Name ______________________________ Period __________ Date ______________

UNIT 2
STUDENT PACKET

Math Links
GRADE 8

REAL NUMBERS AND THE PYTHAGOREAN THEOREM

Parent (or Guardian) signature ______________________________________

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See Student Resources for definitions and examples.

square of a number perfect square (or square number)	square root radical expression (radical sign, radicand)
Pythagorean theorem hypotenuse legs	repeating decimal terminating decimal
The Real Number System (natural numbers, whole numbers, integers, rational numbers, irrational numbers, real numbers)	

OPENING PROBLEM: A RECTANGLE PARADOX

[SMP 1, 3]

Follow your teacher's directions. Use a ruler for drawings. Each small square is one square unit of area.

(1)

(2)

(3)

SQUARES AND SQUARE ROOTS

We will find perfect squares and square roots of whole numbers. We will approximate and compare square roots of numbers that are not perfect squares.

[8.EE.2, 8.NS.2; SMP3, 4, 5]

GETTING STARTED

1. Each small square below represents one square unit of area. Let s represent side length in linear units and A represent area in square units. Draw squares for $s = 1, 2, 3, 4, 5$, and 6, and record the value for A for each.

2. Why do you think the word "squared" is used to refer to a number to the second power?

3. Fill in the table below of perfect squares.

1^2 = ______	2^2 = ______	3^2 = ______	4^2 = ______	5^2 = ______
6^2 = ______	7^2 = ______	8^2 = ______	9^2 = ______	10^2 = ______
11^2 = ______	12^2 = ______	13^2 = ______	14^2 = ______	15^2 = ______
16^2 = ______	17^2 = ______	18^2 = ______	19^2 = ______	20^2 = ______
21^2 = ______	22^2 = ______	23^2 = ______	24^2 = ______	25^2 = ______

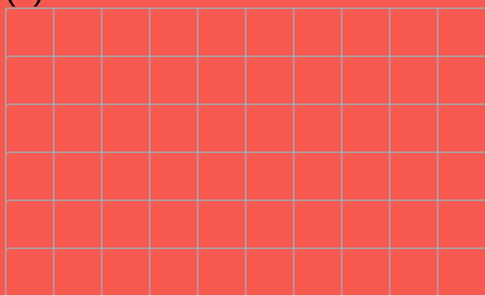
4. Record the meanings of <u>square of a number</u> and <u>perfect square</u> (or <u>square number</u>) in **My Word Bank**.

A RADICAL INVESTIGATION

Follow your teacher's directions. Each small square in the grid below represents one square unit of area.

(1)

(2)



(3)

(4)

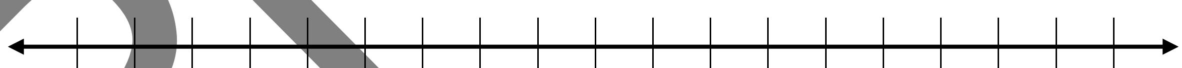
a. _____ is between _____ and _____, but is closer to _____.

b. _____ is between _____ and _____, but is closer to _____.

c. _____ is between _____ and _____, but is closer to _____.

d. _____ is between _____ and _____, but is closer to _____.

(5)



Whole number part:	Fractional part:	Calculator check (nearest hundredth)

PRACTICE 1

1. Record the meanings of <u>square root</u> and <u>radical expression</u> in **My Word Bank**.

2. Write the whole number that is equivalent to each radical expression. If not possible, write "no whole number." Use the table on **Getting Started** as a reference.

a. $\sqrt{121}$	b. $\sqrt{225}$	c. $\sqrt{275}$	d. $\sqrt{420}$	e. $\sqrt{625}$

Estimate each radical expression below using linear interpolation.

3. Square root form: ______ < $\sqrt{5}$ < ______; $\sqrt{5}$ is closer to ______

   Whole numbers: ______ < $\sqrt{5}$ < ______; $\sqrt{5}$ is closer to ______

   Estimate the fractional part of $\sqrt{5}$ as a fraction.

   Calculator checks → fraction form: ___________; square root form: ___________

   A square with area = 5 units^2 has side length that is approximately _____units.

4. Square root form: ______ < $\sqrt{14}$ < ______; $\sqrt{14}$ is closer to ______

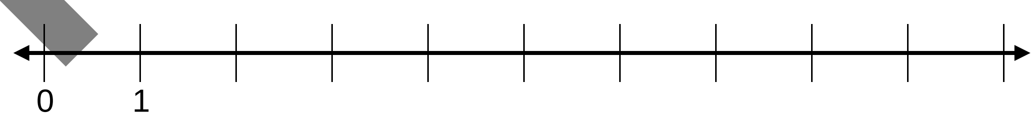
   Whole numbers: ______ < $\sqrt{14}$ < ______; $\sqrt{14}$ is closer to ______

   Estimate the fractional part of $\sqrt{14}$ as a fraction.

   Calculator checks → fraction form: ___________; square root form: ___________

   A square with area = 14 units^2 has side length that is approximately _____units.

5. Locate estimates from problems 3 and 4 on the number line below.



PRACTICE 2

1. Alicia is working with her group and says to them, “There is no square root of 40.” Is Alicia’s statement precise?

2. Between which two consecutive integers is $\sqrt{40}$?

Use fractions and decimals to approximate each square root in the table below. Use the table on the **Getting Started** page for reference.

Number in square root form	**Between consecutive square roots of perfect squares and their integer equivalents**		**About** (fraction and decimal)		**Calculator check**
3. $\sqrt{11}$	$\sqrt{9}$ and ______	3 and ______	$3\frac{\square}{\square}$	3.______	
4. $\sqrt{20}$					
5. $\sqrt{78}$					
6. $\sqrt{130}$					
7. $\sqrt{220}$					

For their house, Greg and Lauren bought a square rug with an area of 20 square feet. Explain all answers below.

8. If the dimensions of their front entry is 5 feet by 5 feet, will the rug fit?	9. Greg decides he would rather put the rug in front of the kitchen sink, which is a space 4 feet wide. Will the rug fit in that space?
10. Lauren thinks the rug will look great in the hallway, which is $4\frac{1}{2}$ feet wide. Will the rug fit?	11. Greg measured the hallway again, and discovered it is actually 4 feet 4 inches wide. Will the rug fit?

PYTHAGOREAN THEOREM

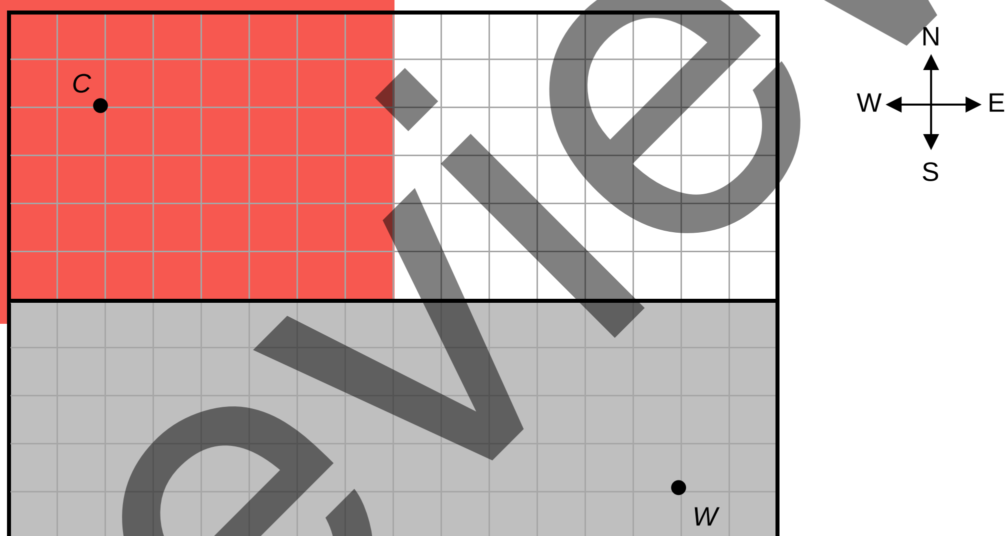
We will explore the relationship among the side lengths of right triangles and then understand a proof of the Pythagorean theorem. Then we will use this theorem to solve problems.

[8.G.6, 8.G.7, 8.G.8; SMP1, 2, 3, 4, 6]

GETTING STARTED

MAXIE AND MINNIE: Maxie and Minnie are at their campsite (point *C*) and want to hike to Wilshire Waterfall (point *W*). Maxie wants to hike due south and then due east in the shortest way possible. Minnie wants to do the same, but head east first and then south. On the map below, the unshaded portion represents smooth, even terrain, and the shaded portion represents rougher terrain with some rocks.

Each small square represents $\frac{1}{4}$ mi × $\frac{1}{4}$ mi



1. Are the distances traveled by Maxie and Minnie the same or different? Explain.

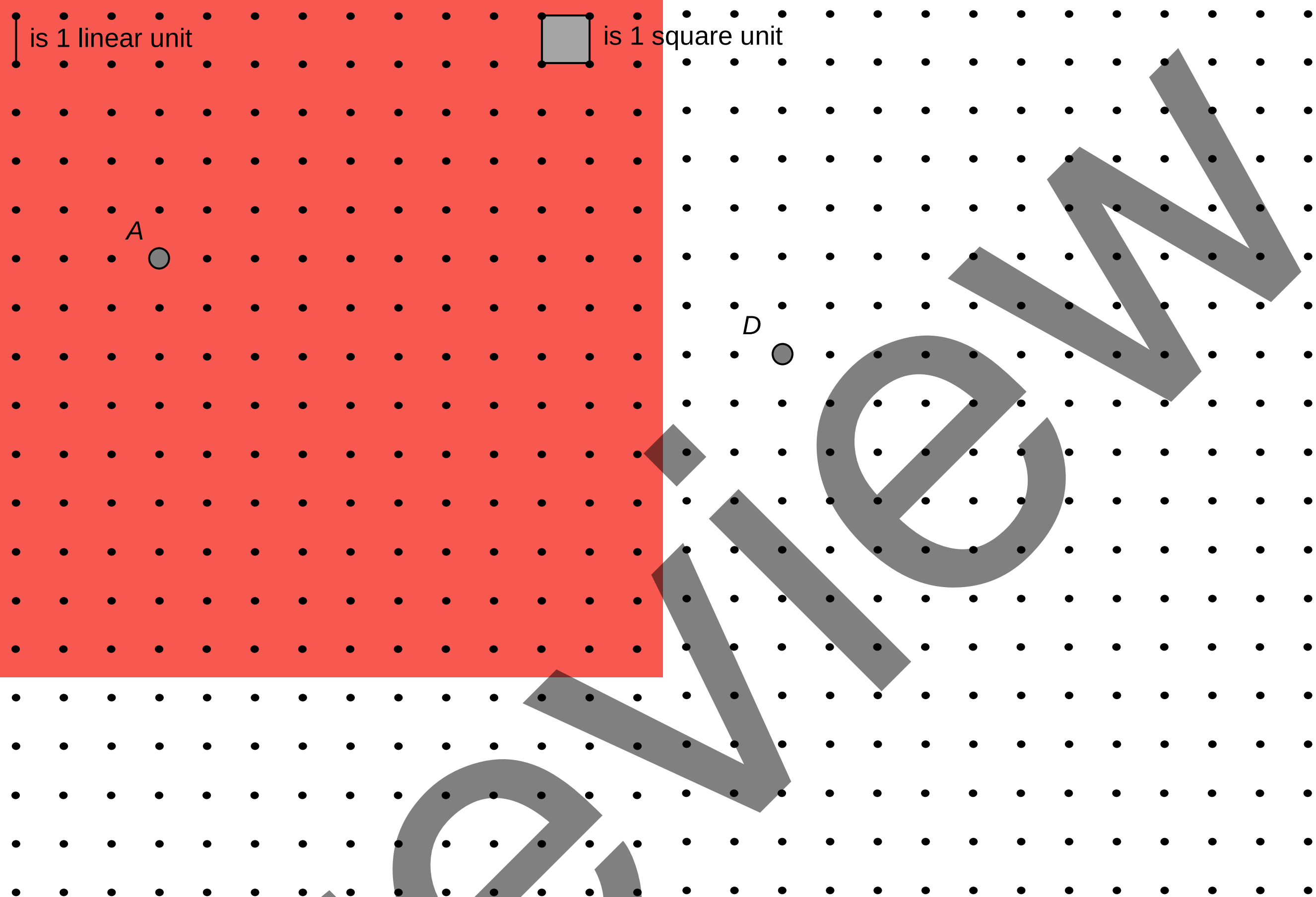
2. If they can both hike at constant rates of 4 miles per hour on the smooth terrain and 2 miles per hour on the rough terrain, whose route is faster? Explain.

A RIGHT TRIANGLE INVESTIGATION

Follow your teacher's directions for (1) – (2).

(1)

(2)

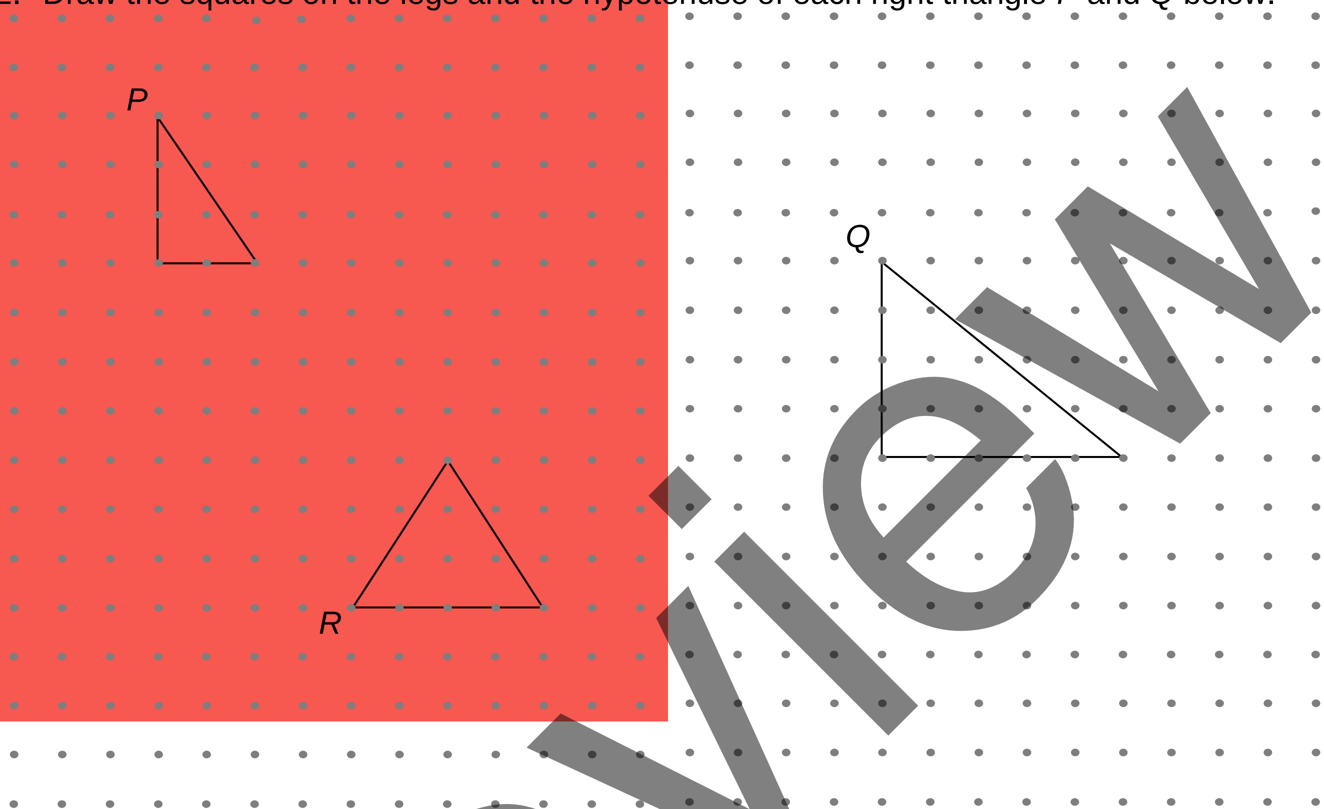


	(1) Δ*ABC*	(2) Δ*DEF*
Length of the shorter leg		
Length of the longer leg		
Area of the square on the shorter leg		
Area of the square on the longer leg		
Area of the square on the hypotenuse		
Length of the hypotenuse		

3. Write a conjecture about the relationship between the area of the square on the hypotenuse and the area of the squares on the legs of a right triangle.

PRACTICE 3

1. Record the meanings of <u>legs</u> and <u>hypotenuse</u> in **My Word Bank**.
2. Draw the squares on the legs and the hypotenuse of each right triangle *P* and *Q* below.



3. Find the area of each square on the triangles' legs and hypotenuse and fill in the blanks for the area equations in the table below. Find the length of the legs and the hypotenuse of each triangle and fill in the blanks for the side length equations.

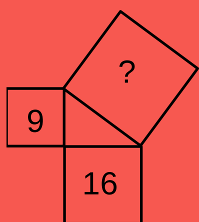
Triangle *P*	**Triangle *Q***
Area equation: ______ + ______ = ______	Area equation: ______ + ______ = ______
Side length equation: $(\_\_\_\_\_)^2 + (\_\_\_\_\_)^2 = (\_\_\_\_\_)^2$	Side length equation: $(\_\_\_\_\_)^2 + (\_\_\_\_\_)^2 = (\_\_\_\_\_)^2$

PRACTICE 3
Continued

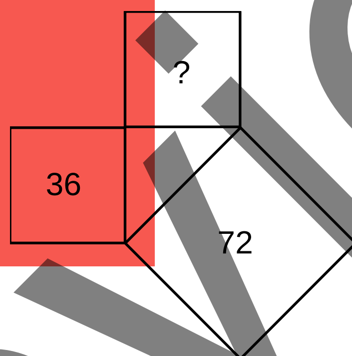
4. Draw squares on the sides of triangle *R*, find the areas of the squares, and demonstrate that the relationship in problems 2 and 3 does NOT hold for this triangle (which is NOT a right triangle).

5. For each right triangle below, write the missing square area (in square units) and side lengths (in units, listed from shortest side to longest). Leave numbers in exact square root form if not a whole number.

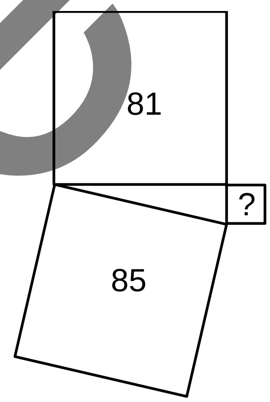
a.



b.



c.



a. missing area: ________________ side lengths: ________________

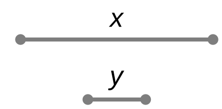
b. missing area: ________________ side lengths: ________________

c. missing area: ________________ side lengths: ________________

6. Using areas, explain the relationship between the legs and hypotenuse of a right triangle.

LENGTHS AND AREAS

To the right are line segments of lengths x and y.



Your teacher will give you some cards.

Match the cards to the expressions below and complete the table below.

Given Expression	Matching card(s)	Linear or area relationship	Simplified expression
1. $x + x$			
2. $x \bullet x$			
3. $x + y$			
4. $xy + xy$			
5. $\frac{1}{2}x + \frac{1}{2}x$			
6. $\frac{1}{2}x \bullet \frac{1}{2}x$			
7. $\frac{1}{2}xy$			
8. $\frac{xy}{2}$			
9. $\frac{1}{2}xy + \frac{1}{2}xy$			

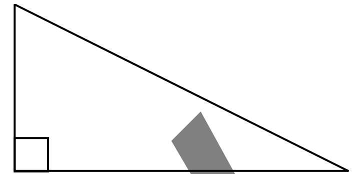
Simplify each expression by combining like terms.

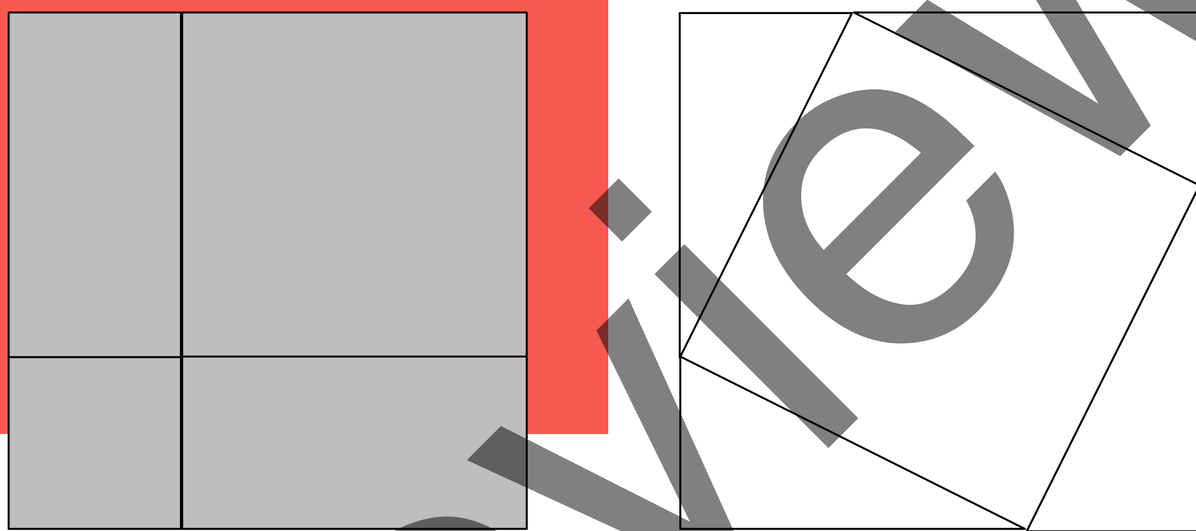
10. $\frac{1}{4}y + 2.5y$	11. $-5(x - 4) + 5$	12. $2x - 4y + 5x - 6y$
13. $3(x - 2) + 4\left(\frac{1}{2} - x\right)$	14. $\frac{x+1}{4} + \frac{1}{8}$	15. $-(-x - 2)$

A FAMOUS THEOREM

Follow your teacher's directions for (1) – (3).

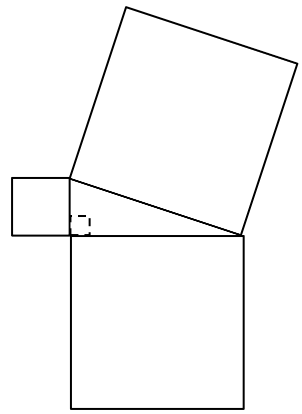
(1)





(2)

(3)



4. Record the meanings of <u>Pythagorean theorem</u> in **My Word Bank**.

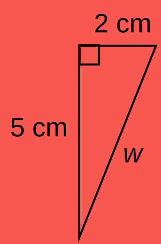
PRACTICE 4

On this page, if the result is not equivalent to a whole number, write it in both square root form and as a decimal rounded to the tenths place.

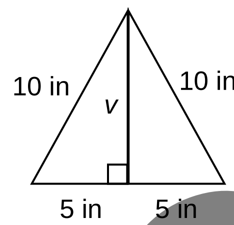
1. A common slogan used for the Pythagorean theorem is _____ + _____ = _____.

Find the missing length in each triangle using the formula above.

2.



3.



Sketch and label a diagram for each description below. Then find the missing length.

4. Find the length of the diagonal, *d*, of a square whose side is 10 cm long.

5. Find the height, *h*, of an isosceles triangle with equal sides that each measure 12 inches and a base that is 18 inches long.

6. To get from home to work every day, Samos drives about 7 miles south on Avenue A, and then drives east on Avenue B. He knows that the straight-line distance from his home to his place of work is about 20 miles. How many miles does he drive east on Avenue B?

   If Samos could drive in a straight line, "as the crow flies," about how much shorter would his daily commute be?

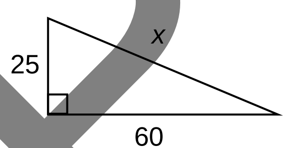
PRACTICE 5: EXTEND YOUR THINKING

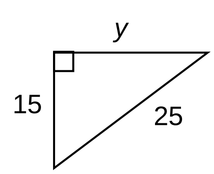
1. The first right triangle we investigated on dot paper had sides equal to 3, 4, and 5 units of length. It is in Set 1 in the table below. Calculate the missing lengths for Set 1 and Set 2.

	Right Triangle Side Lengths (in linear units)			work space
	Leg 1	**Leg 2**	**Hypotenuse**	
Set 1	3	4	5	
Set 1	6	8		
Set 1	9		15	
Set 1		16	20	
Set 2	5	12		
Set 2	10		26	
Set 2		36	39	
	20	48		

2. What patterns do you notice?

3. When the three sides of a right triangle all have whole number lengths, we refer to these numbers as "Pythagorean triples." Find the missing side lengths below based upon the patterns in the sets above. (Hint: one triangle below relates to Set 1, the other to Set 2.)



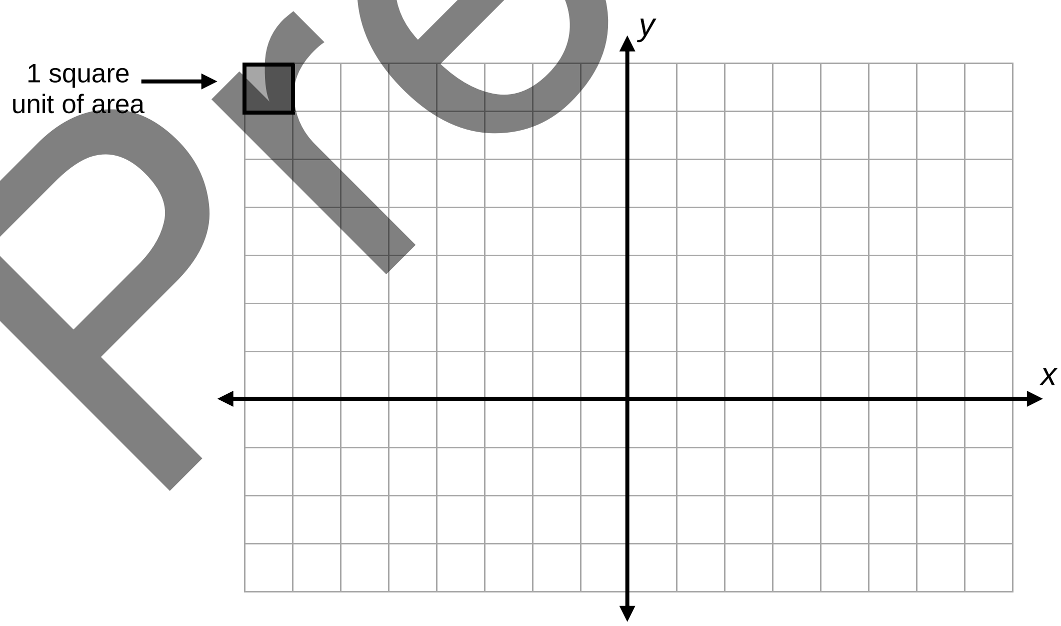


4. Challenge: find another Pythagorean triple that would not belong with either Set 1 or Set 2.

FINDING DISTANCES

Graph each figure below and use the grid at the bottom of the page to help you find the given lengths. If a length is not equivalent to a whole number, write it in both square root form and as a decimal approximation.

1. ΔABC vertices: A (2,6), B (2,2), C (5,2). Find $\|AB\|$: ____________ $\|BC\|$: ____________ $\|AC\|$: ____________	2. ΔDEF vertices: D (-2,6), E (-6,6), F (-6,4) Find $\|ED\|$: ____________ $\|EF\|$: ____________ $\|FD\|$: ____________
3. $\square JKLM$ vertices: J (-3,0), K (7,0), L (7,-3), M (-3,-3), Find $\|JL\|$: ____________	4. Find $\|BD\|$: ____________ $\|CK\|$: ____________

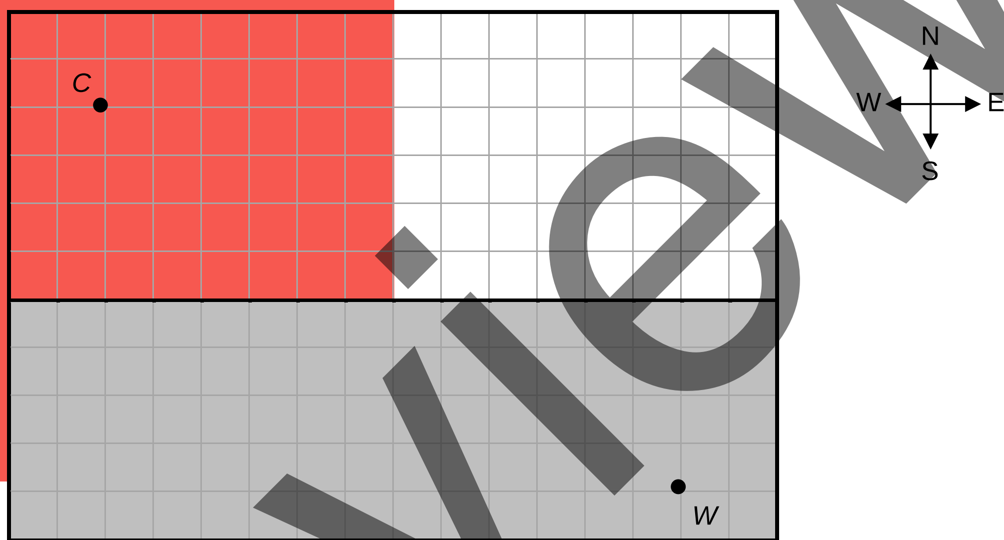


REVISITING MAXIE AND MINNIE

Refer back to **Getting Started** to complete this page. Maxie and Minnie are at their campsite (point *C*) and want to hike to Wilshire Waterfall (point *W*). Recall that the unshaded portion represents smooth terrain (they can hike it at 4 mi/hr), and the shaded portion represents rougher terrain (they can hike it at 2 mi/hr). After making the hike once each, they both think that they could have done it in less time.

Find at least two different pathways, showing clearly that they take less time than both of the ways done previously.

Each small square represents $\frac{1}{4}$ mi × $\frac{1}{4}$ mi



REVISITING A RECTANGLE PARADOX

Follow your teacher's directions. Revisit the opening problem, **A Rectangle Paradox**, and explain why the first rectangle cannot be reconfigured to form the second rectangle. Use a ruler for drawings. Each small square is one square unit of area.

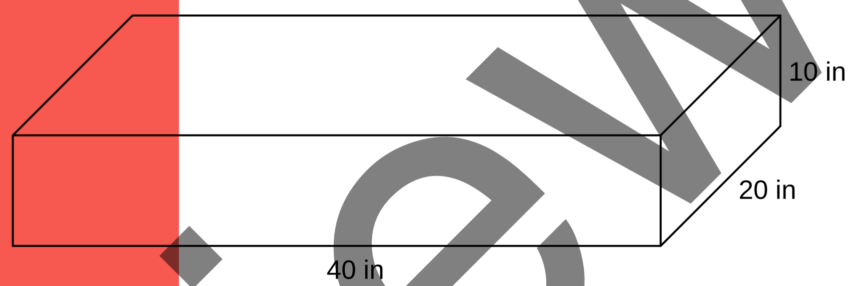
(1)

(2)

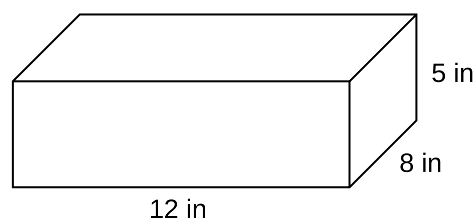
THE CLUB AND THE BOX

1. Dorie is an avid golfer. Lorie has recently taken up the sport and Dorie wants to send Lorie one of her old golf clubs. Dorie's club length is 45 inches, and she needs to figure out the smallest box she can buy to mail it to Lorie so that postage is not too high. Dorie finds a box (pictured below), but thinks it's too small. She tells Lorie the dimensions, and after making some calculations, Lorie thinks the club will fit.

   Draw on the figure, show calculations, and write a sentence to support either Dorie's claim or Lorie's claim.



2. Find the length of the longest stick that could fit inside the shoe box pictured here.



EXTEND YOUR THINKING: ANOTHER PROOF

Pythagorean theorem	Converse of the Pythagorean theorem
For a right triangle, the sum of the squares of the lengths of the two legs is equal to the square of the length of the hypotenuse.	If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle.

1. Answer each of the following questions, which explains why the converse of the Pythagorean theorem is true.

   a. How many triangles can be drawn with side lengths 3, 4, and 5?

   b. Is there a right triangle with side lengths 3, 4, and 5?

   c. Two given facts about a particular triangle (called triangle T):
      - T has side lengths a, b, and c.
      - For T, we know that $a^2 + b^2 = c^2$.

      Must T be a right triangle?

2. Solve each problem below. Then state whether it requires the use of the Pythagorean theorem or its converse.

a. How long is the hypotenuse of a right triangle with legs equal to 4 cm and 5 cm?	b. Why is a triangle with side lengths equal to 4 cm, 6 cm, and 9 cm not a right triangle?
c. How long are the legs of an isosceles right triangle with hypotenuse equal to 10 cm?	d. How do we know that a triangle with side lengths equal to 5 cm, 7 cm and $\sqrt{74}$ cm is a right triangle?

COMPLETING THE REAL NUMBER SYSTEM

We will learn that rational numbers and irrational numbers make up the real number system. We will change rational numbers that are repeating decimals to fractions. We will revisit two familiar irrational numbers, π and $\sqrt{2}$.

[8.NS.1, 8.NS.2; SMP3, 6, 7, 8]

GETTING STARTED

Compute.

Multiply each number by	1. 0.456	2. 0.0808	3. 0.4444
10			
100			
1,000			

4. Record the meanings of <u>natural numbers</u>, <u>whole numbers</u>, <u>integers</u>, and <u>rational numbers</u> in **My Word Bank**.

5. Numbers like 19, -5, and 1.4 are NOT quotients of integers, but they ARE rational numbers. Why?

6. Write each of the following numbers in ALL places below where it belongs.

$6 \quad -12 \quad \frac{5}{1} \quad -\frac{3}{7} \quad 0 \quad 2.7 \quad \pi \quad \frac{-24}{-3} \quad -9.1$

natural numbers	whole numbers	integers	rational numbers

A RATIONAL NUMBERS INVESTIGATION

Continue each pattern below, and describe it in words. Use a calculator to check as needed.

1. Ninths as decimals:

$\frac{0}{9} = 0$
$\frac{1}{9} = 0.1111\ldots = 0.\overline{1}$
$\frac{2}{9} = 0.2222\ldots = 0.\overline{2}$
$\frac{3}{9}$
$\frac{4}{9}$
$\frac{5}{9}$
$\frac{6}{9}$
$\frac{7}{9}$
$\frac{8}{9}$
$\frac{9}{9}$
Pattern description:

2. Elevenths as decimals:

$\frac{0}{11} = 0$
$\frac{1}{11} = 0.0909\ldots =$
$\frac{2}{11} = 0.1818\ldots =$
$\frac{3}{11}$
$\frac{4}{11}$
$\frac{5}{11}$
$\frac{6}{11}$
$\frac{7}{11}$
$\frac{8}{11}$
$\frac{9}{11}$
$\frac{10}{11}$
$\frac{11}{11}$
Pattern description:

3. Sevenths as decimals:

$\frac{0}{7} = 0$
$\frac{1}{7} = 0.\overline{142857}\ldots$
$\frac{2}{7} = 0.\overline{285714}\ldots$
$\frac{3}{7}$
$\frac{4}{7}$
$\frac{5}{7}$
$\frac{6}{7}$
$\frac{7}{7}$
Pattern description:

4. The fraction $\frac{a}{b}$ poses the division problem $a \div b$. Reason why any fraction that is converted to a decimal by division must have a pattern for which a repeat bar can be used. (Recall that $\frac{1}{2}$, though terminating, *can be* written as $0.5 = 0.50000\ldots = 0.5\overline{0}$.

HOW CAN 0.9999… = 1?

Follow your teacher's directions for (1) – (8).

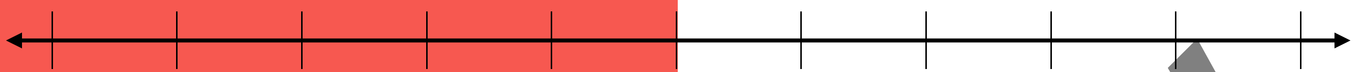
(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

9. Write the meanings of <u>terminating decimal</u> and <u>repeating decimal</u> in **My Word Bank**.

THE REAL NUMBER SYSTEM

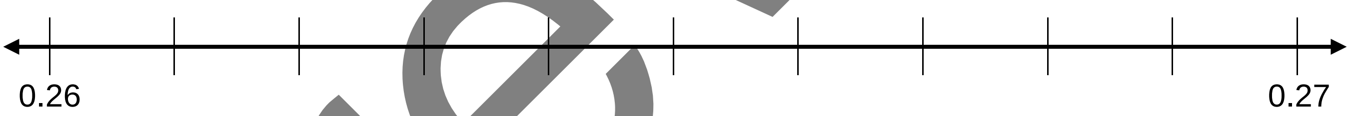
Follow your teacher's directions for (1) – (4).

(1) – (4)



	Given number (write at least 10 decimal places)	**Write with a repeat bar** (if possible)	**Does the decimal terminate?**	**Write as a fraction** (if possible)
S				
T				
E				
M				

5. Number the tick marks on the line below. Continue the decimal pattern for each given number. Complete the table. Graph the points.



	Given number (continue each pattern)	**Write with a repeat bar** (if possible)	**Does the decimal terminate?**	**Write as a fraction** (if possible)
Q	0.2626 ___ ___ ___ ___ ___ …			
U	0.2666 ___ ___ ___ ___ ___ …			
A	0.26000 ___ ___ ___ ___ ___ …			
D	0.262262226 ___ ___ ___ ___ ___ …			

6. Record the meanings of <u>irrational numbers</u> and <u>real numbers</u> in **My Word Bank**.

PRACTICE 6

1. Write a made-up an irrational number below (we will call it *Y*). Create *Y* so that it's between 3.14 and 3.15 and its pattern is like numbers *M* and *D* on the previous page. Write about 10 digits of the number like so that the pattern is clear.

   Y: ______________________________

   Explain what the pattern is and how you know this number is irrational.

2. Use a calculator to write several digits of pi (π).

   Explain the meaning of pi in your own words.

3. Line up pi under your value for *Y* and state which is greater. What decimal place verifies your decision?

4. Elijah says that π is a rational number because it is equal to 3.14. Why is this statement incorrect?

PRACTICE 6
Continued

5. Some different approximations for pi are attributed to ancient civilizations. Here are a few.

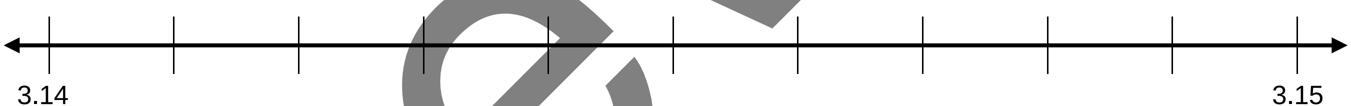
   a. Write each as a decimal to at least seven places:

R	Roman	$\frac{377}{120}$	
C	Chinese	$\frac{355}{113}$	
B	Babylonian	$\frac{25}{8}$	

   b. Which of these is the best approximation for pi?

   c. Explain why none of these can be exactly equal to pi.

6. Number the tick marks below and estimate the locations of *Y* from the previous page and *R*, *C*, and *B* from above. If any cannot fit on this portion of the number line, explain why not.



7. Use a calculator to complete the table for values of pi and its square.

Round 3.14159… to the nearest:	**If π is:**	**Then π^2 is (or is about):**
Whole number		
Tenth		
Hundredth		
Thousandth		

ANOTHER WELL-KNOWN IRRATIONAL NUMBER

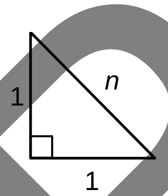
1. Fact: $(\sqrt{2})^2$ = ______.

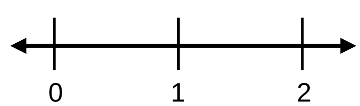
   Use guess and check and the multiplication operation only on your calculator to find a good approximation for $n = \sqrt{2}$ by squaring different values of n.

Approximations for *n*	1.5	1.3							
Find n^2	2.25								
Is n^2 too high or too low?	HI								

2. Dakotah used a calculator and found that $(1.4142135)^2 = 2.00$. Does this mean that Dakotah found an exact value for $\sqrt{2}$? Explain.

3. Calculate the length of the hypotenuse, n. Leave it in square root form. Use the hypotenuse to estimate the location of n on the number line.





Fill in the blank with >, <, or =.

4. $\sqrt{2} \bullet \sqrt{2}$ ____ $\sqrt{4}$	5. $\sqrt{2} + \sqrt{2}$ ____ $\sqrt{4}$	6. $\sqrt{2} + \sqrt{18}$ ____ $\sqrt{10} + \sqrt{10}$

REVIEW

POSTER PROBLEMS:
REAL NUMBERS AND THE PYTHAGOREAN THEOREM

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _____.
- Each group will have a different colored marker. Our group marker is _____.

Part 2: Do the problems on the posters by following your teacher's directions.

For each problem, two side lengths of a right triangle are given in linear units. Find the **two** different possible unknown side lengths for each problem.

Poster 1 (or 5)	Poster 2 (or 6)	Poster 3 (or 7)	Poster 4 (or 8)
sides lengths: 2 and 5	sides lengths: 5 and 7	sides lengths: 5 and 8	sides lengths: 3 and 8

A. Make TWO sketches, using *x* for the shorter unknown side length in one diagram and *y* for the longest unknown side length (hypotenuse) in the other diagram.

B. Write both equations.

C. Solve both equations. Leave both solutions in square root form.

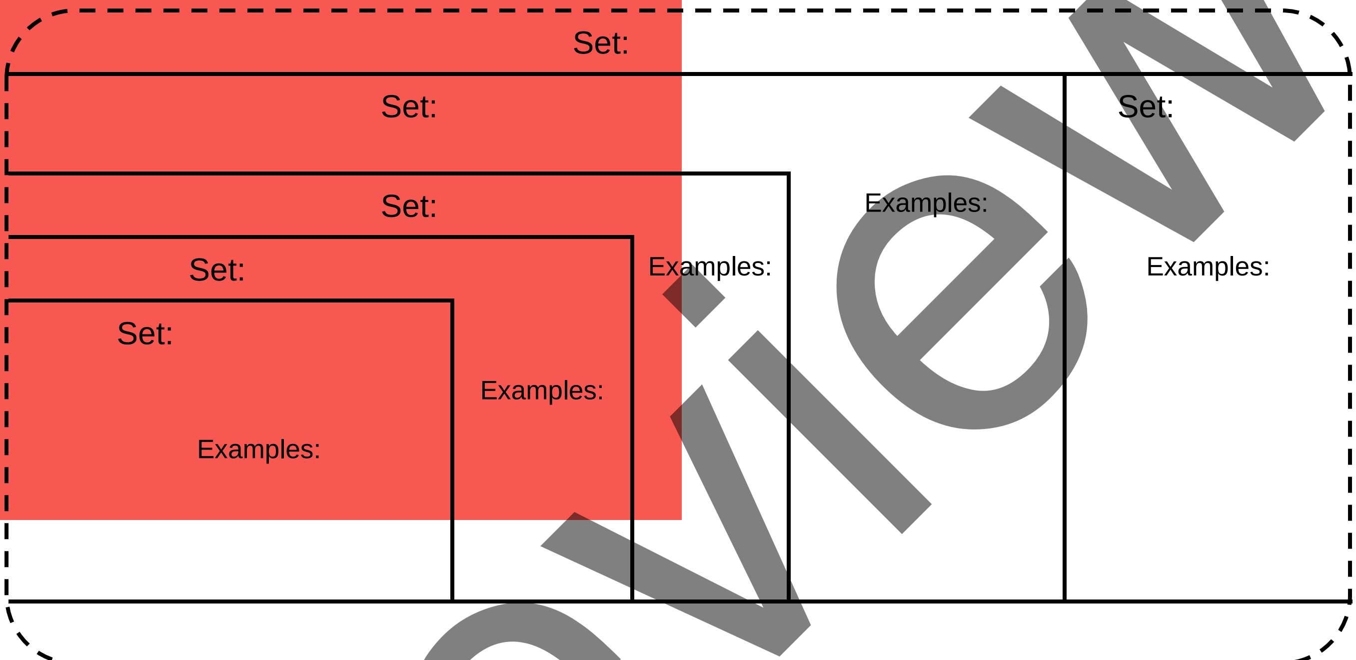
D. Estimate each side length to the nearest whole number.

Part 3: Return to your seats. Work with your group, and show all work.

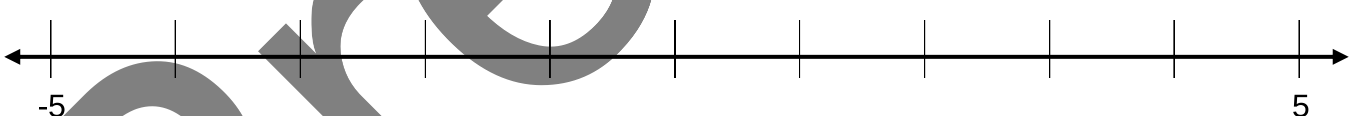
1. A triangle has side lengths 5, 12, and 13 linear units. Why is this a right triangle?	2. A triangle has side lengths 6, 10, and 12 linear units. Why is this NOT a right triangle?	3. How are these two problems structurally different than the poster problems above?

SORT AND MATCH

1. Your teacher will give you six cards that name number sets and 16 cards with numbers on them. Match each number with the "smallest" or "most restrictive" set. For example, 25 belongs to ALL sets except irrational numbers, but its smallest, most restrictive set is the natural numbers. Another example, $\frac{2}{3}$, is a rational number and a real number, and so its smallest, most restrictive set is the rational numbers. Then record all card information into the graphic organizer below.



2. Label the tick marks on the number line.



3. Complete the table below and locate each point on the number line.

Find	Label the point	Write the number
Any whole number that is not a natural number	*P*	
Any a rational number between 2 and 3	*Q*	
Any rational number between -5 and -4	*R*	
Any irrational number between 3 and 4	*V*	
Any irrational number between -3 and -2	*W*	

WHY DOESN'T IT BELONG?: REAL NUMBERS AND THE PYTHAGOREAN THEOREM

1. Choose one of these decimal numbers A – D to the right and explain why it doesn't belong with the others.

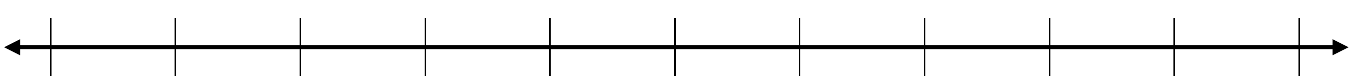
A 0.181818…	**B** 0.1888…
C 0.18	**D** 0.181181118…

2. Choose another number and explain why it does not belong with the others in the space above.

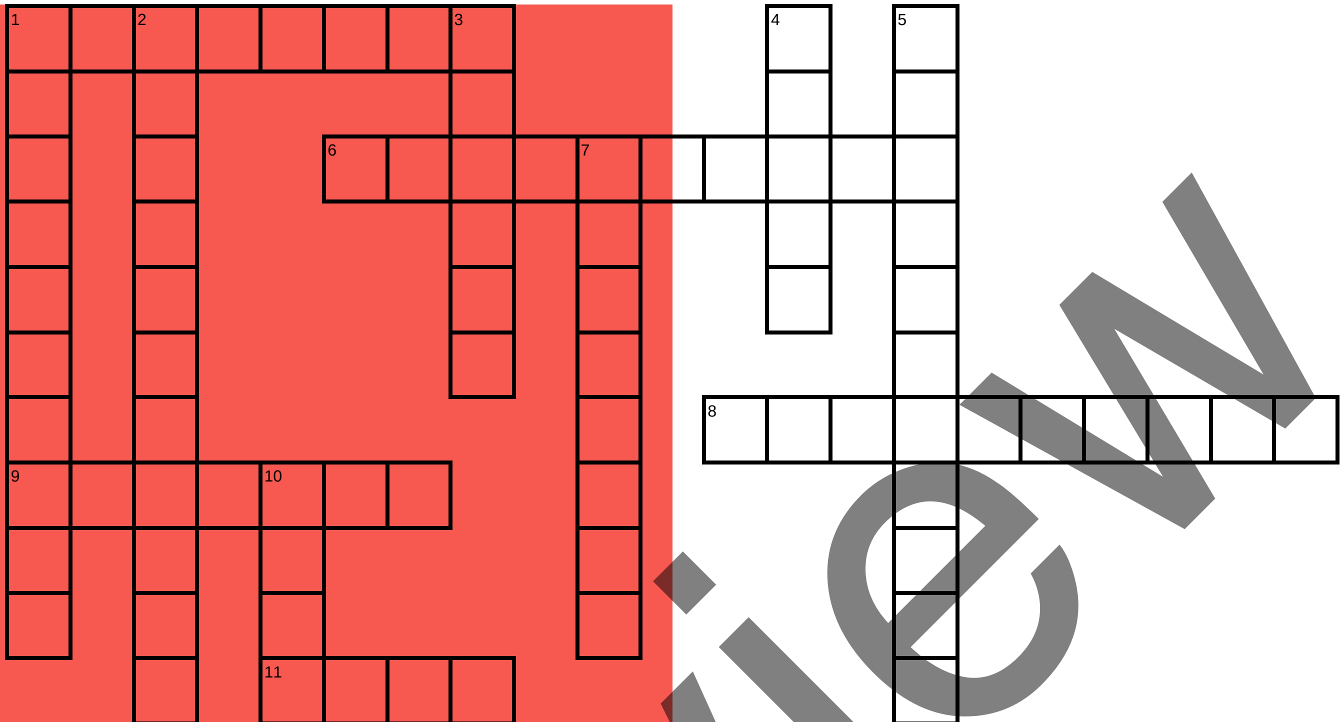
3. Write each number above with a repeat bar and then change it to a quotient of integers. If it cannot be done, explain why not.

A	B	C	D

4. Locate each number from problem 1 on the number line. When estimating the placement, use the letters A – D. Be sure to label all tick marks.



VOCABULARY REVIEW



Across

1 Whole numbers and their opposites

6 The inverse of squaring a number (2 words)

8 The longest side of a right triangle

9 Counting numbers are also called ____ numbers

11 The shorter sides of a right triangle

Down

1 A nonrepeating decimal

2 Decimal numbers that repeat with infinite zeros at the end

3 A whole number to the 2^{nd} power is a perfect ___

4 0, 1, 2, 3, … are ____ numbers

5 The famous theorem regarding right triangles

7 A repeating decimal

10 The rational and the irrational numbers together

SPIRAL REVIEW

1. **Alge-Grid: What's the *a*?** Each clue gives the value of a corresponding cell. Use clues to find *a*, which has the same value in all cells. Once evaluated, the cells will contain the whole numbers 1 – 9, exactly once each.

The Alge-Grid

$a^8 + a^7$	$2a^6 + a^5 + a^4 + 2$	$a + 3$
$3\sqrt{a}$	$8a$	$a \div 4 + 6.75$
$(a + 9) \div 2$	$a \div a$	$a \div a^2 + 8$

The Clues

Number of cups in a pint	
	Number of arms on an octopus
Number of toes on each foot	

2. Solve each equation below.

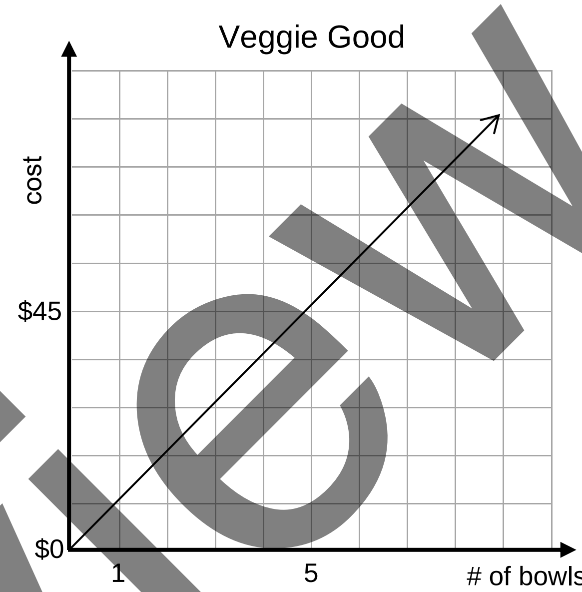
a. $2x - 8 = 60$	b. $36 = 3f + 6$	c. $\frac{y}{6} = 7$
d. $14 - 3r = 5$	e. $-16 = 4y + 4$	f. $24 = -6(s + 4)$

SPIRAL REVIEW

Continued

3. Innik is choosing from two different vegan bowl restaurants, Viva Las Vegan and Veggie Good. Innik created tables and graphs to show how much he would have to pay for different amounts of bowls. He spilled water on parts of his tables, and the graph for Viva Las Vegan and the information was erased.

VIVA LAS VEGAN		**VEGGIE GOOD**	
# of bowls (*x*)	**cost (*y*)**	**# of bowls (*x*)**	**cost (*y*)**
1	8	0	
2	16	1	
3	24	2	
4		3	
5		4	



a. Complete Innik's tables and draw a graph for Viva Las Vegan.

b. Do both graphs represent proportional relationships? Explain.

c. What is the price per bowl for Viva Las Vegan? Write an equation to represent the relationship between cost and number of bowls.

d. What is the price per bowl for Veggie Good? Write an equation to represent the relationship between cost and number of bowls.

e. Which bowl is the better buy? Explain.

4. Solve for *x*.

a. $x + 3y = 12$	b. $4(x - y) + x = y + 10$	c. $5 - x + y = 9$

REFLECTION

1. **Big Ideas**. Shade all circles that describe big ideas in this unit. Draw lines to show connections that you noticed.

Use transformational geometry to investigate congruence and similarity (8.G.A)

Extend applications of volume to cylinders, cones, and spheres (8.G.C)

Explore bivariate data (8.SP.A)

Complete the real number system (8.NS.A)

Solve linear equations in one variable and linear systems in two variables (8.EE.C)

Discover and apply properties of lines, angles, and triangles, including the Pythagorean theorem (8.G.B)

Create, analyze, and use linear functions in problem solving (8.EE.B, 8.F.AB)

Explore exponents and roots, and very large and very small quantities (8.EE.A)

Give an example from this unit of one of the connections above.

2. **Unit Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before or something you would still like to work on.

3. **Mathematical Practice.** Explain how you persevered to make sense of a difficult problem [SMP1]. Then circle one more SMP on the back of this packet that you think was addressed in this unit and be prepared to share an example.

4. **Making Connections**: Explain how you took wholes apart or put parts together to make sense of concepts or problems.

STUDENT RESOURCES

Word or Phrase	Definition
converse of the Pythagorean theorem	The converse of the Pythagorean theorem states that if the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle. See Pythagorean theorem. If the lengths of the sides of a triangle are 3, 4, and 5 units respectively, then the triangle is a right triangle, because $3^2 + 4^2 = 5^2$.
hypotenuse	The hypotenuse of a right triangle is the side of the triangle opposite the right angle. It is the longest side in a right triangle.
integers	The integers are the whole numbers and their opposites. They are the numbers 0, 1, 2, 3, … and -1, -2, -3, … .
irrational numbers	Irrational numbers are real numbers whose decimal expansions continue infinitely without continuously repeating the same block of digits. The irrational numbers are the real numbers that are not rational. $\sqrt{2}$, , and 0.101001000100001... are irrational numbers and cannot be written as quotients of integers.
legs	The legs of a right triangle are the two sides of the triangle adjacent to the right angle.
natural numbers	The natural numbers are the numbers 1, 2, 3, … .Natural numbers are also referred to as counting numbers.
perfect square	A perfect square, or square number, is a number that is the square of a natural number. The area of a square with a natural number side-length is a perfect square. The perfect squares are $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, $16 = 4^2$, $25 = 5^2$, … .
Pythagorean theorem	The Pythagorean theorem states that the sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse. See converse of the Pythagorean theorem. $a^2 + b^2 = c^2$ If the lengths of the legs of a right triangle are 5 and 12 units respectively, then the hypotenuse has length 13 units, because $13^2 = 5^2 + 12^2$.
radical expression	A radical expression is an expression involving a root, such as a square root. In a radical expression, the symbol $\sqrt{\ }$ is called a radical sign, and the number under the radical sign is called the radicand. $\sqrt{5}$ is a radical expression. The radicand is 5.

Word or Phrase	Definition
rational numbers	Rational numbers are quotients of integers. Rational numbers can be expressed as $\frac{m}{n}$, where m and n are integers and $n \neq 0$. $\frac{3}{5}$ is rational because it *is* a quotient of integers. 4, $2\frac{1}{3}$, 0.7, and $0.\overline{25}$ are rational numbers because they *can be* expressed as quotients of integers ($4 = \frac{4}{1}$; $2\frac{1}{3} = \frac{7}{3}$; $0.7 = \frac{7}{10}$; $0.\overline{25} = 0.2525\ldots = \frac{25}{99}$). $\sqrt{2}$ and π are NOT rational numbers. They *cannot be* expressed as a quotient of integers.
real numbers	Real numbers refer to the rational numbers and irrational numbers together. Each real number has a decimal name (address) locating it on the real number line.
repeating decimal	A repeating decimal is a decimal that ends in repetitions of the same block of digits. A "repeat bar" can be placed above the digits that repeat. A terminating decimal is regarded as a repeating decimal that ends in all zeros. Repeating decimals represent rational numbers. $\frac{2}{9} = 0.22222\ldots = 0.\overline{2}$ $\frac{2}{11} = 0.181818\ldots = 0.\overline{18}$ these repeating decimals do NOT terminate $\frac{1}{2} = 0.50000\ldots = 0.5\overline{0} = 0.5$ $\frac{3}{4} = 0.750000\ldots = 0.75\overline{0} = 0.75$ these repeating decimals do terminate
square of a number	The square of a number is the product of the number with itself. The square of 5 is 25, since $5^2 = (5)(5) = 25$. The square of -5 is also 25, since $(-5)^2 = (-5)(-5) = 25$. This is different than $-5^2 = -(5)(5) = -25$.
square root	A square root of a number n is a number whose square is equal to n, that is, a solution of the equation $x^2 = n$. The positive square root of a number n, written $\sqrt{n}$, is the positive number whose square is n. Except where otherwise noted, the term "the square root of n" refers to the positive square root. $\sqrt{25} = 5$, because $5^2 = (5)(5) = 25$
terminating decimal	A terminating decimal is a repeating decimal whose digits are eventually a repeating 0 from some point on. The final 0's in the expression for a terminating decimal are usually omitted. 4.6200000… = 4.62. It is a terminating decimal with value $4 + \frac{6}{10} + \frac{2}{100}$.
whole numbers	The whole numbers are the natural numbers together with 0. They are the numbers 0, 1, 2, 3, … .

Numbers Squared

Why do we say that a number raised to the second power is "squared"? The reason has to do with the area formula for squares. The area of a square of side length s is given by

$$\text{area} = s \bullet s = s^2.$$

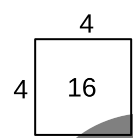
A square with side length 4 units has area "4 squared" = $4^2 = 16$ square units.

What about "square root" – where does that term come from?

Here the reason is that a "root" can also refer to the solution of an equation. A "square root" has to do with finding the side length of a square of a given area; that is, of solving the equation $s^2 = A$. For a given area A, the side length s of the square with area A is

side length = $s = \sqrt{A}$ = "square root of A."

A square with area 16 square units has side length $\sqrt{16}$ = 4 units.



$\rightarrow$ $4^2 = 16$ and $\sqrt{16} = 4$

Square Roots: Estimates Versus Exact Value

Q: What is the square root of 9? *A*: We know that $3^2 = 9$, so $\sqrt{9} = 3$.

Q: What is the square root of 7? *A*: We know of no rational number that, when squared, is equal to 7.

Using the square root function on a simple calculator, we get an approximation to several decimal places: $\sqrt{27} \approx 5.196152$, and then by multiplication: $(5.196152)^2 = (5.196152)(5.196152) = 26.9999956$.

Find another approximation for $\sqrt{27}$ using a calculator with greater capacity, then square that number, and it will still not be exactly equal to 27.

We know that $\sqrt{27}$ is an irrational number and its decimal expansion is infinite with no block of digits that repeats.

So how do we write $\sqrt{27}$? The only way to write it exactly is to leave it in square root form.

If we choose to approximate $\sqrt{27}$, the simplest way may be to state which two consecutive integers it is between. We know that $5^2 = 25$ and $6^2 = 36$, and we also know that 27 is between 25 and 36, so $\sqrt{27}$ is between 5 and 6.

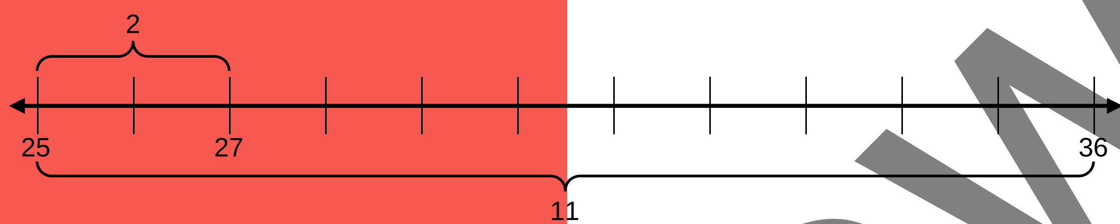
See the next box for an estimation method from lesson 2.1 that is more accurate.

Estimating Square Roots with Greater Accuracy: Linear Interpolation

The following two strategies may be helpful for square root estimation. Note: the method illustrated below is referred to as "linear interpolation."

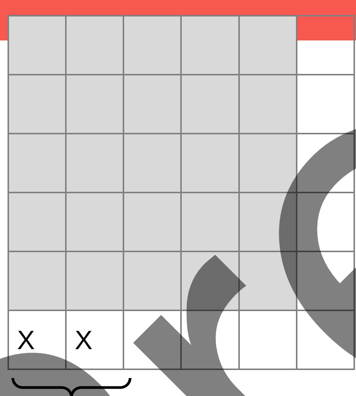
Find an estimate of $\sqrt{27}$ as a mixed number and as a whole number with decimal remainder.

<u>Strategy 1</u>: Since $\sqrt{27}$ is between $\sqrt{25}$ and $\sqrt{36}$ (25 and 36 are perfect squares), provide a "magnified" portion of a number line from 25 to 36.



The distance from 25 to 27 is $\frac{2}{11}$ of the distance from 25 to 36. Hence, the distance from $\sqrt{25}$ to $\sqrt{27}$ should be about $\frac{2}{11}$ of the distance from $\sqrt{25}$ to $\sqrt{36}$. In other words, the distance from 5 to $\sqrt{27}$ should be about $\frac{2}{11}$ of the distance from 5 to 6. Thus $\sqrt{27} \approx 5\frac{2}{11}$.

<u>Strategy 2</u>: Since 25 and 36 are perfect squares, on grid paper, draw a 5 x 5 square inside a 6 x 6 square as illustrated below.



The two X's represent the 26th and 27th unit squares.

- The larger square is $6 \times 6 = 6^2 = 36$ unit squares.
- The smaller, shaded square inside is $5 \times 5 = 5^2 = 25$ unit squares.

So, $\frac{27-25}{36-25} = \frac{2}{11}$

Therefore, $\sqrt{27}$ is about $5\frac{2}{11}$. (calculator check: $5\frac{2}{11} = 5.1\overline{8}$ and $\sqrt{27} \approx 5.196$)

A Clever Procedure: Writing a Repeating Decimal as a Quotient of Integers

Any repeating decimal can be written as a quotient of integers. Therefore, all repeating decimals are rational. The following algebraic idea is used to change a repeating decimal to a quotient of integers.

Example 1: Change $0.1\overline{6} = 0.16666\ldots$

$10x = 1.66666\ldots$ (2)

Let $\boldsymbol{x} = \mathbf{0.16666\ldots}$ **(1)**

$9x = 1.5$ (3)

$x = \frac{1.5}{9} = \frac{15}{90} = \frac{1}{6}$ $\frac{7}{9}$ (4) $\frac{7}{9}$

- Notice that step 2 is above step 1
- The "trick" is to multiply both sides of the equation in step 1 by a power of 10 that will "line-up" the repeating portion of the decimal.
- Subtract the expressions in step 1 from step 2. This results in a step 3 equation that has a terminating decimal. Solve for x in step 4 and simplify the result so it is a quotient of integers

Example 2: Change $0.\overline{7} = 0.77777\ldots$

$10x = 7.777777\ldots$ (1)

Let $\boldsymbol{x} = \mathbf{0.7777777\ldots}$ **(2)**

$9x = 7.000000\ldots$ (3)

$x = \frac{7}{9}$ $\frac{7}{9}$ (4) $\frac{7}{9}$

Ask yourself:

How many digits are repeating?
one

What do I multiply both sides by?
10 (a power of 10 with one zero)

Example 3: Change $0.\overline{45} = 0.454545\ldots$

$10x = 45.454545\ldots$ (1)

Let $\boldsymbol{x} = \mathbf{0.45454545\ldots}$ **(2)**

$99x = 45.00000\ldots$ (3)

$x = \frac{45}{99} = \frac{15}{33} = \frac{5}{11}$ (4)

Ask yourself:

How many digits are repeating?
two

What do I multiply both sides by?
100 (a power of 10 with two zeros)

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
8.EE.A	**Work with radicals and integer exponents.**
8.EE.2	~~Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number~~. Evaluate square roots of small perfect squares ~~and cube roots of small perfect cubes~~. Know that $\sqrt{2}$ is irrational.
8.G.B	**Understand and apply the Pythagorean theorem.**
8.G.6	Explain a proof of the Pythagorean theorem and its converse.
8.G.7	Apply the Pythagorean theorem to determine the unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
8.G.8	Apply the Pythagorean theorem to find the distance between two points in a coordinate system.
8.NS.A	**Know that there are numbers that are not rational, and approximate them by rational numbers.**
8.NS.1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
8.NS.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of* $\sqrt{2}$, *show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

STANDARDS FOR MATHEMATICAL PRACTICE	
SMP1	Make sense of problems and persevere in solving them.
SMP2	Reason abstractly and quantitatively.
SMP3	Construct viable arguments and critique the reasoning of others.
SMP4	Model with mathematics.
SMP5	Use appropriate tools strategically.
SMP6	Attend to precision.
SMP7	Look for and make use of structure.
SMP8	Look for and express regularity in repeated reasoning.

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