Name ______________________________ Period __________ Date ______________

UNIT 4
STUDENT PACKET

MathLinks GRADE 8

INTRODUCTION TO FUNCTIONS

Parent (or Guardian) signature ___________________________________

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

function	graph of a function
input-output rule	proportional proportional relationship
unit rate	*y*-intercept

OPENING PROBLEM: SLIDES AND JUMPS

[SMP 1, 2, 4, 5, 8]

Follow your teacher's directions for (1) – (2).

(1)

Level #	
1	
2	
3	
4	
5	

(2)

3. Record the missing values in the table below. Show your work on this page as needed.

Level #	**# of Slides**	**# of Jumps**	**Total # of Moves**
10			
	40		
25			
	100		
		10,000	
1,000			
			$n^2 + 2n$

MULTIPLE REPRESENTATIONS

We will use words, pictures, tables of numbers, and graphs to represent, describe, and analyze situations involving area and money.

[8.EE.5, 8.F.2, 8.F.3, 8.F.4; SMP1, 2, 3, 4, 7, 8]

GETTING STARTED

Fill in missing numbers and blanks based on the suggested numerical patterns. In the tables below, the *x*-value is considered the input value (independent variable) and the *y*-value is the output value (dependent variable).

Table I

1.

x		1	2		4		6
y		4	8	12		20	

a. Rate of change: for every increase of x by 1, y increases by ___.

b. Input-output rule (words): multiply the *x*-value by ___ to get the corresponding *y*-value.

c. Input-output rule (equation): y = __________. When $x = 0$, y = ___.

Table II

2.

x		1		3		5	6
y		5	9		17		25

a. Rate of change: for every increase of x by 1, y increases by ___.

b. Input-output rule (words): multiply the *x*-value by ___, then __________ to get the corresponding *y*-value.

c. Input-output rule (equation): y = __________. When $x = 0$, y = ___.

Table III

3.

x		1	2		4	5	
y		3		11		19	23

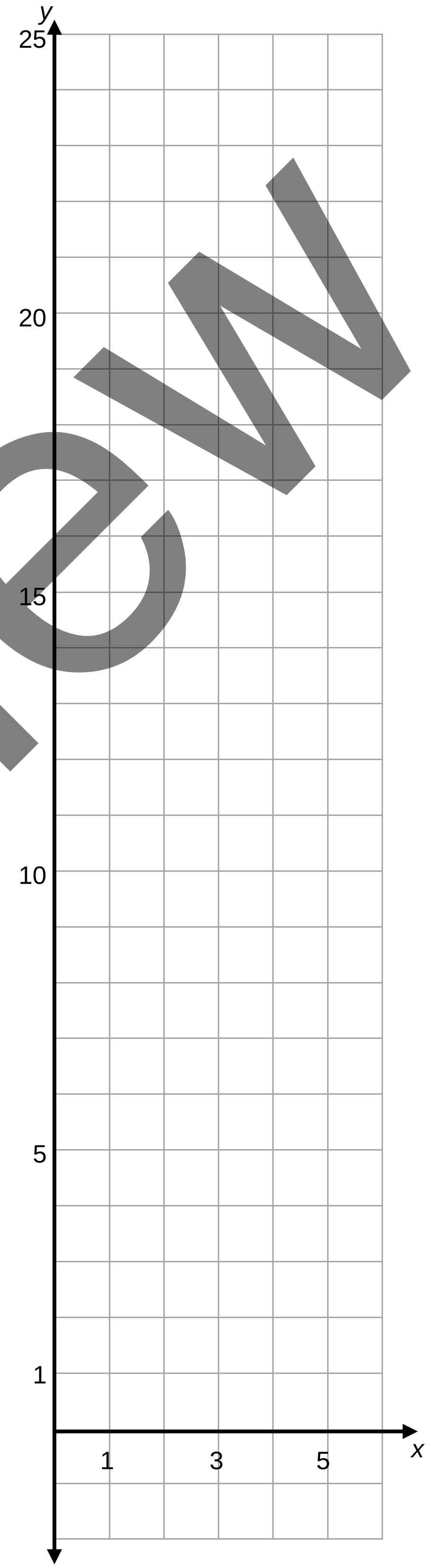
a. Rate of change: for every increase of x by 1, y increases by ___.

b. Input-output rule (words): multiply the *x*-value by ___, then __________ to get the corresponding *y*-value.

c. Input-output rule (equation): y = __________. When $x = 0$, y = ___.

4. Record the meaning of <u>input-output rule</u> in **My Word Bank**.

INTERPRETING TABLES, EQUATIONS AND GRAPHS



1. Graph each of the tables of numbers from **Getting Started** and connect the points with a line.

   Clearly label all three lines.

2. Describe each table of numbers. What is…
   - the same:
   - different:

3. Describe each equation. What is…
   - the same:
   - different:

4. Describe each graph. What is…
   - the same:
   - different:

5. Explain which table of values/equation/graph describes a proportional relationship.

6. Record the meanings of <u>proportional</u> and <u>proportional relationship</u> in **My Word Bank.**

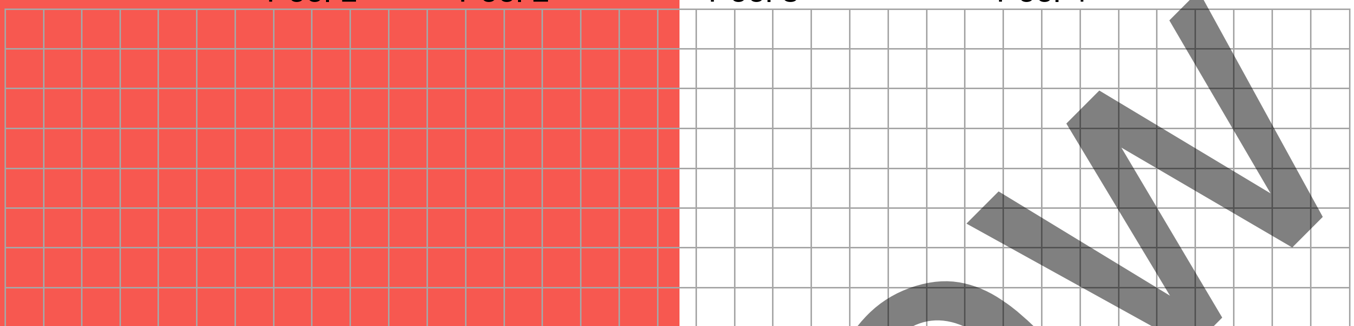
THE POOL PROBLEM

Follow your teacher's directions.

(1)

(2) Picture

Pool 1 Pool 2 Pool 3 Pool 4



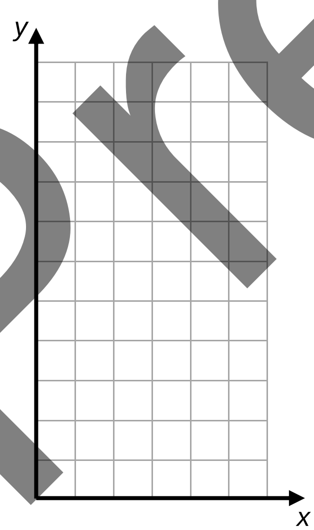
(3) Table

1	
2	
3	
4	
5	

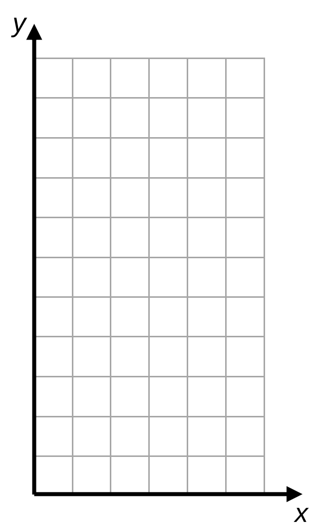
(3) Table

1	
2	
3	
4	
5	

(4) Graph



(4) Graph



ANALYZING THE POOL PROBLEM

Refer to **The Pool Problem**.

1. Draw and label "Pool 0" in the space to the left of Pool 1. How is it different from the other pools?

2. In the top row of the tables, write the entries for Pool 0. Graph the points for Pool 0.

3. When does the Border Pattern have more squares than the Water Pattern?

   When does the Water Pattern have more squares?

4. Write equations to represent the number of squares for each pool number.

   a. Border: b. Water:

5. Write the number of squares for each pattern for Pool 20.

   a. Border: b. Water:

6. Which pool has 48 border squares and 121 water squares?

7. Explain what (0, 4) and (0, 0) represent in the context of **The Pool Problem**.

   Where are these points found on the graphs?

8. Does the Border Pattern grow at a constant rate? Explain.

   Does it represent a proportional relationship? Explain.

9. Does the Water Pattern grow at a constant rate? Explain.

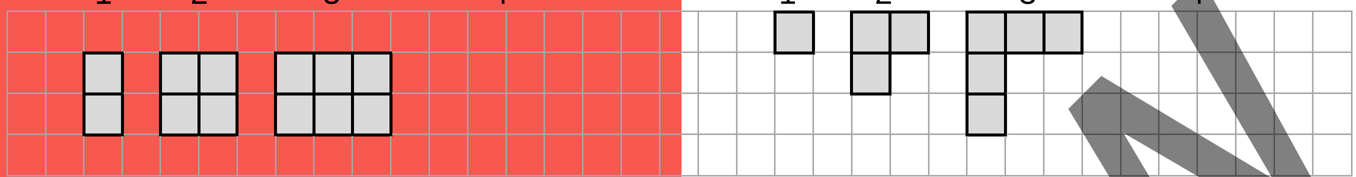
   Does it represent a proportional reationship? Explain.

PRACTICE 1

1. Below are two different square patterns. Draw the 4^{th} step for each pattern. Fill in the tables. Draw graphs with titles and labels.

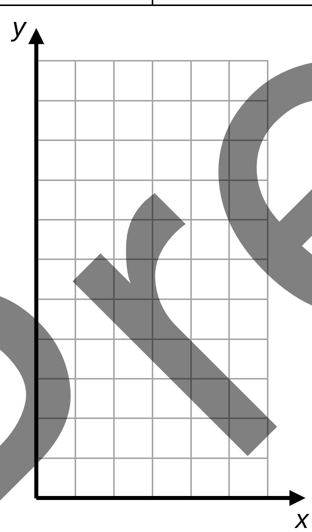
Pattern A
Step numbers:

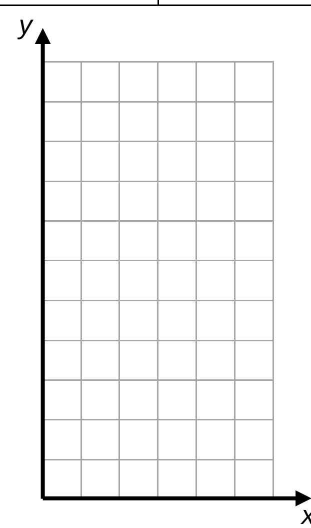
Pattern B
Step numbers:



Pattern A	
step # (*x*)	(*y*)
1	
2	
3	
4	
5	

Pattern B	
step # (*x*)	(*y*)
1	
2	
3	
4	
5	





2. If not already done, write the entries for step 0 in the top row of the tables.

3. Given the pictures and numbers, does "step 0" make sense for either pattern?

4. Graph a point for step 0 only if it makes sense for the pattern.

PRACTICE 1
Continued

5. Write equations to represent the number of squares for each pattern.

   a. Pattern A:

   b. Pattern B:

6. For Pattern A, find…

   a. The number of squares in step 80

   b. The step number for 200 squares:

7. For Pattern B, find…

   a. The number of squares in step 70

   b. The step number for 101 squares:

8. Consider the tables, graphs, and rules used to represent both patterns.

   a. List some things that are the same for both.

   b. List some things that are different for both.

9. Why does Pattern A represent a proportional relationship, while Pattern B does NOT?

10. For both patterns:

    a. In the tables, as the x-value increases by 1, the y-value increases by _____.

    b. On the graphs, the y-coordinate moves up by _____ as the x-coordinate moves right by 1.

    c. For the equations, the coefficient of x is _____.

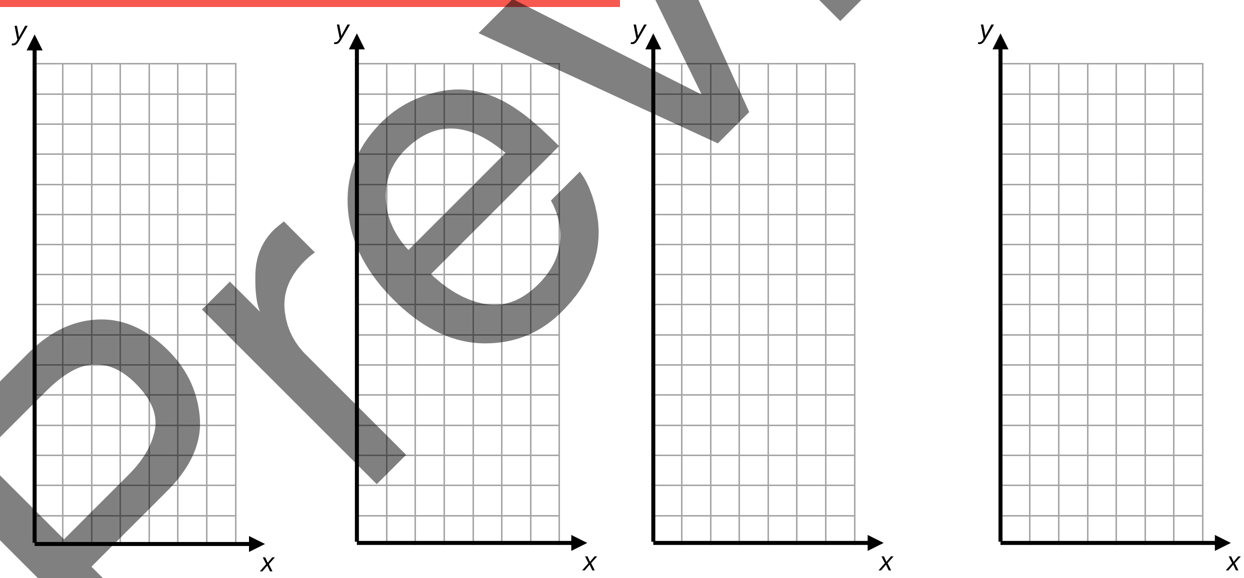
SAVING VS SPENDING

Follow your teacher's directions for (1) – (3).

(1)

	Mateo	Dion	Talia	Ayla
0	0		140	
1	20	30		130
2	40	50		105
		70	80	80
4			60	
				30
	120	130	20	5
7	140	150		

(2)



(3)

4. Record the meaning of <u>*y*-intercept</u> in **My Word Bank**.

ANALYZING SAVING VS SPENDING

Refer to **Saving vs Spending** on the previous page. As a convention, we read graphs from left to right.

1. For which students do table values and graphs appear to be increasing?

   Decreasing?

2. Compare tables and graphs for pairs of students.

Compare:	How are they the same?	How are they different?
Mateo and Dion		
Dion and Talia		
Talia and Ayla		

3. How do initial amounts (in these cases, this is Month 0) appear to be shown in the equations?

4. How do rates of change (slope) appear in the equations?

5. Do any of these situations represent a proportional relationship? Explain.

PRACTICE 2

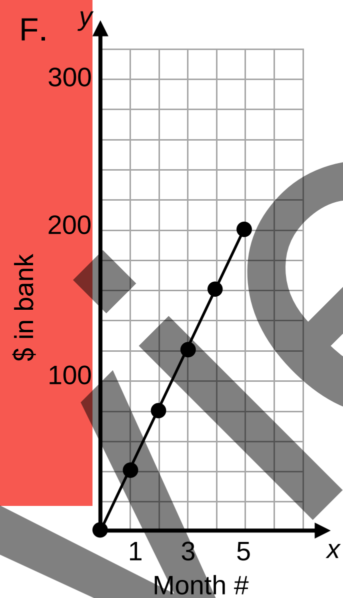
Use the representations A – G to fill in the table below.

A. With an initial value of \$260, Jocelyn's bank account balance goes down at a constant rate of \$30 per month.	B. With an initial value of \$50 Jayson's bank account balance goes up at a constant rate of \$20 per month.
C. $y = 25x + 100$ (let x be month #, and y be \$ in bank)	D. $y = -30x + 280$ (let x be month #, and y be \$ in bank)

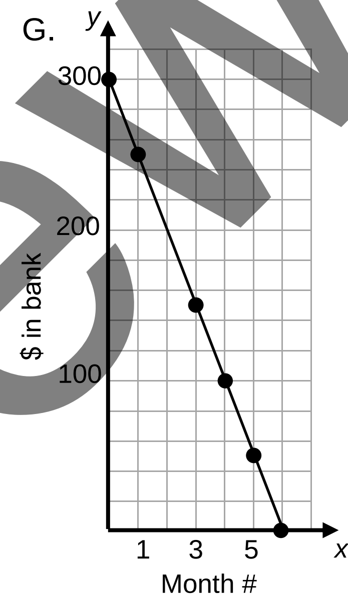
E.

Month # (x)	**\$ in bank (y)**
0	260
1	220
2	180
3	140
4	100
5	60
6	20

F.



G.



	An increase or a decrease?	**Initial value (\$ at month 0)**	**Rate of change (slope)**	**A proportional relationship?**
A.		\$260		
B.			up \$20/mo (20)	
C.	increase			
D.				no
E.				
F.				
G.				

FUNCTION REPRESENTATIONS

We will explore the concept of a function. We will define function and graph of a function. We will describe examples of functions and examples of non-functions.

[8.F.1, 8.F.3, 8.F.5; SMP1, 2, 4, 5, 6]

GETTING STARTED

1. Refer to the table to the right about professional football in the 1970s.

Year	Football Champion
1970	Kansas City
1971	Baltimore
1972	Dallas
1973	Miami
1974	Miami
1975	Pittsburgh
1976	Pittsburgh
1977	Oakland
1978	Dallas
1979	Pittsburgh

   a. Which cities won exactly once?

   b. Which cities won exactly twice?

   c. Which city won exactly three times?

   d. If you are given a specific year (input), can you always tell which team won (output)? _____ Give an example.

   e. If you are given a specific team (input), can you always tell which one year they won (output)? _____ Give an example.

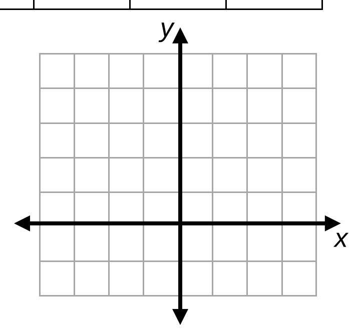
2. For the equation $y = x + 1$, fill in the table, write ordered pairs to correspond with table entries, and draw a graph.

Table:

input (x)	3	2	1	0	-1	-2	-3
output (y)							

Ordered Pairs:

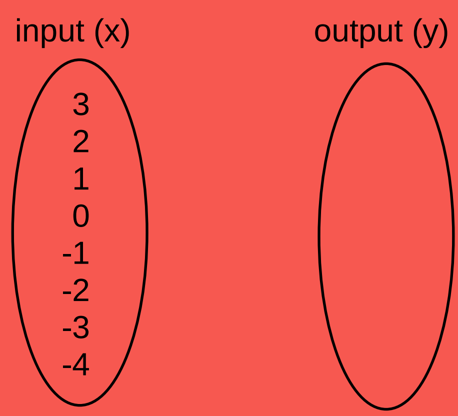
Graph:



WHAT IS A FUNCTION?

Follow your teacher's directions for (1) – (6).

(1)



(2)

(3)

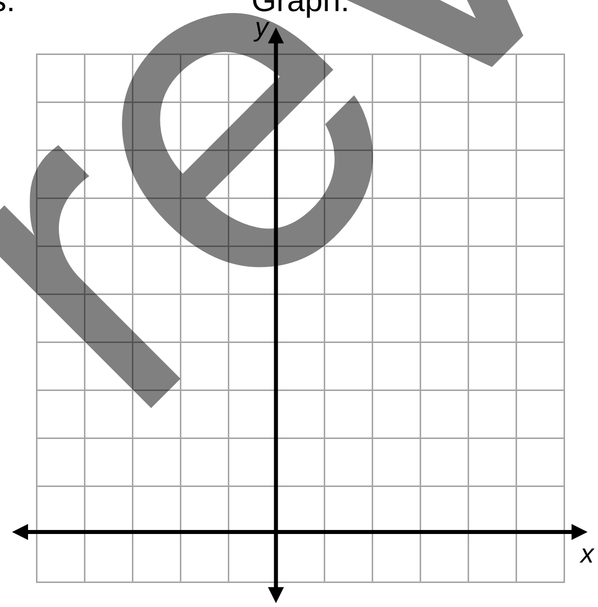
Equation:

Table:

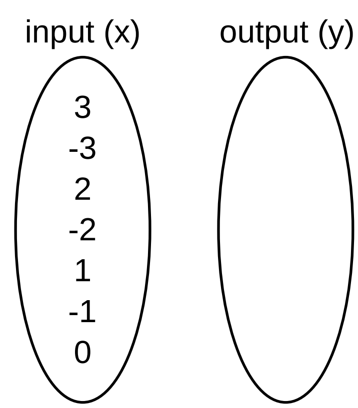
input (*x*)	3	2	1	0	-1	-2	-3
output (*y*)							

Ordered Pairs:

Graph:



Mapping Diagram:



(4)

WHAT IS A FUNCTION?
Continued

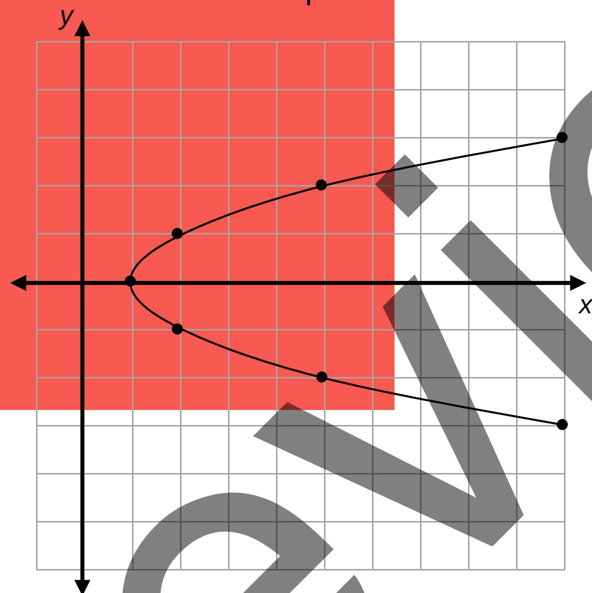
(5)

Equation:

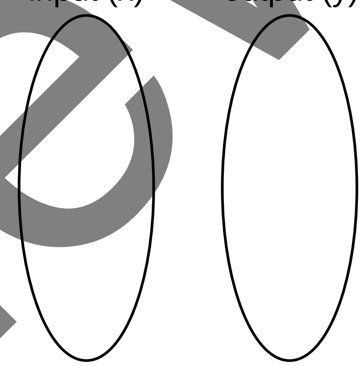
Table:

input (*x*)							
output (*y*)							

Ordered Pairs: Graph: Mapping Diagram:



input (x) output (y)



(6)

7. Go back to the football champion problem in **Getting Started**. Does the situation represented by the ordered pairs (year, champion) represent a function? _____

   How about (champion, year)? ______

   Explain your reasoning.

8. Record the meanings of <u>function</u> and <u>graph of a function</u> in **My Word Bank**.

PETS AND APARTMENTS

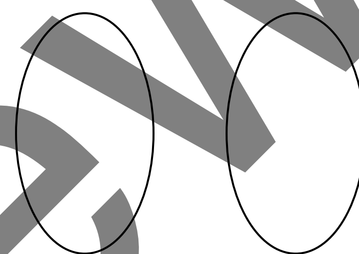
Mary has three pets, Kerry has one pet, and both Larry and Barry have no pets.

Let friend names be the input values.
Let the number of pets they each own be the output values.

1. Represent this situation with an input-output table, ordered pairs, and a mapping diagram.

Input Output Ordered pairs Mapping diagram

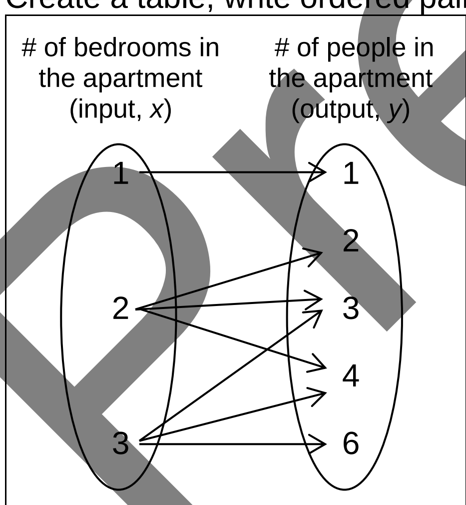
Name	
Mary	
Kerry	
Larry	
Barry	



2. Explain why this situation represents a function.

The mapping diagram below shows the number of bedrooms in an apartment building and the number of people who live in the apartment.

3. Create a table, write ordered pairs, and draw a graph for this situation.



4. Does this situation represent a function? Explain.

PRACTICE 3

1. Which of the following input-output tables could represent functions when x is used for the input value and y for the output value?

a.

x	y
0	4
3	6
6	4
9	6
12	4

b.

x	y
0	10
0	9
2	8
2	7
4	6

c.

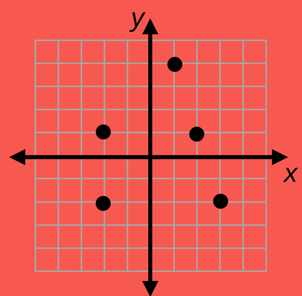
x	9	9	9	9	9
y	1	2	3	4	5

d.

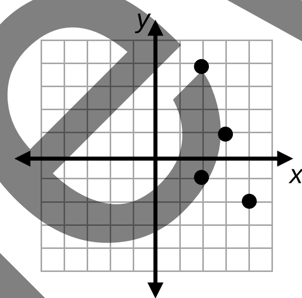
x	1	2	3	4	5
y	9	9	9	9	9

2. Which of the following graphs of points could represent functions?

a.



b.



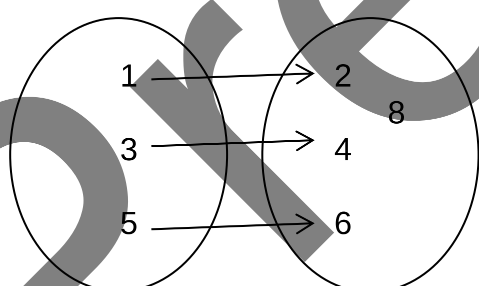
3. Which of the following sets of ordered pairs could represent functions?

a. (1, 5), (2, 6), (3, 5), (4, 6)

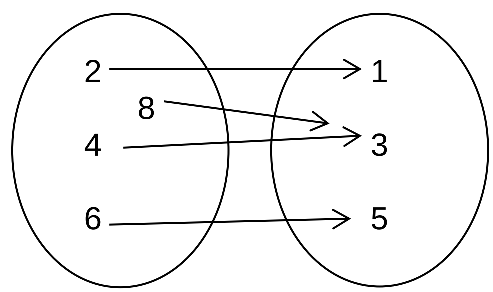
b. (10, -20), (-20, 10), (-10, -5), (10, 5)

4. Which of the following mapping diagrams could represent functions?

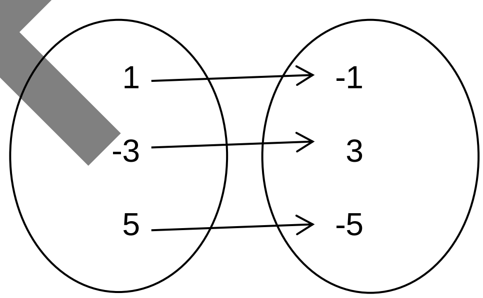
a.



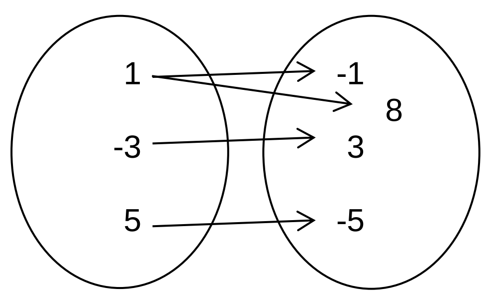
b.



c.



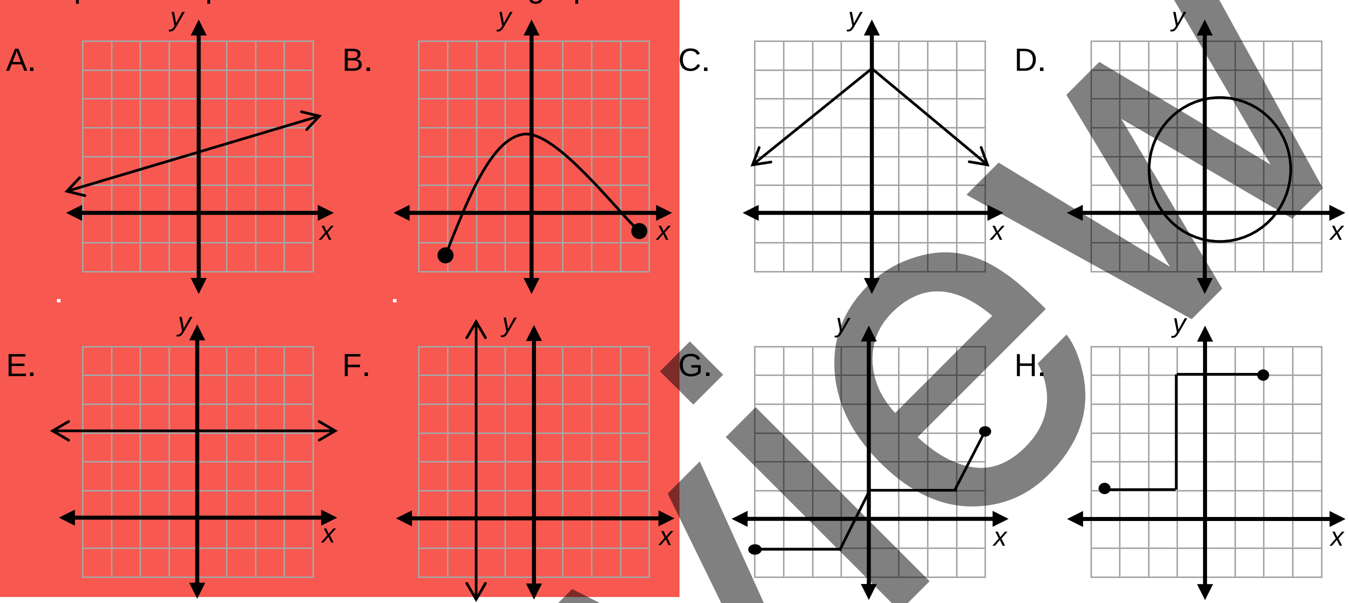
d.



PRACTICE 4

We say a function is **increasing** if a graph of the output values increases from left to right, and **decreasing** if a graph of the output values decreases from left to right. **Piece-wise** linear functions are considered to be linear.

Complete the problems below about graphs A – H.



1. List all graphs that are:
   a. Linear, but NOT piece-wise linear. ____________________
   b. Linear, but NOT a function. ____________________
   c. A function, but nonlinear. ____________________
   d. Nonlinear and not a function. ____________________
   e. An increasing function that is not decreasing anywhere. ____________________

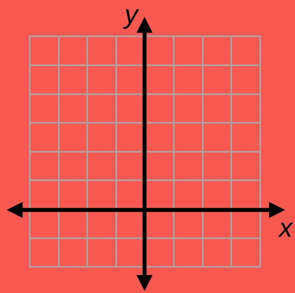
2. Describe where the graph of C is increasing and where it is decreasing.

3. The graph of G has four line segments. How many of them are increasing? Circle those segments on the graph.

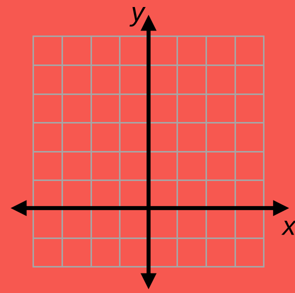
PRACTICE 5

Draw the four graphs as described.

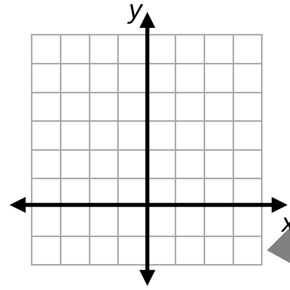
1. A decreasing linear function



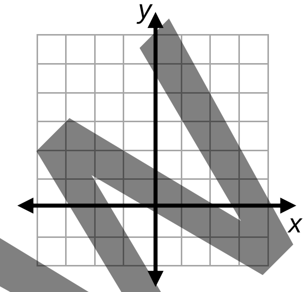
2. An increasing nonlinear function



3. A linear non-function



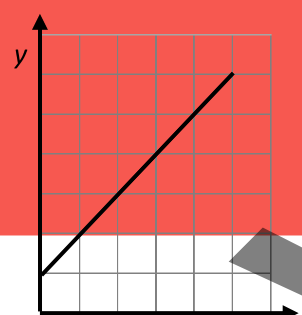
4. A nonlinear non-function



Estimate appropriate ordered pairs for each graph. Circle one **bold** choice for each bulleted statement.

5.

x	***y***

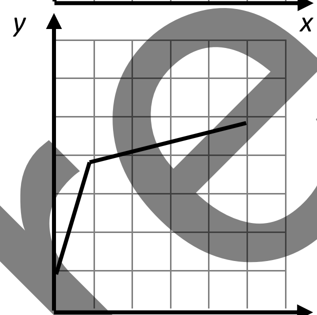


This graph:

- **could** **could not** represent a function
- is **increasing** **decreasing**
- is **linear** **nonlinear**

6.

x	***y***

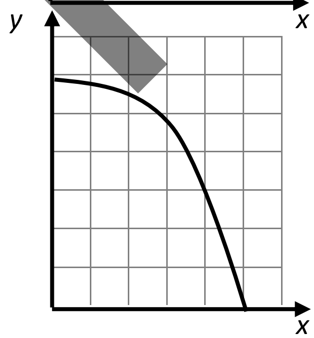


This graph:

- **could** **could not** represent a function
- is **increasing** **decreasing**
- is **linear** **nonlinear**

7.

x	***y***



This graph:

- **could** **could not** represent a function
- is **increasing** **decreasing**
- is **linear** **nonlinear**

8. Describe the change you observe in the table and graph in problem 7.

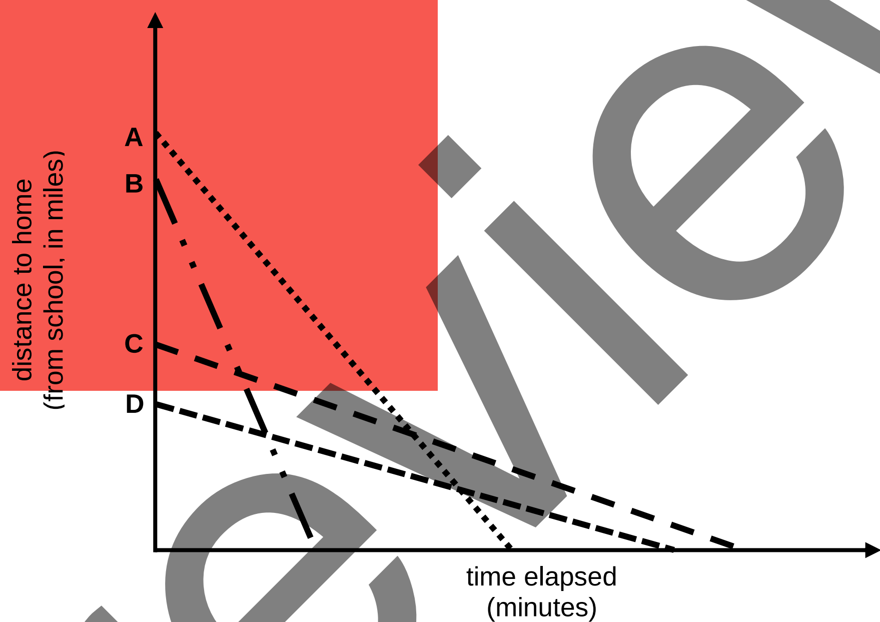
RATE REPRESENTATIONS

We will use words, tables of numbers, equations, and graphs to represent rates. We will compare representations of functions.

[8.EE.5, 8.F.1, 8.F.2, 8.F.3, 8.F.4, 8.F.5; SMP1, 2, 3, 4, 5, 7, 8]

GETTING STARTED

Andre, Bethany, Claudia, and Derek all walked home from John Baxter Taylor Middle School today to each of their respective homes. The graph below shows the distance from school to home, and the time elapsed during their walks. Use the letters A, B, C, and D to identify each person.



1. Why do all walkers appear to have constant rates of speed?

2. Who started farthest from home? _____ Closest to home? _____

3. Who got home first? _____ Last? _____

4. Who walked at the fastest rate? _____ Slowest? _____

5. Which pair appears to walk at the closer rate of speed, A-B, or C-D? _____

6. Could all of these graphs represent functions?

7. Which of these graphs are increasing and which are decreasing?

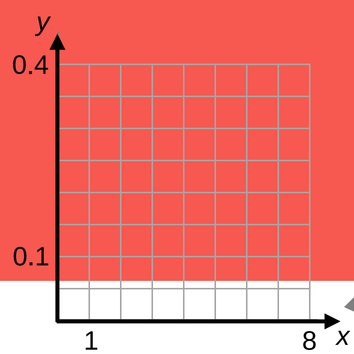
TO SCHOOL AND BACK HOME

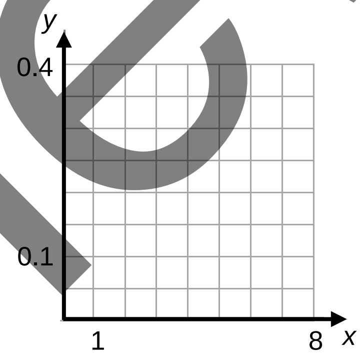
Nellie walks to Marjorie Lee Brown Middle School each morning at a constant rate of 0.05 miles per minute, and jogs home in the afternoon at a constant rate of 0.08 miles per minute.

School and home are $\frac{4}{10}$ of a mile apart.

1. Fill in both columns in the table below and draw graphs based upon the given data.

Minutes Elapsed (*x*)	0	1	2	3	4	5	6	7	8
To School: Miles from home (*y*)	0								
To Home: Miles from home (*y*)	0.4								





2. Why does it make sense to draw lines for the graphs with this context?

3. Which graph is increasing? ______________________ Decreasing? _____________________

4. Does either one of these situations represent a proportional relationship? Explain.

5. For walking to school, what is the unit rate?

6. Write an equation for each situation.

   Walking to school: ________________ Jogging home: ________________

TO SCHOOL AND BACK HOME

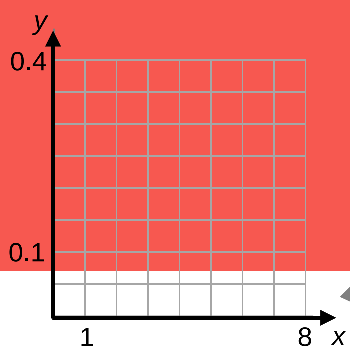
Continued

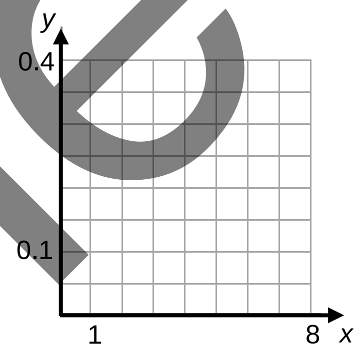
Nellie's brother, Willie, lives in the same house, goes to the same school, and he also walks there and jogs back.

- His "jogging home" data is in the table below
- The equation that represents his walk to school is $y = 0.04x$

7. Fill in the rest of the table below and draw graphs based on the given data.

Minutes Elapsed (*x*)	0	1	2	3	4	5	6	7	8
To School: Miles from home (*y*)	0								
To Home: Miles from home (*y*)	0.4	0.3	0.2	0.1	0	N/A	N/A	N/A	N/A





8. For walking to school, what is the unit rate?

   What does an initial value of $x = 0$ mean?

9. Write an equation for jogging home. ____________________________

10. Who walked to school at a faster rate? Explain.

11. Who jogged home at a faster rate? Explain.

PRACTICE 6

Below is information for four different cyclists, Tamika, Vinnie, Wanda, and Zach, all of whom ride at constant rates of speed. Use the letters T, V, W and Z to identify each person.

Let x be time elapsed in hours (hr) and y be distance traveled in miles (mi).

One representation is given for each. Complete the remaining representations in any order

1. Word descriptions.

T	Tamika rides at a constant rate of 18 miles per hour.
V	
W	
Z	

2. Entries in the table and equations.

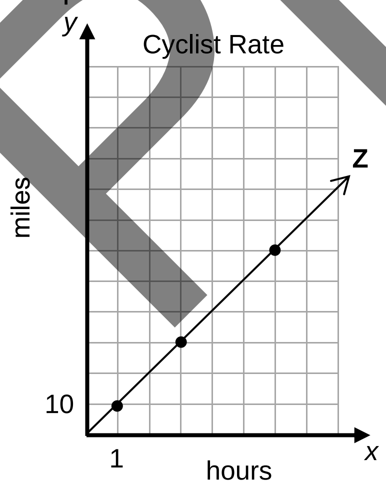
T	x	0	1	2	3	4	5	
	y							

V	x	0	1	2	3	4	5	
	y	0	15	30	45	60	75	

W	x	0	1	2	3	4	5	$y = 12x$
	y							

Z	x	0	1	2	3	4	5	
	y							

3. Graphs



4. What are the initial values (y-intercepts) for each rider?

5. What are the speeds in miles per hour (rates of change) for each rider? Where do you see these rates in the equations?

6. How are the graphs of the fastest and slowest riders different?

PRACTICE 7

Chris went jogging in Malala Yousafzai Park on Monday.

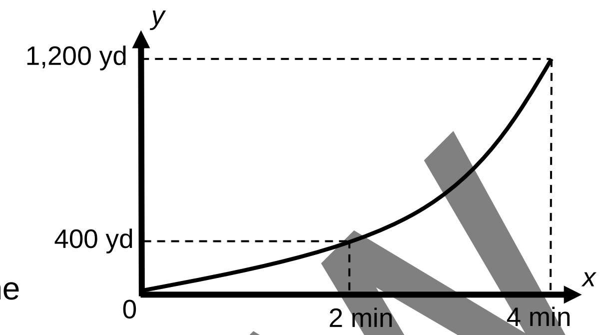
1. Could Chris' graph represent a function?

2. Does it appear to be linear?

3. Is it increasing or decreasing?

4. Use the Monday graph to the right to complete the table below.

Monday



Time Period	Number of Minutes	Yards Traveled	Average Rate of Speed
From 0 to 2 minutes			
From 2 to 4 minutes			
From 0 to 4 minutes			

5. In what part of the jog did Chris run faster, the initial two minutes or the last two minutes? Explain by referencing numbers and the shape of the graph.

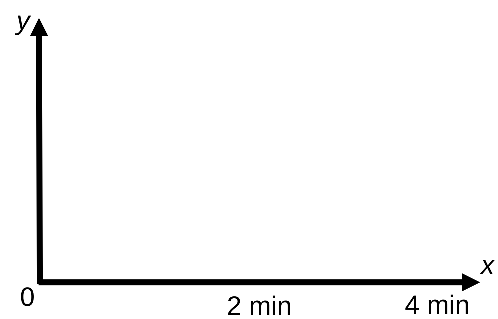
Chris went jogging in the park again on Wednesday.

6. Complete the table below, and sketch and label a graph.

Time Period	Number of Minutes	Yards Traveled	Average Rate of Speed
From 0 to 2 minutes	2	600	
From 2 to 4 minutes	2	200	
From 0 to 4 minutes			

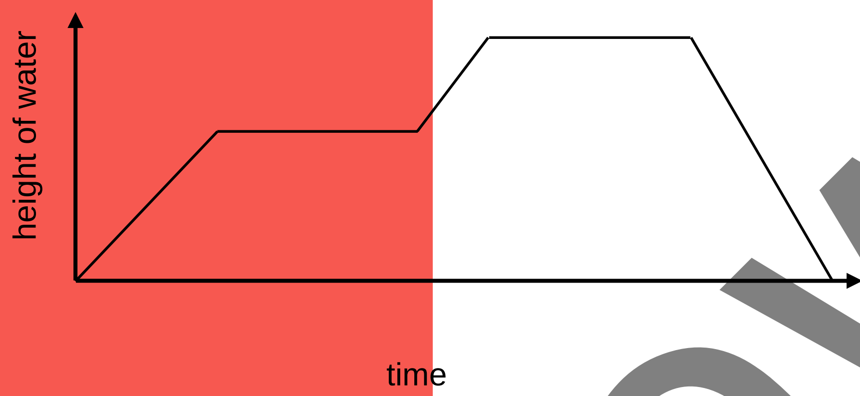
7. In what part of the jog did Chris run faster, the initial two minutes or the last two minutes? Explain by referencing numbers and the shape of the graph to the right.

Wednesday



THE BATH GRAPH

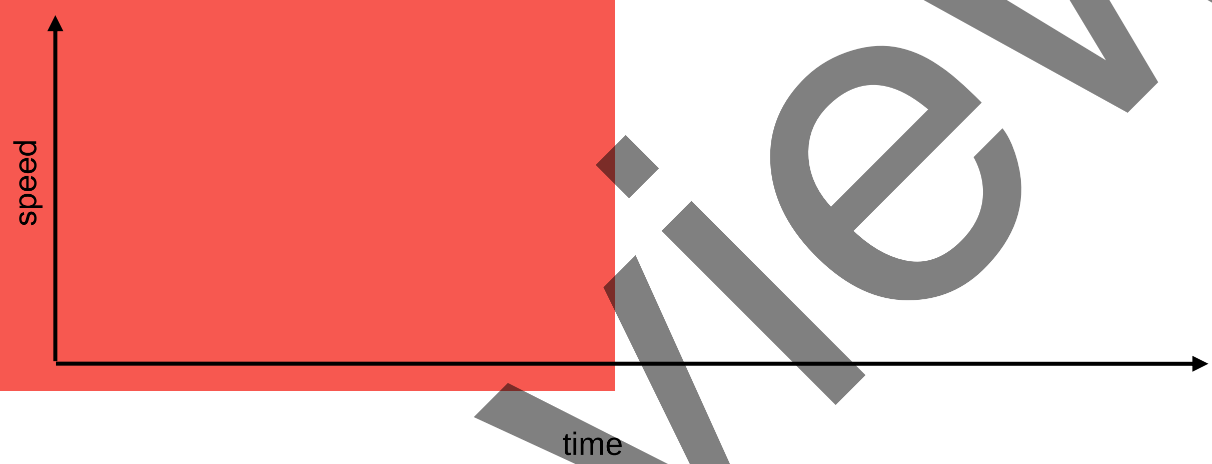
Write several sentences to explain what story this graph could be telling. Explain in the context of the story why this graph must represent a function.



THE ROLLERCOASTER

1. Draw a reasonable graph for a typical rollercoaster ride, based on the following information. Label each section by letter (each segment or curved portion of your graph) based upon the descriptions below, A – F. Note that the vertical axis represents speed, NOT HEIGHT.

   A. The rollercoaster starts slowly and gradually builds speed.
   B. It comes to a hill and climbs up slowly.
   C. It races downhill.
   D. It does a full loop.
   E. It continues at a constant speed.
   F. It gradually comes to a stop.



2. Write a few sentences to summarize your work and explain how you know you have drawn a good depiction of the rollercoaster ride described. Include in your explanation if this graph could represent a function and why.

REVIEW

$y = 3x + 4$

1. Your teacher will give you some cards.

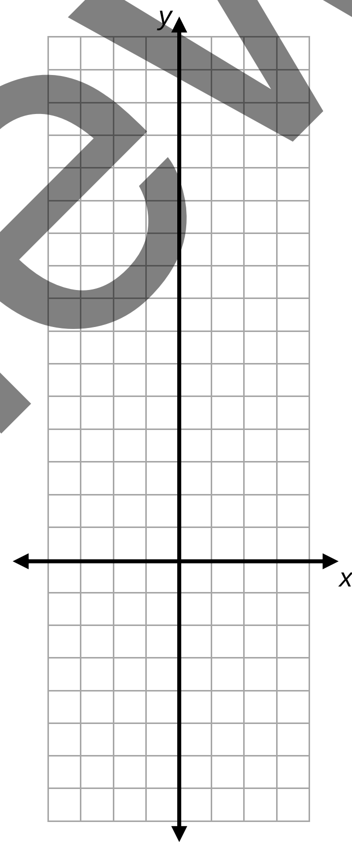
   Sort them into two piles and list the appropriate cards below.

   DOES model y = 3x + 4: ________________

   DOES NOT model y = 3x + 4: ________________

2. On the grid to the right, graph and label:

   - $y = 3x + 4$
   - a different equation that has the same y-intercept as $y = 3x + 4$
   - a different equation that is parallel to $y = 3x + 4$



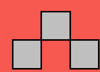
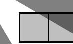
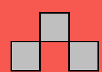
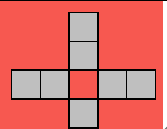
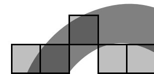
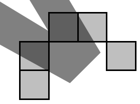
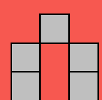
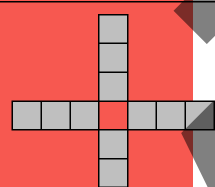
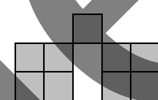
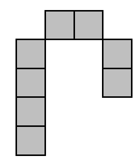
3. Use different representations (numbers, words, and pictures) to describe at least four features of the equation $y = 3x + 4$.

POSTER PROBLEMS: INTRODUCTION TO FUNCTIONS

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _______.
- Each group will have a different colored marker. Our group marker is ________.

Part 2: Do the problems on the posters by following your teacher's directions.

	Poster 1 (or 5)	**Poster 2 (or 6)**	**Poster 3 (or 7)**	**Poster 4 (or 8)**
Step 1				
Step 2				
Step 3				

A. Copy steps 1 – 3 onto the poster and draw step 4. Explain your step 4 in words.

B. Make a table, label it appropriately, and record values for steps 0 through 5. Make note of the initial value and the rate of increase.

C. Make a graph and label it appropriately.

D. Write an input-output rule that relates the total number of tiles to the step number, and then find the number of tiles for step 100.

Part 3: Return to your seats. Work with your group, and show all your work.

1. The ordered pair (1, 1) is on both Posters 1 and 3. What does it represent?

2. Find the step number that has exactly 155 tiles in poster 2.

3. Find the step number that has exactly 185 tiles in poster 4.

4. What does the *y*-intercept mean in each poster?

WHY DOESN'T IT BELONG?: INTRODUCTION TO FUNCTIONS

A. **Table:**

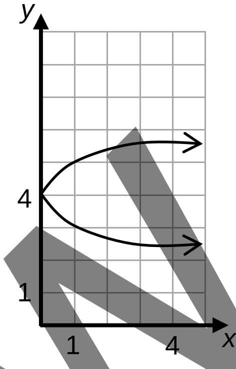
Input (x)	**Output (y)**
0	0
1	1
2	4
3	9
4	16

B. **Equation:** $y = -2x + 1$

C. **Context:**

Sal skateboards to and from work every day at an average rate of 6 miles per hour. He uses this information to keep track of how far he travels after any number of hours.

D. **Graph**

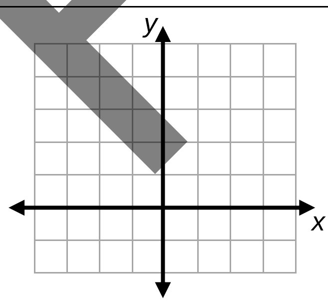


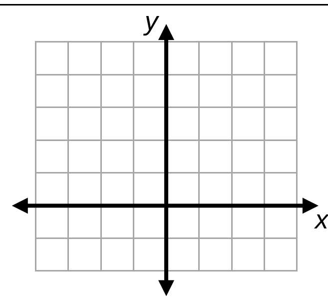
Avoid the obvious differences, such as "It's a graph."

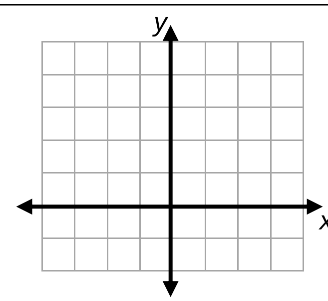
1. Choose one representation A – D above and explain why it does not belong with the others.	2. Now choose a different representation and explain why it does not belong.

Graph each of the described situations below, answer the questions, and explain.

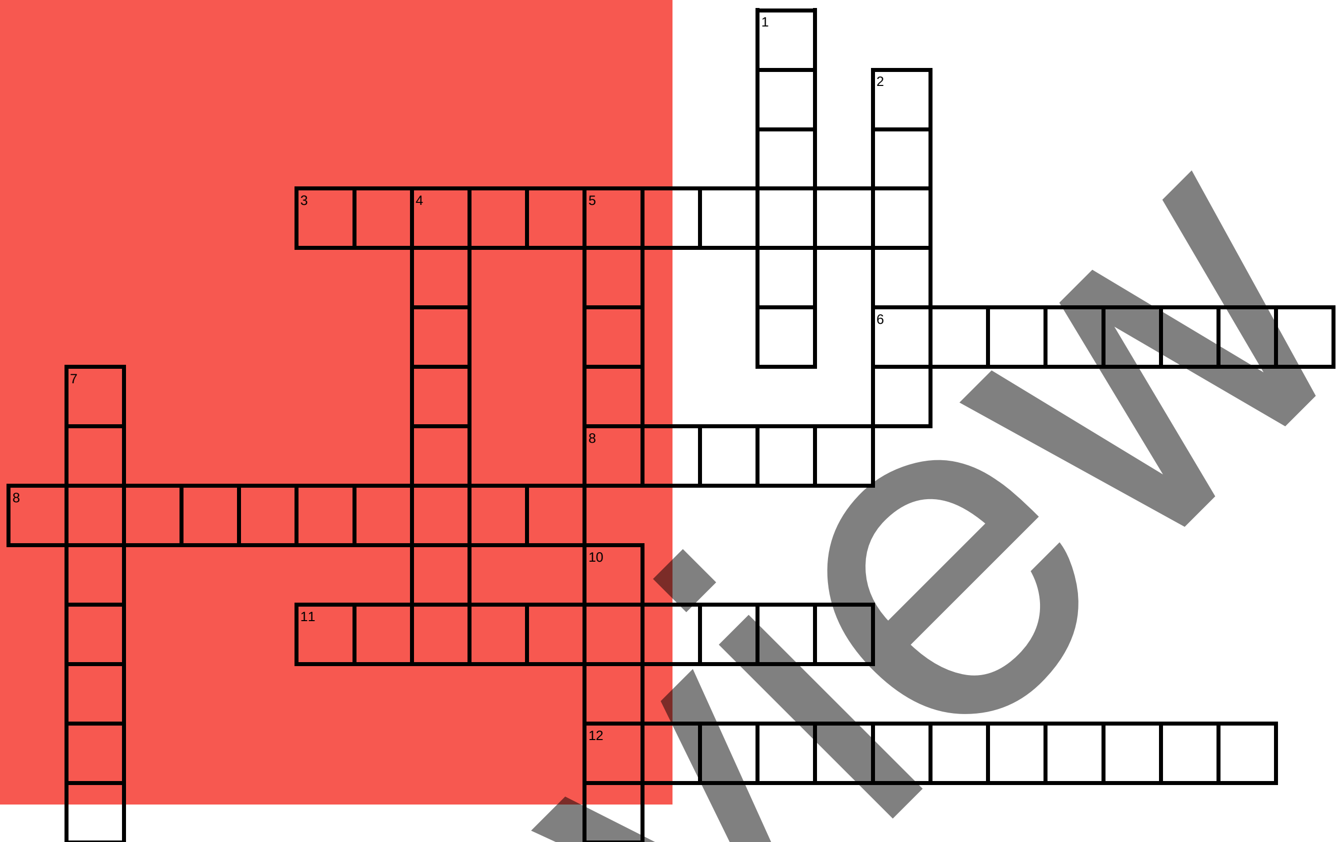
3. I am a linear function. Two of my points are located at (-2, 0) and (2, 4). My y-intercept is: _____ Am I increasing or decreasing? Explain.	4. I am a line. Two of my points are (-3, 4) and (-3, -1). Am I a function? Explain.	5. Graph and connect my three points in this order: (-3, 4), (1, 2), and (-3, 0). Am I a function? Explain.







VOCABULARY REVIEW



Across

3 In the equation $y = 4x + 2$, the rate of change is the ___ of x.

6 in a proportional relationship, the rate of change per unit (two words)

8 Entries in this organized chart represent ordered pairs of numbers.

9 The graph of a function that has a positive rate of change is ___.

11 number where the graph crosses the y-axis (hyphenated word)

12 relationship where each output is a constant multiple of the input

Down

1 function represented by a straight line

2 dependent variable, y-value

4 symbolic representation of a mathematical rule

5 independent variable, x-value

7 rule where every input has a unique output

10 visual representation of a mathematical rule (usually on a grid)

SPIRAL REVIEW

1. **Alge-Grid: What's the *a*?** Each clue gives the value of a corresponding cell. Use clues to find *a*, which has the same value in all cells. Once evaluated, the cells will contain the whole numbers 1 – 9, exactly once each.

The Alge-Grid

$\frac{3}{4}a + 0.5$	a^2	$(a + 1)^2$
$3a$	a^0	$a^2 - 1$
$a^2 + 1$	$a^2 + 2a - 1$	a^3

The Clues

Perfect number	Factor of all numbers	
		$(\text{Prime})^3$

2. Solve each equation below.

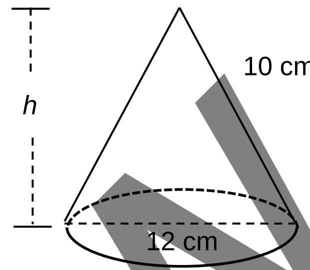
a. $6(g - 7) = 18$	b. $6 = \frac{x}{4} - 2$
c. $\frac{m + 6}{3} = -5$	d. $3(p - 5) + 2p = -55$

SPIRAL REVIEW

Continued

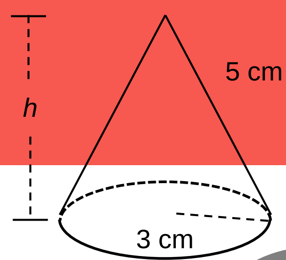
Figures not drawn to scale.

3. Find the height and the volume of the cone to the right if the diameter is 12 cm and the slanted edge is 10 cm. Round to the nearest tenth. Use $\pi = 3.14$.

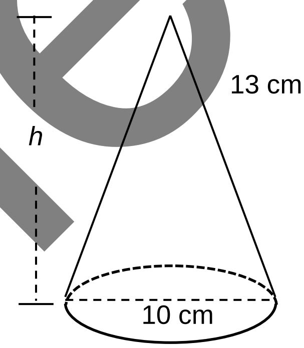


4. Find the height and the volume of each cone below. Round to the nearest tenth. Use $\pi = 3.14$.

a.

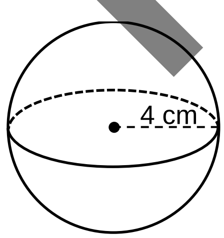


b.

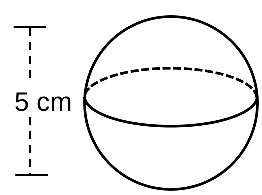


5. Find the volume of each sphere below. Use $\pi = 3.14$. Round to the nearest hundredth.

a.



b.



SPIRAL REVIEW

Continued

6. Solve and show your work.

A store is having a sale on bikes. The original cost of a bike that Epic wants to buy is $190.

a. If the bike is marked down to $110, what is the percent discount?	b. Sales tax in this location is 9.6%. What is the sales tax amount for the discounted bike?

c. Epic has $125 to spend on the bike. Will this be enough money? Explain.

7. Compute

a. $4^{-2} \bullet 4^{3}$	b. $5^{7} \bullet 5^{-7}$	c. $3^{-2} \bullet 3^{-3} \bullet 3^{8}$
d. $\left(2^{3}\right)^{4} \bullet \left(4^{2}\right)^{-3}$	e. $\frac{10^{5} \bullet 10^{8}}{10^{10}}$	f. $\sqrt[3]{27}$
g. $\sqrt[3]{-27}$	h. $\left(\sqrt{25}\right)\left(\sqrt{4}\right)$	i. $\left(\frac{\sqrt{36}}{\sqrt{16}}\right)$

REFLECTION

1. **Big Ideas**. Shade all circles that describe big ideas in this unit. Draw lines to show connections that you noticed.

Use transformational geometry to investigate congruence and similarity (8.G.A)

Extend applications of volume to cylinders, cones, and spheres (8.G.C)

Explore bivariate data (8.SP.A)

Complete the real number system (8.NS.A)

Solve linear equations in one variable and linear systems in two variables (8.EE.C)

Discover and apply properties of lines, angles, and triangles, including the Pythagorean theorem (8.G.B)

Create, analyze, and use linear functions in problem solving (8.EE.B, 8.F.AB)

Explore exponents and roots, and very large and very small quantities (8.EE.A)

Give an example from this unit of one of the connections above.

2. **Unit Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before.

3. **Mathematical Practice.** Explain how you used multiple representations to model and analyze relationships [SMP4]. Then circle one more SMP on the back of this packet that you think was addressed in this unit and be prepared to share an example.

4. **Making Connections.** Give examples of how a function helps us to explore changing quantities and predict what might happen.

STUDENT RESOURCES

<table>
<tr><th>Word or Phrase</th><th>Definition</th></tr>
<tr><td>coefficient</td><td>A <u>coefficient</u> is a number or constant factor in a term of an algebraic expression.

In the expression $3x + 5$, 3 is the coefficient of the term $3x$, and 5 is the constant term.</td></tr>
<tr><td>dependent variable</td><td>A <u>dependent variable</u> is a variable whose value is determined by the values of the independent variables. See <u>independent variable</u>.</td></tr>
<tr><td>function</td><td>A <u>function</u> is a rule that assigns to each input value exactly one output value.

For $y = 3x + 6$, any input value, say $x = 10$, has a unique output value, in this case $y = 36$.

For $y = x^2 + 1$, $x = 2$ has the unique output value $y = 2^2 + 1 = 5$.</td></tr>
<tr><td>graph of a function</td><td>The <u>graph of a function</u> is the set of all ordered pairs (x, y) where y is the output for the input value x. If x and y are real numbers, then we can represent the graph of a function as points in the coordinate plane.</td></tr>
<tr><td>independent variable</td><td>An <u>independent variable</u> is a variable whose value may be specified. Once specified, the values of the independent variables determine the values of the dependent variables.

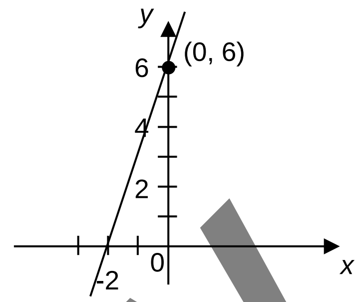
For the equation $y = 3x$, y is the dependent variable and x is the independent variable. We may assign a value to x. The value assigned to x determines the value of y.</td></tr>
<tr><td>input-output rule</td><td>An <u>input-output rule</u> for a sequence of values is a rule that establishes explicitly an output value for each given input value.

<table>
<tr><td>input value (x)</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>x</td></tr>
<tr><td>output value (y)</td><td>1.5</td><td>3</td><td>4.5</td><td>6</td><td>7.5</td><td>$1.5x$</td></tr>
</table>

In the table above, the input-output rule could be $y = 1.5x$. To get the output value, multiply the input value by 1.5. If $x = 100$, then $y = 1.5(100) = 150$.</td></tr>
<tr><td>proportional</td><td>Two variables are <u>proportional</u> if the values of one are the same constant multiple of the corresponding values of the other. The variables are said to be in a <u>proportional relationship</u>, and the constant is referred to as the <u>constant of proportionality</u>.

If Wrigley eats 3 cups of kibble each day, then the number of cups of kibble is proportional to the number of days. If x is the number of days, and y is the number of cups of kibble, then $y = 3x$. The constant of proportionality is 3.</td></tr>
<tr><td>unit rate</td><td>The <u>unit rate</u> associated with a ratio $a : b$ of two quantities a and b, $b \neq 0$, is the number $\frac{a}{b}$, to which units may be attached. This is sometimes referred to as the <u>value of the ratio</u>.

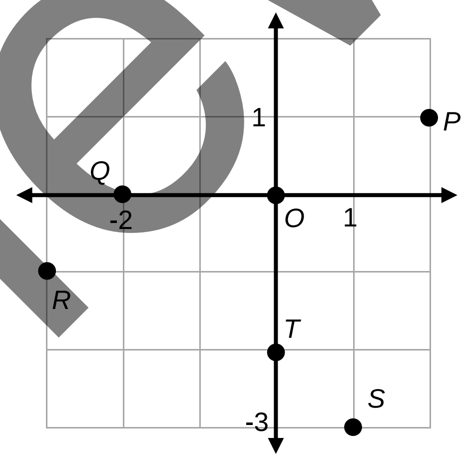
The ratio of 40 miles for every 5 hours has a unit rate of 8 miles per hour.</td></tr>
</table>

Word or Phrase	Definition
y-intercept	The <u>*y*-intercept</u> of a line is the *y*-coordinate of the point at which the line crosses the *y*-axis. It is the value of *y* that corresponds to $x = 0$. The *y*-intercept of the line $y = 3x + 6$ is 6. If $x = 0$, then $y = 6$. 

The Coordinate Plane

A coordinate plane is determined by a horizontal number line (the *x*-axis) and a vertical number line (the *y*-axis) intersecting at the zero on each line. The point of intersection (0, 0) of the two lines is called the origin. Points are located using ordered pairs (*x*, *y*).

- The first number (*x*-coordinate) indicates how far the point is to the right or left of the *y*-axis.
- The second number (*y*-coordinate) indicates how far the point is above or below the *x*-axis.



Point, coordinates, and interpretation

O (0, 0) → This is the intersection of the axes (origin).

P (2, 1) → start at the origin, move 2 units right, then 1 unit up

R (-3, -1) → start at the origin, move 3 units left, then 1 unit down

S (1, -3) → start at the origin, 1 unit right, then 3 units down

Q (-2, 0) → start at the origin, move 2 units left, then 0 units up or down

T (0, -2) → start at the origin, 0 units right or left, then 2 units down

Functions

Some ways to represent rules in mathematics are input-output tables, mapping diagrams, ordered pairs, equations, and graphs.

Examples that are Functions

Input-Output Table

x **input**	***y*** **output**
1	1
3	3
5	5
7	7
9	9

This table lists input values with unique output values.

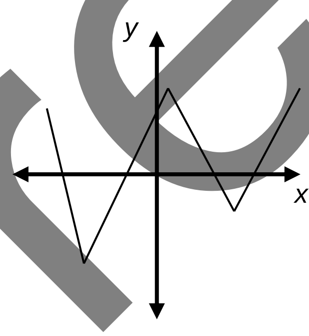
Ordered Pairs

(0, 2), (1, -2), (2, 2), (3, -2)

In this set of ordered pairs, each input value is assigned to a unique output value. Note that different input values may be assigned the same output value. In this example, both 1 and 3 are assigned the output value -2.

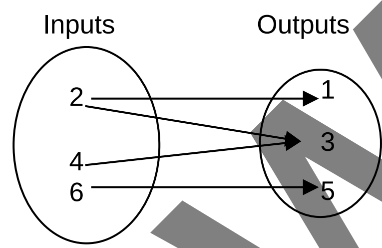
Graph

This graph represents a function because every vertical line through it intersects at most one point of the graph. In other words, each possible *x*-value corresponds to a unique *y*-value.



Examples that are NOT Functions

Mapping Diagram



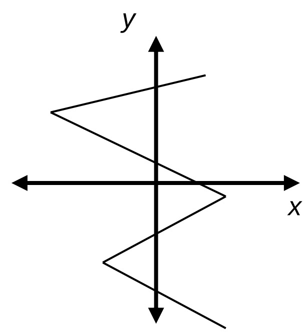
This mapping diagram is not a function. It is not permissible for the same input value (in this case 2) to be assigned two different output values. However, all other input-output mappings above are fine.

Equation (with Ordered Pairs)

Consider the set of pairs (x, y) that satisfy $x = y^2$, such as (0, 0), (25, 5), and (25, -5). Since the input value, x = 25, corresponds to two different output values (y = 5 and y = -5), the y-values are not a function of the x values.

Graph

This graph does not represent a function because some vertical lines (for example, the *y*-axis) intersect the graph in more than one point. In other words, some *x*-values correspond to more than one *y*-value.



Using Multiple Representations to Describe Linear Functions

Here are four representations commonly used to approach a math problem:

- Numbers (numerical approach, as by making a table)
- Pictures (visual approach, as with a picture or graph)
- Symbols (approaching the problem using algebraic symbols)
- Words (verbalizing a solution, orally or in writing)

Each approach may lead to a valid solution. Collectively they should lead to a complete and comprehensive solution, one that is readily accessible to more people and that provides more insight.

Example 1: Describe this pattern of hexagons using numbers, pictures, words, and symbols.

Numbers

Step #	**number of segments**	**Breaking apart numbers sometimes helps you see an input-output rule.**	
1	6	6	= 6 + (0)5
2	11	6 + 5	= 6 + (1)5
3	16	6 + 5 + 5	= 6 + (2)5
4	21	6 + 5 + 5 + 5	= 6 + (3)5
5	26	6 + 5 + 5 + 5 + 5	= 6 + (4)5
n	5*n* + 1	5*n* + 1	= 6 + (*n* – 1)5

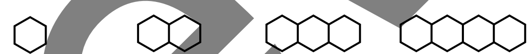
Words

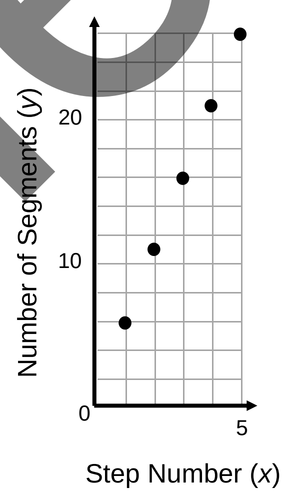
One way to describe the hexagonal pattern is to start with 6 segments and add 5 more segments at each subsequent step. Notice that the number of 5's added at each step is equal to 1 less than the step number.

Symbols

A rule for finding the number of segments at step n is $6 + (n - 1)5$, which can be simplified to $5n + 1$.

Pictures





Note: we consider a graph to be a picture.

Using Multiple Representations to Describe Linear Functions (Continued)

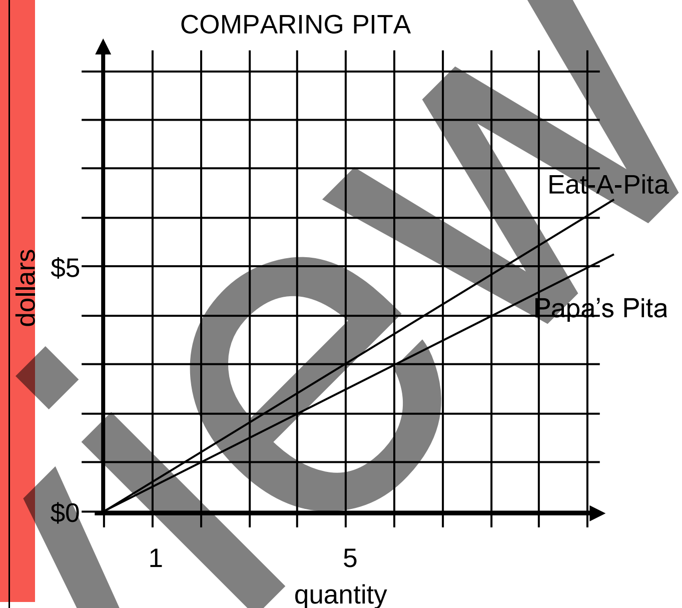
Example 2: At Papa’s Pitas, 2 pitas cost \$1.00. At Eat-A-Pita, 5 pitas cost \$3.00. Assuming a proportional relationship between the number of pitas and their cost, use multiple representations to explore which store offers the better buy for pitas.

Numbers (make a table)

PAPA’S PITAS	
# of pitas (x)	cost (y)
2	\$1.00
4	\$2.00
6	\$3.00
8	\$4.00
10	\$5.00

EAT-A-PITA	
# of pitas (x)	cost (y)
5	\$3.00
10	\$6.00
15	\$9.00
20	\$12.00
25	\$15.00

Pictures (make a graph)



Words (write sentences)

Based on the table, Papa’s Pitas is the better buy.

At Papa’s Pitas, you get 6 pitas for \$3.00. This means the unit price (cost for one pita) is \$0.50.

At Eat-A-Pita you only get 5 pitas for \$3.00. This means the unit price (cost for one pita) is \$0.60.

Symbols (write equations to relate the number of pitas to cost)

PAPA’S PITAS $y = 0.5x$

EAT-A-PITA $y = 0.6x$

Notice that \$0.50 is the cost of one pita at Papa’s Pita. This corresponds to the point (1, 0.5) on the graph.

Notice that \$0.60 is the cost of one pita at Eat-A-Pita. This corresponds to the point (1, 0.6) on the graph.

The equations above are both in the form $y = mx$. This equation form represents a proportional relationship because y is a constant multiple of x. Graphs of equations in this form are always lines going through the origin. They will be explored more in the next unit and contrasted with equations in the form $y = mx + b$.

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
8.EE.B	**Understand the connections between proportional relationships, lines, and linear equations.**
8.EE.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
8.F.A	**Define, evaluate, and compare functions.**
8.F.1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
8.F.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
8.F.3	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*
8.F.B	**Use functions to model relationships between quantities.**
8.F.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
8.F.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

STANDARDS FOR MATHEMATICAL PRACTICE	
SMP1	Make sense of problems and persevere in solving them.
SMP2	Reason abstractly and quantitatively.
SMP3	Construct viable arguments and critique the reasoning of others.
SMP4	Model with mathematics.
SMP5	Use appropriate tools strategically.
SMP6	Attend to precision.
SMP7	Look for and make use of structure.
SMP8	Look for and express regularity in repeated reasoning.

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