Name ______________________________ Period __________ Date ______________

UNIT 1
STUDENT PACKET

Math Links GRADE 8

PLANE AND SOLID FIGURES

Parent (or Guardian) signature ______________________________

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

alternate exterior angles	alternate interior angles
cone	corresponding angles
cylinder	exterior angle of a triangle
parallel transversal	sphere

OPENING PROBLEM: PAPER SOLIDS

[SMP 1, 2, 4, 5, 6]

Follow your teacher's directions for (1) – (4) for each solid.

	Sketch #1:	Sketch #2:	Sketch #3:
(1)			
(2)			
(3)			
(4)			

Recording space for **Practice 2**.

Sketch #4:	Sketch #5:	Sketch #6:

VOLUME OF CYLINDERS

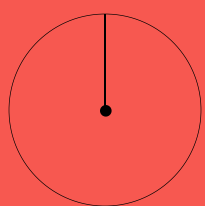
We will develop the formula for the volume of a cylinder and use it to solve problems.

[8.G.9; SMP1, 2, 3, 4, 5, 6, 7]

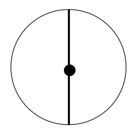
GETTING STARTED

Find the area of each circle with the given radius (r) or diameter (d) measures.

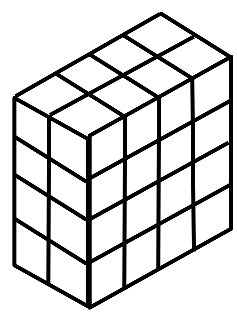
1. r = 6 mm (Use π = 3.14.)



2. d = 8 mm (Leave in terms of π.)



Consider the rectangular prism pictured to the right to complete the following.



3. How many squares are in the rectangular base (B) of this prism (either top or bottom)? _____

4. How many cubes are in the top (or bottom) horizontal "layer" of this rectangular prism? _____

5. How many horizontal layers are there? _____

6. How many cubes are there in all (the total volume)? _____

7. Write the length, width, and height of this prism: ℓ = _____, w = _____, h = ____

8. Write a formula to find the volume of a rectangular prism using ℓ, w, and h:

   V = __________

9. Write a second version of this formula using B as the area of the base:

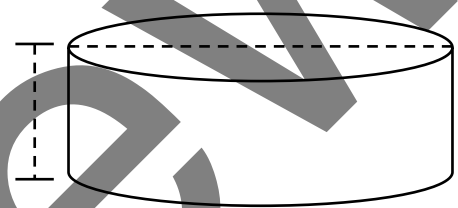
   V = __________

VOLUME OF A CYLINDER

Follow your teacher's directions for (1) – (7). Use π = 3.14 as needed.

(1)	(2)
	If this is 1 unit of __________, then this is 1 _________ unit of __________, and this is 1 _________ unit of __________.

(3)



(4)

(5)

(6)

(7)

Find the volume of each cylinder described below.

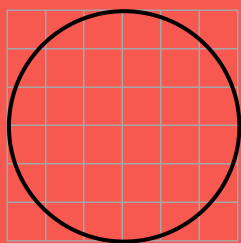
8. Use π = 3.14, h = 10 cm, d = 4 cm	9. Use $\pi = \frac{22}{7}$, $h = \frac{7}{8}$ in, $r = \frac{1}{2}$ in	10. Leave in terms of π, h = 6 mm, $C = 24\pi$ mm

11. Record the meaning of <u>cylinder</u> in **My Word Bank**.

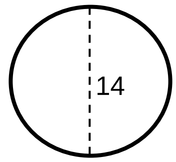
PRACTICE 1

Find the volume of each cylinder described below.

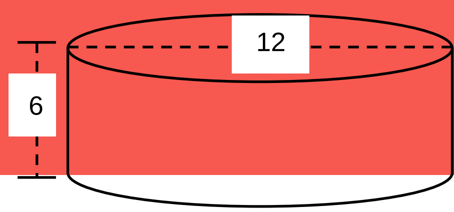
1. The base is pictured below.
   Each small square is 1 square unit.
   The height is 10 units.
   Use π = 3.14.



2. The base is pictured below.
   The diameter is given in millimeters.
   The height is 20 mm.
   Use π = 3.14.



3. Units given in centimeters.
   Use π = 3.14.



4. The base radius is 6 feet. The height is 30 feet. Leave in terms of π.

5. The base radius is $3\frac{1}{2}$ cm. The height is 4 cm. Use $\pi = \frac{22}{7}$.

Find the height of each cylinder described below.

6. A cylinder with circumference of 62.8 inches has volume = 628 cubic inches.
   Use π = 3.14.

7. A cylinder with diameter = 8 cm has volume = 400 cubic cm.
   Write in terms of π and use π = 3.14.

PRACTICE 2: EXTEND YOUR THINKING

In the opening problem, you created models for a square prism, triangular prism, and cylinder from an 8.5 × 11-inch piece of paper with a height of 8.5 inches. Then you found the volumes.

1. Go back to **Paper Solids** and update or correct your work if needed.

2. Suppose you created models for a square prism, triangular prism, and cylinder from an 8.5 × 11-inch piece of paper with a height of 11 inches. Which of the six models do you think would have the greatest volume? Why?

3. In the space provided on **Paper Solids**, sketch the following with 11-inch heights:

   - Sketch #4, a square prism,
   - Sketch #5, a triangular prism, and
   - Sketch #6, a cylinder.

   Then find the areas of the bases and the volumes.

4. Write conclusions based on your work. Compare volumes based on the height or shape of the base. Include which has the greatest volume and the least volume in your explanation.

5. A soup can is measured and found to have a radius of about 3.7 cm and a height of about 7.3 cm. The label on the can lists the volume as 310.52 mL. Is this a reasonable volume of soup? Explain. (1 cubic cm is equivalent to 1 mL.)

A COIN PROBLEM

To the right is some information about coins.

	Dime	Quarter
Weight (grams)	2.27	5.67
Thickness (mm)	1.35	1.75
Diameter (mm)	17.9	24.26

Suppose you had a $10 stack of dimes and a $10 stack of quarters.

1. Make some predictions:

- Which stack will have the greatest weight?
- Which stack will have the greatest height?
- Which stack will have the greatest volume?

2. Compute the weight, height, and volume for the $10 stacks of coins.

	weight	height	volume
dime			
quarter			

3. Compare your results to your predictions. Were there any surprises?

VOLUME OF CONES AND SPHERES

We will develop formulas for the volume of a cone and a sphere and apply them to solve problems.
[8.G.9; SMP2, 3, 4, 3, 5, 6, 7, 8]

GETTING STARTED

1a. Write the formula for the volume of a cylinder in words and using symbols.	2a. Write the formula for the circumference of a circle in words and using symbols.
1b. Find the volume of a cylinder with a height of 20 cm and diameter of 14 cm. Express an exact answer in terms of π and an approximate answer using $\pi = \frac{22}{7}$.	2b. Find the diameter of a circle with a circumference of 40 in. Express an exact answer in terms of π and an approximate answer using $\pi = 3.14$.

VOLUME OF A CONE AND A SPHERE

Follow your teacher's directions for (1) – (5). Use π = 3.14 as needed.

(1)

(2)

(3)

(4)

$V_{cylinder}$ = ____________

V_{cone} = (______) • $V_{cylinder}$

$\boxed{V_{cone} = \text{\_\_\_\_\_\_\_\_\_\_\_\_}}$

(5)

$V_{cylinder}$ = ____________

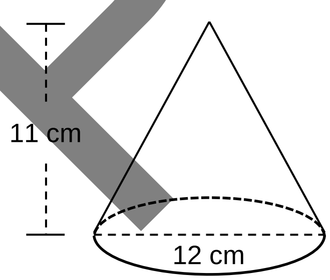
V_{sphere} = (______) • $V_{cylinder}$

V_{sphere} = ______________________________

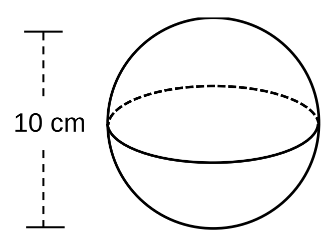
$\boxed{V_{sphere} = \text{\_\_\_\_\_\_\_\_\_\_\_\_}}$

Find the volume of each figure below. Use π = 3.14. Round appropriately.

6.



7.

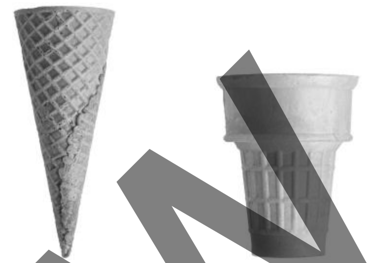


8. Record the meanings of <u>cone</u> and <u>sphere</u> in **My Word Bank**.

ICE CREAM CONES

An ice cream store has two different kinds of cones. For a single scoop, they fill the cone with ice cream and then put a dome (half sphere) of ice cream on the top. Below are the dimensions and prices for one scoop. *Remember that $d = 2r$.

	Height	Top Diameter*	Bottom Diameter*	Cost
Sugar Cone	$4\frac{5}{8}$ inches	2 inches	0 inches	$3.50
Cake Cone	3 inches	$2\frac{1}{2}$ inches	$1\frac{1}{2}$ inch	$3.50


Sugar Cone
Cake Cone

1. Predict which option you think will have the most ice cream: ________________

2. Rank the amount of ice cream from least to greatest. Show formulas and substitutions.

Sugar Cone	**Cake Cone** Assume two cylinders. The bottom has twice the height as the top.

3. Which is the best buy? Which would you choose? Explain your reasoning.

LINES, ANGLES, AND TRIANGLES

We will establish facts about angles in the interior and on the exterior of a triangle. We will introduce vocabulary and facts related to angles formed when two parallel lines are intersected by another line. We will use properties of parallel lines to solve problems.

[8.G.5; SMP2, 3, 5, 6, 7, 8]

GETTING STARTED

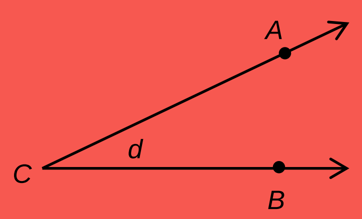


Figure 1

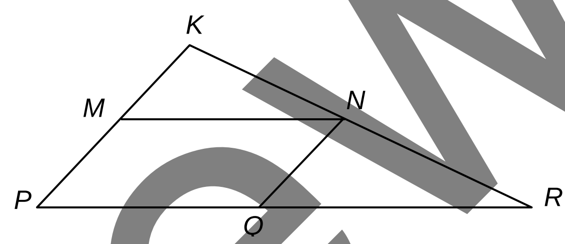


Figure 2

Refer to Figure 1 for problems 1 – 2.

1. Which one of the labeled points represents the vertex of the angle? _____

2. Circle all the names below that can correctly be used to name this angle.

$\angle A$ $\angle B$ $\angle ABC$ $\angle ACB$

$\angle C$ $\angle d$ $\angle BAC$ $\angle BCA$

Refer to Figure 2 for problems 3 – 5.

3. Name three different triangles that appear to be scaled copies of one another.

4. Name one quadrilateral that appears to be a parallelogram.

5. Name one quadrilateral that is not a parallelogram.

We will use absolute value notation for measures of geometric objects.

Measure of $\angle d \longrightarrow |\angle d|$

Use a protractor to find angle measures in degrees for the figures above.

6. $\lvert\angle d\rvert$	7. $\lvert\angle P\rvert$	8. $\lvert\angle KMN\rvert$
9. $\lvert\angle PQN\rvert$	10. $\lvert\angle PRK\rvert$	11. $\lvert\angle PKR\rvert$

TWO INVESTIGATIONS ROLLED INTO ONE

Follow your teacher's directions.

(1)
(2) – (4)
(5) A straight angle measures _______. Sketch: The sum of the measures of the interior angles in a triangle is _______.

ANGLES

1. Your teacher will give you some cards. Match vocabulary to descriptions and pictures.

Vocabulary	A	B	C	D	E	F	G	H
Description								
Picture								

2. Which card(s) were difficult to match up? Explain.

3. Find the missing angle measures in the figure below.

a ______

b ______

c ______

d ______

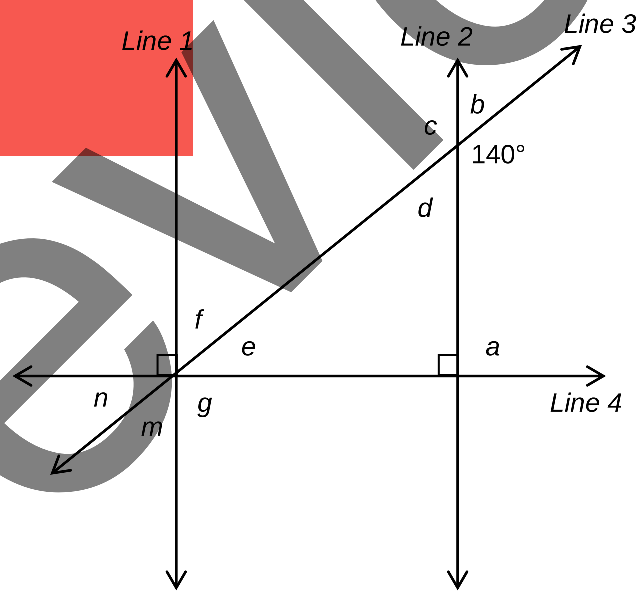
e ______

f ______

g ______

m ______

n ______



4. Refer back to your measurements in **Getting Started** for this lesson.

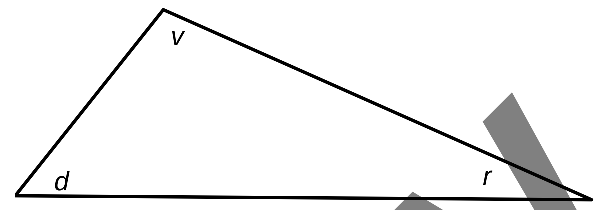
   Does $\left|\angle P\right| + \left|\angle K\right| + \left|\angle R\right| = 180$? Explain.

ANGLE RELATIONSHIPS

Follow your teacher's directions for (1) – (5).

(1)

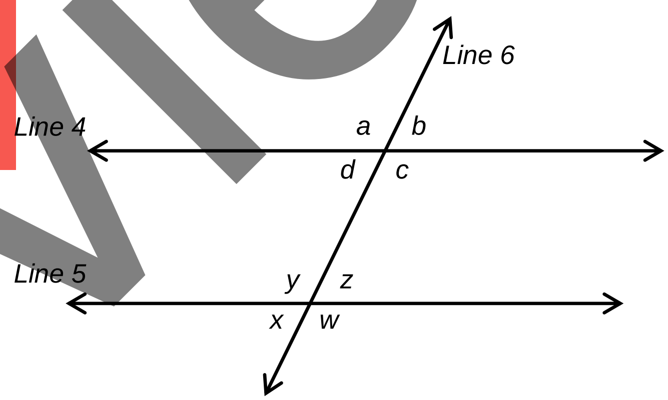
(2)



(3)

(4)

(5)



$|\angle a|$ = ______ $|\angle b|$ = ______

$|\angle c|$ = ______ $|\angle d|$ = ______

$|\angle w|$ = ______ $|\angle x|$ = ______

$|\angle y|$ = ______ $|\angle z|$ = ______

6. List all pairs of vertical angles that have the same measures.

7. List all pairs of corresponding angles that have the same measures.

8. List all pairs of alternate interior angles that have the same measures.

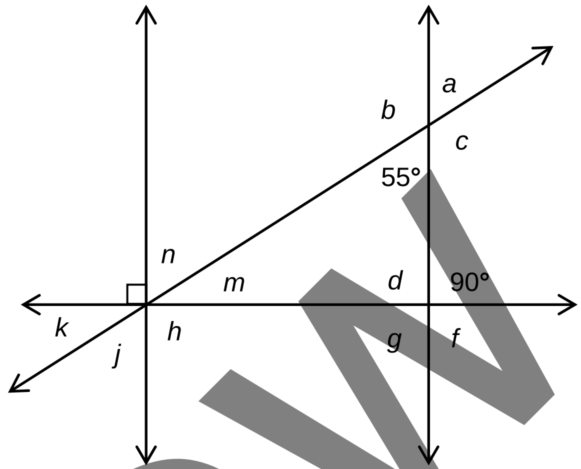
9. List all pairs of alternate exterior angles that have the same measures.

10. Record the meanings of <u>parallel</u> and <u>transversal</u> in **My Word Bank**.

PRACTICE 3

1. If two lines are cut by a transversal, and the three statements below are true, what do we know about the two lines?

   - corresponding angles have equal measure
   - alternate interior angles have equal measure
   - alternate exterior angles have equal measure



2. In the figure to the right, assume lines that appear to be parallel are parallel. Label the parallel lines using arrow notation. Then find the measures of all labeled angles.

$|\angle a|$ = ______ $|\angle b|$ = ______ $|\angle c|$ = ______ $|\angle d|$ = ______

$|\angle f|$ = ______ $|\angle g|$ = ______ $|\angle m|$ = ______ $|\angle n|$ = ______

$|\angle h|$ = ______ $|\angle j|$ = ______ $|\angle k|$ = ______

Name one pair of each of the following types of angles from the figure above.

3. Acute, vertical angles	4. Right, vertical angles	5. Obtuse, vertical angles
6. Form a straight angle	7. Corresponding angles with the same measures	8. Alternate exterior angles with the same measures
9. Alternate interior angles with the same measures	10. Adjacent, supplementary angles	11. Vertical, supplementary angles

12. When do corresponding angles have equal measures?

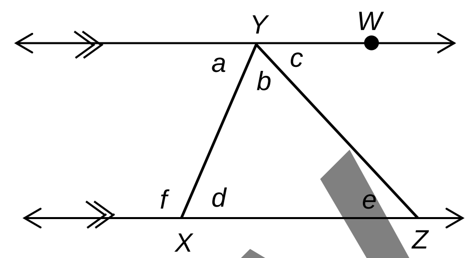
13. For the triangle in the figure above, name two exterior angles whose measures are equal to the sum of $|\angle m| + |\angle d|$.

14. Record the meanings of <u>corresponding angles</u>, <u>alternate interior angles</u>, <u>alternate exterior angles</u>, and <u>exterior angle of a triangle</u> in **My Word Bank**.

ANGLE FACTS RELATED TO TRIANGLES

Use the cards given to you by your teacher and the figure to the right to establish two important facts about angles related to triangles.

Match each fact statement 1 – 8 below with a fact card. Some cards may be used more than once, and some not at all.



	Statement	Card
1.	$\overleftrightarrow{WY}$ is parallel to $\overleftrightarrow{XZ}$	
2.	$\lvert\angle a\rvert + \lvert\angle b\rvert + \lvert\angle c\rvert = 180°$	
3.	$\lvert\angle a\rvert = \lvert\angle d\rvert$	
4.	$\lvert\angle c\rvert = \lvert\angle e\rvert$	
5.	$\lvert\angle d\rvert + \lvert\angle b\rvert + \lvert\angle e\rvert = 180°$	
6.	$\lvert\angle f\rvert + \lvert\angle d\rvert = 180°$	
7.	$\lvert\angle b\rvert + \lvert\angle d\rvert + \lvert\angle e\rvert = \lvert\angle f\rvert + \lvert\angle d\rvert$	
8.	$\lvert\angle b\rvert + \lvert\angle e\rvert = \lvert\angle f\rvert$	

9. Which statement (problem) establishes that:

The sum of the measures of the interior angles of a triangle equals 180°.

10. Which statement (problem) establishes that:

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

11. Choose any three **unused** cards so that you can draw a figure that incorporates all three of them. Describe your examples.

Cards chosen: Bold words on cards:

Figure and descriptions:

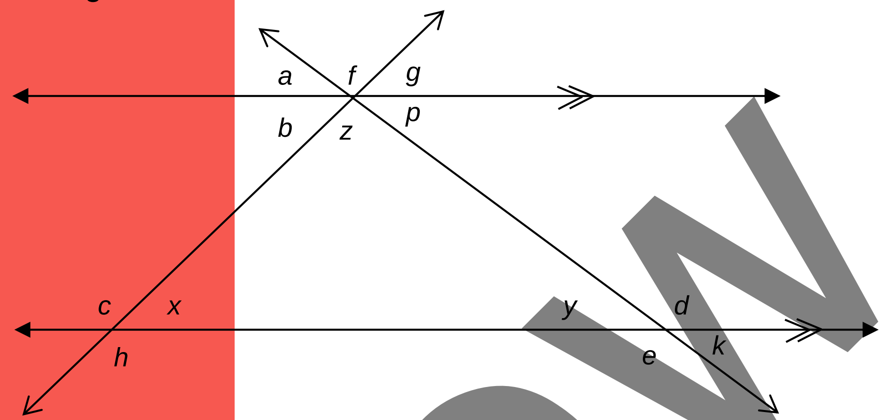
PRACTICE 4

Use this figure for the problems below.
Write important information into the figure as needed.

Let $|\angle x| = n°$

$|\angle y| = (n - 10)°$

$|\angle z| = (2n + 10)°$



1. Find n.

2. Find the measures of each angle in the triangle.

$\|\angle x\|$	$\|\angle y\|$	$\|\angle z\|$

3. The vertical angle to $\angle z$ is _____ and it measures _____.

4. An angle adjacent to $\angle x$ is _____ and it measures _____.

5. The alternate interior angle to $\angle x$ is _____ and it measures _____.

6. The measure of $\angle h$ is _____ and it is an alternate exterior angle to the sum of angles _____ and _____.

7. The measure of $\angle c$ is _____ and it corresponds to the sum of angles _____ and _____.

8. The angle that corresponds to $\angle k$ is _____ and it measures _____.

9. The exterior angle of the triangle that is adjacent to $\angle y$ is _____ and it measures _____.

10. The two angles in the interior of the triangle that have the same sum as $\angle c$ are angles _____ and _____.

11. Corresponding angles in the figure above have the same measure. Under what condition do corresponding angles NOT have the same measure?

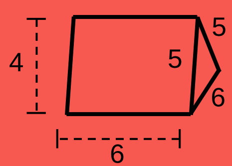
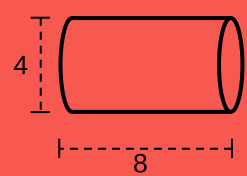
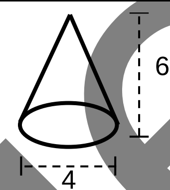
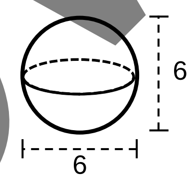
REVIEW

POSTER PROBLEMS: PLANE AND SOLID FIGURES

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _______.
- Each group will have a different colored marker. Our group marker is _________.

Part 2: Do the problems on the posters by following your teacher's directions.

Poster 1 (or 5)	**Poster 2 (or 6)**	**Poster 3 (or 7)**	**Poster 4 (or 8)**
			

Use π = 3.14 and round results to two decimal places as needed.

A. Write the name of the solid figure and its volume formula. If a second, equivalent formula exists, include it.

B. Find the volume of the solid.

C. Find the volume if the height of the solid is doubled.

D. Find the volume of the solid if ALL given measures on the figure are doubled.

Part 3: Return to your seats. Work with your group, and show all work.

Compare the three volume measures in parts B – D for your "start problem." Record what you notice below. Be ready to share with the class.

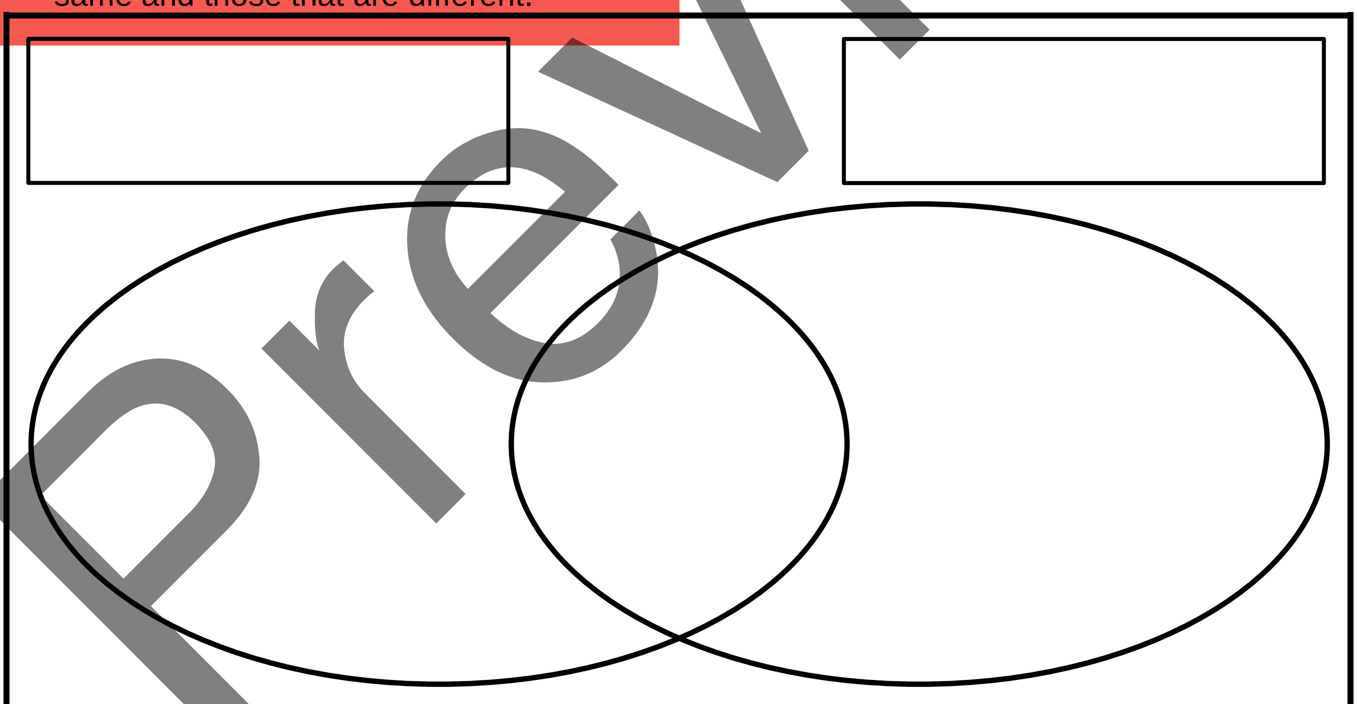
MATCH AND COMPARE SORT: PLANE AND SOLID FIGURES

Your teacher will give you some cards. Cut them out.

1. Individually, match words with descriptions. Record results.

Card set △			Card set ○		
Card number	**word**	**Card letter**	**Card number**	**word**	**Card letter**
I			**I**		
II			**II**		
III			**III**		
IV			**IV**		

2. Partners, choose a pair of numbered matched cards and record the attributes that are the same and those that are different.



3. Partners, choose another pair of numbered matched cards and discuss the attributes that are the same and those that are different.

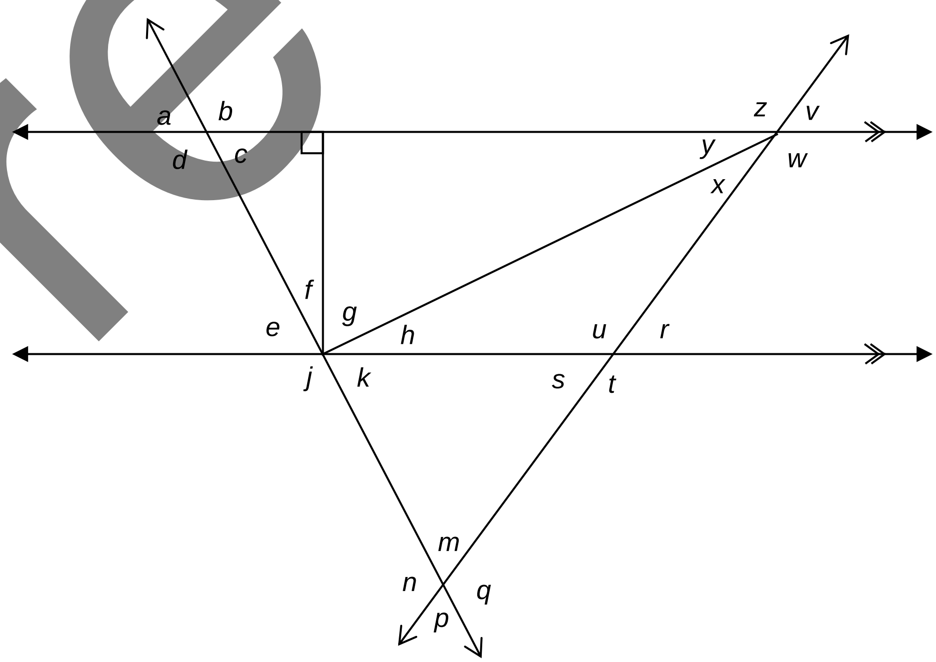
A BIG PUZZLE

Use the figure below for all problems. For problems 1 – 8, name an angle that is:

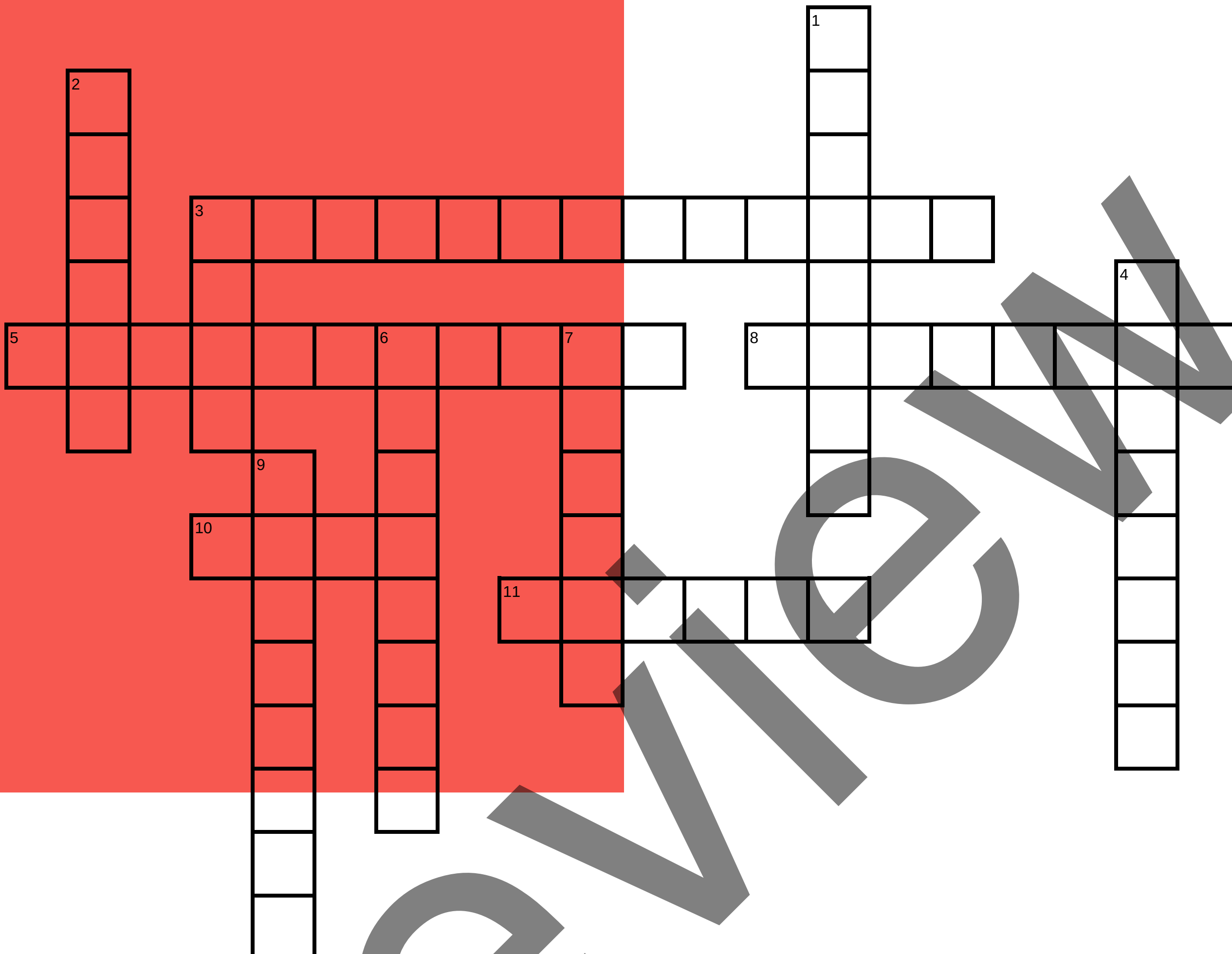
1. adjacent to $\angle m$.	2. vertical to $\angle r$.	3. supplementary to $\angle m$.
4. corresponding to $\angle z$.	5. complementary to $\angle e$.	6. alternate interior to $\angle u$.
7. alternate exterior to $\angle j$.	8. an exterior angle of a triangle and equal to $\lvert\angle h\rvert + \lvert\angle x\rvert$.	

Find the angle measures in any order. Notice that $\lvert\angle a\rvert$, $\lvert\angle h\rvert$ and $\lvert\angle z\rvert$ are given.

9. $\lvert\angle a\rvert = 60°$
10. $\lvert\angle b\rvert =$ _____
11. $\lvert\angle c\rvert =$ _____
12. $\lvert\angle d\rvert =$ _____
13. $\lvert\angle e\rvert =$ _____
14. $\lvert\angle f\rvert =$ _____
15. $\lvert\angle g\rvert =$ _____
16. $\lvert\angle h\rvert = 30°$
17. $\lvert\angle j\rvert =$ _____
18. $\lvert\angle k\rvert =$ _____
19. $\lvert\angle m\rvert =$ _____
20. $\lvert\angle n\rvert =$ _____
21. $\lvert\angle p\rvert =$ _____
22. $\lvert\angle q\rvert =$ _____
23. $\lvert\angle r\rvert =$ _____
24. $\lvert\angle s\rvert =$ _____
25. $\lvert\angle t\rvert =$ _____
26. $\lvert\angle u\rvert =$ _____
27. $\lvert\angle v\rvert =$ _____
28. $\lvert\angle w\rvert =$ _____
29. $\lvert\angle x\rvert =$ _____
30. $\lvert\angle y\rvert =$ _____
31. $\lvert\angle z\rvert = 125°$



VOCABULARY REVIEW



Across

3 Angles in the same relative location

5 A line that intersects two or more other lines

8 Two angles that share a common side

10 One of the parallel faces of a cylinder or prism

11 The point where two rays of an angle meet

Down

1 Solid with two parallel circular bases

2 Tennis ball is an example

3 Solid with circular base and an apex

4 Alternate angles on the inside of lines on opposite sides of a transversal

6 Angle adjacent to a triangle's interior angle

7 Measured in degrees

9 Lines that never meet

SPIRAL REVIEW

1. **READY-X.** Solve for the values of R, E, A, D, Y, X. Sums of rows and columns are indicated at the end of each row and column.

COLUMNS

ROWS

X	X	D	**17**
A	Y	D	**16**
Y	X	D	**14**
E	R	D	**17**
14	**14**	**36**	

R = ______ E = ______ A = ______ D = ______ Y = ______ X = ______

2. Solve each equation below.

a. $4 - 6a = 22$	b. $36 = 3(7 - x)$

SPIRAL REVIEW
Continued

3. Write an expression for each word description.

   a. 3 more than 4 times a number

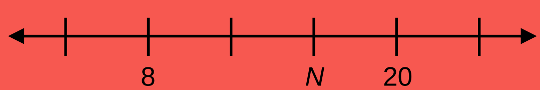
   b. The difference of twice a number and 20

   c. 2 less than the sum of a number and 8

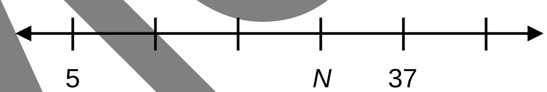
   d. The quotient of a number and the sum of 5 and 7

4. Find the value of *N*.

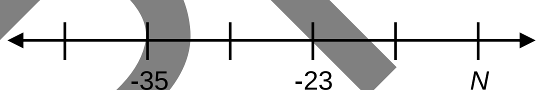
a.



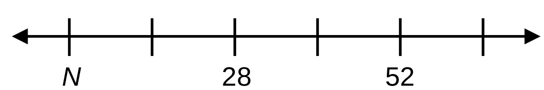
b.



c.



d.



REFLECTION

1. **Big Ideas**. Shade all circles that describe big ideas in this unit. Draw lines to show connections that you noticed.

- Use transformational geometry to investigate congruence and similarity (8.G.A)
- Explore bivariate data (8.SP.A)
- Solve linear equations in one variable and linear systems in two variables (8.EE.C)
- Create, analyze, and use linear functions in problem solving (8.EE.B, 8.F.AB)
- Extend applications of volume to cylinders, cones, and spheres (8.G.C)
- Complete the real number system (8.NS.A)
- Discover and apply properties of lines, angles, and triangles, including the Pythagorean theorem (8.G.B)
- Explore exponents and roots, and very large and very small quantities (8.EE.A)

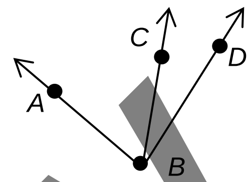
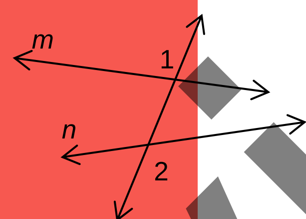
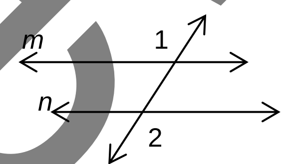
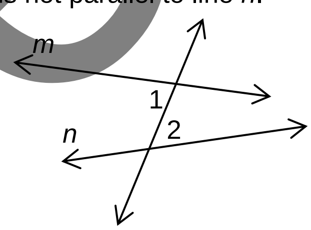
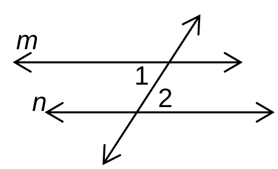
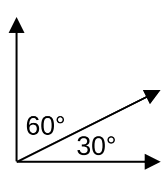
Give an example from this unit of one of the connections above.

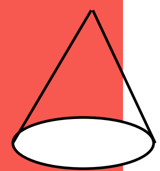
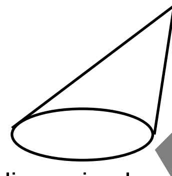
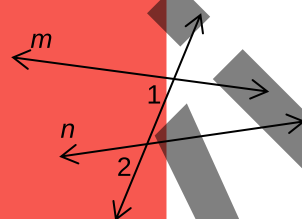
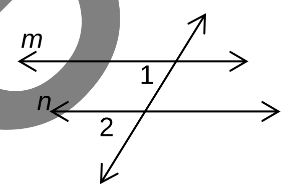
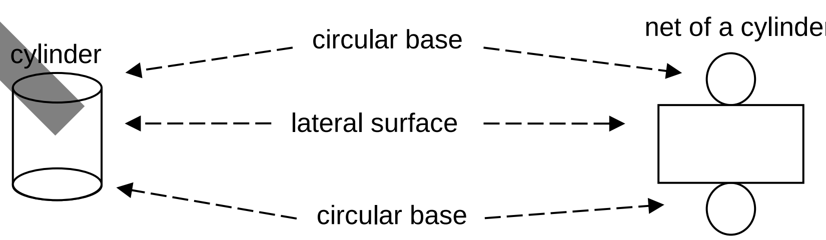
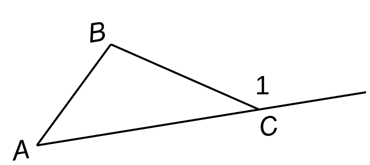
2. **Unit Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before or something you would still like to work on.

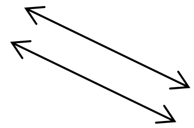
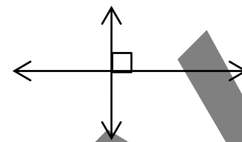
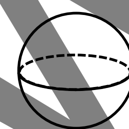
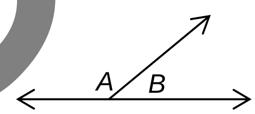
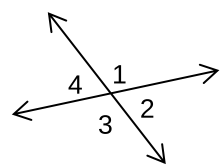
3. **Mathematical Practice.** Explain how the structure of a previously learned concept helped you learn a new one [SMP7]. Then circle one more SMP on the back of this packet that you think was addressed in this unit and be prepared to share an example.

4. **Making Connections.** Describe something new that you learned about shapes and space.

STUDENT RESOURCES

Word or Phrase	Definition
adjacent angles	Two angles are <u>adjacent</u> if they have the same vertex and share a common ray, and they lie on opposite sides of the common ray. $\angle ABC$ and $\angle CBD$ are adjacent angles. 
alternate exterior angles	When two lines in a plane are cut by a transversal, two angles on opposite sides of the transversal and outside the two lines are referred to as <u>alternate exterior angles</u>. When parallel lines are cut by a transversal, alternate exterior angles have the same measure. Line *m* is not parallel to line *n*.  $\angle 1$ and $\angle 2$ are alternate exterior angles. Line *m* is parallel to line *n.*  $\angle 1$ and $\angle 2$ are alternate exterior angles. $\lvert\angle 1\rvert = \lvert\angle 2\rvert$
alternate interior angles	When two lines in a plane are cut by a transversal, two angles on opposite sides of the transversal and between the two lines are referred to as <u>alternate interior angles</u>. When parallel lines are cut by a transversal, alternate interior angles have the same measure. Line *m* is not parallel to line *n*.  $\angle 1$ and $\angle 2$ are alternate interior angles. Line *m* is parallel to line *n.*  $\angle 1$ and $\angle 2$ are alternate interior angles. $\lvert\angle 1\rvert = \lvert\angle 2\rvert$
complementary angles	Two angles are <u>complementary</u> if the sum of their measures is 90°. Two angles that measure 30° and 60° are complementary. 

Word or Phrase	Definition
cone	A circular cone is a figure in space consisting of a circle in a plane (called the base of the cone), a point off the plane (called the vertex of the cone), and all the straight line segments joining the vertex to the base. If the line joining the vertex of the cone to the center of its base is perpendicular to the base, the cone is a right circular cone. Otherwise it is an oblique circular cone.  right circular cone  oblique circular cone
corresponding angles	When two lines in a plane are cut by a transversal, two angles that appear on the same side of the transversal in the same relative location are referred to as corresponding angles. When parallel lines are cut by a transversal, corresponding angles have the same measure. Line *m* is not parallel to line *n*.  $\angle 1$ and $\angle 2$ are corresponding angles. Line *m* is parallel to line *n*.  $\angle 1$ and $\angle 2$ are corresponding angles. $\lvert\angle 1\rvert = \lvert\angle 2\rvert$
cylinder	A (right circular) cylinder is a figure in three-dimensional space that has two parallel circular bases. These circles are connected by a curved surface, called the lateral surface, which is a "rolled up" rectangle. Most soup cans have the shape of a right circular cylinder. 
exterior angle of a triangle	An exterior angle of a triangle is an angle formed by a side of the triangle and an extension of its adjacent side. $\angle 1$ is an exterior angle of ΔABC. 

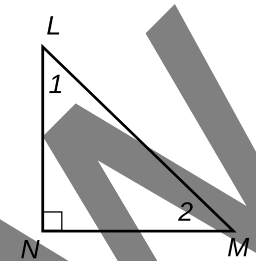
Word or Phrase	Definition
parallel	Two lines in a plane are <u>parallel</u> if they do not meet. Two line segments in a plane are parallel if the lines they lie on are parallel. 
perpendicular	Two lines are <u>perpendicular</u> if they intersect at right angles. 
sphere	A <u>sphere</u> is a closed surface in three-dimensional space consisting of all points at a fixed distance (the radius) from a specified point (the center). 
supplementary angles	Two angles are <u>supplementary</u> if the sum of their measures is 180°. Any two right angles are supplementary, because the sum of their measures is 90° + 90° = 180°. Angles *A* and *B* are supplementary because they determine a straight line, or 180°. 
transversal	A <u>transversal</u> is a line that passes through two or more other lines.
vertical angles	Two angles are <u>vertical angles</u> if they are the opposite angles formed by a pair of intersecting lines. When two lines intersect at a point, they form two pairs of vertical angles with vertex at the point. $\angle 1$ and $\angle 3$ are vertical angles. $\angle 2$ and $\angle 4$ are vertical angles. 

Some Properties of Equality
Properties of equality govern the manipulation of equations (mathematical sentences). For any three numbers *a*, *b*, and *c*: ✓ Addition property of equality (Subtraction property of equality) If $a = b$ and $c = d$, then $a + c = b + d$. ✓ Multiplication property of equality (Division property of equality) If $a = b$ and $c = d$, then $ac = bd$ ✓ Reflexive property of equality $a = a$ ✓ Symmetric property of equality If $a = b$, then $b = a$ ✓ Transitive property of equality (Substitution property) If $a = b$, and $b = c$, then $a = c$

Geometry Notation

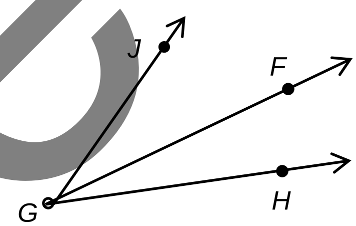
Here are some geometry notations used in these lessons.

- Points are named by capital letters.
- The symbol for triangle is Δ.
- The symbol for angle is $\angle$.
- Absolute value signs are used to denote nonnegative quantities that measure the "size" of something, such as length or angle measure.



The measure of an angle called $\angle N$ is denoted by $|\angle N|$. The small square at N indicates that $\angle LNM$ is a right angle, that is, that $|\angle LNM| = 90°$.

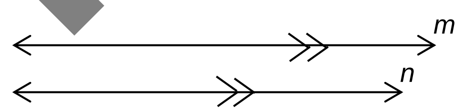
In naming a triangle, vertices may be listed in either a clockwise or counter - clockwise direction. For example, the triangle may be named ΔLMN or ΔLNM.



In naming an angle, vertices may be listed in either a clockwise or counterclockwise direction. In the triangle above, the angle at the top can be denoted by $\angle NLM$, $\angle MLN$, $\angle L$ or $\angle 1$.

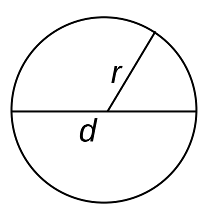
The pair of adjacent angles to the right are $\angle FGJ$ and $\angle HGF$. Using $\angle G$ to name an angle is unclear They share the common ray $\overrightarrow{GF}$.
The two adjacent angles together form the angle $\angle JGH$.

The arrows on the lines m and n indicate that they are parallel.



Formulas for Circles

Let r = radius of a circle.
Let d = diameter of a circle.



Circumference: $C = \pi d$ or $C = 2\pi r$

Area: $A = \pi r^2$

Volume Formulas

Here are some volume formulas from this unit.

Volume of a Rectangular Prism

Let ℓ = length and w = width of rectangular base.

$$V = Bh$$

Area of base (B) = ℓw

Therefore, $V = \ell wh$

Volume of a Cylinder

Let r = radius of the circular base.

$$V = Bh$$

Area of base (B) = πr^2

Therefore, $V = \pi r^2 h$

Volume of a Cone

Through experimentation, observe that the volume of a cone is $\frac{1}{3}$ of the volume of a cylinder with the same height and base.

Let r = radius of the circular base

$$V = \frac{1}{3}Bh$$

Area of base (B) = πr^2

Therefore, $V = \frac{1}{3}\pi r^2 h$

Volume of a Sphere

Through experimentation, observe that the volume of a sphere is $\frac{2}{3}$ of the volume of a cylinder whose diameter and height are the same as the diameter of the sphere. Use substitution to derive the formula of a sphere.

Let r = radius of the sphere and cylinder

Then height (h) of cylinder = $2r$
Volume of cylinder = $\pi r^2 (2r) = 2\pi r^3$

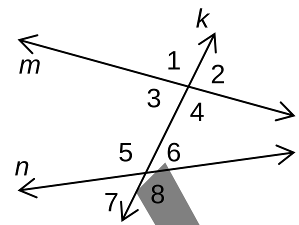
Observe that volume of sphere is $\frac{2}{3}$ of the volume of a cylinder.

Therefore, $V_{sphere} = \frac{2}{3} \bullet 2\pi r^3 = \frac{4}{3}\pi r^3$

Transversals and Parallel Lines

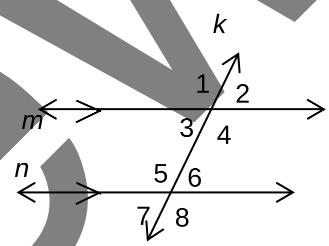
In this figure, line k is a transversal. Lines m and n are NOT parallel.

When two lines in a plane are cut (crossed) at two points by a transversal, eight angles are created. Some of these pairs of angles have special names.



corresponding angles		alternate interior angles	alternate exterior angles
$\angle 1$ and $\angle 5$	$\angle 2$ and $\angle 6$	$\angle 3$ and $\angle 6$	$\angle 1$ and $\angle 8$
$\angle 3$ and $\angle 7$	$\angle 4$ and $\angle 8$	$\angle 4$ and $\angle 5$	$\angle 2$ and $\angle 7$

Here are three important properties of the angles formed when a transversal cuts two parallel lines.



1. If two parallel lines are cut by a transversal, then alternate interior angles have the same measure.
   Example: $|\angle 3| = |\angle 6|$ and $|\angle 4| = |\angle 5|$

2. If two parallel lines are cut by a transversal, then alternate exterior angles have the same measure.
   Example: $|\angle 1| = |\angle 8|$ and $|\angle 2| = |\angle 7|$

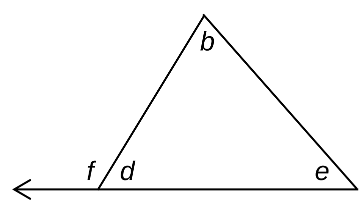
3. If two parallel lines are cut by a transversal, then corresponding angles have the same measure.
   Example: $|\angle 2| = |\angle 6|$ and $|\angle 4| = |\angle 8|$

Interior and Exterior Angles in Triangles

Here are two important facts about angle sums in triangles.

1. The sum of the measures of the angles in a triangle is equal to 180°.

   $|\angle d| + |\angle b| + |\angle e| = 180°$



2. The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

   $|\angle b| + |\angle e| = |\angle f|$

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
8.G.A	**Understand congruence and similarity using physical models, transparencies, or geometry software.**
8.G.5	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, ~~and the angle-angle criterion for similarity of triangles~~. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
8.G.C	**Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**
8.G.9	Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

STANDARDS FOR MATHEMATICAL PRACTICE	
SMP1	Make sense of problems and persevere in solving them.
SMP2	Reason abstractly and quantitatively.
SMP3	Construct viable arguments and critique the reasoning of others.
SMP4	Model with mathematics.
SMP5	Use appropriate tools strategically.
SMP6	Attend to precision.
SMP7	Look for and make use of structure.
SMP8	Look for and express regularity in repeated reasoning.

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