Name _____ Period _____ Date _____

# UNIT 9
# STUDENT PACKET

Math Links
GRADE 6

## AREA AND VOLUME


| | | Monitor Your Progress | Page |
|---|---|---|---|
| | **My Word Bank** | | 0 |
| | **My Formula Bank** | | 1 |
| **9.0** | **Opening Problem: Which Rug is Bigger?** | | 2 |
| **9.1** | **Area of Polygons**<br>• Derive formulas for the areas of parallelograms, triangles, and trapezoids.<br>• Find areas of irregular polygons.<br>• Solve problems that apply area and algebra. | 3 2 1 0<br>3 2 1 0<br>3 2 1 0 | 3 |
| **9.2** | **Surface Area of Prisms and Pyramids**<br>• Explore properties of prisms and pyramids.<br>• Use nets to create prisms and pyramids and find their surface areas.<br>• Solve problems apply surface area and algebra. | 3 2 1 0<br>3 2 1 0<br>3 2 1 0 | 12 |
| **9.3** | **Volume of Prisms**<br>• Derive formulas for the volume of a right rectangular prism.<br>• Use cubes and formulas to compare volumes of right prisms with fractional edge lengths.<br>• Solve problems that apply volume and algebra. | 3 2 1 0<br>3 2 1 0<br>3 2 1 0 | 19 |
| | **Review** | | 25 |
| | **Student Resources** | | 33 |




Parent (or Guardian) signature _____

# MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

| | |
|---|---|
| area<br>(base, height)<br>Use *b* for length of base. | net |
| polygon | prism |
| pyramid | solid figure |
| surface area | volume<br>(Base, height)<br>Use *B* for area of base. |

# MY FORMULA BANK

Fill in the table below as instructed.

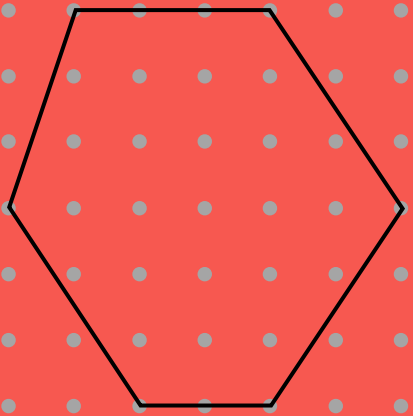
| Definition / Description / Properties | Sketch | Formula(s) |
|---|---|---|
| 1. Rectangle: | | Area: |
| 2. Square: | | Area: |
| 3. Parallelogram: | | Area: |
| 4. Triangle: | | Area: |
| 5. Trapezoid: | | Area: |
| 6. Rectangular Prism | | Surface Area:<br><br>Volume: |

# OPENING PROBLEM: WHICH RUG IS BIGGER?

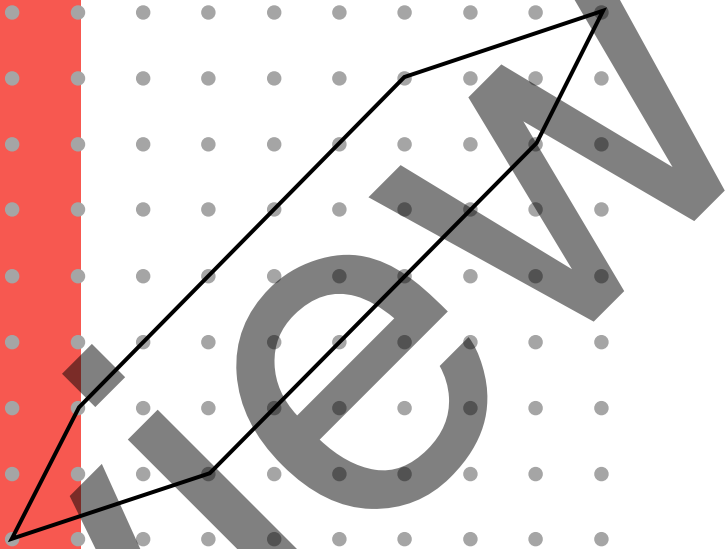
[6.G.1, 6.EE.2c; SMP 1, 2, 3, 5, 7]

Follow your teacher's directions. Save the lower part of the sheet for later.

Sarina's Rug



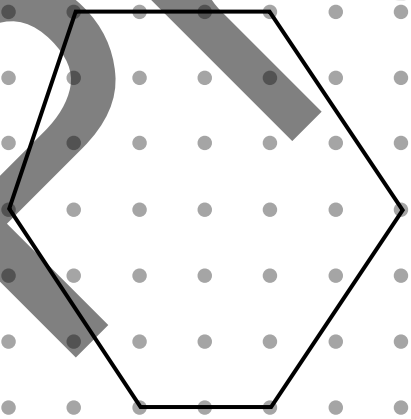
Jianna's Rug

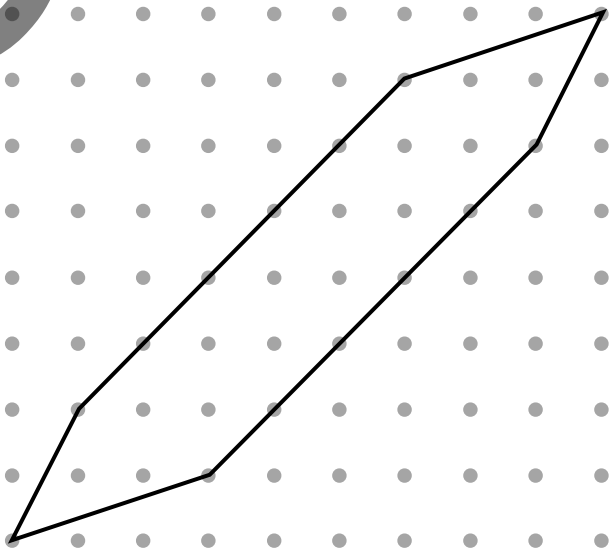


(1) At first glance, I think _____ because…

(2) After discussion, I think _____ because…

Use the figures below when revisiting this problem on **Practice 3**.





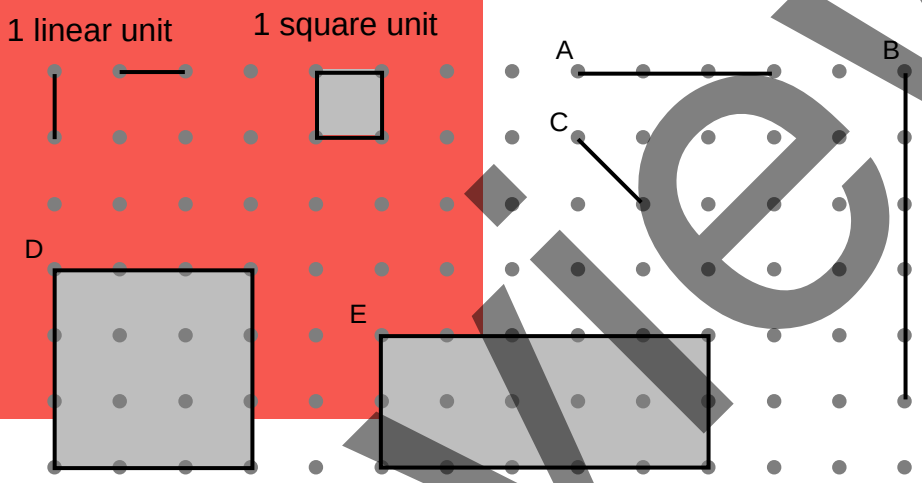
# AREA OF POLYGONS

We will derive the area formulas for parallelograms, triangles, and trapezoids. We will apply formulas to solve real world and mathematical problems and review number and algebra concepts. We will find areas of irregular polygons.

[6.G.1, 6.EE.2ac, 6.EE3, 6.EE4, 6.EE6, 6.EE9; SMP2, 5, 6, 7]

## GETTING STARTED

One unit of length (1 linear unit) and one unit of area (1 square unit) are defined below.




1. Find the length of the segments labeled: A. _____ B. _____

   Is the length of the segment labeled C equal to 1 linear unit? ____ Explain how you know.

2. Label a base and height of figure D. The area of figure D is _____.

3. Label a base and height of figure E. The area of figure E is _____.

4. Figure F (not shown) is a rectangle that is 12 units by 10 units. Explain how to find the area of figure F without drawing it and counting squares.

   The area of figure F is _____.

5. Complete the rows for <u>rectangle</u> and <u>square</u> in **My Formula Bank**.

# A TANGRAM PUZZLE

Follow your teacher's directions for (1) – (2).
(1)
(2)

| Piece | (1) Name | (2) Estimated Area | Area using formulas Complete after **Area of a Triangle** |
|---|---|---|---|
| **A** | | | |
| **B, D** | | | |
| **C, E** | | | |
| **F** | | | |
| **G** | | | |

3. Order the tangram pieces A, B, C, F, and G from least area to greatest. Do any of these have the same area? Explain.

## PRACTICE 1

Use the tangram pieces you created for **A Tangram Puzzle** to answer the following questions.

1. Create a square using the small triangles.

   The area of a small triangle is _____ the area of the square.

2. Create a parallelogram using the small triangles.

   The area of the parallelogram is _____ the area of a small triangle.

   The area of the parallelogram is _____ the area of the square.

3. How many small triangles exactly cover a large triangle (without gaps or overlaps)? ____
   Sketch a diagram of this.

   Therefore, the area of the large triangle is _____ the area of the small triangle.

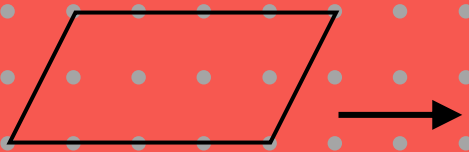
4. Adrian says that the area of A is 9 square units because the length of each side of the square is 3 units. Critique her reasoning.

5. Jamala says that the area of B is equal to the area of A and G combined. Mahzi disagrees and says the area of B is greater. Who is correct? Explain with words and diagrams.

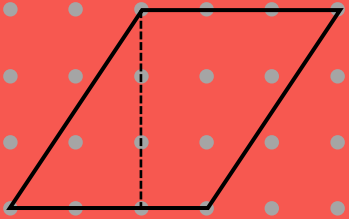
# AREA OF A PARALLELOGRAM

Follow your teacher's directions for (1) – (3).

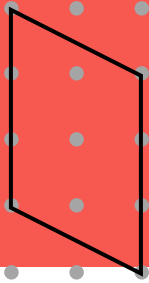
(1)



(2)



(3)



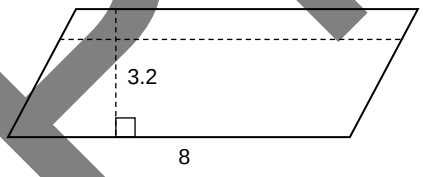
4. Explain why a rectangle is also a parallelogram.

5. Record the meaning of area in **My Word Bank**.

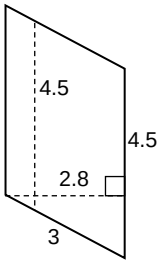
6. Complete the row for parallelogram in **My Formula Bank**.

Find the area of each parallelogram below using the formula. All measures are in centimeters.

7.



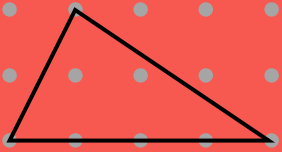

8.



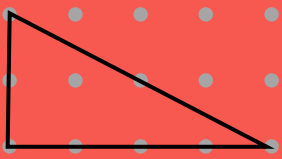
# AREA OF A TRIANGLE

Follow your teacher's directions for (1) – (3).

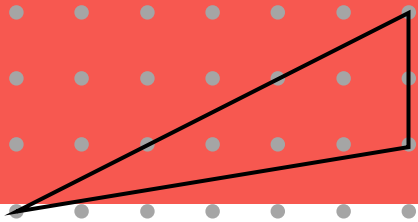
(1)



(2)



(3)

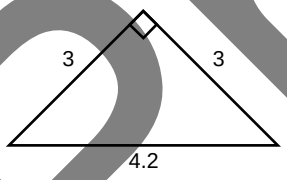


4. What is the relationship between the area of the original triangle and the area of the parallelogram created by the cut-up strategy?

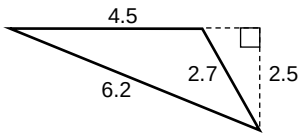
5. Complete the row for <u>triangle</u> in **My Formula Bank**.

Find the area of each triangle below using the formula. All measures are in cm.

6.




7.

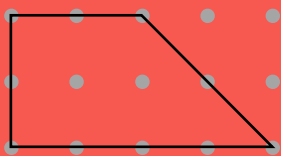



8. Go back to **A Tangram Puzzle** and find the area of each of the tangram puzzle shapes using formulas. Record in the last column of the table. Why might the calculations between your original estimates and the areas you found using the formulas be slightly different?

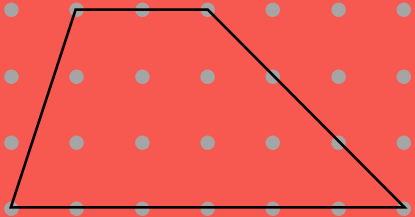
# AREA OF A TRAPEZOID

Follow your teacher's directions for (1) – (3).

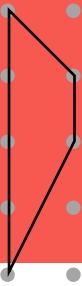
(1)



(2)



(3)

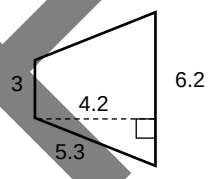


4. Describe how the two bases of a trapezoid ($b_1$ and $b_2$) relate to the base of the parallelogram formed by combining the two copies of the trapezoid.

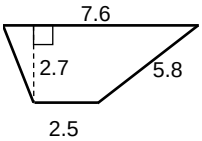
5. Complete the row for trapezoid in **My Formula Bank**.

Find the area of each trapezoid using the formula. All measures are in cm.

6.




7.

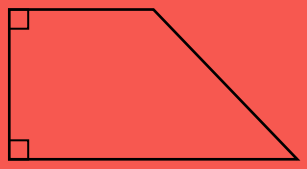


# PRACTICE 2

For each problem:

- Identify the polygon and the corresponding area formula.
- Measure and label the relevant dimensions to the nearest tenth of a cm (mm).
- Substitute values into the formula and evaluate to find the area.
- Use appropriate units in answers.

1. Polygon Name: _____



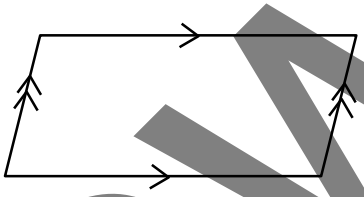
Area formula: _____

Substitute: _____

*A* = _____

What do the little squares mean in the corners of the polygon?

2. Polygon name: _____



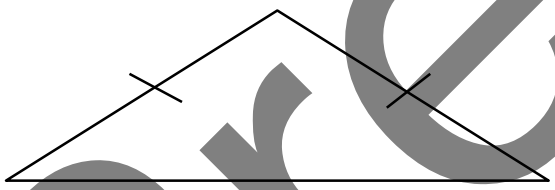
Area formula: _____

Substitute: _____

*A* = _____

What do the arrows mean on the sides of the polygon?

3. Polygon name: _____



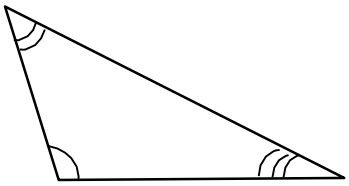
Area formula: _____

Substitute: _____

*A* = _____

What do the tick marks mean on the sides of the polygon?

4. Polygon Name: _____



Area formula: _____

Substitute: _____

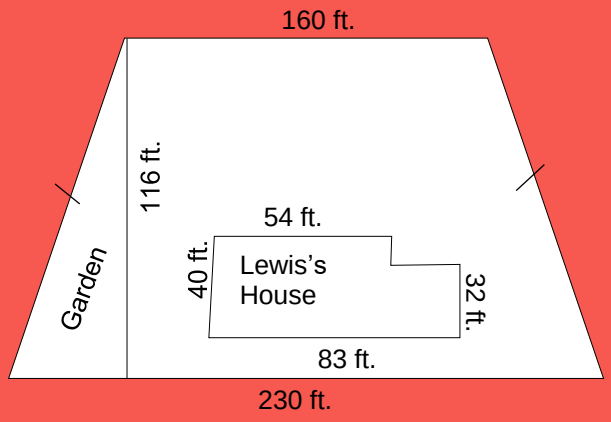
*A* = _____

What do the curved markings mean inside the angles of the polygon?

5. Record the meaning of polygon in **My Word Bank**.

## PRACTICE 3: EXTEND YOUR THINKING

Lewis's house sits on a piece of land that is shaped like an isosceles trapezoid.



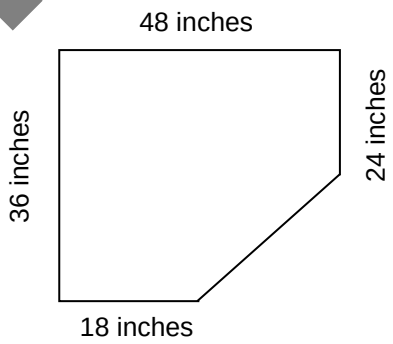

This diagram is not to scale.

1. What is the area of Lewis's garden?

2. What is the area of Lewis's house?

3. What is the area of Lewis's entire property?

4. Lewis has a corner desk in his house. Find the area of the desk in square feet. The diagram is not to scale.




5. Go back to the **Which Rug is Bigger?** and use formulas to find the area of Sarina's and Jianna's rugs. Show your work on that page. Whose rug has a larger area?

## PRACTICE 4: EXTEND YOUR THINKING

Use your knowledge of geometry formulas and algebra procedures to solve these problems. Find the height of each polygon described below. Draw pictures if helpful.

| **Figure** | 1. Rectangle | 2. Parallelogram | 3. Triangle | 4. Trapezoid |
|---|---|---|---|---|
| **Facts** | Area = 3,000 $u^2$<br>base = 120 u | Area = 215.25 $u^2$<br>base = 20.5 u | Area = 108 $u^2$<br>base = 27 u | Area = 135 $u^2$<br>$base_1$ = 8.4 u<br>$base_2$ = 6.6 u |
| **Area formula** | | | | |
| **Substitute values and solve for *h*** | | | | |

Fill in the heights (*h*) for given base lengths (*b*) for the indicated polygons with given areas.

5. Parallelograms with area equal to 36 square units.

| ***b*** | ***h*** |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 6 | |
| 9 | |
| 12 | |
| 18 | |
| 36 | |
| Rule (equation): *h* = | |

6. Triangles with area equal to 12 square units.

| ***b*** | ***h*** |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 6 | |
| 8 | |
| 12 | |
| 24 | |
| 0.5 | |
| Rule (equation): *h* = | |

# SURFACE AREA OF PRISMS AND PYRAMIDS

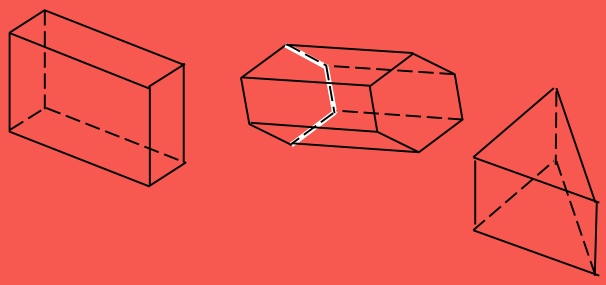
We will use nets to construct two types of solid figures, prisms and pyramids, and find their surface areas. We will solve real-world and mathematical problems that involve surface area, and review concepts from number and algebra.

[6.G.4, 6.EE.2ac, 6.EE6, 6.EE9; SMP2, 3, 4, 5, 6, 8]

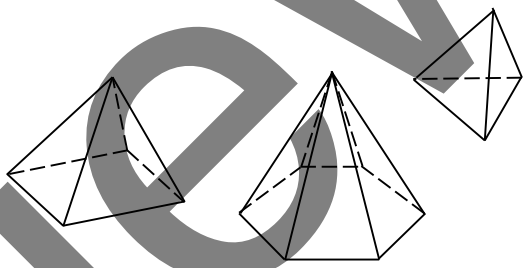
## GETTING STARTED

Prisms and pyramids are two types of solid figures.

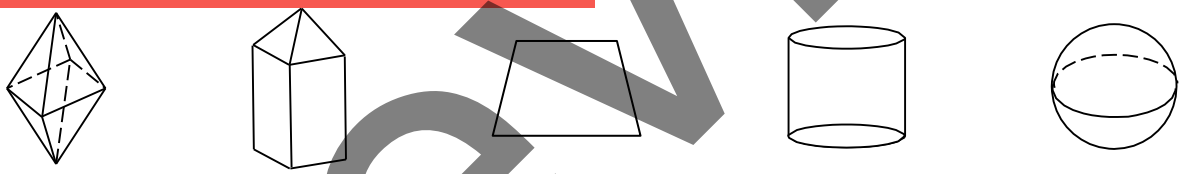
These represent prisms



These represent pyramids

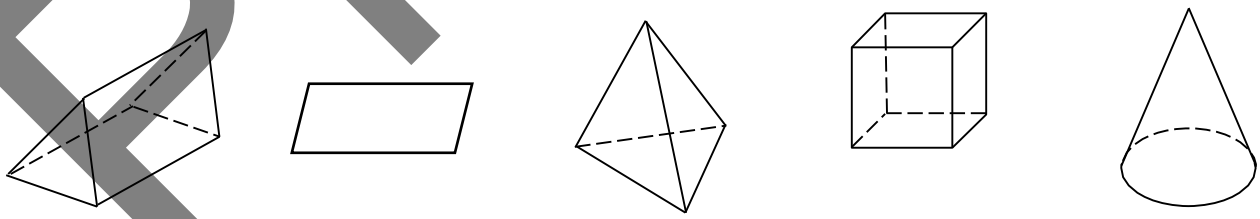


These represent neither prisms nor pyramids



1. What appear to be some properties of prisms and pyramids?

2. Label each shape as a prism, pyramid, or neither.

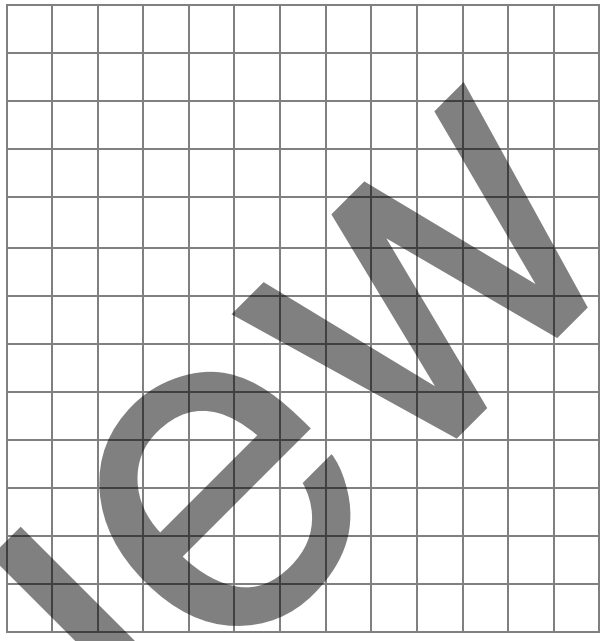


3. Record the meanings of solid figure, prism and pyramid in **My Word Bank**.

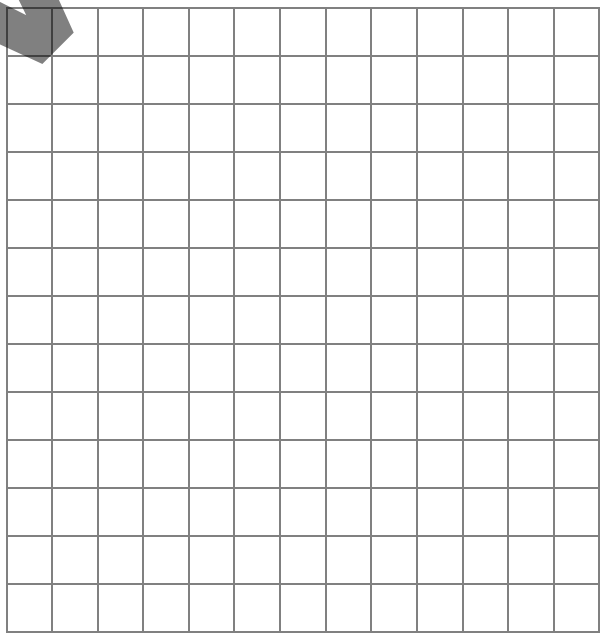
# FINDING SURFACE AREA USING NETS

Follow your teacher's directions.

(1) Shape 1



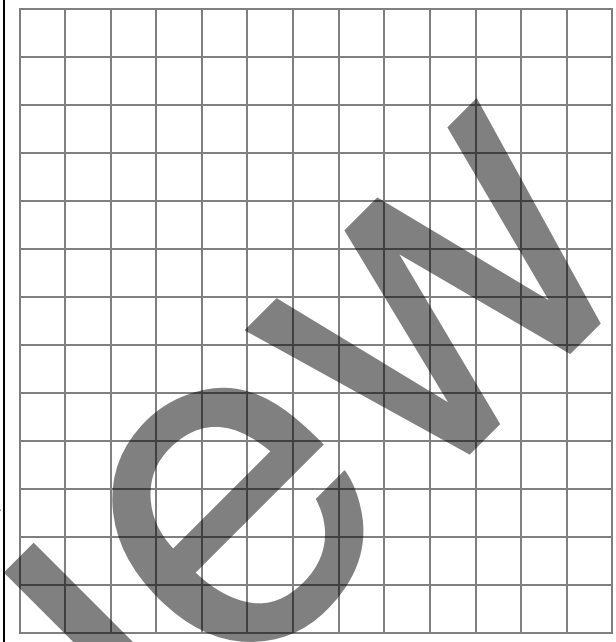
(2) Shape 2



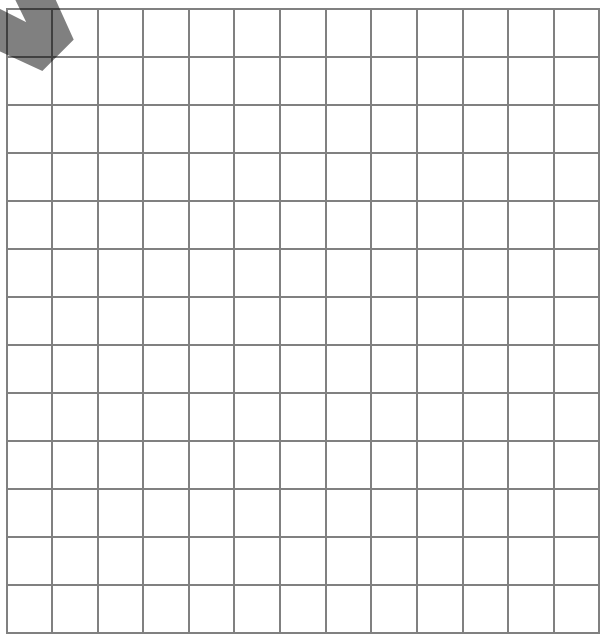
## FINDING SURFACE AREA USING NETS

**Continued**

(3) Shape 3



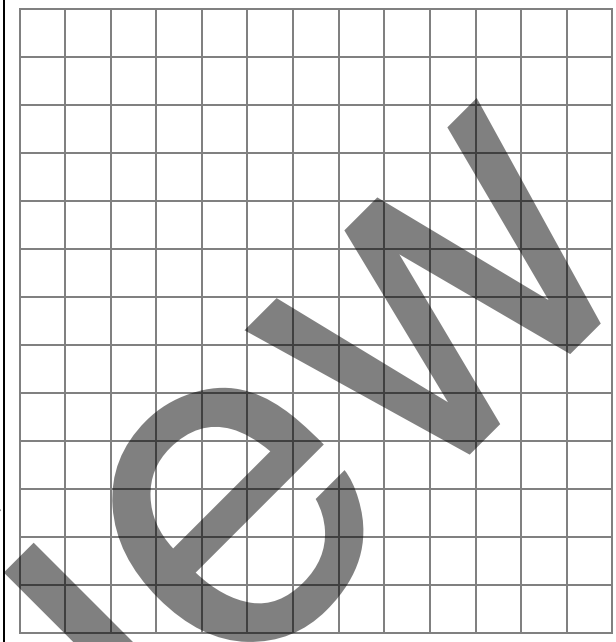
(4) Shape 4



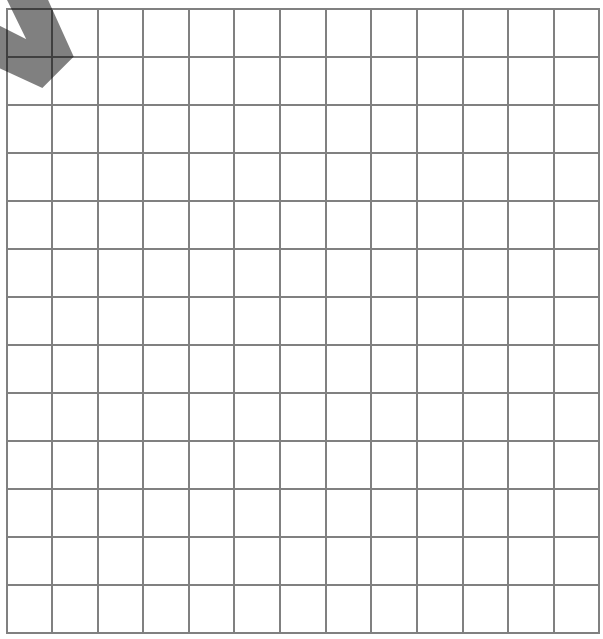
## FINDING SURFACE AREA USING NETS

**Continued**

(5) Shape 5

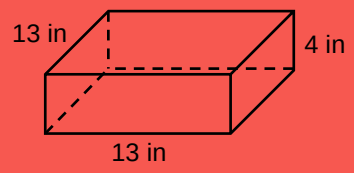
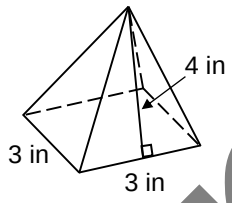


(6) Shape 6



## PRACTICE 5

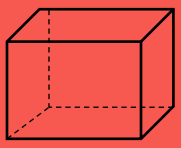
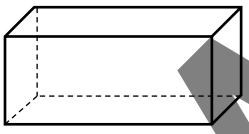
Sketch the faces for each object separately or as a net. Find the surface area. Then answer the related question. Diagrams are not to scale.

| | |
|---|---|
| 1. Gift box<br>  <br><br>Count the numbers of<br>Faces: _____<br>Edges: _____<br>Vertices: _____ | Sketch the faces and find the surface area. |
| Will a piece of wrapping paper that is 18 inches by 18 inches be large enough to wrap the gift box? Explain. | |
| 2. Candle<br>  <br><br>Count the numbers of<br>Faces: _____<br>Edges: _____<br>Vertices: _____ | Sketch the faces and find the surface area. |
| How many candles will fit inside the gift box in problem 1? Explain. | |

3. Record the meanings of net and surface area in **My Word Bank**.

## WHO NEEDS MORE PAINT?

Follow your teacher's directions for (1) – (2).
(1)
(2)

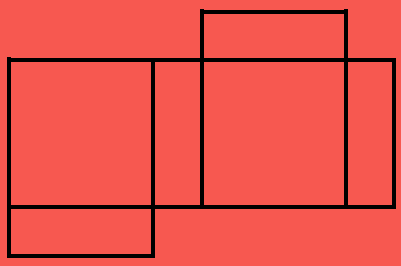
| **Robin's Bedroom** | **Gerry's Bedroom** |
|---|---|
|  |  |
| Assumptions: | |
| Calculations: | Calculations: |
| Answer the question: | |

3. Complete the row for rectangular prism in **My Formula Bank**.

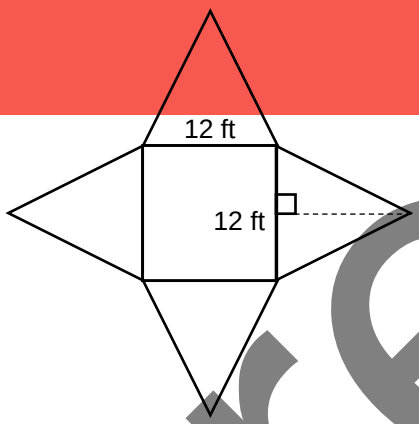
## PRACTICE 6: EXTEND YOUR THINKING

Sketch the figure represented by the net for these geometry problems that connect to algebra. List the numbers of faces, edges, and vertices, and then answer the question.

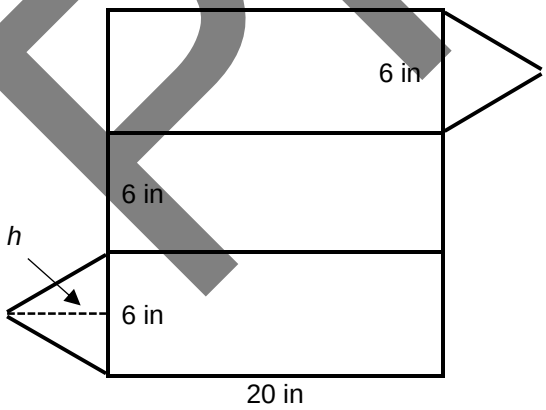
1. A puzzle box has a surface area of 96 square inches. The top and bottom faces of the box are squares with side lengths of 6 inches. What is the height of the box?



2. This camping tent has a surface area of 432 square feet. The base of the tent is a square with side lengths equal to 12 feet. The lateral faces are isosceles triangles. What is the height of a triangular face?




3. The surface area of this shipping container is 391.2 square inches. What is its height?

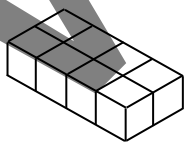


# VOLUME OF PRISMS

We will derive formulas for the volume of a right rectangular prism. We will explore volumes of right rectangular prisms with fractional edge lengths, and apply a volume formula to these figures. We will solve real-world and mathematical problems that involve volume, and review concepts from number and algebra.

[6.G.2, 6.EE.2ac, 6.EE3, 6.EE4, 6.EE6, 6.EE9, 6.RP.3; SMP1, 2, 4, 6, 7]

## GETTING STARTED

| | Prism A | Prism B | Prism C |
|---|---|---|---|
| The volume of each small cube is 1 cubic unit (also 1 cu u or 1 $u^3$) |  |  |  |

1. Find the volume of Prism A. Explain.

2. Find the volume of Prism B. Explain.

3. Find the volume of Prism C. Explain.

4. How does knowing the volume of the bottom layer and the number of layers help determine the number of cubes (volume) in the figure?

5. How does knowing the height of a right rectangular prism and the area of the top surface help determine the number of cubes (volume) in the prism?

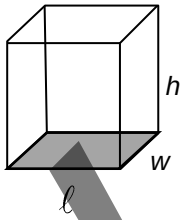
6. Record the meaning of volume in **My Word Bank**.

# STRATEGIES FOR FINDING VOLUME

Follow your teacher's directions for (1) – (2).

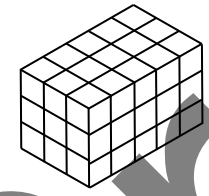
(1) Write Kim's strategy using symbols. $V$ = _____

(2) Label the Base with a $B$ on the rectangular prism to the right and fill in the area formula. Then write Mateo's strategy using symbols.



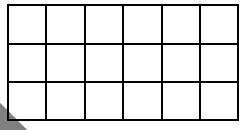

$A_{base}$ = _____ $V$ = _____

Find the volume of each right rectangular prism using:

| | Kim's strategy | Mateo's strategy |
|---|---|---|
| 3.  | length _____<br>width _____<br>height _____<br>$V$ = _____ | area of Base _____<br>height _____<br>$V$ = _____ |
| 4.  | | |
| 5.  | | |

Find the volume of each right rectangular prism described below without counting all of the cubes in the layers individually.

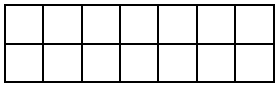
6. Top view of the prism:



Height of the prism: 12 units

Volume of the prism: _____

7. Top view of the prism:



Height of the prism: 7 units

Volume of the prism: _____

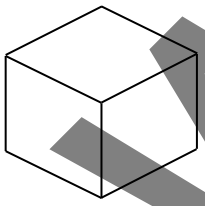
## WHAT IF?

For this page, let the little cube have a side length of $\frac{1}{2}$ inch.

Follow your teacher's directions for (1) – (3).

(1)

(2)



(3)

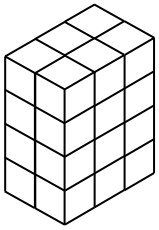
4a. Volume of little cube _____

Number of little cubes _____

Find the volume by counting the cubes.

4b. $\ell$ = _____ $w$ = _____ $h$ = _____

Find the volume using a formula.



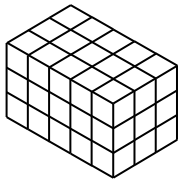
5a. Volume of little cube _____

Number of little cubes _____

Find the volume by counting the cubes.

5b. $\ell$ = _____ $w$ = _____ $h$ = _____

Find the volume using a formula.

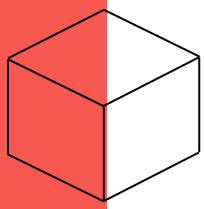


# PRACTICE 7

For problems 1 - 5, let the edge length of the little cube equal $\frac{1}{3}$ inch.

1. Label the edge lengths of the little cube.

2. This represents 1 $in^3$. Draw little cubes in it. How many little cubes fit inside?



3. What is the volume of the little cube?

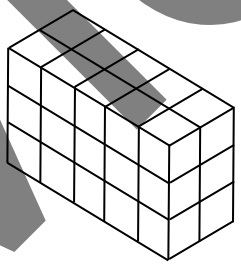
4. Volume of a little cube _____

   Number of little cubes in the prism to the right _____

   Find the volume by counting the cubes.

5. $\ell$ = _____ $w$ = _____ $h$ = _____

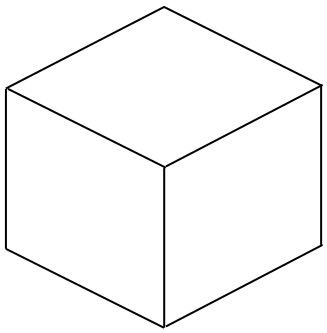
   Find the volume using a formula.



6. Baseball collectors keep autographed baseballs in cubes that are 3 inches on an edge.

   a. Describe the dimensions of the cube in terms of feet.

   b. How many baseball cubes will fit into a box that is 1 foot on each edge?



7. Complete this exponent pattern involving cubes.

   $1 = 1^3$ 8 = _____ 27 = _____ 64 = _____ 125 = _____

8. The numbers 1, 8, 27, 64, 125,… are called “cubic” numbers. What is their relationship to geometry and measurement?

## THE FOOD DRIVE

At Tecumseh Middle School, the student council led a food drive effort to feed needy families. Enough food was donated for 200 families, so they will fill boxes at school and transport them to their local regional food bank.

They will purchase 200 boxes at $1.75 each (taxes included). These boxes are in the shape of cubes, 18 inches on each edge.

They will rent a truck from U-Move for $19.95, plus mileage and taxes. The distance from school to the food bank is about 10 miles. The truck has inside dimensions that are 10’ long (or deep) × 6’ wide × 8’ high for storage space.

1. What additional information do you need to determine the cost to pack and deliver the boxes?

   If possible, either research the unknowns, or agree as a class, and record reasonable estimates here.

2. Convert all measurements in the table before making calculation decisions.

| | **Box** | | **Truck** | |
|---|---|---|---|---|
| | **dimensions** | **volume** | **dimensions** | **volume** |
| **inches; cubic inches** | | | | |
| **feet; cubic feet** | | | | |
| **yards; cubic yards** | | | | |

## THE FOOD DRIVE
**Continued**

3. Use measurements, drawings, and reasoning to determine how many trips are needed.

4. Make calculations to estimate the cost to pack and deliver the boxes. Use your assumptions from question 1 on the previous page for the cost analysis.

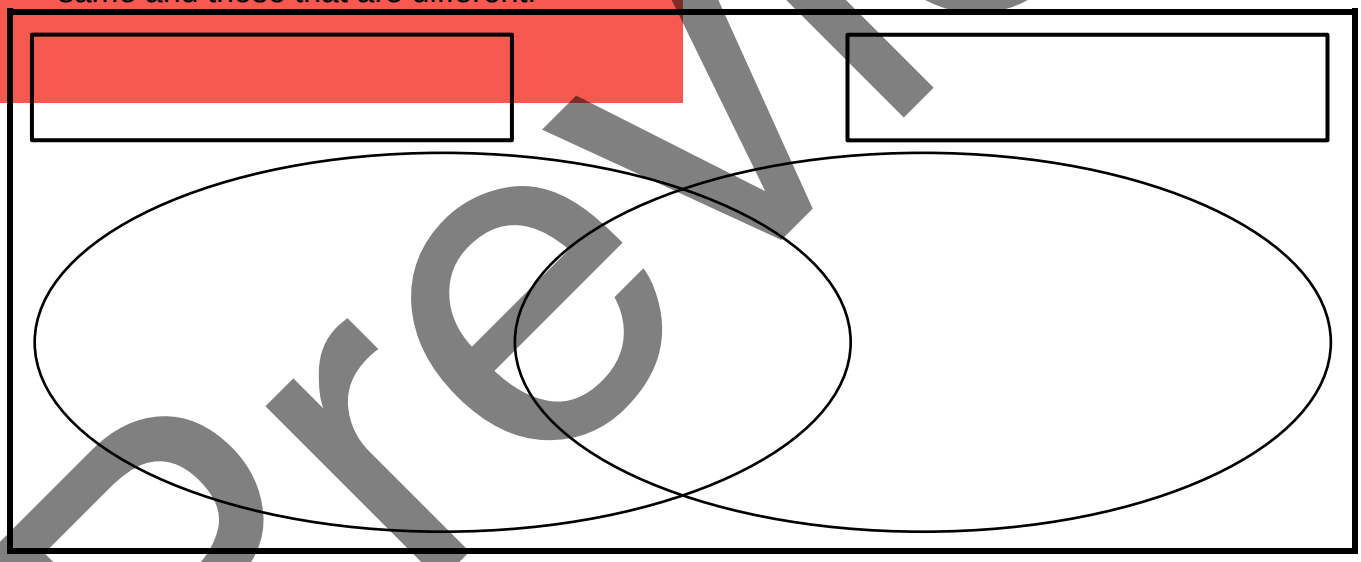
# REVIEW

## MATCH AND COMPARE SORT: AREA AND VOLUME

1. Individually, match words with descriptions. Record results.

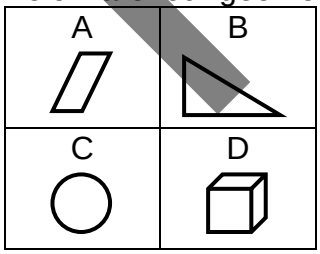
| Card set △ | | | Card set ◯ | | |
|---|---|---|---|---|---|
| **Card number** | **word** | **Card letter** | **Card number** | **word** | **Card letter** |
| **I** | | | **I** | | |
| **II** | | | **II** | | |
| **III** | | | **III** | | |
| **IV** | | | **IV** | | |

2. Partners, choose a pair of numbered matched cards and record the attributes that are the same and those that are different.



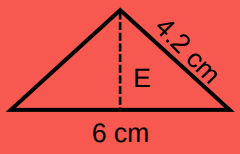
## WHY DOESN'T IT BELONG?: AREA AND VOLUME

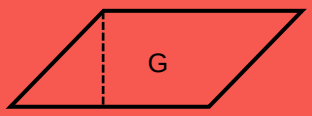
Below are four geometric figures. Explain why one does not belong with the other three.

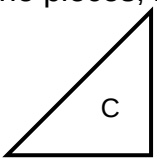


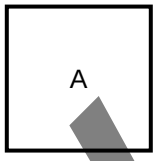
# TANGRAM AREA

1. Below are four pieces from a set of tangrams. Some lengths of the small triangle (E) are marked. Write the side lengths and heights for all the pieces, based on piece E.



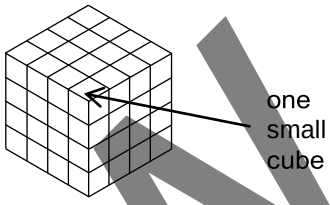




Build figures using the number of pieces indicated. Use each piece no more than once when building a figure. Sketch each figure, label the tangram pieces, and find its total area using an appropriate formula.

| | |
|---|---|
| 2. 2-piece triangle | 3. 2-piece trapezoid |
| 4. 3-piece trapezoid | 5. 3-piece rectangle |
| 6. 4-piece trapezoid | 7. 4-piece parallelogram |

# POSTER PROBLEMS: AREA AND VOLUME

Part 1: Your teacher will divide you into groups.

| | |
|---|---|
| • Identify members of your group as A, B, C, or D.<br>• Each group will start at a numbered poster. Our group start poster is _____.<br>• Each group will have a different colored marker. Our group marker is _____. | Each large cube described in Part 2 looks similar to the one pictured here.<br>  <br> |

Part 2: Do the problems on the posters by following your teacher's directions. Refer to the large cube above.

| Poster 1 (or 5) | Poster 2 (or 6) | Poster 3 (or 7) | Poster 4 (or 8) |
|---|---|---|---|
| Its small cubes each have edge length equal to $\frac{1}{4}$ in. | Its small cubes each have edge length equal to $\frac{3}{4}$ in. | Its small cubes each have edge length equal to $\frac{5}{4}$ in. | Its small cubes each have edge length equal to $\frac{3}{2}$ in. |

A. Copy the small cube's edge length and find its volume.

B. Write the total number of small cubes in the large cube. Then multiply this number by the volume computed in Part A.

C. Write the length, width, and height of the large cube.

D. Write the formula for the volume of a rectangular prism and use the information from part C to find the total volume of the large cube.

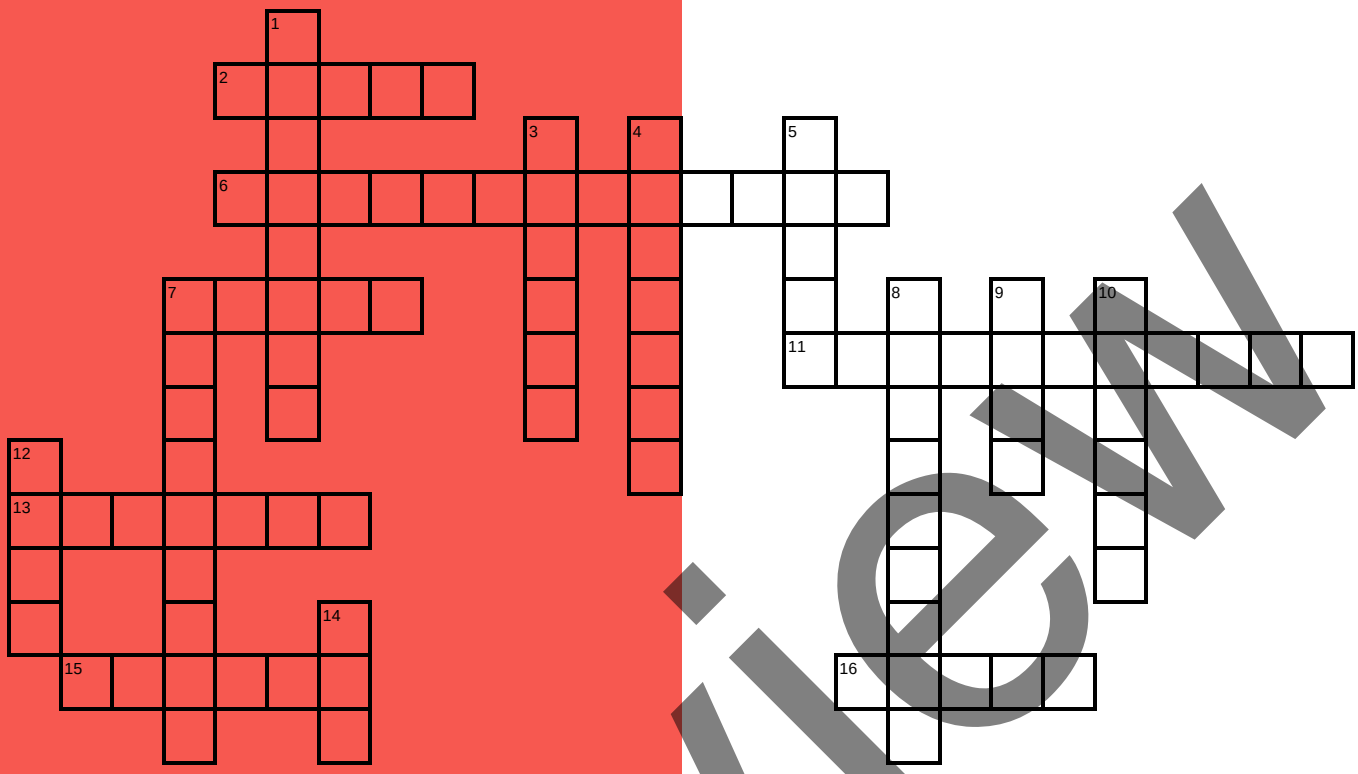
Part 3: Review your original poster. Work with your group and show all work.

1. Compare results from Parts B and D. Are they the same?

2. Write the formula for the volume of a cube with side length equal to *s*.

3. Use your small cube side length and find the surface area of the large cube.

# VOCABULARY REVIEW



**Across**

2 a solid figure whose faces are polygons and with two parallel, identical bases

6 a quadrilateral in which opposite sides are parallel and have equal length

7 a _____ rectangular prism has 6 rectangular faces, and opposite faces are parallel

11 measure of the size of the total surface of a three-dimensional figure (two words)

13 a solid figure where the lateral faces are triangles that meet at “the top” (the apex).

15 measured in cubic units

16 one of the measures of a side of a rectangle (also see 10 down)

**Down**

1 a three-sided polygon

3 the distance between two parallel sides of a parallelogram

4 a closed figure made up of a chain of line segments laid end to end

5 A rectangular prism has 6 _____ that are all rectangles.

7 a quadrilateral with four right angles

8 a polygon with at least one pair of parallel sides

9 a pre-chosen side of a figure, usually the “bottom”

10 one of the measures of a side of a rectangle (see also 16 across)

12 the highest point of a pyramid, (if its base is, say, on a table) – see 13 across

14 a two-dimensional pattern for a three- dimensional figure

## SPIRAL REVIEW

1. **Computational Fluency Challenge**: This paper and pencil exercise will help you gain fluency with multiplication and division. Try to complete this challenge without any errors. No calculators!

   a. Start with 1.25. Multiply by 20. Multiply the result by 5. Multiply the result by 0.75. Multiply the result by 4. Now you have a "big number". My big number is _____.

   b. Start with your big number. Divide it by 25. Divide the result by 1.5. Divide the result by 8. What is the final result? _____

2. Evaluate each numerical expression below.

| a. $7^2 - 9 - 2$ | b. $\frac{(9-7)^2}{2}$ | c. $(9 + 7)\,2^3$ |
|---|---|---|
| | | |

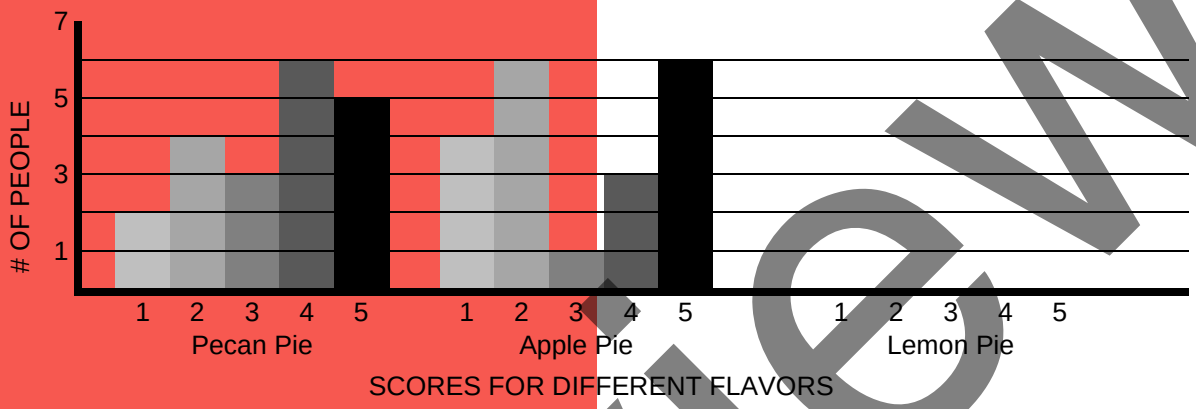
3. Complete the table.

| Fraction | Decimal | Percent | Percent of $200 |
|---|---|---|---|
| $\frac{8}{5}$ | | | |
| | | | $136 |
| | 0.46 | | |

## SPIRAL REVIEW
**Continued**

4. Lucca is going to sell pies at his school for Pi Day on March 14th. He conducted a survey to see which flavor of pie students prefer. He asked people to rate the flavors 1 – 5 (1 is dislike very much and 5 is like very much).

   The graph below illustrates some results of the survey.




   a. Complete the graph above for Lemon Pie with the survey results below.

   5, 4, 3, 5, 1, 1, 1, 2, 4, 3, 1, 5, 2, 3, 1, 3, 3, 3, 4, 3

   b. Complete each row in the table below for each pie.

| | Mean | Median | Mode | Range |
|---|---|---|---|---|
| **Pecan Pie** | | | | |
| **Apple Pie** | | | | |
| **Lemon Pie** | | | | |

   c. Lucca must pick one flavor to sell at his school. What do you recommend that he choose and why?

## SPIRAL REVIEW
**Continued**

5. Stellina is going to play games in the arcade. Her parents give her $8 to spend.

   a. She spends 40% of her money on a game called Toad in the Road. How much money did she have left?

   b. After spending money on candy, she has $1.50 left. This is $\frac{2}{3}$ of the amount that her sister has left. How much money does her sister have left?

6. Circle all equations below that have a solution of 4.

   $2x = 6$ $2 + x = 6$ $x - 2 = 6$ $8 = \frac{x}{32}$ $8 = \frac{32}{x}$

7. Solve each equation.

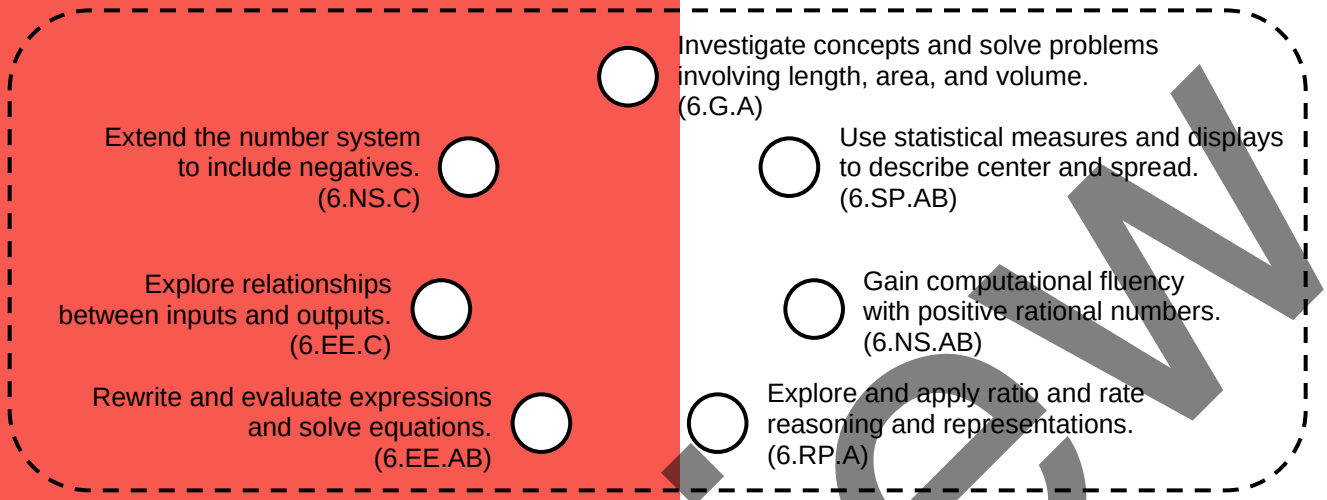
| | | |
|---|---|---|
| a. $4g = 48$ | b. $h + 19 = 42$ | c. $15y = 60$ |
| d. $s - 34 = 12$ | e. $\frac{x}{5} = 10$ | f. $12 = \frac{x}{6}$ |

8. Evaluate each expression.

| | | |
|---|---|---|
| a. $20\left(\frac{4}{5} - \frac{1}{10}\right)^2$ | b. $20\left(\frac{4}{5}\right) - \left(\frac{1}{10}\right)^2$ | c. $\left(\frac{4}{5}\right) - (20)\left(\frac{1}{10}\right)^2$ |

## REFLECTION

1. **Big Ideas**. Shade all circles that describe big ideas in this unit. Draw lines to show connections that you noticed.



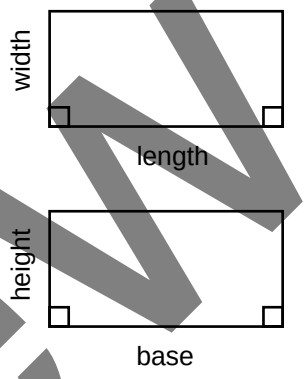
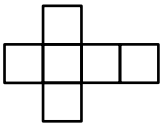
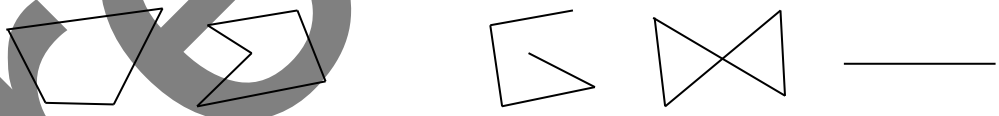

Give an example from this unit of one of the connections above.

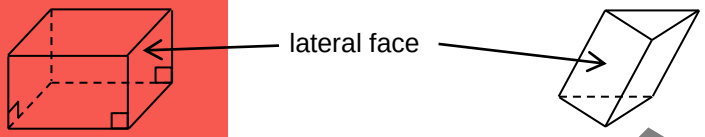
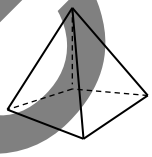
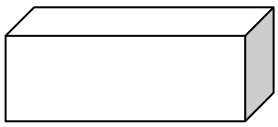
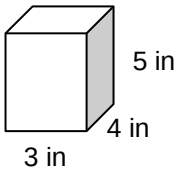
2. **Unit Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before or something you would still like to work on.

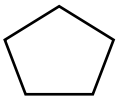
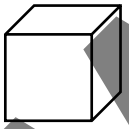
3. **Mathematical Practice.** What tools did you find useful as your explored relationships of shapes and space [SMP5]? Then circle one more SMP on the back of this packet that you think was addressed in this unit and be prepared to share an example.

4. **Making Connections.** You learned formulas for the areas of rectangles, triangles, parallelograms, and trapezoids. Choose two of these figures and explain how the areas are related.

# STUDENT RESOURCES

| Word or Phrase | Definition |
|---|---|
| area | The <u>area</u> of a two-dimensional figure is a measure of the size of the figure, expressed in square units.<br><br>The area of a rectangle is the product of its length and width (Area = length • width).<br>or<br>The area of a rectangle is the product of its base and height (Area = base • height).<br><br>If this rectangle has a length of 12 inches and a width of 5 inches, then:<br><br>$A = \ell w$ or $A = bh$<br>$A = (12)(5)$ or $A = (12)(5)$<br>$A = 60$ square inches or $A = 60$ square inches<br><br>  |
| net | A <u>net</u> for a three-dimensional figure is a two-dimensional pattern for the figure.<br><br>If cut from a sheet of paper, for example, a net forms one connected piece which can be folded with the edges joined to form the given figure.<br><br>cube<br>  <br>net of a cube<br>  |
| plane | A <u>plane</u> is a flat, two-dimensional surface without holes that extends to infinity in all directions. |
| polygon | A <u>polygon</u> is a special kind of figure in a plane made up of a chain of line segments laid end-to-end to enclose a region.<br><br>  <br>polygons not polygons |

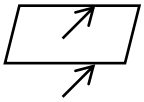
| Word or Phrase | Definition |
|---|---|
| *prism* | *A <u>prism</u> is a solid figure in which two faces (the bases) are identical parallel polygons, and the other faces (referred to as the lateral faces) are parallelograms.*<br><br>*If the lateral faces are perpendicular to the bases, the prism is a right prism. Otherwise, the prism is an oblique prism.*<br><br>  <br><br>A right rectangular prism is a right prism whose bases are rectangles and whose faces are rectangles.<br><br>An oblique triangular prism is a prism whose bases are triangles and whose faces are parallelograms. |
| pyramid | A <u>pyramid</u> is a solid figure in which one face (the base) is a polygon, and the other faces (referred to as lateral faces) are triangles with a common vertex (referred to as the apex).<br><br>The Egyptian pyramids are square pyramids since they have square bases.<br><br>  |
| right rectangular prism | A <u>right rectangular prism</u> is a six-sided solid figure in which all the faces are rectangles.<br><br>A rectangular box is a right rectangular prism.<br><br>  |
| solid figure | A <u>solid figure</u> refers to a figure in three-dimensional space such as a prism or a cylinder.<br><br>  <br>cube triangular prism rectangular pyramid cylinder |
| surface area | The <u>surface area</u> of a three-dimensional figure is a measure of the size of the surface of the figure, expressed in square units. If the surface of the three-dimensional figure consists of two-dimensional polygons, the surface area is the sum of the areas of the polygons.<br><br>If this rectangular box has a length of 3 inches, a width of 4 inches, and a height of 5 inches, then<br><br>  <br><br>$SA = 2(\ell w) + 2(\ell h) + (wh)$<br>$SA = 2(3 \bullet 4) + 2(3 \bullet 5) + 2(4 \bullet 5)$<br>$SA = 94$ square inches |

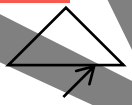
| Word or Phrase | Definition |
|---|---|
| vertex | A vertex (plural of vertices) of a polygon or solid figure is a point where two edges meet. See polygon, solid figure.<br>A pentagon has five vertices.  |
| volume | The volume of a three-dimensional figure is a measure of the size of the figure, expressed in cubic units. The volume of a right rectangular prism is the product of its length, width, and height.<br>If this cube has a side length of 3 units, then  <br>$V = \ell wh$<br>$V = 3 \bullet 3 \bullet 3$<br>$V = 27$ cubic inches |

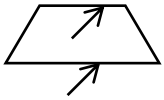
**Base of a Polygon (*b*) Versus Base of a Solid Figure (*B*)**

The base of a polygon is a predesignated side of the figure. It is typically denoted with a "*b*."

The base is usually regarded as the "bottom" of the polygon. The top is also a base, if it is parallel to the bottom.

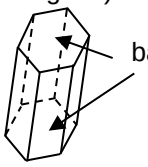
Any side of a parallelogram may be the base. 

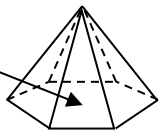
Any side of a triangle may be chosen as the base. 

A trapezoid has two bases. They are the parallel sides. 

The base of a solid figure is a predesignated face of the figure. It is typically denoted with a *"B*."

The base is usually regarded as the "bottom" of the figure, on which it is standing. The "top" of a figure is sometimes also referred to as a base if it is identical and parallel to the "bottom."

This right prism has two parallel bases (hexagons). 

This right pyramid has two one base (a hexagon). 

base

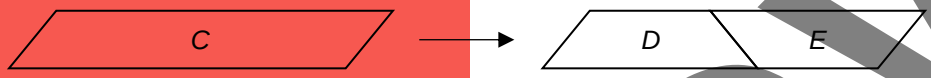
## Composing and Decomposing Shapes: "Cut-Up Strategies"

Composing shapes refers to joining geometric shapes without overlaps to form other shapes.

Here are two identical triangles (*A* and *B*). When joining *A* and *B* (after rotating *B*), the result is a parallelogram with twice the area of each of the given triangles.

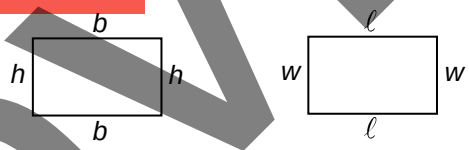
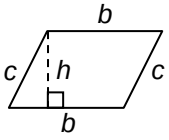
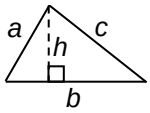
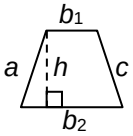

Decomposing shapes refers to taking a given geometric shape, and identifying geometric shapes that meet without overlap to form that given shape.

Given parallelogram *C*, we can identify a segment that creates two identical trapezoids *D* and *E*, each with one-half the area of *C*.




Composing and decomposing shapes are useful strategies for finding area formulas for common polygons derived from ones we already know. We refer to these methods collectively as "cut-up" strategies. For example, first we learn the formula for area of a rectangle. Then we can use a cut-up strategy to find the formula for area of a parallelogram. Then we can use other cut-up strategies to find the formulas for area of a triangle and area of a trapezoid.

## Summary of Area Formulas

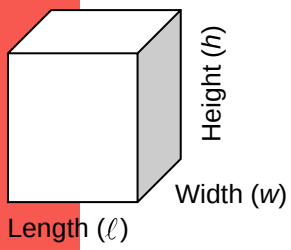
| Shape/Definition | Diagram | Area |
|---|---|---|
| **Rectangle**<br>a quadrilateral with 4 right angles |  | $A = bh$<br>or<br>$A = \ell w$ |
| **Square**<br>a rectangle with 4 sides of equal length |  | $A = b^2$<br>or<br>$A = s^2$ |
| **Parallelogram**<br>a quadrilateral with opposite sides parallel |  | $A = bh$ |
| **Triangle**<br>a polygon with three sides |  | $A = \frac{1}{2}bh$ |
| **Trapezoid**<br>a quadrilateral with at least one pair of parallel sides |  | $A = \frac{1}{2}(b_1 + b_2)h$ |

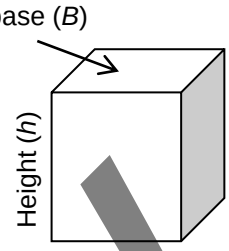
## Volume and Surface Area of Right Rectangular Prisms

A right rectangular prism is identified by its length, width, and height.

The area of the base is the product of the length and width ($B = \ell w$).

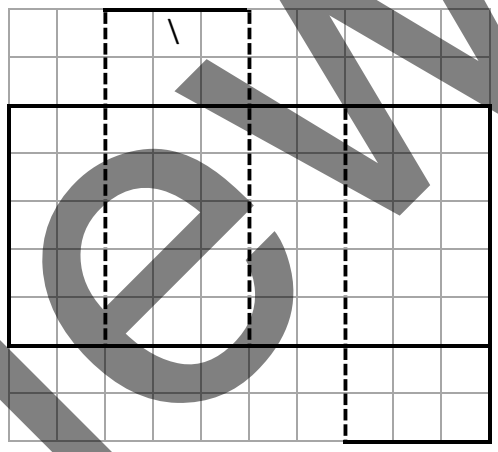
If the top and bottom rectangular faces are chosen as the bases, then the other rectangles are referred to as the lateral faces.




Right rectangular prism made with 30 cubes

Net of the same prism



### Volume

The volume of a prism may be computed by counting layers of unit cubes. In the prism above, each layer has 10 cubes ($5 \times 2$). There are 3 layers.

The volume is $(5 \times 2)(3) = 10(3) = 30$ cubic units.

In general, multiply the area of the base (B) by the height.

$V = \ell wh$ OR $V = Bh$

### Surface Area

The surface area may be computed by creating a net that shows the areas of each face of the prism. In this prism there are two faces with dimensions $2 \times 5$, two faces with dimensions $3 \times 2$, and two faces with dimensions $3 \times 5$.

The surface area is
$2(2 \times 5) + 2(3 \times 2) + 2(3 \times 5)$
$= 20 + 12 + 30$
$= 62$ square units.

In general, find the area of each rectangular face.

$SA = \ell w + \ell w + wh + wh + \ell h + \ell h$ OR
$SA = 2\ell w + 2wh + 2\ell h$ OR
$SA = 2(\ell w + wh + \ell h)$

# COMMON CORE STATE STANDARDS

| STANDARDS FOR MATHEMATICAL CONTENT | |
|---|---|
| **6.G.A** | **Solve real-world and mathematical problems involving area, surface area, and volume.** |
| 6.G.1 | Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. |
| 6.G.2 | Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. |
| 6.G.4 | Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. |
| **6.EE.A** | **Apply and extend previous understandings of arithmetic to algebraic expressions.** |
| 6.EE.2 | Write, read, and evaluate expressions in which letters stand for numbers: |
| a. | Write expressions that record operations with numbers and with letters standing for numbers. |
| c. | Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6\ s^2$ to find the volume and surface area of a cube with sides of length s = 1/2.* |
| 6.EE.3 | Apply the properties of operations to generate equivalent expressions. |
| 6.EE.4 | Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). |
| **6.EE.B** | **Reason about and solve one-variable equations and inequalities.** |
| 6.EE.6 | Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. |
| **6.EE.C** | **Represent and analyze quantitative relationships between dependent and independent variables.** |
| 6.EE.9 | Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs ~~and tables~~, and relate these to the equation. |
| **6.RP.A** | **Understand ratio concepts and use ratio reasoning to solve problems.** |
| 6.RP.3 | Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations: |
| d. | Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. |

| STANDARDS FOR MATHEMATICAL PRACTICE | |
|---|---|
| SMP1 | Make sense of problems and persevere in solving them. |
| SMP2 | Reason abstractly and quantitatively. |
| SMP3 | Construct viable arguments and critique the reasoning of others. |
| SMP4 | Model with mathematics. |
| SMP5 | Use appropriate tools strategically. |
| SMP6 | Attend to precision. |
| SMP7 | Look for and make use of structure. |
| SMP8 | Look for and express regularity in repeated reasoning. |

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