

Name \_\_\_\_\_

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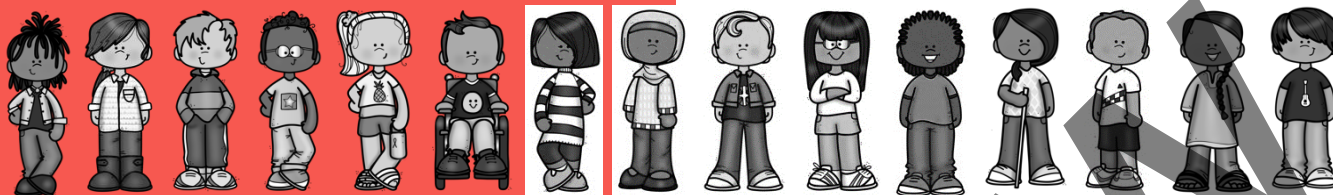
Date \_\_\_\_\_

## UNIT 8

### STUDENT PACKET

# MathLinks

## GRADE 7



## PLANE AND SOLID FIGURES

		Monitor Your Progress	Page
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<b>8.1</b>	<b>Angles</b> <ul style="list-style-type: none"> <li>Understand facts about supplementary, complementary, vertical, and adjacent angles</li> <li>Use facts about angles to solve problems</li> <li>Write and solve equations involving angle measures</li> </ul>	3 2 1 0 3 2 1 0 3 2 1 0	2
<b>8.2</b>	<b>Geometric Drawings</b> <ul style="list-style-type: none"> <li>Draw figures freehand, with a ruler and protractor, and using technology</li> <li>Construct triangles given side lengths and angle measures</li> <li>Recognize when conditions determine a unique triangle, more than one triangle, or no triangle</li> </ul>	3 2 1 0 3 2 1 0 3 2 1 0	8
<b>8.3</b>	<b>Cross Sections</b> <ul style="list-style-type: none"> <li>Identify and describe two-dimensional cross sections of three-dimensional figures</li> </ul>	3 2 1 0	15
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Parent (or Guardian) signature \_\_\_\_\_

## MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

adjacent angles

complementary angles

cross section

plane

prism

pyramid

supplementary angles

vertical angles

**OPENING PROBLEM: TEAR IT UP**

[7.G.5; SMP 1, 5, 6, 7]

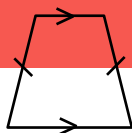
Follow your teacher's directions.

(1) The sum of the measures of the interior angles in a quadrilateral is \_\_\_\_\_.

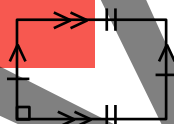
(2) The sum of the measures of the interior angles in a triangle is \_\_\_\_\_.

(3)

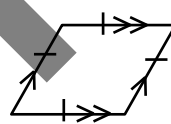
A



B

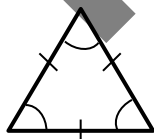


C

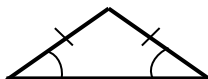


(4)

A



B



C



## ANGLES

We will use patterns to learn facts about angles. We will use these facts to write equations and solve for unknown angle measures in diagrams.

[7.G.5, 7.EE.4a; SMP1, 2, 3, 5, 6, 7]

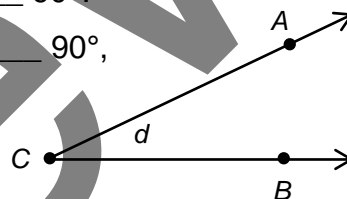
### GETTING STARTED

Fill in each blank space with **more than**, **less than**, or **exactly**.

1. A straight angle is an angle that measures \_\_\_\_\_  $180^\circ$ .
2. A right angle is an angle that measures \_\_\_\_\_  $90^\circ$ .
3. An acute angle is an angle that measures \_\_\_\_\_  $90^\circ$ .
4. An obtuse angle is an angle that measures \_\_\_\_\_  $90^\circ$ , but less than  $180^\circ$ .

Refer to the angle to the right for problems 5 – 6.

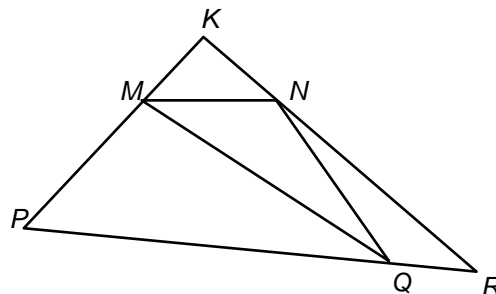
5. Which one of the labeled points represents the vertex of the angle? \_\_\_\_\_
6. Circle all names that can correctly be used to name this angle.



$\angle A$      $\angle B$      $\angle C$      $\angle d$      $\angle ABC$      $\angle ACB$      $\angle BAC$      $\angle BCA$

Refer to the diagram to the right for problems 7 and 8.

7. A triangle has \_\_\_\_\_ sides and \_\_\_\_\_ angles.  
Name three different triangles in the diagram.
8. A quadrilateral has \_\_\_\_\_ sides and \_\_\_\_\_ angles.  
Name two different quadrilaterals in the diagram.



We will use absolute value notation to describe angle measures and lengths of line segments.

Measure of  $\angle d \rightarrow |\angle d|$     Length of  $\overline{AC} \rightarrow |AC|$

9. Use a ruler (mm) and protractor (degrees) to find measures in the diagrams above.

a. $ \angle d $	b. $ \angle MNQ $	c. $ AC $	d. $ PK $

# AN ANGLE INVESTIGATION

Follow your teacher's directions for (1) – (6). If shading angles, do so lightly.

(1) – (2)



(3)

(4)

(5)

(6)

7. Record the meanings of complementary angles, supplementary angles, vertical angles, and adjacent angles in **My Word Bank**.

Suppose  $|\angle x| = 35^\circ$  in the diagram above. Find the following measures.

8. $ \angle w $	9. $ \angle p $	10. $ \angle c $	11. $ \angle a $
12. $ \angle x  +  \angle d  +  \angle t $	13. $ \angle r  +  \angle c  +  \angle w $		

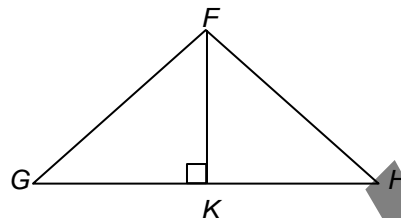
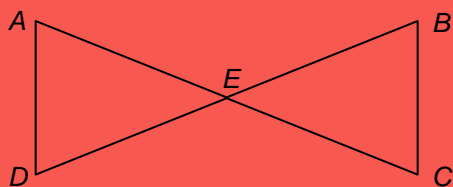
14. Complete each statement below and explain why it is true for the diagram above.

a.  $|\angle c| + |\angle x|$  must have a sum of...

b.  $|\angle a| + |\angle p| + |\angle w|$  must have a sum of...

c.  $|\angle x| + |\angle q| + |\angle b|$  must have a sum of...

## PRACTICE 1



Use the diagrams above (not to scale) to name **two** pairs of each type of angles.

1. adjacent angles	2. vertical angles	3. complementary angles	4. supplementary angles
--------------------	--------------------	-------------------------	-------------------------

- If two complementary angles have the same measure, each angle measures \_\_\_\_\_.
- If two supplementary angles have the same measure, each angle measures \_\_\_\_\_.
- Vanessa thinks that  $\angle AED$  and  $\angle BEC$  cannot be vertical angles because they are in a “horizontal” orientation. Why is Vanessa incorrect?

Find the missing angle measures for each diagram below.

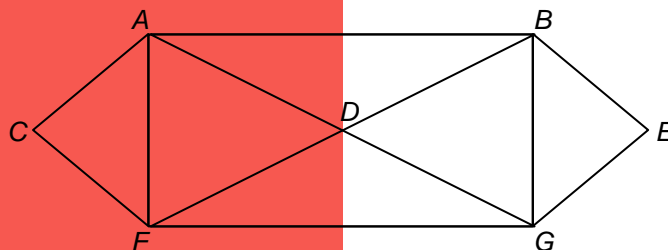
8. $ \angle r $ 	9. $ \angle p $ 	10. $ \angle n $ 	11. $ \angle v $ $ \angle w $ 
---------------------	---------------------	----------------------	--------------------------------------

Fill in each blank with adjacent, vertical, complementary, or supplementary.

- In problem 8, the  $45^\circ$  angle is \_\_\_\_\_ and also \_\_\_\_\_ to  $\angle r$ .
- In problem 9, the right angle is \_\_\_\_\_ and also \_\_\_\_\_ to  $\angle p$ .
- In problem 10, the  $70^\circ$  angle is \_\_\_\_\_ to  $\angle n$ .
- In problem 11, the  $145^\circ$  angle is \_\_\_\_\_ and also \_\_\_\_\_ to  $\angle v$ .

Also,  $\angle v$  is \_\_\_\_\_ to  $\angle w$ .

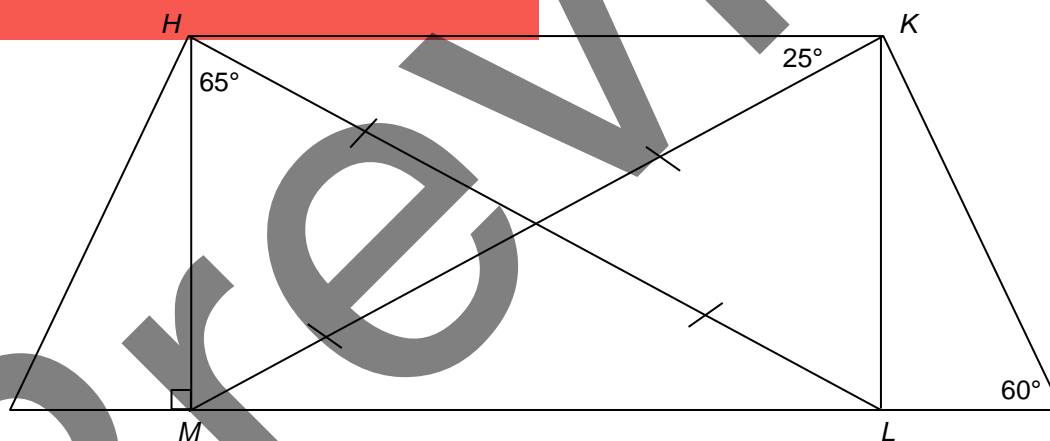
## PRACTICE 2



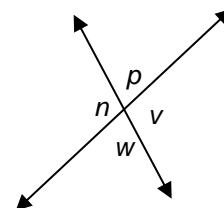
$ABGF$  is a rectangle and triangles that appear identical are identical. Name **two** pairs of each type of angles in this figure.

1. adjacent angles	2. vertical angles	3. complementary angles	4. supplementary angles

5. Triangles are often used to make structures stronger. Below is a diagram (not to scale) of a trestle bridge that can support trains. This bridge is an isosceles trapezoid.  $HKLM$  is a rectangle. Write in the measures of all the angles in the interior of the diagram.

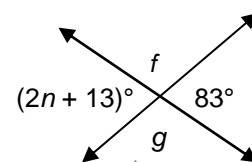


6. If you know  $|\angle p|$ , how do you know the measures of the other three angles? Use appropriate mathematical vocabulary in your explanation.



## USING ALGEBRA TO FIND ANGLE MEASURES

1. Find the measures of  $\angle f$  and  $\angle g$  in the diagram below. The diagram is not to scale. Explain your reasoning or show your work.



2. Refer to the diagram above. Write two different equations that could be used to find the value of  $(2n + 13)$ . Solve for  $n$  in both equations, and write the value of  $(2n + 13)$ .

Equation:

$n \rightarrow$  \_\_\_\_\_

$2n + 13 \rightarrow$  \_\_\_\_\_

Check:

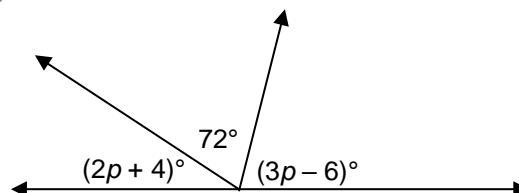
Equation:

$n \rightarrow$  \_\_\_\_\_

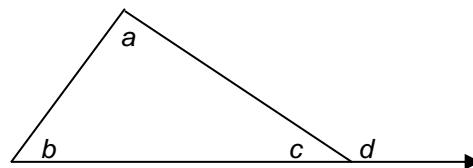
$2n + 13 \rightarrow$  \_\_\_\_\_

Check:

3. Use an equation to find the measure of the two angles in this diagram that are represented by variable expressions. The diagram is not to scale. Show your work and check your results.



4. Explain why, without knowing any specific angle measures, that  $|\angle a| + |\angle b|$  must be equal to  $|\angle d|$ .





PRACTICE 3

Find the missing values below by writing and solving equations. The diagram is not to scale.

<p>1. Find <math>x</math>.</p>     <p>Find <math>(x - 60)^\circ</math>.</p>	<p>2. Find <math>y</math>.</p>     <p>Find <math>(2y - 10)</math>.</p>	
--	---	--

Find the measure of each angle using the diagram above and support each answer with an explanation or calculation.

3. $ \angle a $	4. $ \angle b $
5. $ \angle c $	6. $ \angle d $
7. $ \angle e $	8. $ \angle f $

**GEOMETRIC DRAWINGS**

We will draw figures freehand, with rulers and protractors, and using technology. We will observe conditions that make a triangle unique, and conditions for which it is impossible to draw a triangle.

[7.G.2; SMP1, 3, 4, 5, 6, 7]

**GETTING STARTED**

Draw the following.

- |   |   |
|---|---|
| 1. Draw a square freehand with side lengths greater than 2 cm, but less than 6 cm.                      | 2. Draw a scalene triangle freehand that takes most of this space.  |
| 3. Draw an equilateral triangle with a ruler and/or protractor so that the sides are $2\frac{1}{2}$ in. | 4. Draw a parallelogram with a ruler and protractor so that one pair of sides are 1 inch each; the other pair of sides are $1\frac{3}{4}$ inches each; one pair of angles is $45^\circ$ . |

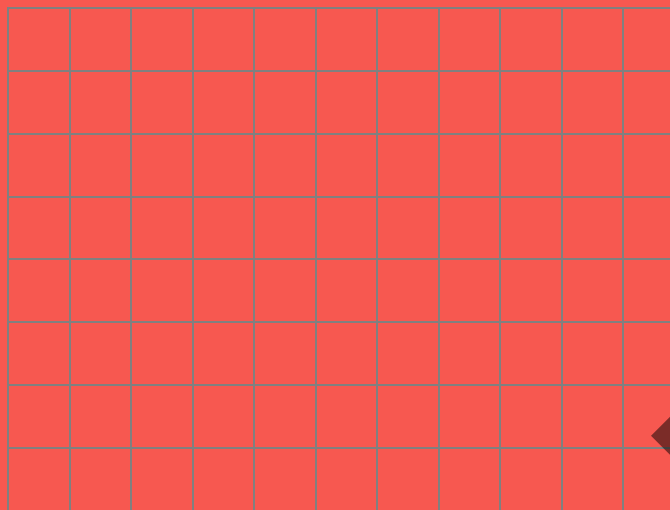
Each angle must measure \_\_\_\_\_.

The other pair of angles is \_\_\_\_\_ each.

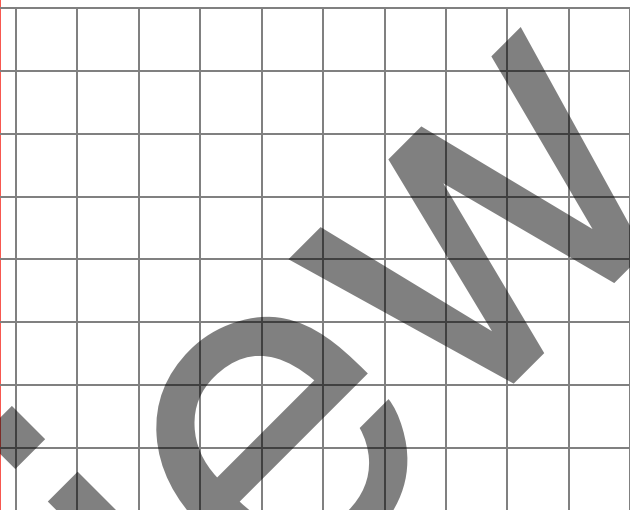
**SKETCHING FIGURES**

Follow your teacher's directions for (1) – (4). Draw freehand, or use a straightedge if desired.

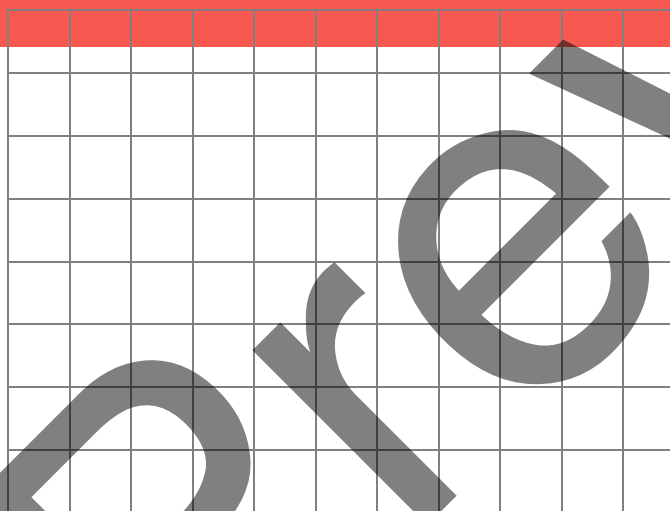
(1)



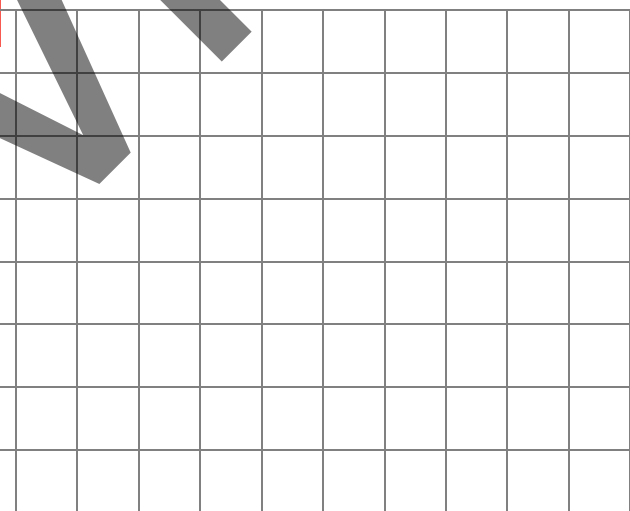
(2)



(3)



(4)



5. Why might we say that the figure described for problem 2 is “unique”?
6. What does it mean for two geometric figures to exactly cover one another?

**A POLYGON INVESTIGATION**

Use several “sticks” of lengths 1, 2, 3, 4, and 5-inches. Make a sketch of the description and state if it is unique, if there are many possibilities, or if it is impossible. If it cannot be created, explain why not.

1. Build a triangle with three 4-in sticks.	2. Build a triangle with two 3-in sticks and one 2-in stick.	3. Build a triangle with one 5-in stick, one 3-in stick, and one 1-in stick.
4. Build a triangle with three 3-in sticks.	5. Build a triangle with two 1-in sticks and one 3-in stick.	6. Build a triangle with one 5-in stick, one 4-in stick, and one 2-in stick.
7. Build a triangle with two right angles.	8. Build an equilateral triangle with an obtuse angle.	9. Build an isosceles triangle with three acute angles, two of which have the same measure.

**A POLYGON INVESTIGATION**

Continued

10. Build a scalene triangle with one obtuse angle and one right angle.	11. Build a scalene triangle with one obtuse angle and two acute angles.	12. Build an isosceles triangle with three acute angles, none of which have the same measure.
13. Build a quadrilateral with four 4-in sticks.	14. Build a quadrilateral with two 2-in sticks and two 4-in sticks.	15. Build a quadrilateral with four sticks of different lengths.
16. Look at problems 1 and 13, and their answers. How are they the same? How are they different?		
17. Matai wants to build a triangular dog pen for his dog, Aima. He has three pieces of fence. One is 6 feet, one is 3 feet, and one is 2 feet. How might he build the dog pen?		

**PROTRACTOR AND RULER DRAWINGS**

Follow your teacher's directions. Measure and label all three sides and angles of each triangle.

(1)

Triangle names (side and angle): \_\_\_\_\_ Is the triangle unique? \_\_\_\_\_

(2)

Triangle names (side and angle): \_\_\_\_\_ Is the triangle unique? \_\_\_\_\_

**PROTRACTOR AND RULER DRAWINGS**

Continued

(3)

Triangle names (side and angle): \_\_\_\_\_ Is the triangle unique? \_\_\_\_\_

(4)

Triangle names (side and angle): \_\_\_\_\_ Is the triangle unique? \_\_\_\_\_

## PRACTICE 4

Fill in the blanks and use appropriate tools to draw.

1. Each of the angles in an equilateral triangle measures \_\_\_\_\_. Draw an equilateral  $\triangle XYZ$  with sides measuring 4.5 cm each.

2. An isosceles triangle that is not equilateral has \_\_\_\_\_ equal side lengths and \_\_\_\_\_ equal angle measures.

Draw an isosceles triangle with  
 $|\angle U| = |\angle V| = 40^\circ$  and  $|\angle W| =$  \_\_\_\_\_;  
 $|VU| = 5.5$  cm and  
 $|VW| = |UW| =$  \_\_\_\_\_ cm.

3. A right trapezoid that is not a parallelogram has \_\_\_\_\_ sides, \_\_\_\_\_ of which are parallel. Draw right trapezoid  $QRST \rightarrow$  bases:  $|QR| = 7$  cm;  $|TS| = 5$  cm; height = 2 cm. Label all four side lengths (measure only as needed). Measure the four angles and write them inside the figure.

4. Under what conditions do you think you can draw a unique triangle? Explain.

5. Using a computer program of your choice, draw each figure listed below and name it with words and symbols.

line	right angle	equilateral triangle	right triangle	segment	rhombus
ray	obtuse angle	isosceles triangle	obtuse triangle	rectangle	trapezoid
square	acute angle	scalene triangle	acute triangle	parallelogram	hexagon



## CROSS SECTIONS

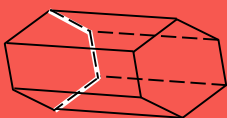
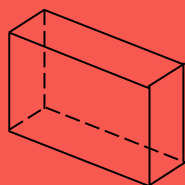
We will describe two-dimensional figures that result from slicing three-dimensional figures by planes.

[7.G.3; SMP5, 8]

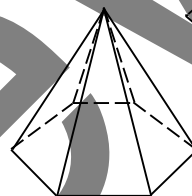
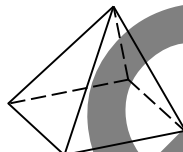
### GETTING STARTED

Below are some three-dimensional figures.

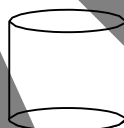
These solid figures are prisms.



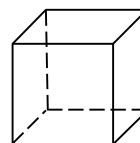
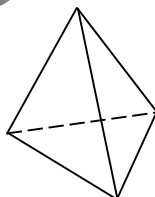
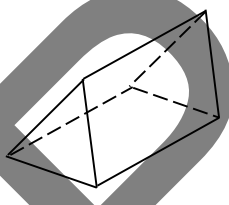
These solid figures are pyramids.



These solid figures are neither prisms nor pyramids.



1. Record the meanings of prism and pyramid in **My Word Bank**.
2. Label each figure below as a prism, pyramid, or neither. If neither, state why.

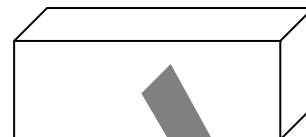


**A STICK OF BUTTER**

1. A stick of butter is 8 cm long and has a square face with sides of length 3.5 cm. Find the following.

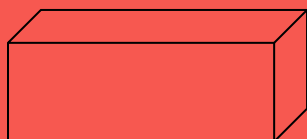
a. Volume:

b. Surface Area:



2. Draw a slice that cuts a stick of butter in half. Do this in two different ways.

a. A “shorter” (vertical) cut:



b. A “longer” (horizontal) cut:



3. Determine which half-stick has the greater volume (shorter or longer cut).

4. Determine which half-stick has the greater surface area (shorter or longer cut).

5. When making the shorter cut, what type of polygon is the inner face of the butter? What are its dimensions?

6. When making the longer cut, what type of polygon is the inner face of the butter? What are its dimensions?

7. What other type of polygons (inner face) can be made with a different type of slice? Name the polygons and describe the slice if possible. Slicing in half is not required.

**CROSS SECTIONS 1: PRISMS**

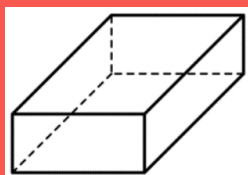
Follow your teacher's directions for (1) – (2).

(1)

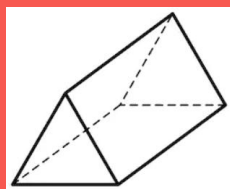
(2)

Explore cross sections using your spatial ability, hands-on tools, or a computer application. Make sketches of the polygon faces that could be cross sections of the figures below when sliced by a plane.

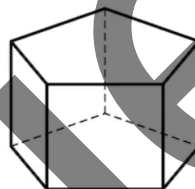
3. Rectangular prism



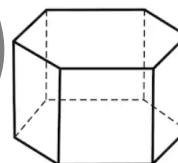
4. Triangular prism



5. Pentagonal prism



6. Hexagonal prism



7. Each cross section (polygon) you drew above is created by a plane intersecting edges of the prism. Write observations about the number of edges intersected for each cross section created.

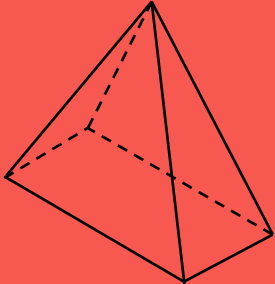
8. Record the meanings of plane and cross section in **My Word Bank**.

CROSS SECTIONS 2: PYRAMIDS


Follow your teacher’s directions for (1).  
(1)

Explore cross sections using your spatial ability, hands-on tools, or a computer application. Label the figures below and draw sketches of the polygon faces that could be cross sections of the figures when sliced by a plane.

2. Name of figure:



3. Name of figure:



4. Each cross section (polygon) you drew above is created by a plane intersecting edges of the pyramid. Write observations about the number of edges intersected for each cross section created.

5. For each solid figure listed below, write the first letter of each polygon listed that you think *cannot* be one of its cross sections.

Polygon choices			
Triangle	Quadrilateral	Pentagon	Hexagon
a. Triangular prism _____		b. Rectangular prism _____	
c. Pentagonal prism _____		d. Hexagonal prism _____	
e. Triangular pyramid _____		f. Rectangular pyramid _____	

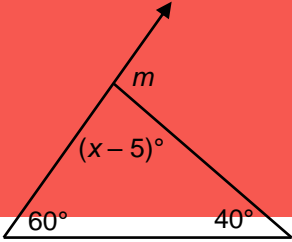
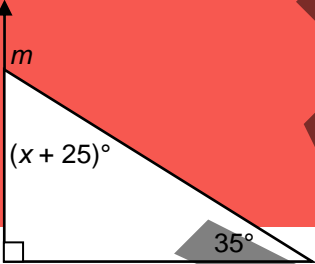
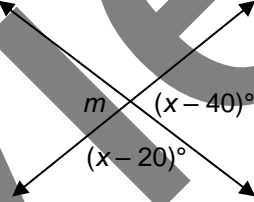
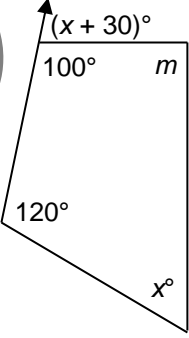
REVIEW

POSTER PROBLEMS: PLANE AND SOLID FIGURES

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is \_\_\_\_\_.
- Each group will have a different colored marker. Our group marker is \_\_\_\_\_.

Part 2: Do the problems on the posters by following your teacher’s directions. Pictures may not be drawn to scale.



Poster 1 (or 5)	Poster 2 (or 6)	Poster 3 (or 7)	Poster 4 (or 8)
			
A. Sketch the picture.		C. Solve for $x$ .	
B. Write a correct equation that involves $x$ .		D. Find the measure of angle $m$ .	

Part 3: Return to your seats. Work with your group, and show all work.

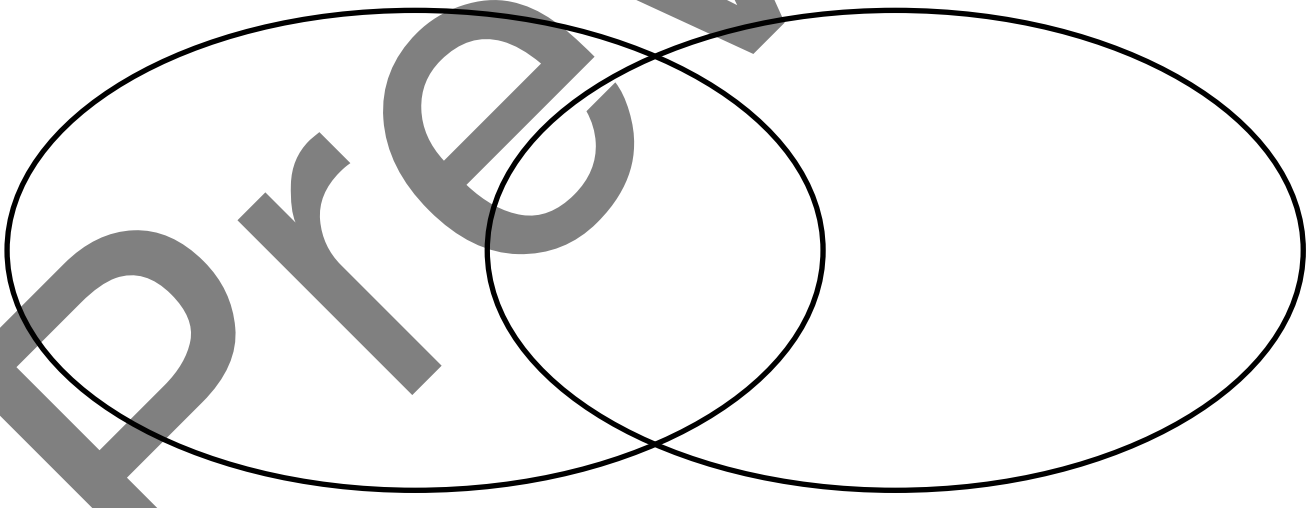
1. List facts about triangles and angles that are important for solving posters 1 and 2 (or 5 and 6).
2. List facts about angles and intersecting lines that are important for solving poster 3 (or 7).
3. List facts about quadrilaterals that are important for solving poster 4 (or 8).

MATCH AND COMPARE SORT: PLANE AND SOLID FIGURES

1. Individually, match words with descriptions. Record results.

Card set 			Card set 		
Card number	word	Card letter	Card number	word	Card letter
I			I		
II			II		
III			III		
IV			IV		

2. Partners, choose a pair of numbered matched cards and record the attributes that are the same and those that are different.



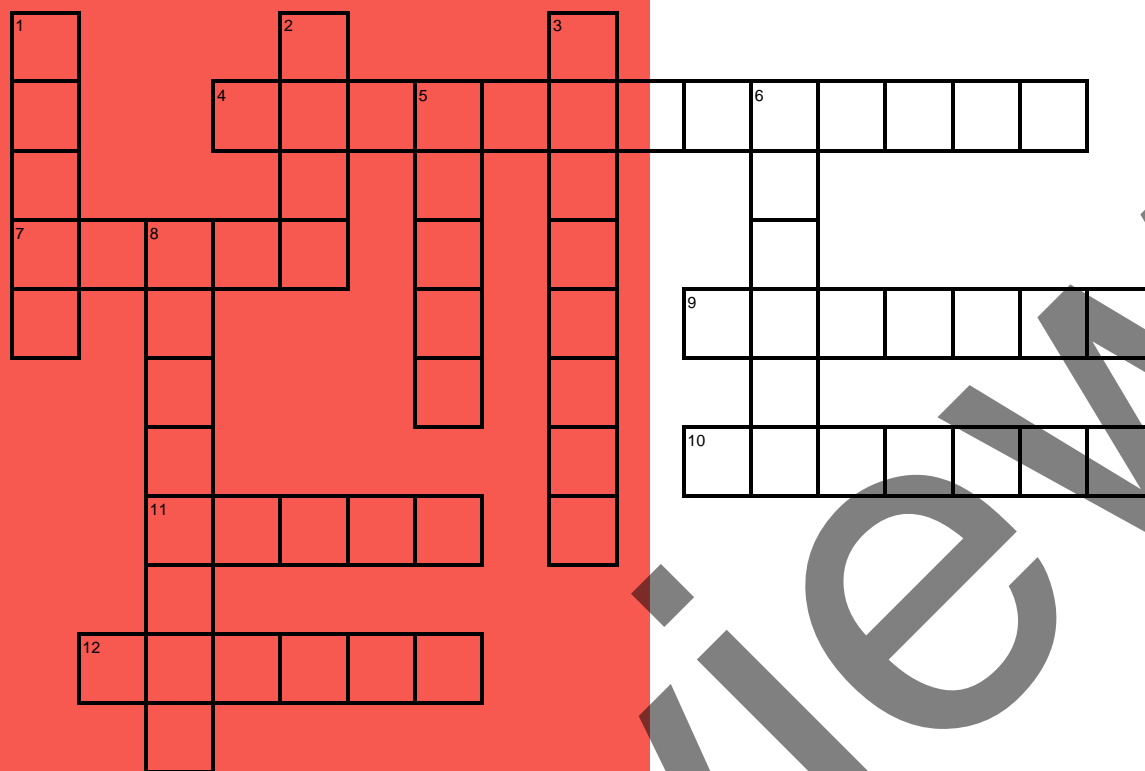
3. Partners, choose another pair of numbered matched cards and discuss the attributes that are the same and those that are different.

**TRUE-FALSE-EXPLAIN**

Your teacher will assign you to work with one or more partners. State whether each statement is true or false. Then write an explanation, give an example, or create a drawing to support your assertion.

1. Any two triangles with the same base and same height have the same area.	2. Any two triangles with the same base and the same height are identical to one another.
3. A triangle can be made with side lengths equal to 5 units, 6 units, and 7 units.	4. A triangle can be made with side lengths equal to 3 units, 4 units, and 9 units.
5. A triangle can be made with two obtuse angles and one acute angle.	6. It is impossible for a triangle to be made with three acute angles.

## VOCABULARY REVIEW

**Across**

- 4 two angles whose sum is  $180^\circ$
- 7 sides of proportional figures have the same \_\_\_\_ factor
- 9 portion of a line
- 10 solid figure with one base and triangular faces
- 11 when a plane slices a solid figure, it creates a \_\_\_\_ section
- 12 one of a kind

**Down**

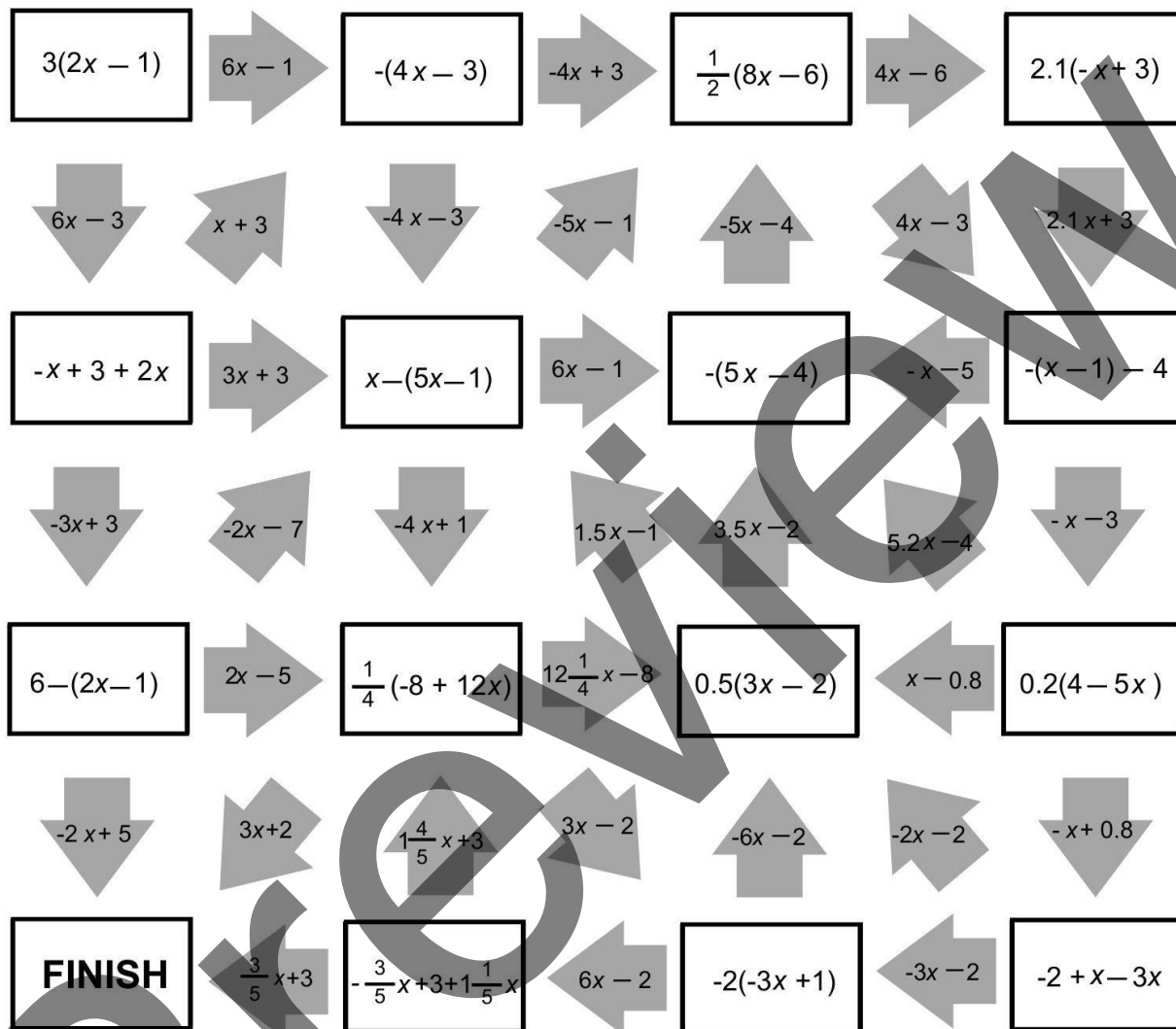
- 1 solid figure with parallel polygon bases
- 2 prism made of square faces
- 3 angles that are “opposite” each other
- 5 infinite two-dimensional flat surface
- 6 number of degrees in complementary angles
- 8 angles that share a vertex and are next to each other



# SPIRAL REVIEW

1. Follow the math path to computational fluency.

**START**



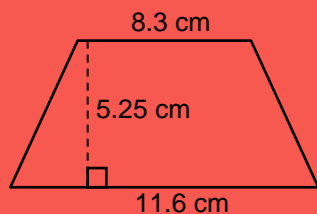
2. Complete the table. Round to the nearest cent.

	10%	25%	2.5%	0.5%	150%
\$45					
\$33.50					
\$12.75					

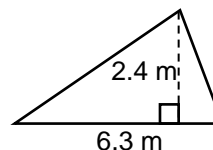
**SPIRAL REVIEW**  
Continued

3. Find the area of each polygon below. Drawings are not to scale.

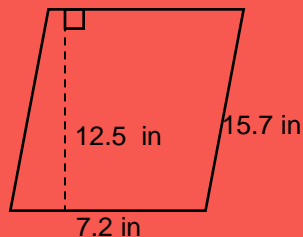
a.



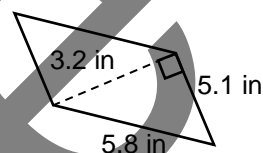
b.



c.



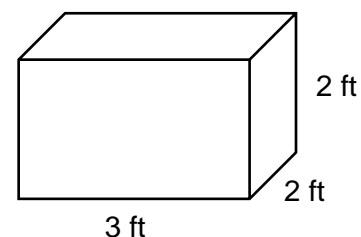
d.



4. Zell is making hacky sacks to sell. Zell puts each hacky sack in a cubic box that has a length of  $\frac{1}{3}$  ft.

a. How many hacky sacks will fit into a cubic box that is 1 foot on each edge?

b. Zell wants to pack them in the shipping box pictured to the right. How many hacky sacks can fit inside this box?



c. What is the volume of the shipping box to the right?

**SPIRAL REVIEW**

Continued

5. Solve each rate problem.

- a. A coffee shop took  $\frac{1}{5}$  of an hour to use  $\frac{1}{6}$  of a package of coffee cups. At this rate, how many hours would it take to use the entire package?
- b. A fun run fundraiser goes through  $1\frac{3}{4}$  boxes of completion medals for  $\frac{1}{10}$  of the participants. How many boxes of medals will they need for all the participants?
- c. It takes  $2\frac{2}{3}$  gallons of paint to completely paint  $1\frac{1}{5}$  rooms. How many gallons would it take to paint 6 similar size rooms?

6. A group of friends are at the beach to play 2-on-2 volleyball. They've already split up into teams of two, but to ensure the games are fair, they discuss their heights before setting up the matches. Below is a list of teams and heights of players.

a. Fill in the table.

Team	A	B	C	D
Players' heights (inches)	60 and 72	64 and 64	65 and 67	53 and 75
Numerical expression for average team height (inches)				
Average team height (inches)				

b. If you were creating two matches with the four teams above, which teams would you match up? Explain your reasoning.

## REFLECTION

1. **Big Ideas.** Shade all circles that describe big ideas in this unit. Draw lines to show connections that you noticed.

Sample to understand populations with statistics. ☐

Solve problems involving measurements of geometric figures. ☐

Develop spatial reasoning in two- and three-dimensions. ☐

Find the likelihood of events with probability. ☐

Apply proportional reasoning to ratios, rates, percent and scale. ☐



Operate with rational numbers and solve problems. ☐

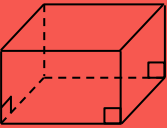
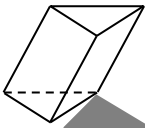
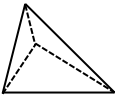
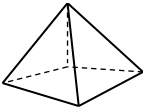
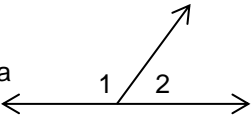
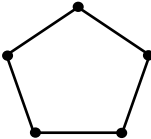
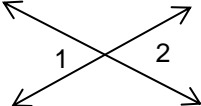
Use algebra as a problem-solving tool. ☐

Give an example from this unit of one of the connections above.

2. **Unit Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before.
3. **Mathematical Practice.** Choose a few tools that were essential to completing your work in this unit [SMP5]. Then circle one more SMP on the back of this packet that you think was addressed in this unit and be prepared to share an example.
4. **Making Connections.** Describe a new insight you have about shapes in space based upon the work you did with 3-dimensional figures in this unit.

## STUDENT RESOURCES

Word or Phrase	Definition
adjacent angles	Two angles are <u>adjacent</u> if they have the same vertex and share a common ray, and they lie on opposite sides of the common ray.  $\angle ABC$ and $\angle CBD$ are adjacent angles.
complementary angles	Two angles are <u>complementary</u> if the sum of their measures is $90^\circ$ .  Two angles that measure $30^\circ$ and $60^\circ$ are complementary.
complementary angles	Two angles are <u>complementary</u> if the sum of their measures is $90^\circ$ .  Two angles that measure $30^\circ$ and $60^\circ$ are complementary.
cross section	The intersection of a solid figure with a plane is a <u>cross section</u> of the figure.
parallel	Two lines in a plane are <u>parallel</u> if they do not meet. Two line segments in a plane are <u>parallel</u> if the lines they lie on are parallel.
perpendicular	Two lines are <u>perpendicular</u> if they intersect at right angles.
plane	A <u>plane</u> refers to a flat two-dimensional surface that has no holes and that extends to infinity in all directions.
polygon	<p>A <u>polygon</u> is a special kind of figure in a plane made up of a chain of line segments laid end-to-end to enclose a region. Each endpoint of a segment of the polygon meets one other segment, otherwise the segments do not meet each other. The line segments are the <u>sides</u> (or <u>edges</u>) of the polygon, and the endpoints of the line segments are the <u>vertices</u> of the polygon. A polygon divides the plane into two regions, an "inside" and an "outside." The region inside a polygon may also be referred to as a <u>polygon</u>.</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  <p>polygons</p> </div> <div style="text-align: center;">  <p>not polygons</p> </div> </div>

Word or Phrase	Definition
prism	<p>A <u>prism</u> is a solid figure in which two faces (the <u>bases</u>) are identical parallel polygons, and the other faces (referred to as the lateral faces) are parallelograms.</p> <p>If the lateral faces are perpendicular to the bases, the prism is a right prism. Otherwise, the prism is an oblique prism.</p> <div style="display: flex; align-items: center; justify-content: center;">  <div style="margin: 0 10px;">← lateral face →</div>  </div> <p>A right rectangular prism is a right prism whose bases are rectangles and faces are rectangles.</p> <p>An oblique triangular prism is a prism whose bases are triangles and faces are parallelograms.</p>
pyramid	<p>A <u>pyramid</u> is a solid figure in which one face (the <u>base</u>) is a polygon, and the other faces are triangles with a common vertex (the <u>apex</u>). Each edge of the base is the side of a triangular face with the opposite vertex at the apex.</p> <p>A <u>triangular</u> pyramid is a pyramid with a triangular base.</p> <p>A <u>square pyramid</u> is a pyramid with a square base. The Egyptian pyramids are examples of square pyramids.</p> <div style="display: flex; align-items: center; justify-content: center;">   </div>
solid figure	A <u>solid figure</u> refers to a figure in three-dimensional space such as a prism or a cylinder.
supplementary angles	<p>Two angles are <u>supplementary</u> if the sum of their measures is <math>180^\circ</math>.</p> <p>Angles 1 and 2 are supplementary because they determine a straight line, or <math>180^\circ</math>.</p> 
vertex	<p>A <u>vertex</u> (pl. vertices) of a polygon or solid figure is a point where two edges meet.</p> <p>A pentagon has five vertices.</p> 
vertical angles	<p>Two angles are <u>vertical angles</u> if they are opposite angles formed by a pair of intersecting lines.</p> <p><math>\angle 1</math> and <math>\angle 2</math> are vertical angles.</p> 

### Symbols and Conventions for Geometry Notation

Below are some geometry notations we will use. Note that we use absolute values to denote lengths of segments and measures of angles. This is consistent with more advanced levels of mathematics.

Points are named by capital letters.

The line segment from  $P$  to  $Q$  is denoted by  $\overline{PQ}$ .



The length of the line segment from  $P$  to  $Q$  is denoted by  $|PQ|$ , which is shorthand for  $|\overline{PQ}|$ .

The symbol for triangle is  $\triangle$ .

- The triangle in Figure 1 below may be denoted by  $\triangle LMN$ , or also by  $\triangle LNM$ . Vertices may be listed in either a clockwise or counterclockwise direction starting from any of the three vertices.

The symbol for angle is  $\angle$ .

- The angle at the top of Figure 1 below can be denoted by  $\angle NLM$ , or by  $\angle a$  or by  $\angle L$ .
- The pair of adjacent angles in Figure 2 below are  $\angle FGJ$  and  $\angle HGF$ . They share the common ray  $\overrightarrow{GF}$ . The two adjacent angles together form the angle  $\angle JGH$ .

**Error alert:** Using " $\angle G$ " to name the angle below is ambiguous. We do not know if it refers to  $\angle JGF$ ,  $\angle FGH$ , or  $\angle JGH$ .

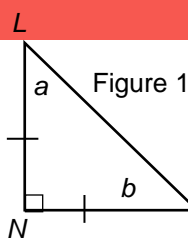
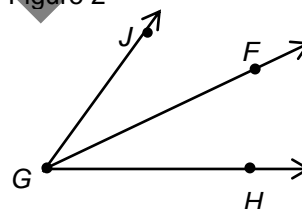


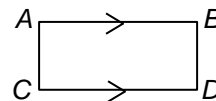
Figure 2



The measure of an angle  $\angle N$  is denoted by  $|\angle N|$ . The small square at  $N$  indicates that  $\angle LNM$  is a right angle, that is, that  $|\angle LNM| = 90^\circ$ .

The single hash marks on the segments  $\overline{LN}$  and  $\overline{NM}$  indicate that the segments have equal length, that is,  $|\overline{LN}| = |\overline{NM}|$ .

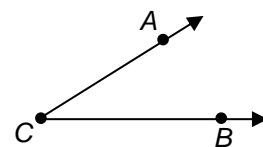
The arrow marks on the segments  $\overline{AB}$  and  $\overline{CD}$  indicate that the segments are parallel.



### Classifying Angles by their Degree Measure

An angle is a geometric shape formed by two (distinct) rays that share a common endpoint (the vertex of the angle).

The angle in the figure to the right can be named any one of the following:


 $\angle ACB$ 

or

 $\angle BCA$ 

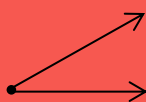
or

 $\angle C$ 

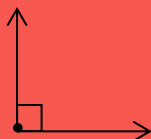
The point  $C$  is the vertex of the angle. The rays  $\overrightarrow{CA}$  and  $\overrightarrow{CB}$  meet at  $C$  and form the sides of the angle.

To each angle is assigned a degree measure between 0 and 180 degrees, which indicates the size of the angle. Angles may be classified by their degree measure.

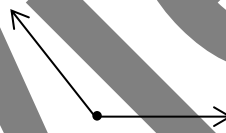
- An acute angle is an angle whose measure is less than  $90^\circ$ .
- A right angle is an angle whose measure is exactly  $90^\circ$ .
- An obtuse angle is an angle whose measure is between  $90^\circ$  and  $180^\circ$ .
- A straight angle is an angle whose measure is  $180^\circ$ . The sides of a straight angle are opposite rays that form a straight line.



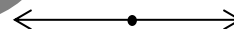
acute angle



right angle



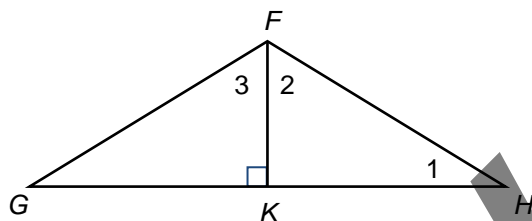
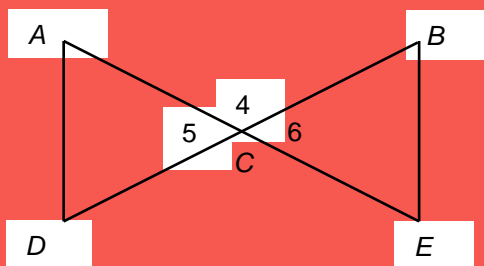
obtuse angle



straight angle



## Special Angle Pairs


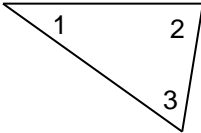


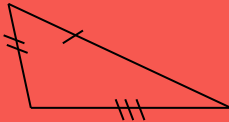
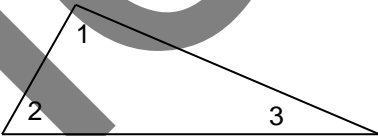


Angle Pairs	Defining Properties	Examples
complementary angles	sum of degree measures is $90^\circ$	$\angle KHF$ and $\angle KFH$ ( $\angle 1$ and $\angle 2$ )
supplementary angles	sum of degree measures is $180^\circ$	$\angle ACB$ and $\angle BCE$ ( $\angle 4$ and $\angle 6$ )
adjacent angles	two angles that share a common vertex and ray, and lie on opposite sides of the ray	$\angle GFK$ and $\angle KFH$ ( $\angle 3$ and $\angle 2$ )
vertical angles	opposite angles formed when two lines intersect	$\angle ACD$ and $\angle BCE$ ( $\angle 5$ and $\angle 6$ )

Some facts about angles:

Any two right angles are supplementary. This is because a right angle measures  $90^\circ$ , so any two right angles have measures with a sum of  $180^\circ$ .

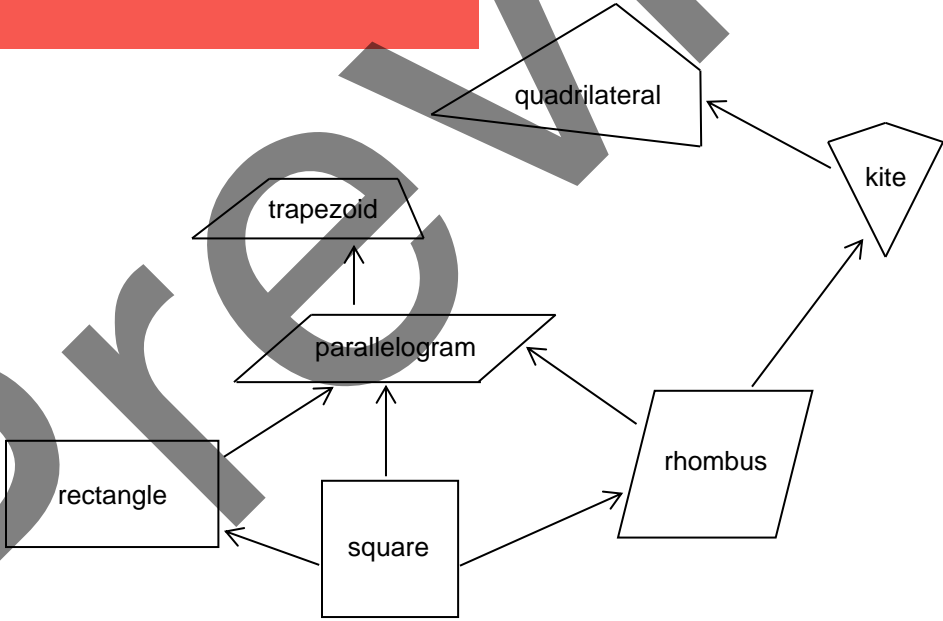
In a right triangle, the two lesser angles are always complementary. This is because the sum of the measures of the angles of a triangle is  $180^\circ$ . Since the right angle measures  $90^\circ$ , the sum of the other two angles must be  $90^\circ$ .

Classifying Triangles	
A triangle is a three-sided polygon. Triangles may be classified by their sides or by their angles.	
Classification by Sides	Classification by Angles
<p>An <u>equilateral</u> triangle is a triangle with three congruent sides.</p> 	<p>An <u>acute</u> triangle is a triangle with three acute angles.</p>  <p><math> \angle 1  &lt; 90^\circ</math>      <math> \angle 2  &lt; 90^\circ</math>      <math> \angle 3  &lt; 90^\circ</math></p>
<p>An <u>isosceles</u> triangle is a triangle with at least two congruent sides.</p> 	<p>A <u>right</u> triangle is a triangle with one right angle.</p>  <p>The square in the corner indicates that the angle measures <math>90^\circ</math>.</p>
<p>A <u>scalene</u> triangle is a triangle with no congruent sides.</p> 	<p>An <u>obtuse</u> triangle is a triangle with one obtuse angle.</p>  <p><math> \angle 1  &gt; 90^\circ</math>      <math> \angle 2  &lt; 90^\circ</math>      <math> \angle 3  &lt; 90^\circ</math></p>
Note that an equilateral triangle is also <u>equiangular</u> because all three angles measure $60^\circ$ .	

Some Properties of Quadrilaterals

A quadrilateral is a four-sided polygon. Some of the common types of quadrilaterals are:

rectangle	A quadrilateral with four right angles. Opposite sides of a rectangle are parallel and have the same length.
square	A quadrilateral with four congruent sides and four right angles. A square is a rectangle.
parallelogram	A quadrilateral in which opposite sides are parallel. Opposite sides of a parallelogram have the same length, and opposite angles have the same measure.
rhombus	A quadrilateral whose four sides have the same length. A square is a rhombus, but a rhombus is not necessarily a square. (The plural of “rhombus” is either “rhombuses” or “rhombi.”)
trapezoid	A quadrilateral with at least one pair of parallel sides.
kite	A quadrilateral whose four sides can be grouped in two pairs of adjacent sides of the same length. The two vertices where the congruent sides meet determine a line of symmetry of the kite.



## COMMON CORE STATE STANDARDS

### STANDARDS FOR MATHEMATICAL CONTENT

<b>7.EE.B</b>	<b>Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</b>
7.EE.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities:
a.	Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$ , where $p$ , $q$ , and $r$ are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. <i>For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?</i>
<b>7.G.A</b>	<b>Draw, construct and describe geometrical figures and describe the relationships between them.</b>
7.G.2	Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
7.G.3	Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
<b>7.G.B</b>	<b>Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.</b>
7.G.5	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

### STANDARDS FOR MATHEMATICAL PRACTICE

SMP1	Make sense of problems and persevere in solving them.
SMP2	Reason abstractly and quantitatively.
SMP3	Construct viable arguments and critique the reasoning of others.
SMP4	Model with mathematics.
SMP5	Use appropriate tools strategically.
SMP6	Attend to precision.
SMP7	Look for and make use of structure.
SMP8	Look for and express regularity in repeated reasoning.

