Name ______________________________ Period __________ Date ______________

UNIT 5
STUDENT PACKET

MathLinks
GRADE 7

RATIONAL NUMBER MULTIPLICATION AND DIVISION

Parent (or Guardian) signature ______________________________

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

exponential notation	integers
inverse operation	product
quotient	rational numbers

OPENING PROBLEM: MORE OF MR. MORTIMER'S MAGIC

[SMP 1, 2]

Merrimack Mortimer is at it again. He decides that he wants to heat up and cool down his liquids faster by putting in and removing pre-made packages of magic cubes. Remember, each cube changes the temperature by 1 degree.

Explain how the temperature of the liquid changes in each of the following situations. Remember that each situation is totally independent.

1. Mortimer puts in 2 packs of 4 hot cubes.

2. Mortimer puts in 5 packs of 4 cold cubes.

3. Mortimer removes 4 packs of 3 hot cubes.

4. Mortimer removes 3 packs of 5 cold cubes.

5. Describe four different ways for Mortimer to make a liquid 24 degrees hotter using pre-made packs.

Mortimer puts in:	Mortimer puts in:
Mortimer removes:	Mortimer removes:

MULTIPLYING AND DIVIDING INTEGERS

We will use a counter model to generalize rules for integer multiplication and extend these rules to integer division. We will use these rules to multiply and divide integers.

[7.NS.1d, 7.NS.2ac, 7.NS.3, 7.EE.3; SMP3, 5, 6, 7, 8]

GETTING STARTED

Compute.

1. 5 + 5 + 5	2. -5 + (-5) + (-5)	3. 0 + (-10) + 10
4. 10 + (-10) + 2 + (-2)	5. 9 – 3 – 3 – 3	6. 0 – (-3) – (-3) – (-3)
7. -4 – (-4)	8. -4 – (-4) + (-4)	9. -4 – (-4) + (-4) – (-4)

For problems 10 – 11 make a drawing as directed. For problems 12 – 13, alter the given drawings as directed.

10. Add 2 groups of 5 **positive** counters to the work space.

    What is the resulting value?

11. Add 2 groups of 5 **negative** counters to the work space.

    What is the resulting value?

12. Subtract 2 groups of 5 **positive** counters from the work space.

    What is the resulting value?

    \+ + + + +
    – – – – –

    \+ + + + +
    – – – – –

13. Subtract 2 groups of 5 **negative** counters from the work space.

    What is the resulting value?

    \+ + + + +
    – – – – –

    \+ + + + +
    – – – – –

MULTIPLYING INTEGERS WITH COUNTERS 1

Use these sentence frames to help think through integer multiplication. **Do not write in these.**

- Begin with a work space that has a value equal to 0.
- The first factor is **positive**. We will place _____ groups on the work space.
- The second factor is ____________ (positive/negative), so each group will contain _____ ____________ (positive/negative) counters.
- The result is _____ ____________ (positive/negative) counter(s).

Follow your teacher's directions for (1) – (2).

(1)	(2)

Compute each product. Record drawings using positive symbols (+) and negative symbols (–).

3. (2) • (4)	4. (2) • (-4)
5. (3) • (2)	6. (3) • (-2)

7. Refer to problems 1 – 6 above to complete these statements.
   - The product of a positive number and a positive number is a ____________ number.
   - The product of a positive number and a negative number is a ____________ number.
   - Putting in packs of hot cubes makes a liquid ____________.
   - Putting in packs of cold cubes makes a liquid ____________.
8. Record the meanings of <u>product</u> and <u>integers</u> in **My Word Bank**.

MULTIPLYING INTEGERS WITH COUNTERS 2

Use these sentence frames to help think through integer multiplication. **Do not write in these.**

- Begin with a work space that has a value equal to 0.
- The first factor is **negative**. We will remove _____ groups from the work space.
- The second factor is ____________ (positive/negative), so each group will contain _____ ____________ (positive/negative) counters.
- Introduce _____ zero pairs to remove these groups.
- The result is _____ ____________ (positive/negative) counter(s).

Follow your teacher's directions for (1) – (2).

(1)	(2)

Compute each product. Record drawings using positive symbols (+) and negative symbols (–).

3. (-2) • (4)	4. (-2) • (-4)
5. (-3) • (2)	6. (-3) • (-2)

7. Refer to the problems above to complete these statements.

- The product of a negative number and a positive number is a ______________ number.
- The product of a negative number and a negative number is a ______________ number.
- Taking out packs of hot cubes makes a liquid ____________.
- Taking out packs of cold cubes makes a liquid ____________.

PRACTICE 1

Compute. Refer to the script from the previous pages and draw pictures as desired.

1. (4) • (-5)	2. (-4) • (3)	3. (-3) • (-5)
4. (3) • (-1)	5. (-5) • (2)	6. (-1) • (-2)

7. Summarize the rules for integer multiplication.

   The product of two positive numbers is ____________.

   The product of two negative numbers is _____________.

   The product of one positive and one negative number is _____________.

Compute without using counters or drawing pictures. If NOT done mentally, show your work.

8. (-3) • (-10)	9. (3) • (-10)	10. (-3) • (10)
11. (-30) • (-10)	12. (-3) • (100)	13. (30) • (-100)
14. (-3) • (17)	15. (-3) • (-241)	16. (-31) • (25)
17. -3 + (-10)	18. 3 + (-10)	19. -3 + 10

RELATING MULTIPLICATION AND DIVISION

1. Record the meanings of quotient and inverse operation in **My Word Bank**.

2. Use the fact that division is the inverse of multiplication to fill in the blanks.

a.	(4) • (_____) = 12	(12) ÷ (4) = _____ (12) ÷ (_____) = 4
b.	(4) • (_____) = -12	(-12) ÷ (4) = _____ (-12) ÷ (_____) = 4
c.	(_____) • (2) = -8	(_____) ÷ (2) = _____ (_____) ÷ (_____) = 2
d.	(-4) • (_____) = 8	(_____) ÷ (-4) = _____ (_____) ÷ (_____) = -4

We will use the shorthand **pos** for a positive number and **neg** for a negative number. Circle the correct result.

3. pos ÷ pos → pos neg	4. neg ÷ neg → pos neg
5. pos ÷ neg → pos neg	6. neg ÷ pos → pos neg

7. How do the rules for multiplying integers compare to the rules for dividing integers?

Compute.

8. -14 ÷ 7	9. 15 ÷ (-3)	10. -25 ÷ (-5)
11. $\frac{-20}{-4}$	12. $\frac{24}{-6}$	13. $\frac{-170}{10}$

PRACTICE 2

Compute.

1. $(-3) \bullet 7$	2. $(-27) \div 3$	3. $\dfrac{-12}{4}$
4. $(-5)(-9)$	5. $(-40) \div (-10)$	6. $\dfrac{-26}{-13}$
7. $(-4)(-40)$	8. $(-200) \div (-50)$	9. $\dfrac{-77}{-11}$
10. $(-600)7$	11. $320 \div -80$	12. $\dfrac{-560}{-70}$
13. $5 - 10$	14. $-5 - 10$	15. $-5 - (-10)$

16. Silvia hides some counters in her left hand and some more in her right hand. Each hand below has either all negatives or all positives. She challenges you to answer each question. Clearly explain your answers.

a. "The product of the amounts in my hands is 50, and the sum is -15. What do I have in each hand?"

b. "The product of the amounts in my hands is -36, and the sum is 9. What do I have in each hand?"

PRACTICE 3

1. During a cold week in Wisconsin, the temperature each day at noon in Fahrenheit was 4°, -6°, -1°, 3°, and 0°. Write a numerical expression that can be used to find the average noontime temperature for the week and simplify the expression.	2. During the same cold week in Wisconsin, the temperature each day at midnight in Fahrenheit was -4°, -6°, -10°, -3°, and -7°. Write a numerical expression that can be used to find the average midnight temperature for the week and simplify the expression.
3. A fish is swimming 15 feet below sea level. a. What number represents the fish's elevation when zero represents sea level? b. A dolphin is swimming 3 times as deep as the fish. What numerical expression represents the elevation that is 3 times the depth of the fish? c. What number represents the elevation of the dolphin?	4. The elevation of water in a lake rose 15 inches per month for 3 months and then dropped 2 feet per month for 4 months. a. Write a numerical expression that can be used to describe the elevation change in inches. Then simplify the expression. b. After 7 months, was the elevation of the lake higher or lower than the starting elevation? c. By how much?

MULTIPLYING AND DIVIDING RATIONAL NUMBERS

We will use number lines, and the inverse relationship between multiplication and division, to extend the multiplication and division rules for integers to the set of rational numbers. We will explore products and quotients involving rational numbers in more depth.

[7.NS.2abc, 7.NS.3; SMP2, 3, 6, 7, 8]

GETTING STARTED

1. Write the following integers as fractions or mixed numbers.

a. 7	b. -13
c. 0.7	d. -1.3

2. Write the following mixed numbers as improper fractions.

a. $3\frac{2}{5}$	b. $-2\frac{1}{6}$

3. Write the following improper fractions as mixed numbers.

a. $\frac{11}{4}$	b. $\frac{-20}{3}$

4. Record the meaning of rational numbers in **My Word Bank**.

5. Why are all of the numbers in problems 1 – 3 above rational?

NUMBER LINE MULTIPLICATION

Follow your teacher's directions for (1) – (15).

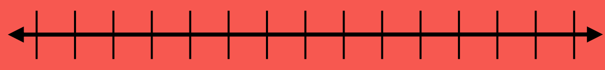
(1) 0 1	(2) 0 0.2	(3) 0 2 4
(4) -1 0	(5) -0.3 0	(6) -4 -3 0
(7) -1 0	(8) -0.3 0	(9) -4 -3 0
(10) 0 1	(11) 0 0.2	(12) 0 2 4
(13) 0 1	(14) -1 0	(15) -1 0

16. Do the multiplication rules we learned in previous lessons hold for (1) – (15) above? ____
    Do you think that these rules hold for all rational number multiplication?

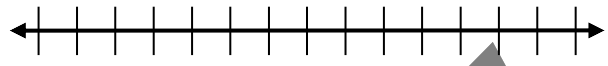
PRACTICE 4

Predict each product. Then compute using arrows. Number the tick marks appropriately.

1. $(2)(3)$

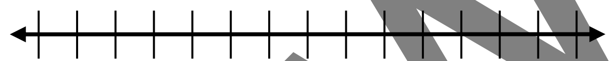


2. $(2)(-3)$

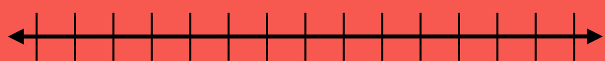


3. $(-2)(3)$

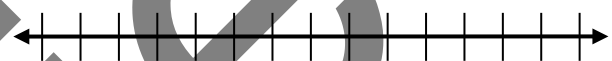
4. $(-2)(-3)$



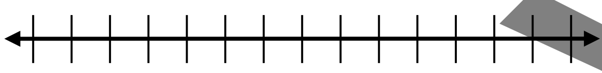
5. $(4)(0.25)$



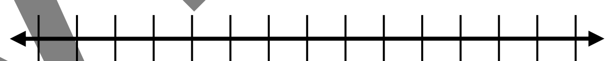
6. $(-2)\left(\frac{3}{4}\right)$



7. $\left(\frac{1}{2}\right)(-5)$

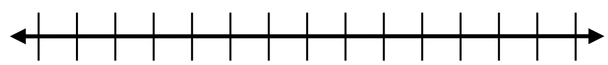


8. $\left(-\frac{1}{3}\right)(-0.6)$



9. $(-4)(-0.25)$

10. $\left(-\frac{1}{3}\right)(0.6)$



11. Use the fact that multiplication and division are inverse operations, along with the results from problems 5, 6, 7, and 8 above, to find each quotient.

$1 \div 0.25$ $\qquad -\frac{3}{2} \div (-2)$ $\qquad -\frac{5}{2} \div \frac{1}{2}$ $\qquad -0.2 \div 0.6$

12. Focus on the signs of the dividend, divisor, and quotient in the examples above. Why do these equations suggest that the rules for integer division hold for all rational numbers?

PRACTICE 5

Complete the puzzle below using the given expression. Then find total sums of rows and columns (exclude the gray numbers). Round decimals to the nearest 100th. Make sure the sums are equal for the very bottom row and far right column.

			b		
	(*a*)(*b*)	**-12**		**0.2**	**TOTAL SUMS (ROWS)**
		7.2			
a	$3\frac{2}{3}$				
		12,000	500		
	TOTAL SUMS (COLUMNS)				**Check** (row sums total equals column sums total)

DETERMINING THE SIGN OF A PRODUCT

Compute each product.

1. (-1) • (-2) • (3)	2. (-1) • (-2) • (-3)	3. (-1) • (-2) • (3) • (-4)
4. (-1) • (-2) • (-3) • (-4)	5. (-1) • (2) • (-3) • (4)	6. (-1) • (0) • (-3) • (-4)

7. Make conjectures about multiplying **nonzero** numbers.

   a. If there are an odd number of negative factors, the product is ______________.
   positive / negative

   b. If there are an even number of negative factors, the product is ______________.
   positive / negative

Without computing, determine whether each product is positive, negative, or zero.

8. (-7)(-9)(11)(-24)	9. (-0.7)(1.9)(0.8)(-2.6)	10. (-8.02)(-3.9)(0)(-5.24)

Write <, >, or = for each.

11. (-1) • (-1) • (1) _____ (-1) • (-1) • (-1)	
12. (-2) • (-3) • (-4) • (10) _____ -2 • (-3) • (-4) • (-10)	
13. 6(-5)(-2) _____ (-6)(5)(2)	
14. (-2) • (-3) • (-4) • (10) _____ -2 • (3) • (-4) • (-10)	
15. $\frac{-40}{10}$ _____ $\frac{-40}{-10}$	16. $\frac{-36}{-12}$ _____ $-\left(-\frac{36}{12}\right)$
17. -4 + (-8) _____ -4 – (-8)	18. -2 – 6 _____ -2 + (-6)

19. Compute. $\left(-\frac{2}{3}\right)\left(-1\frac{1}{5}\right)\left(-2\frac{1}{8}\right)$

DETERMINING THE SIGN OF A QUOTIENT

Divide each fraction below. Determine whether the quotient is positive or negative based upon integer division rules. If the quotient is not an integer, write it as a fraction in simplest form.

1. $-\frac{10}{5}$	2. $\frac{-10}{-5}$	3. $\frac{-10}{5}$	4. $\frac{10}{-5}$	5. $-\left(-\frac{10}{5}\right)$
6. $-\frac{4}{16}$	7. $\frac{-4}{-16}$	8. $\frac{-4}{16}$	9. $\frac{4}{-16}$	10. $-\left(-\frac{4}{16}\right)$

11. For the expressions below, a and b are positive integers. Circle the expressions below that represent negative numbers.

$\frac{a}{b}$ $\frac{-a}{-b}$ $\frac{-a}{b}$ $\frac{a}{-b}$ $-\left(\frac{a}{b}\right)$

12. Mariam says that $\frac{-2}{-7}$ and $-\frac{2}{7}$ represent the same number. Is Mariam correct? _____
Explain.

13. Yunus says that $\frac{-12}{-42}$ and $\frac{12}{42}$ represent the same number. Is Yunus correct? _____
Explain.

14. How do you know whether the quotient of two integers will be a positive number?

15. How do you know whether the quotient of two integers will be a negative number?

16. How do you know whether the quotient of two integers will be an integer?

WRITING RATIONAL NUMBERS IN DIFFERENT FORMS

Write each rational number below in at least three different equivalent forms.

1. $\frac{-8}{16}$	2. $\frac{-8}{-6}$	3. $-\frac{13}{5}$
4. $\frac{0}{13}$	5. $\frac{18}{-2}$	6. $\frac{-60}{20}$

Write each number in the form described in the definition of rational number to show that they are rational.

7. -12	8. 4.75	9. $-3\frac{1}{2}$

10. Brecken says that $-5\frac{2}{3}$ and $\frac{17}{-3}$ represent the same number. Is Brecken correct? _____
    Explain.

11. For the expressions below, *a* is a positive integer and *b* is a negative integer. Circle the expressions below that represent negative numbers.

$\frac{a}{b}$ $\frac{-a}{-b}$ $\frac{-a}{b}$ $\frac{a}{-b}$ $-\left(\frac{a}{b}\right)$

12. Choose one of the positive expressions from problem 11 (un-circled) and explain how you know it is positive. Use a numerical example.

13. Choose one of the negative expressions from problem 11 (circled) and explain how you know it is negative. Use a numerical example.

EXPLORING DIVISION INVOLVING ZERO

Fill in the blanks and answer the questions in the table below.

	Statement/Question	Division Expression	Does the question make sense mathematically? What is the answer?
1.	Four friends are equally sharing 16 grapes. How many grapes does each friend get?	$\frac{\square}{\square}$	
2.	Four friends are equally sharing 0 grapes. How many grapes does each friend get?	$\frac{\square}{\square}$	
3.	Four friends are equally sharing 2 strawberries. How many strawberries does each friend get?	$\frac{\square}{\square}$	
4.	Zero friends are equally sharing 15 strawberries. How many strawberries does each friend get?	$\frac{\square}{\square}$	

Mathematically, we say that division *by* zero is **undefined**.

Fill in each box with a solution if one exists, an **N** if no solution exists, or an **I** if an infinite number of solutions exist.

5a. $2 \bullet \square = 8 \quad \rightarrow \quad \frac{8}{2} = \square$	5b. $0 \bullet \square = 8 \quad \rightarrow \quad \frac{8}{0} = \square$
6a. $10 \bullet \square = 0 \quad \rightarrow \quad \frac{0}{10} = \square$	6b. $0 \bullet \square = 0 \quad \rightarrow \quad \frac{0}{0} = \square$

PRACTICE 6

Compute, if possible.

1. $-20 \bullet (-30) \bullet (-200)$	2. $-80 \div 10$	3. $(-10)(-20)(30)$
4. $64 \div (-8)$	5. $(-1)(-2)(-3)(-4)(-5)$	6. $-60 \div (-30)$
7. $(-12)(0)(-13)(210)$	8. $0 \div 10$	9. $20 \div 0$
10. $(-17)(53)(0)(-27)$	11. $-120 + 20$	12. $-80 + (-40)$
13. $-30 + 70$	14. $100 – (-200)$	15. $100 – 200$
16. $\frac{0}{3}$	17. $-100 – (-200)$	18. $\frac{3}{0}$
19. $\frac{-45}{-9}$	20. $-\left(\frac{-36}{6}\right)$	21. $-\left(-\frac{28}{7}\right)$

22. Why is $\frac{-10}{5}$ not equal to $\frac{-10}{-5}$?

23. If the product of six integers is negative, at most how many of the integers can be negative?

24. L.D. hid some counters in each hand. Each hand had either all negatives or all positives. L.D. said to some friends, “The sum of the amounts in my hands is -12 and the product is -28. What do I have in each hand?” How should her group respond?”

ORDER OF OPERATIONS

We will make sense of the order of operations conventions and solve problems involving rational numbers.

[7.NS.1d, 7.NS.2abc, 7.NS.3, 7.EE.3; SMP2, 3, 6]

GETTING STARTED

Put the following statements in an order you think makes the most sense. Then predict whether you think most of your classmates will agree with you or not.

1. _____ tie your shoelaces

   _____ put on your socks

   _____ put on your shoes

   Prediction:

2. _____ eat dinner

   _____ do homework

   _____ do something recreational like playing basketball or drawing a picture.

   Prediction:

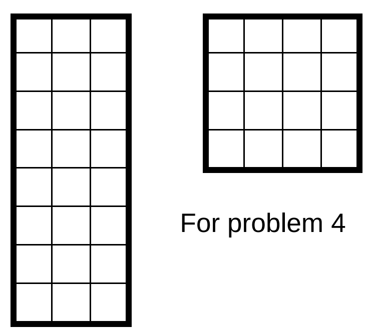
You **do not** need to calculate anything for the following problems. Place operation symbols between the symbols to make numerical expressions that are correct translations of the situation.

3. The cost of buying 2 bottles of juice for $1.50 each and 3 bags of pretzels for $2.00 each.

   2 _____ 1.5 _____ 3 _____ 2

4. The total area of the two rectangles to the right combined.

   3 _____ 8 _____ 4 _____ 4



For problem 4

EXPONENTS

1. Record the meaning of <u>exponential notation</u> in **My Word Bank**.

Write each expression as an appropriate product. Then compute.

2. 5^2 = ______ • ______ = ___________

3. 3^4 = ______________________________________ = ___________

4. 17^1 = ______________________________________ = ___________

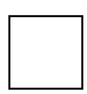
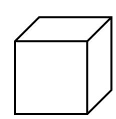
5. Horton thinks that $2^3 = 6$. Explain Horton's mistaken thinking.

Compute.

6. 4^2	7. 2^5	8. 4^3
9. 6^2	10. $3^2 + 3^4$	11. $2^3 \bullet 1^7$

Write each of the following as a base with an exponent.

12. $2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2$	13. $6 \bullet 6$	14. $0 \bullet 0 \bullet 0$

15. Label the side lengths of this square, which are each equal to 4 cm. Write an expression for the area of the square using an exponent. 	16. Label the edge lengths of this cube, which are each equal to 4 cm. Write an expression for the volume of the cube using an exponent. 

17. Why do you think we call a number to the second power "squared," and a number to the third power "cubed?"

THE ORDER OF OPERATIONS CONVENTIONS

Follow your teacher's directions for (1) – (5).

(1)	(2)
ORDER OF OPERATIONS (3) A. Simplify expressions that are _______________. B. Compute expressions with _______________. C. Perform _______________ and _______________ from left to right. D. Perform _______________ and _______________ from left to right.	
(4)	(5)

6. Simplify the expression: $\dfrac{9}{1-4} + \dfrac{-20 + 3 \bullet 2^2}{10-(1+5)}$

PRACTICE 7

Compute.

1. $\dfrac{(-6)(-2) - 2}{-10}$	2. $\dfrac{-1 + (-1) + 4^2}{2}$
3. $\dfrac{2 - (-2)}{-6 - 2}$	4. $\dfrac{-2 - (-2)}{-6 - 2}$
5. $20 + (-8 + 6) - 3^2 \bullet 2$	6. $20 + -8 + (6 - 3^2) \bullet 2$
7. $10 - 14 \div 7 - 3 \bullet 4$	8. $10 - (14 \div 7 - 3 \bullet 4)$

Use all four of the numbers 2, 3, 4, and 5 exactly once in each problem below. Use any of the four operations and any grouping symbols as needed.

9. Write an expression that is equal to 1.	10. Write an expression that is equal to 6.

PRACTICE 8

Here are two equivalent equations for converting between the Celsius and Fahrenheit scales.

Let C = degrees Celsius and F = degrees Fahrenheit

$$F = \frac{9}{5}C + 32 \qquad C = \frac{5}{9}(F - 32)$$

1. A national championship football game played on December 31, 1967 between teams from Green Bay, WI and Dallas, TX became known as the "Ice Bowl." The low temperature for that game was 13 degrees below zero (F).

   a. Write this temperature as an integer.

   b. Choose one of the equations above. Substitute this value to solve for C.

2. The weather report before another football playoff game played on January 5, 2014 (also played in Green Bay) was expected to be 17 degrees below zero (F).*

   a. Write this temperature as an integer.

   b. Choose one of the equations above. Substitute this value to solve for C.

   Is this temperature warmer or colder than the Ice Bowl in 1967? __________

3. A soccer match in Trondheim, Norway in December, 2010 reported a kickoff temperature of -14°C. What is this temperature in degrees Fahrenheit?

4. In Sochi, Russia, the historical average high temperature for January is about 50°F. When they hosted a major winter sports event in 2014, temperatures reached 20°C. Is this temperature higher or lower than the historical average high, and by how much?

(*The temperature that day never actually reached the record low.)

PRACTICE 9: EXTEND YOUR THINKING

Recall that the commutative, associative, and distributive properties allow us to operate on numbers in different orders. Use these properties to make the following calculations easier. Describe your process.

1. $3[-17 + (-11) + 17]$	2. $-\frac{11}{17}\left[-\frac{2}{5} + 2 + \left(-\frac{3}{5}\right)\right]$
3. $2\left(-\frac{1}{3}\right) + 7\left(-\frac{1}{3}\right)$	4. $\left(-\frac{7}{13}\right)(1) - \left(-\frac{7}{13}\right)(4) + \left(-\frac{7}{13}\right)(3)$

Prove whether each expression represents a rational number or not. In other words, show whether the expression **can be** written in the form $\frac{a}{b}$, a and b are both integers, and $b \neq 0$.

5. $-17 \div (-3)$	6. $\frac{-12(-3)}{1 - 7}$
7. $\frac{5^2 - 25}{-2\frac{1}{4} - 3\frac{3}{4}}$	8. $\frac{5 - 18}{-\frac{1}{2} + \frac{2}{3} - \frac{1}{6}}$

REVIEW

OPEN MIDDLE: RATIONAL NUMBER MULTIPLICATION AND DIVISION

Your teacher will turn over 4 integer cards.

Record their values: _______ _______ _______ _______

For each problem below, write an expression using the four numbers above exactly once each. Show your work. You may use any of the four operations and any grouping symbols you know.

1. Write an expression with a value as close to 1 as possible. Expression: ______________________	2. Write an expression with a value as close to -1 as possible. Expression: ______________________
3. Write an expression with the greatest value possible. Expression: ______________________	4. Write an expression with the least value possible. Expression: ______________________

POSTER PROBLEMS: RATIONAL NUMBER MULTIPLICATION AND DIVISION

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is __________.
- Each group will have a different colored marker. Our group marker is __________.

Part 2: Do the problems on the posters by following your teacher's directions.

Round 1:

Poster #	1	2	3	4	5	6	7	8
Start #	1	2	3	4	5	6	7	8

A. Create a 4-column chart like the one below to use for steps 1 – 7:

step number	step directions	Round 1 work	Round 2 work

B. Do Step 1: Copy your start number from the table above onto your chart.
   Do Step 2: Multiply the start number by -4.

C. Do Step 3: Add -10 to the result.
   Do Step 4: Subtract -6 from the result.

D. Do Step 5: Divide the result by -4.
   Do Step 6: Subtract the given start number from the result.

E. Do Step 7: Add -1 to this result. Circle this number.

(For **Round 2**, change A – D roles, and start over with the opposite reciprocal of your start number. For example, a group that started with 12 in Round 1, would now start with $-\frac{1}{12}$.)

Part 3: Return to your seats. Work with your group.

1. Was the circled number on every poster the same? _____

2. If not, use a start number given to you by your teacher and rework the problem.

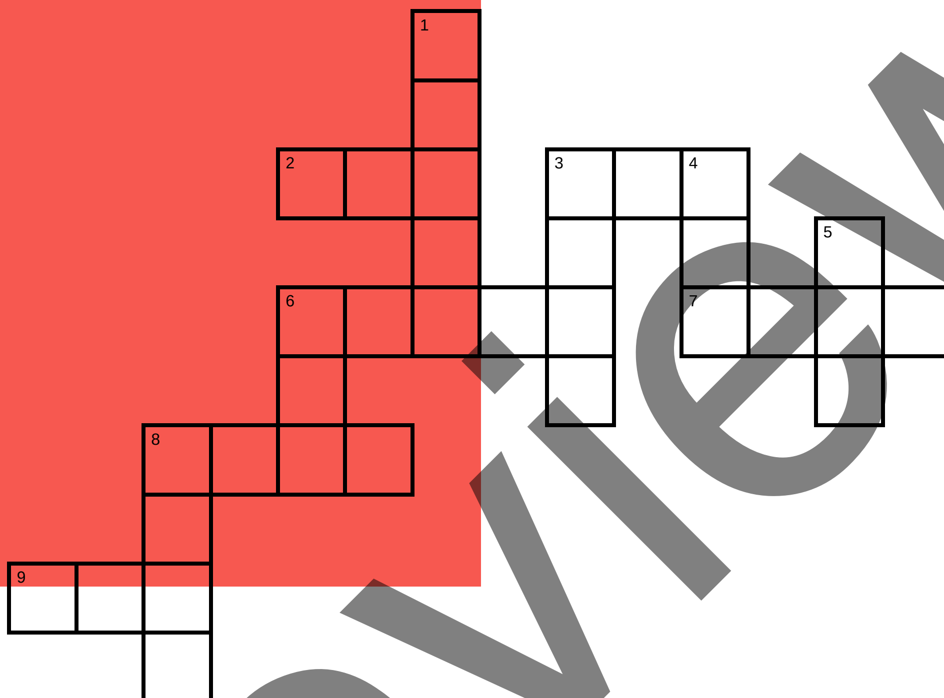
ORDER OF OPERATIONS PAIR SHARE

Partner A

- Do “across” problems on another piece of paper.
- Check B’s work on the “down” problems using a calculator.

Partner B

- Do “down” problems on another piece of paper.
- Check A’s work on the “across” problems using a calculator.



Across

2. $\left(\dfrac{-8-2}{-4-6}\right) + 379$

3. $\left(\dfrac{12-18}{-3}\right) + 245$

6. $\left(\dfrac{-3-4 \bullet 2-5}{1-2}\right) \bullet (1{,}000)$

7. $-(-600 - 800 - 200)$

8. $\left(15-\dfrac{21}{3}+12\right) \bullet (80)$

9. $-4 - (-3)(-6) + 22 + 150$

Down

1. $[100 + (-2)(-2)(-2)] \bullet (1{,}000)$

3. $[-3 - 5(-6)] \bullet (100)$

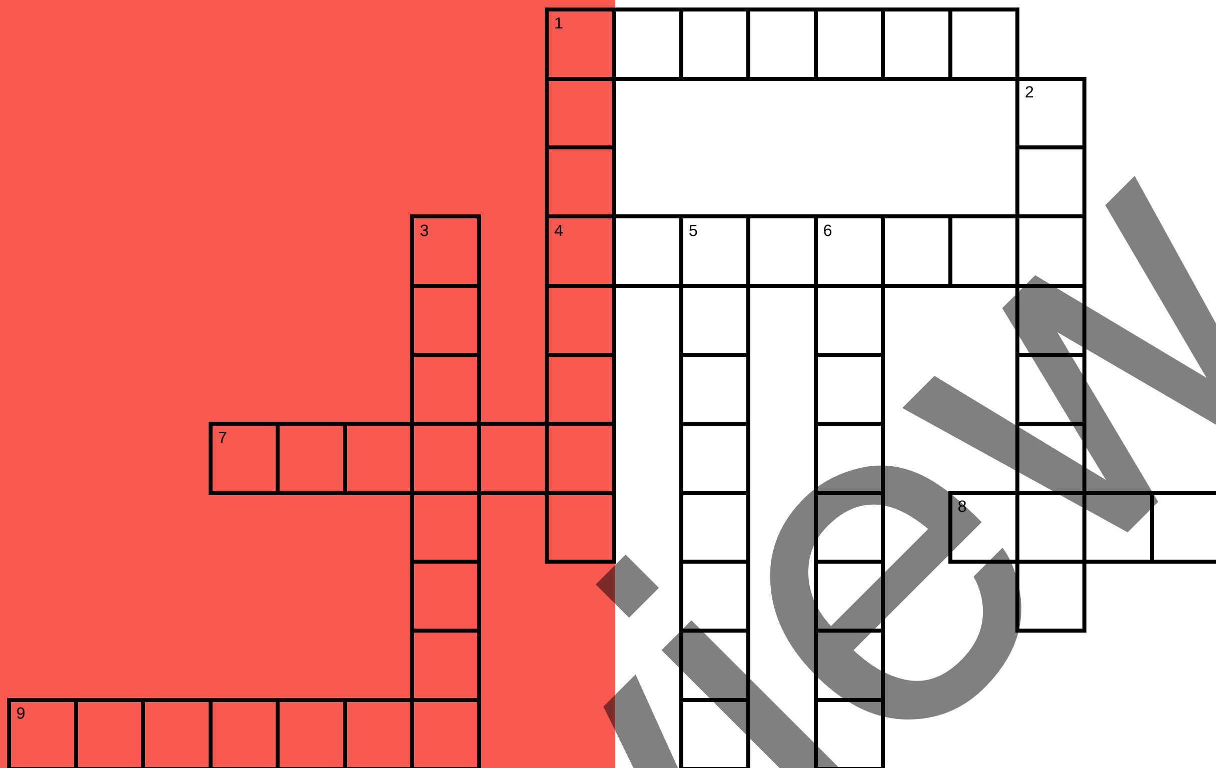
4. $\left(\dfrac{-8+4-6}{-11+1}\right) + 720$

5. $\left(\dfrac{-5-3 \bullet 5}{-7-3}\right) \bullet (200)$

6. $-4 + \left(\dfrac{-16}{2}\right)(-3-1) + 72$

8. $100(24 - 2 \bullet 3)$

VOCABULARY REVIEW



Across

1 multiplication and division are ___ operations

4 in 3^2, 2 is the ___

7 in 4 • 9 = 36, 4 is a ___

8 in 3^2, 3 is the ___

9 in 4 • 9 = 36, 36 is the ___

Down

1 the numbers …, -3, -2, -1, 0, 1, 2, 3, … are called ___

2 a ___ number is a number that can be written in the form $\frac{m}{n}$, where m and n are integers, and $n \neq 0$

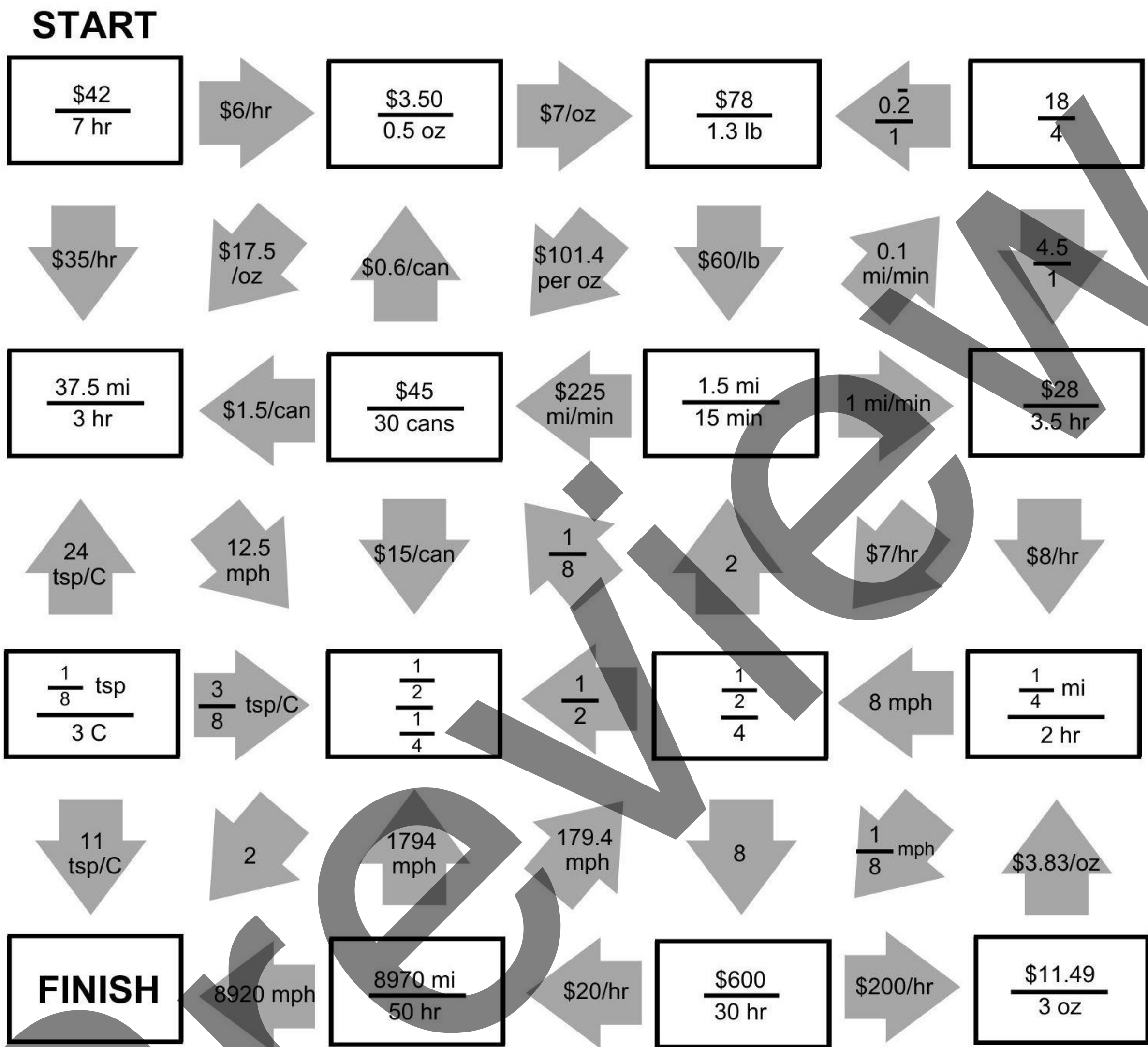
3 in 14 ÷ 2 = 7, 7 is the ___

5 a number that is greater than zero is ___

6 a number that is less than zero is ___

SPIRAL REVIEW

1. Follow the math path to computational fluency.



2. Complete the table.

Fraction			$\frac{25}{20}$			$\frac{6}{64}$
Decimal	0.125			0.064		
Percent		40%			$16.\overline{6}\%$	

SPIRAL REVIEW
Continued

3. An art supply store sells colored pencils in different sets.
   - Set A: \$5.29 for 24 pencils
   - Set B: \$7.69 for 50 pencils
   - Set C: \$5.19 for 18 pencils

   Find the unit rate in pencils per dollars for each set. Clearly show which is the best deal.

4. While exercising Cheyenne walked $\frac{1}{3}$ of a mile in $\frac{1}{8}$ of an hour. At this rate, how far will Cheyenne have traveled in an hour?

5. Solve each equation below.

a. $m + 43 = 91$	b. $15n = 165$
c. $83.5 = x - 12.2$	d. $60.12 = 0.6y$
e. $\frac{1}{6} + m = 5$	f. $12 = \frac{1}{4}a$
g. $y - 2\frac{1}{2} = 3\frac{1}{8}$	h. $2\frac{1}{5}n = 8\frac{4}{5}$

REFLECTION

1. **Big Ideas**. Shade all circles that describe big ideas in this unit. Draw lines to show connections that you noticed.

   - Find the likelihood of events with probability. (7.SP.C)
   - Sample to understand populations with statistics. (7.SP.AB)
   - Apply proportional reasoning to ratios, rates, percent and scale. (7.RP.A)
   - Solve problems involving measurements of geometric figures. (7.G.B)
   - Operate with rational numbers and solve problems. (7.NS.A)
   - Develop spatial reasoning in two- and three-dimensions. (7.G.A)
   - Use algebra as a problem-solving tool. (7.EE.AB)

   Give an example from this unit of one of the connections above.

2. **Unit Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before.

3. **Mathematical Practices.** How did the relationship between multiplication and division help you to make sense of these rational number operations? Give an example [SMP 7, 8]. Then circle one more SMP on the back of this packet that you think was addressed in this unit and be prepared to share an example.

4. **Making Connections.** Look back at the patterns and rules you established for multiplying and dividing negative numbers in Lesson 2. Which pattern did you find most useful or interesting?

STUDENT RESOURCES

Word or Phrase	Definition
distributive property	The <u>distributive property</u> states that $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for any three numbers a, b, and c. $3(4 + 5) = 3(4) + 3(5)$ and $(4 + 5)8 = 4(8) + 5(8)$
exponential notation	The <u>exponential notation</u> b^n (read as " b to the <u>power</u> n ") is used to express n factors of b. The number b is the <u>base</u>, and the number n is the <u>exponent</u>. $2^3 = 2 \cdot 2 \cdot 2 = 8$. The base is 2 and the exponent is 3. $3^2 = 3 \cdot 3 = 9$. The base is 3 and the exponent is 2.
integers	The <u>integers</u> are the whole numbers and their opposites. They are the numbers 0, 1, 2, 3, … and -1, -2, -3, … .
inverse operation	The <u>inverse operation</u> to a mathematical operation reverses the effect of the operation. Addition and subtraction are inverse operations. Multiplication and division are inverse operations.
product	A <u>product</u> is the result of multiplying two or more numbers or expressions. The numbers or expressions being multiplied to form the product are <u>factors</u> of the product. $\underbrace{3}_{\text{factor}} \bullet \underbrace{5}_{\text{factor}} = \underbrace{15}_{\text{product}}$
quotient	In a division problem, the <u>quotient</u> is the result of the division. $\underbrace{12}_{\text{dividend}} \div \underbrace{3}_{\text{divisor}} = \underbrace{4}_{\text{quotient}}$
rational numbers	<u>Rational numbers</u> are numbers expressible in the form $\frac{m}{n}$, where m and n are integers, and $n \neq 0$. $\frac{3}{5}$ is rational because it is a quotient of integers. $2\frac{1}{3}$ and 0.7 are rational numbers because they **can be** expressed as quotients of integers, namely $\frac{7}{3}$ and $\frac{7}{10}$, respectively. $\sqrt{2}$ and π are NOT rational numbers. They cannot be expressed as a quotient of integers. $\frac{7}{0}$ is undefined. It is NOT a rational number.

Symbols for Multiplication

The product of 8 and 4 can be written as:

8 times 4 8×4 $8 \bullet 4$ $(8)(4)$ $\begin{array}{r} 8 \\ \underline{\times\ 4} \end{array}$

The product of 8 and the variable x is written simply as $8x$. We are cautious about using certain symbols for multiplication. The $\times$ could be misinterpreted as the variable x and the $\bullet$ could be misinterpreted as a decimal point.

Symbols for Division

The quotient of 8 and 4 can be written as:

8 divided by 4 $8 \div 4$ $4\overline{)8}$ $\frac{8}{4}$ 8/4

In algebra, the preferred way to show division is with fraction notation.

Mr. Mortimer's Magic Hot and Cold Cubes for Multiplication

Mr. Mortimer discovered an amazing way to control the temperature of liquid. He invented magic hot and cold cubes to change the liquid's temperature. These magic cubes never melt or change in any way. For example, ice cubes melt, but magic cold cubes do not.

Hot Cubes (the basics):

- If you add 1 hot cube to a liquid, the liquid heats up by 1 degree.
- If you remove 1 hot cube from the liquid, the liquid cools down by 1 degree.

For multiplication:

- If you put in packs of hot cubes to a liquid, the liquid heats up.
  For example, adding 2 packs of 10 hot cubes is like adding $2 \bullet 10 = 20$ hot cubes.
  The liquid heats up by 20 degrees.
- If you take out packs of hot cubes from a liquid, the liquid cools down.
  For example, subtracting 2 packs of 10 hot cubes is like subtracting $2 \bullet 10 = 20$ hot cubes.
  The liquid cools down by 20 degrees.

Cold Cubes (the basics):

- If you add 1 cold cube to the liquid, the liquid cools down by 1 degree.
- If you remove 1 cold cube from the liquid, the liquid heats up by 1 degree.

For multiplication:

- If you put in packs of cold cubes to a liquid, the liquid cools down.
  For example, adding 2 packs of 10 cold cubes is like adding $2 \bullet 10 = 20$ cold cubes.
  The liquid cools down by 20 degrees.
- If you take out packs of cold cubes from a liquid, the liquid heats up.
  For example, subtracting 2 packs of 10 cold cubes is like subtracting $2 \bullet 10 = 20$ cold cubes.
  The liquid heats up by 20 degrees.

Counter Multiplication Sentence Frames

- Begin with a workspace that has a value equal to 0.
- **If the first factor is positive**, we will place _____ groups on the workspace.
  If the first factor is negative, we will remove _____ groups on the workspace.
- The second factor is ____________ (positive/negative), so each group will contain _____ ____________ (positive/negative) counter(s).
- Introduce _____ zero pairs to remove these groups (if needed).
- The result is _____ ____________ (positive/negative) counter(s).

Integer Multiplication Using Counters

2(4) = 8

+ + + +
+ + + +

- Start with a work space equal to zero.
- The first factor is positive.
  We will put 2 groups on the workspace.
- The second factor is positive.
  Each group will contain 4 positive counters.
- [No zero pairs needed.]
- The result is 8 positive counters.

2(- 4) = - 8

− − − −
− − − −

- Start with a work space equal to zero.
- The first factor is positive.
  We will put 2 groups on the workspace.
- The second factor is negative.
  Each group will contain 4 negative counters.
- [No zero pairs needed.]
- The result is 8 negative counters

- 2(4) = -8

~~+ + + +~~ − − − −
~~+ + + +~~ − − − −

- Start with a work space equal to zero.
- The first factor is negative.
  We will remove 2 groups from the workspace.
- The second factor is positive. Each group will contain 4 positive counters.
- Introduce at least 8 zero pairs.
- The result is 8 negative counters.

- 2(- 4) = 8

+ + + + ~~− − − −~~
+ + + + ~~− − − −~~

- Start with a work space equal to zero.
- The first factor is negative.
  We will remove 2 groups from the workspace.
- The second factor is negative. Each group will contain 4 negative counters.
- Introduce at least 8 zero pairs.
- The result is 8 positive counters.

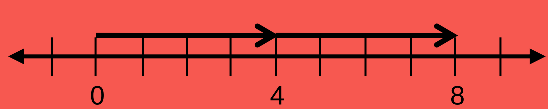
Rules for Multiplication of Integers	
Rule 1:	The product of two numbers with the same sign is a positive number. Think: (+)(+) = (+) and (–)(–) = (+)
Rule 2:	The product of two numbers with opposite signs is a negative number. Think: (+)(–) = (–) and (–)(+) = (–)

Multiplication on a Number Line

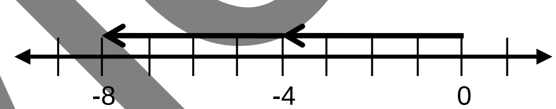
We can use arrows to represent multiplication on a number line. One interpretation for **multiplying** any two numbers is:

- The first factor tells us the number of arrows. The second factor tells us the length of each arrow.
- If the length of the arrow (second factor) is a positive number, then the arrow goes to the right. If the length of the arrow is a negative number, then the arrow goes to the left.
- If the number of arrows (first factor) is positive, then the number line diagram is complete. If the number of arrows is negative, then the entire diagram is reflected!

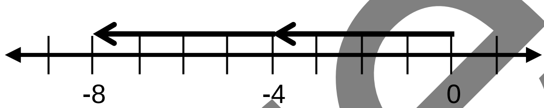
1. 2(4) = 8



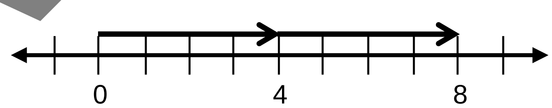
2. 2(-4) = -8



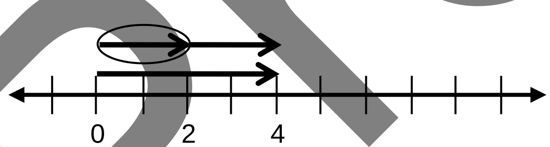
3. -2(4) = -8
(Example 1 reflected)



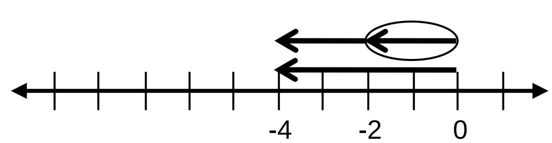
4. -2(-4) = 8
(Example 2 reflected)



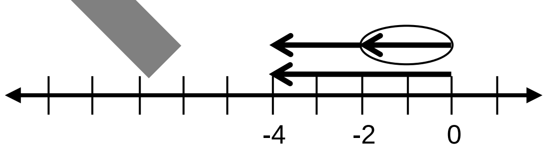
5. $\frac{1}{2}(4) = 2$



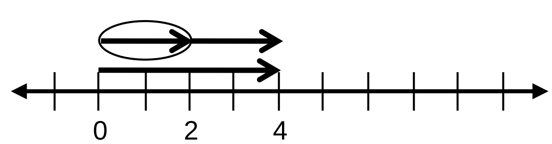
6. $\frac{1}{2}(-4) = -2$



7. $-\frac{1}{2}(4) = -2$
(Example 5 reflected)



8. $-\frac{1}{2}(-4) = 2$
(Example 6 reflected)



Rules for Division of Integers	
Rule 1:	The quotient of two numbers with the same sign is a positive number.
	Think: $\frac{(+)}{(+)} = (+)$ and $\frac{(-)}{(-)} = (+)$
Rule 2:	The quotient of two numbers with opposite signs is a negative number.
	Think: $\frac{(+)}{(-)} = (-)$ and $\frac{(-)}{(+)} = (-)$

Mathematical Separators

Parentheses ( ) and square brackets [ ] are used in mathematical language as separators. The expression inside the parentheses or brackets is considered as a single unit. Operations are performed inside the parentheses before the expression inside the parentheses is combined with anything outside the parentheses.

$$5 - (2 + 1) = 5 - (3) = 2$$

In the example below, operate on the expression in the innermost separator first and work your way out.

$$20 \div [6 - (4 - 8)] = 20 \div [6 - (-4)] = 20 \div 10 = 2$$

The horizontal line used for a division problem is also a separator. It separates the expressions above and below the line, so the numerator and denominator must be simplified completely before dividing.

$$\frac{20 + 10}{5 \bullet 2} = \frac{30}{10} = 3$$

Order of Operations

There are many mathematical conventions that enable us to interpret mathematical notation and to communicate efficiently about common situations. The agreed-upon rules for interpreting mathematical notation, important for simplifying arithmetic and algebraic expressions, are called the order of operations.

1. Do the operations in grouping symbols first (e.g., use rules 2 – 4 inside parentheses).
2. Calculate all the expressions with exponents.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

$$\frac{11+(17-2\bullet 3^2)}{5} = \frac{11+(17-2\bullet 9)}{5} = \frac{11+(17-18)}{5} = \frac{11+(-1)}{5} = \frac{10}{5} = 2$$

There are many times for which these rules make complete sense and are quite natural. Take this case, for example:

You purchase 2 bottles of water for \$1.50 each and 3 bags of peanuts for \$1.25 each. Write an expression for this situation, and simplify the expression to find the total cost.

$$\underbrace{2\bullet(1.50)}_{3.00} + \underbrace{3\bullet(1.25)}_{3.75} = \$6.75$$

In this problem, it is natural to find the cost of the 2 bottles of water and then the cost of the 3 bags of peanuts prior to adding these amounts together. In other words, we perform the multiplication operations before the addition operation.

Note however that if we were to perform the operations in order from left to right (as we read the English language from left to right), we would obtain a different result:

WRONG $\rightarrow$ 2(1.50) = 3 $\longrightarrow$ 3 + 3 = 6 $\longrightarrow$ 6(1.25) = \$7.50

Using Order of Operations to Simplify Expressions		
Order of Operations	**Example** $\dfrac{40 - 2 \bullet 5^2 - (8 - 6)}{4 + 2 \bullet 10}$	**Comments**
Simplify expressions within grouping symbols.	$\dfrac{40 - 2 \bullet 5^2 - 2}{4 + 2 \bullet 10}$	Parentheses are grouping symbols: (8 – 6) = 2 The fraction bar, used for division, is also a grouping symbol, so the numerator and denominator must be simplified completely prior to dividing.
Calculate all the expressions with exponents.	$\dfrac{40 - 2 \bullet 25 - 2}{4 + 2 \bullet 10}$	$5^2 = 5 \bullet 5 = 25$
Perform multiplication and division from left to right.	$\dfrac{40 - 50 - 2}{4 + 20}$	In the numerator: Multiply 2 • 25 = 50. In the denominator: Multiply 2 • 10 = 20.
Perform addition and subtraction from left to right.	$\dfrac{-12}{24}$	In the numerator: Subtract from left to right 40 – 50 – 2 = -12. In the denominator: Add 4 + 20 = 24
	$\dfrac{-1}{2}$ or $-\dfrac{1}{2}$	Now the groupings in both the numerator and denominator have been simplified, so the final division can be performed.

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
7.NS.A	**Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.**
7.NS 1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; ~~represent addition and subtraction on a horizontal or vertical number line diagram~~:
d	Apply properties of operations as strategies to add and subtract rational numbers.
7.NS.2	Apply and extend previous understandings of multiplication and division of fractions to multiply and divide rational numbers:
a	Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
b	Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then -(*p*/*q*) = (-*p*)/*q* = *p*/(-*q*). Interpret quotients of rational numbers by describing real-world contexts.
c	Apply properties of operations as strategies to multiply and divide rational numbers.
7.NS.3	Solve real-world and mathematical problems involving the four operations with rational numbers.
7.EE.B	**Solve real life and mathematical problems using numerical and algebraic expressions and equations.**
7.EE.3	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

SMP1	Make sense of problems and persevere in solving them.
SMP2	Reason abstractly and quantitatively.
SMP3	Construct viable arguments and critique the reasoning of others.
SMP5	Use appropriate tools strategically.
SMP6	Attend to precision.
SMP7	Look for and make use of structure.
SMP8	Look for and express regularity in repeated reasoning.

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