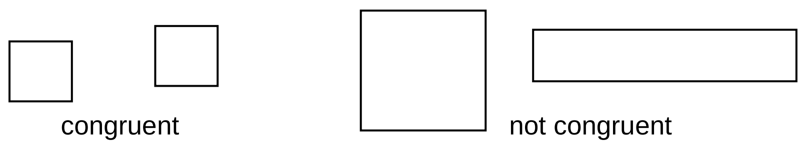
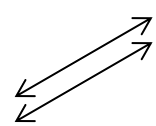
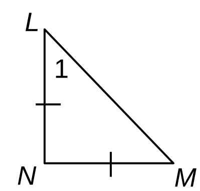
STUDENT RESOURCES

Word or Phrase	Definition
congruent figures	Two figures in the plane are congruent figures if the second can be obtained from the first by a sequence of one or more of translations, rotations, and reflections. Two squares are congruent if they have the same side-length.  congruent not congruent If $\triangle ABC$ is congruent to $\triangle DEF$, we write $\triangle ABC \cong \triangle DEF$.
function	A function is a rule that assigns to each input value exactly one output value. A function may also be referred to as a transformation or mapping. The collection of output values is the image of the function. Consider the function $y = 3x + 6$. For any input value, say $x = 10$, there is a unique output value, in this case $y = 36$. This output value is obtained by substituting the value of x into the equation. This function represents a straight line consisting of all ordered pairs of points (x, y) that satisfy the equation. Consider the transformation $(x, y) \rightarrow (-x, y)$. This transformation maps the x-coordinates in the plane to their opposites, while y-coordinates remain the same. In this case the image is a reflection about the y-axis.
image	The image of a function or transformation is the collection of its output values. The input values are then referred to as the pre-image. See transformation.
parallel	Two lines in a plane are parallel if they do not meet. Two line segments in a plane are parallel if the lines they lie on are parallel. 
reflection	A reflection of a plane through a line L is the transformation that maps each point to its mirror image on the other side of L. The line L is called the line of reflection. The transformation $(x, y) \rightarrow (x, -y)$ is a reflection of the plane through the x-axis.
rigid motion	A rigid motion is a transformation that preserves distances. Any rigid motion of the plane is a sequence of one or more translations, rotations, and reflections. Rigid motions also preserve lengths, angle measures, and parallel lines.
rotation	A rotation of a plane is a transformation that turns it through a given angle about a given point. The given angle is called the angle of rotation, and the given point is called the center point of rotation. The transformation $(x,y) \rightarrow (-y,x)$ is a rotation of the plane about the origin through angle 90°.

Word or Phrase	Definition
transformation	A <u>transformation</u> is a function that maps points in the plane (called the pre-image) to points in the plane (called the image). Rigid motion transformations include translations, rotations, and reflections.
translation	A <u>translation</u> of the plane is the transformation of the plane that maps pre-image points to image point in the same distance and direction. The transformation $(x, y) \longrightarrow (x + 1, y + 2)$ slides all points 1 unit to the right and 2 units up.

Geometry Notation

These are examples of geometry diagrams and notations used in this program.

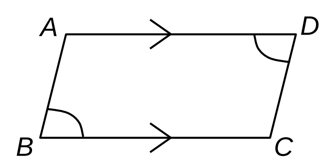


- A point is named using capital letters.

  Example: point M

- A polygon (e.g., triangle, parallelogram) is identified with a small symbol followed by its vertices.

  Examples: $\triangle LMN$, $\square ABCD$



- Line segment from point L to point N *is* named with the endpoints and a "bar" over them.

  Example: $\overline{LN}$

- The length (a measure) of a line segment from point L to point N is distinguished from the line segment (an object) by using absolute value symbols.

  Example: $\left|\overline{LN}\right|$

- An angle is named at its vertex and points on its rays, if needed.

  Example: The angle at L may be denoted $\angle L$, $\angle NLM$, $\angle MLN$, or $\angle 1$.

- The measure of an angle is distinguished from the angle (an object) using absolute value symbols.

  Example: The measure of $\angle L$ is written as $\left|\angle L\right|$.

- The symbol $\|$ indicates parallel lines. Arrows in a diagram indicate parallel segments as well.

  Example: $\overline{AD} \,\|\, \overline{BC}$

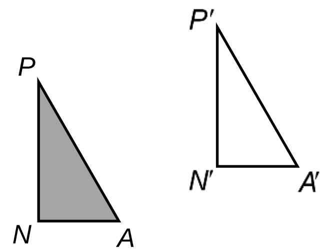
- The symbol $\cong$ indicates congruence. Tick marks on the diagram above indicate congruent segments and arcs indicate congruent angles as well.

  Examples: Line segments $\overline{LN}$ and $\overline{NM}$ have the same length. Therefore, $\overline{LN} \cong \overline{NM}$.

  Angles at B and D have the same measure. Therefore, $\angle B \cong \angle D$.

Transformations of the Plane

A <u>transformation</u> is a function that maps points in the plane (called the spre-image) to points in the plane (called the <u>image</u>).



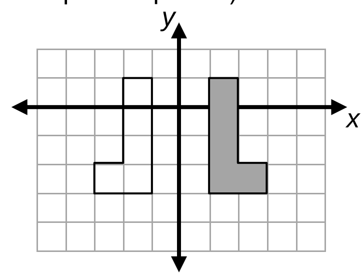
The input values (called the pre-image) are points in the plane. The output values (called the <u>image</u> of the transformation) are also points in the plane.

A transformation can be viewed as a <u>mapping</u> of pre-images (input values) to their corresponding images (output values).

In this figure, shaded triangle $\triangle PAN$ represents input values of a transformation and unshaded triangle $\triangle P'A'N'$ represents its image (output values).

The prime symbol (an apostrophe-like symbol) is often used to distinguish points in a pre-image (input values) from their images (output values).

We use the arrow notation $P \longrightarrow P'$ (read "point *P* is taken to point *P* prime" or "*P* maps to *P* prime") to indicate that the image of the point P under the transformation is P'.

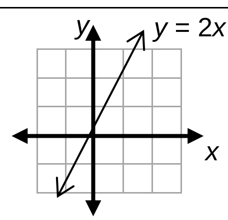
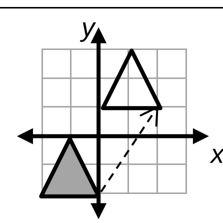


In a coordinate plane, we use the coordinates to describe the transformation as in the following example.

The reflection over the *y*-axis maps the shaded L-figure to a backwards L-figure. We use the arrow notation to describe this transformation. In this example, $(x, y) \longrightarrow (-x, y)$.

Comparison of an Algebraic Function and a Geometric Function

Functions arise in many different contexts. The way we think of them and even the language we use to talk about them may be quite different for different areas of math. Here we compare a typical <u>function</u> we might meet in an algebra course and a typical function (we call it a <u>transformation</u>) that we might study in geometry.

Name of function	Linear function	Translation
Rule	Multiply by 2	Translate 2 units right and 3 units up
Description with symbols	x maps to $2x$ $x \longrightarrow 2x$ $y = 2x$	(x, y) maps to $(x + 2, y + 3)$ $(x, y) \longrightarrow (x + 2, y + 3)$
Graph		
Graph interpretation	The *x*-coordinates represent the inputs and the *y*-coordinates represent the outputs. The set of all input-output pairs is represented by the line.	A figure (shaded triangle - input) and its image (unshaded triangle - output) illustrate what happens to a typical figure in the plane. The translation arrow shows the direction and distance each point is moved.

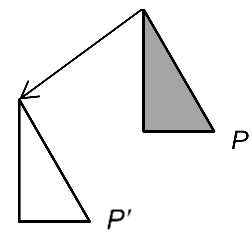
Translations, Rotations, and Reflections

Translations, rotations, and reflections are transformations of the plane that preserve distance between points.

A translation is a transformation that shifts all points the same distance and in the same direction.

This translation maps P to P' ($P \rightarrow P'$).
(read "P maps to P prime").

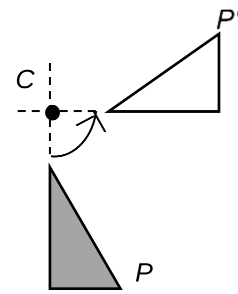
The translation arrow shows the shift



A rotation of a plane is a transformation that turns it through a given angle about a given point. The given point is called the center point of rotation. The given angle is called the angle of rotation.

This rotation maps P to P' ($P \rightarrow P'$).

Point C is the center point of the rotation.



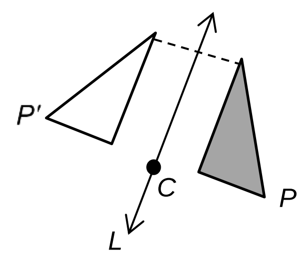
The angle of rotation is 90° (or a quarter counterclockwise).

The reflection of a plane through a line L is the transformation that takes each point to its mirror image on the other side of L.

This reflection maps P to P' ($P \rightarrow P'$).

Line L is the line of reflection.

Line L is the perpendicular bisector of $\overline{PP'}$.



Translations, rotations, and reflections preserve distances between points. Further, translations, rotations, and reflections

- map lines to lines,
- map line segments to line segments of the same length,
- map parallel lines to parallel lines, and
- map angles to angles of the same measure.