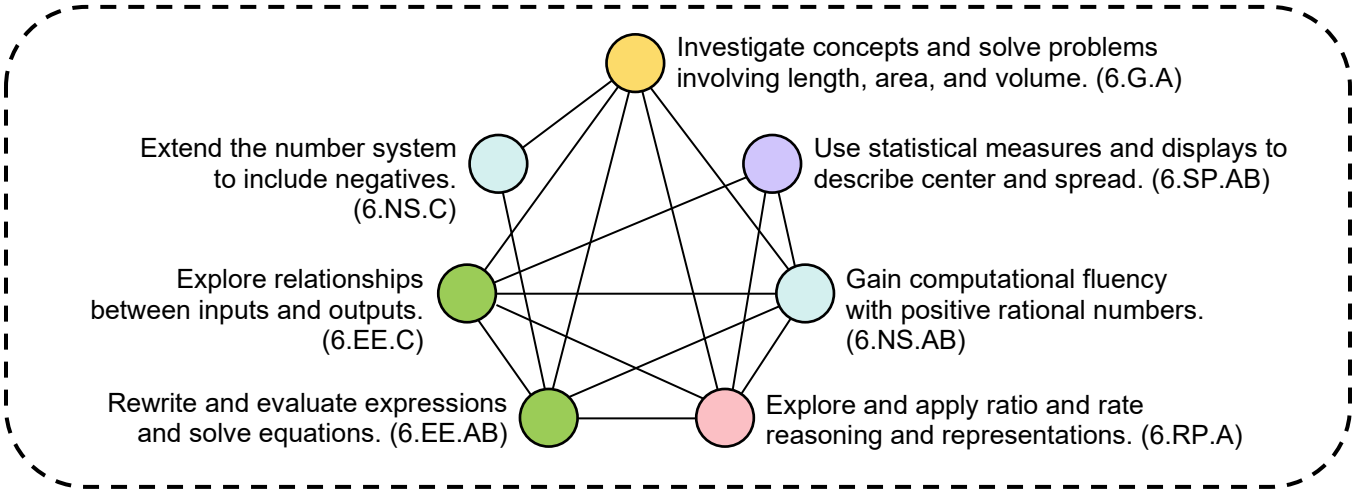
MathLinks intentionally avoids the trap of a "mile wide and inch deep" curriculum that uses standards as a checklist. Rather, we focus students on understanding grade-level mathematics deeply, connecting Big Ideas (see next page), and offering students problems, tasks, and investigations worthy of engagement.

COHERENCE

According to the 2023 California Mathematics Framework, “instructional materials should primarily involve tasks that invite students to make sense of big ideas, elicit wondering in authentic contexts, and necessitate mathematics. Big ideas in math are central to the learning of mathematics, link numerous mathematical understandings into a coherent whole, and provide focal points for students’ investigations.”

Inspired by the work of Jo Boaler (2018) and based on the Content Domains, the *MathLinks* team synthesized the work of the 6th grade into seven Big Ideas. These ideas and their connections within the entire program are shown here.

Grade 6: Big Ideas and Connections



The work in *MathLinks* is organized into 10 units. An expanded dive into the specific Big Ideas, connections, and progressions across grades are included in the front section of each unit of the Teacher Edition. Toward the end of each Student Packet, students identify the Big Ideas within the unit and reflect upon how they are connected to each other.

RIGOR

Achieve the Core (2015) states, “a rigorous math program will pursue with equal intensity conceptual understanding, procedural skill and fluency, and applications.” We now examine the *MathLinks*: Grade 6 program through this lens.

Conceptual Understanding in *MathLinks*

Conceptual understanding, the bedrock of a *MathLinks* course, frequently drives the other two components of rigor. It is a *MathLinks* philosophy to make sure all students have the opportunity to make meaning for every concept presented; we focus on the conceptual development of Big Ideas in depth and make them plausible through investigations, activities, and practice. This is commonly done throughout lessons in all units, oftentimes with the help of teacher Lesson Notes and Slide Decks (or Slide Deck Alternatives).

The table below identifies some examples of concept development in *MathLinks:* Core Grade 6 (unit number and lesson number in parentheses). Typically, these lessons include guided instruction with a slide deck, lesson notes, class discussions, or group work. Opportunities for independent work within a Student Packet appear as Practice pages within lessons, in the Review section as activities, and as Spiral Reviews in subsequent units. Unit Resources on the Teacher Portal also contain Extra Problems, Tasks, and Projects to support conceptual development.

GRADE 6: EXAMPLES OF CONCEPT DEVELOPMENT IN *MATHLINKS*	
CLUSTER DESCRIPTION	**(LESSON NUMBER) CONCEPT DEVELOPMENT ACTIVITY**
Major Clusters	
• 6.RP.A (RP 1, 2, 3) Understand ratio concepts and use ratio reasoning to solve problems.	(3.1) Paint Mixtures (5.3) Percent and Double Number Lines
• 6.NS.A (NS 1) Apply and extend previous understandings of multiplication and division to divide fractions by fractions	(4.3) Exploring Divide Across (4.4) Exploring Multiply by the Reciprocal
• 6.NS.C (NS 5, 6, 7, 8) Apply and extend previous understandings of numbers to the system of rational numbers	(10.1) Opposites (10.1) Distance and Absolute Value
• 6.EE.A (EE 1, 2, 3, 4) Apply and extend previous understandings of arithmetic to algebraic operations.	(6.1) Greatest Common Factor and the Distributive Property (6.3) Perimeter of a Rectangle
• 6.EE.B (EE 5, 6, 7, 8) Reason about and solve one-variable equations and inequalities	(8.2) Solving Equations with Mental Math and Substitution (8.2) Solving Equations with Tapes and Balance
• 6.EE.C (EE 9) Represent and analyze quantitative relationships between dependent and independent variables	(7.1) What Comes Next? (7.1) Input-Output Rules
Supporting and Additional Clusters	
• 6.G.A (G 1, 2, 3, 4) Solve real-world and mathematical problems involving area, surface area, and volume	(9.1) Area of a Parallelogram
• 6.NS.B (NS 2, 3, 4) Compute fluently with multidigit numbers and find common factors and multiples	(2.1) Finding the Greatest Common Factor
• 6.SP.A (SP 1, 2, 3) Develop understanding of statistical variability	(1.2) Name Scores Revisited
• 6.SP.B (SP 4, 5) Summarize and describe distributions	(1.3) Interpreting Box Plots

Ratio and Proportional Relationships **Number Sense** **Expressions and Equations** **Statistics and Probability** **Geometry**

Applications in *MathLinks*

Problem-solving is an important driver of instruction within *MathLinks* courses. In *MathLinks*, we include engaging mathematical problems and applications with accessible entry points for all students, multiple approaches or solutions, and extensions to challenge and enrich. All units begin with an Opening Problem, which introduces a concept or establishes a need to know. In many cases, students require more instruction throughout the unit before they are fully prepared to bring the problem to its conclusion. Substantial problems exist throughout the units as well.

The table below identifies some examples of mathematical problems and applications in *MathLinks:* Grade 6 (unit number and lesson number in parentheses). Some of these problems include guided instruction with a slide deck and lesson notes. Others are appropriate for independent work. Unit Resources on the Teacher Portal (e.g., Tasks, Projects) also contain mathematical problems and applications.

GRADE 6: EXAMPLES OF PROBLEM-SOLVING IN *MATHLINKS*	
CLUSTER DESCRIPTION	**(LESSON NUMBER) MATHEMATICAL PROBLEMS AND APPLICATIONS**
Major Clusters	
• 6.RP.A (RP 1, 2, 3) Understand ratio concepts and use ratio reasoning to solve problems.	(3.0, 3.1) Nana's Chocolate Milk + Nana…Revisited (3.3) The Grain Grocer (5.0, 5.3) Growth Spurts, Growth Spurts Revisited
• 6.NS.A (NS 1) Apply and extend previous understandings of multiplication and division to divide fractions by fractions	(4.3) Practice 8 (Problems 1, 8) (4.4) Practice 11
• 6.NS.C (NS 5, 6, 7, 8) Apply and extend previous understandings of numbers to the system of rational numbers	(10.4) A Basketball Court, Practice 8
• 6.EE.A (EE 1, 2, 3, 4) Apply and extend previous understandings of arithmetic to algebraic operations.	(6.0, 6.1) The Problem of 4's + Problem of 4's Extended
• 6.EE.B (EE 5, 6, 7, 8) Reason about and solve one-variable equations and inequalities	(8.0) Lions and Tigers and Bears (8.1) Mobiles and Balance, Practice 1
• 6.EE.C (EE 9) Represent and analyze quantitative relationships between dependent and independent variables	(7.3) Raising Money for Music, Practice 7 (7.3) Practice 9
Supporting and Additional Clusters	
• 6.G.A (G 1, 2, 3, 4) Solve real-world and mathematical problems involving area, surface area, and volume	(9.0, 9.1) Which Rug is Bigger?, Practice 3 (problem 5) (9.2) Who Needs More Paint? (9.3) The Food Drive
• 6.NS.B (NS 2, 3, 4) Compute fluently with multidigit numbers and find common factors and multiples	(2.1) The Factor Game
• 6.SP.A (SP 1, 2, 3) Develop understanding of statistical variability	(1.0, 1.1) Beach Cleanup, Practice 2 (problem 5)
• 6.SP.B (SP 4, 5) Summarize and describe distributions	(1.3) Practice 5 (Hours Spent Playing Online Games) (7.2) A Committee Decision

Ratio and Proportional Relationships **Number Sense** **Expressions and Equations** **Statistics and Probability** **Geometry**

Procedural Skill and Fluency in *MathLinks*

Procedural skill and fluency require more than just computational speed and accuracy. In a 2014 position paper, NCTM described procedural fluency as "the ability to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another."

In *MathLinks*, we thoughtfully develop new procedural skills and provide opportunities for students to gain fluency throughout the year. Skills practice in each unit is found in the Student Packets in the following ways:

- Practice pages – These pages support concept development.
- Review activities – These pages often include skills practice.
- Spiral Review – These pages have distributed practice of prior skills.
- Math Path Fluency Challenges – This Activity Routine, in the Spiral Review section, utilizes mental math skills and supports fluency development in a puzzle format.

Achieve the Core (2015) explicitly identifies two CCSS-M standards (6.NS.2 and 6.NS.3) where fluency is expected in 6th grade. The EdReports Evidence Guide (2021) also includes 6.EE.1 as a fluency expectation. The table below shows some examples in 6th grade Student Packets where these procedural skills are developed and students have opportunities to gain fluency through independent practice.

GRADE 6: EXAMPLES OF COMPUTATIONAL FLUENCY WORK IN *MATHLINKS*		
Standard / Description	**Lesson**	**Opportunities to gain fluency**
6.NS.2 Fluently divide multidigit numbers using the standard algorithm	(4.1) Whole Number Division	(4.1) Practice 1 – Practice 4 Spiral Review in Packets 1, 2, 3, 5, 6, 7
6.NS.3 Fluently perform operations with multidigit decimals using the standard algorithm.	(4.2) Decimal Division and Rate Problems (5.1) Percent as a Number	(4.2) Practice 5 – Practice 7 (5.1) Practice 3 Spiral Review in Packets 1, 2, 3, 8, 9, 10
6.EE.1 Write and evaluate numerical expressions involving whole number exponents.	(6.0) Problem of 4s (6.1) Numerical Expressions	(6.1) Practice 2, Practice 3 (6.1) Problem of 4s Extended Spiral Review in Packets 7, 8, 9, 10

Number Sense Expressions and Equations

Grade-level skills practice is in each unit, as well as practice to fill in gaps. Both can be found on the Teacher Portal in Other Resources in the following ways:

- Essential Skills – This entire section reviews skills and concepts important for success in a given unit. Activity Routines such as Big Square Puzzles, Open Middle Problems, and Four-in-a-Row games are also in these sections for some units. They provide a practice alternative to "drill and kill."
- Extra Problems – Skills practice by lesson is available for all units.
- Nonroutine Problems – In addition to skills practice that is embedded in nonroutine problems, Big Square Puzzles, Open Middle Problems, and Four-in-a-Row games are located in this section for some units.

Finally, for students who need to fill large gaps in skills, consider Skill Boosters, which are on the Teacher Portal. This resource helps students catch up on below-grade-level work without losing extensive instructional time on grade-level work. A Skill Booster routine is intended to take about 5 – 10 minutes per day. Skill Boosters do not directly connect to any course or unit within a course.

Connecting the Three Aspects of Rigor

Interpretations of rigor are evolving. Achieve the Core described "rigor" as the pursuit of conceptual understanding, procedural skill and fluency, and application with equal intensity. The 2023 California Mathematics Framework added the element of integration, interpreting rigor as "an integrated way in which conceptual understanding, strategies for problem-solving and computation, and applications are learned so that each supports the other." A closer look at two major domains in 6th grade illustrates how this is done in *MathLinks*.

Ratio and Proportional Relationships

The Ratio and Proportional Relationships standards are the focus of four units in *MathLinks*: Grade 6. In Unit 3, **Ratio Representations**, students begin the transition from additive to multiplicative thinking by learning to create and interpret ratios, tape diagrams, tables, equivalent ratios, and double number lines in a variety of contexts. In Unit 4, **Division**, students explore the structure of rate problems and solve rate problems as they gain fluency with division. In Unit 5, **Percent**, students use sense-making strategies, computational procedures, and double number lines to make sense of percent and percent applications. In Unit 7, **Inputs and Outputs**, students informally expand their notion of proportional relationships, specifically the use of unit rates, into the world of input-output rules (functions). Problems such as Nana's Chocolate Milk and Growth Spurts provide contexts for extended explorations.

Expressions and Equations

The Expressions and Equations standards are the focus of four units in *MathLinks*: Grade 6. In Unit 6, **Expressions**, students work with both numerical and algebraic expressions. They learn to manipulate, simplify, and evaluate expressions using the distributive property and the conventions for order of operations. They also translate between words, numbers, and symbols. In Unit 7, **Inputs and Outputs**, students use visuals and contexts to analyze and solve problems with multiple representations. Concepts related to proportional reasoning are reviewed and emphasized. Students' knowledge of expressions enables them to generate equations for relationships relating two variables, called input-output rules. Without explicitly defining function (this is done in grade 8), students begin to develop flexibility when working with variables, expressions, and equations. The problems introduced set the stage for solving a linear equation in one variable since these equations are of the form $x + p = q$ and $px = q$ (i.e., "one-step equations") for cases in which p and q are nonnegative rational numbers. In Unit 8, **Solving Equations**, students learn mental math and substitution strategies and formalize the equation-solving processes, using balance and tape diagrams to work toward traditional equation-solving procedures, as the need grows. In Unit 10, **The Number Lines and the Coordinate Plane**, students graph solutions to simple inequalities.

Problems such as The Problem of 4s and Raising Money for Music provide contexts for explorations and connections.

In Conclusion

The authors of *MathLinks* carefully researched and thoughtfully operationalized focus, coherence, and rigor in designing the program. We integrated mathematical concepts, skills, and applications through contexts and problems in a natural way. The result is an efficient core program that contains just 10 Student Units with 33 lessons that can be completed in about 100 class hours. This leaves ample time for review, intervention, enrichment, assessment, and choice for both teachers and students, using what's available in Student Packets and also Other Resources on the Teacher Portal.