Name ______________________________ Period __________ Date ______________

UNIT 9
STUDENT PACKET

MathLinks
GRADE 8

CONGRUENCE

Parent (or Guardian) signature ______________________________

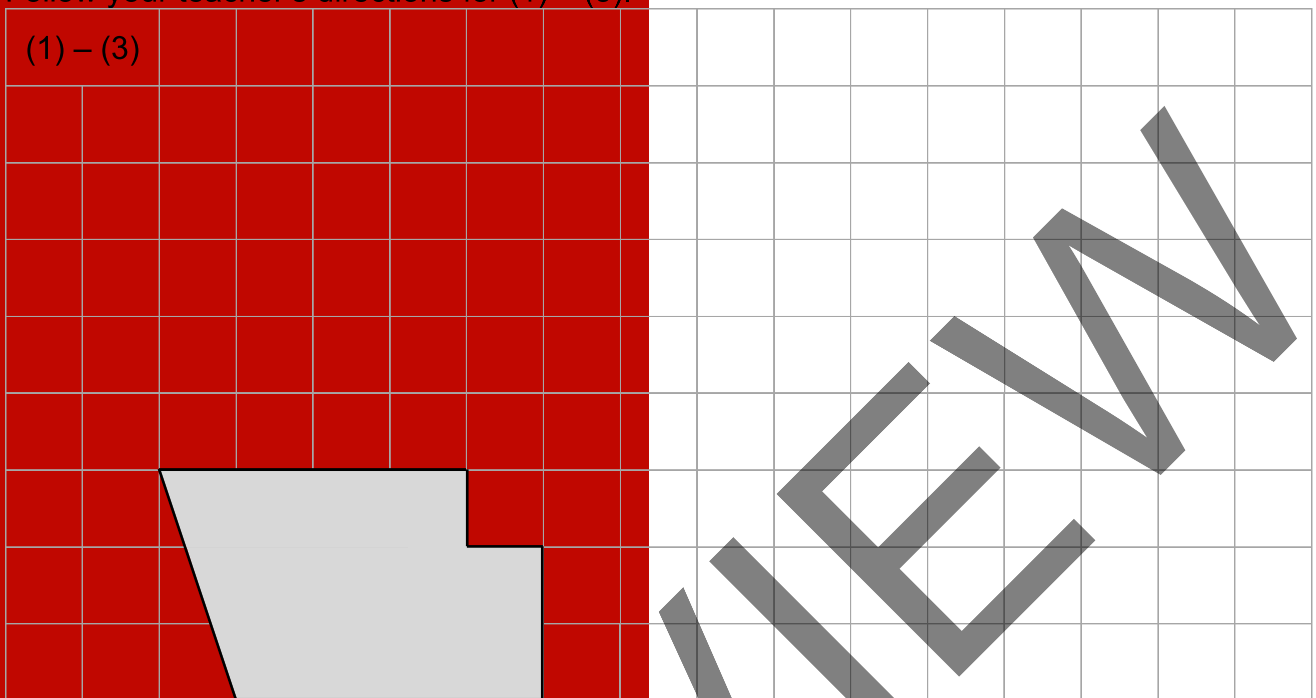
MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

congruent figures	function
image (pre-image)	rigid motion
reflection	rotation
translation	transformation

SLIDES, TURNS, AND FLIPS

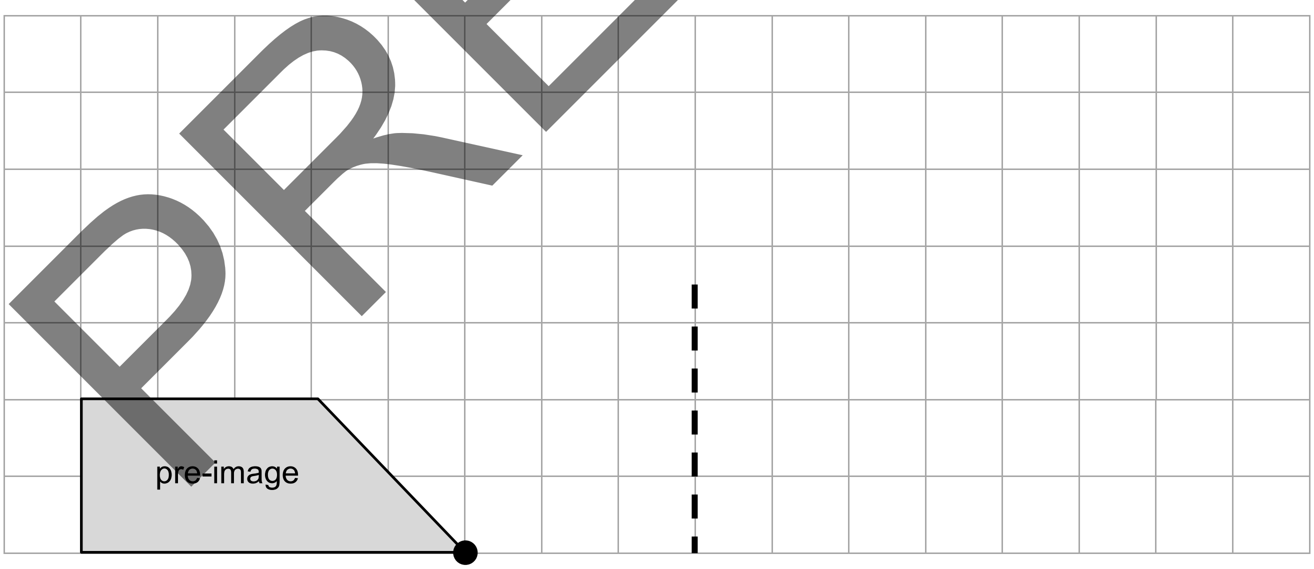
Follow your teacher's directions for (1) – (3).



4. Record the meanings of image, transformation, and function in **My Word Bank**.

5. In a transformation, the input is the ____________ and the output is the __________.

6. On the given pre-image below, perform any slide up and to the right; any clockwise turn around the point; and a flip over the dashed line.



TRANSLATIONS

We will explore translations, which are a type of rigid motion transformation, using patty paper and with coordinates.

[8.G.1a, 8.G.1b, 8.G.1c, 8.G.3, 8.G.7, 8.F.1; SMP1, 2, 3, 5, 6, 7, 8]

GETTING STARTED

Review the box titled **Geometry Notation** in **Student Resources.** Then use symbolic notation for the following.

1. Write two names for a line segment with endpoints *A* and *B*. _______________

2. Write a name for the length of a line segment with endpoints *A* and *B*. _______________

Sometimes we use an apostrophe-like symbol to identify points.
We read $\overline{A'B'}$ as “segment *A* prime *B* prime.”

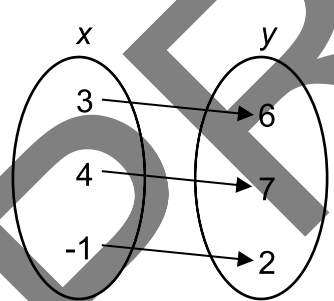
3. Write an equation that states $\overline{AB}$ and $\overline{A'B'}$ have the same length.

4. Write two names for $\angle 1$. _______________

5. Write two names for the measure of $\angle FGJ$. _______________

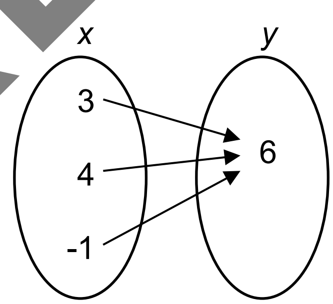
Determine whether each mapping diagram below represents a function.

6.



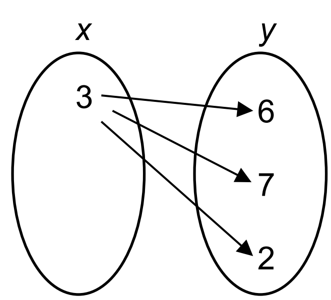
Function?

7.



Function?

8.



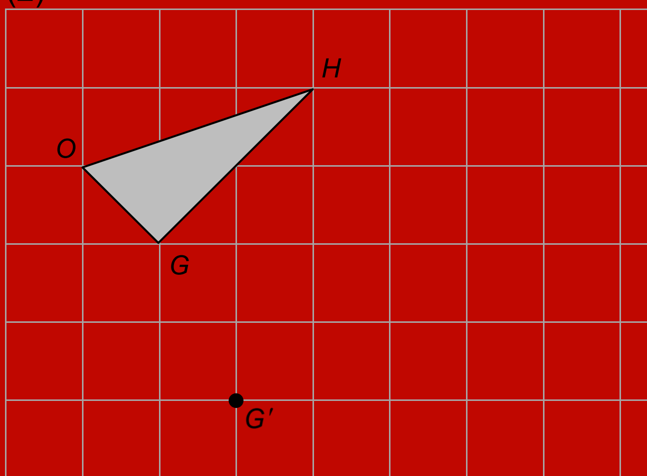
Function?

ABOUT TRANSLATIONS

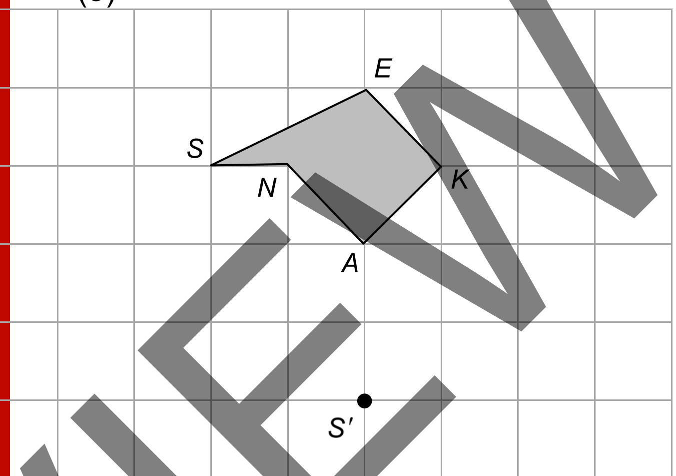
Follow your teacher's directions for (1) – (3).

(1)

(2)



(3)



4. In a translation, line segments are taken to line segments of the same length. For translation $HOG \longrightarrow H'O'G'$, verify that $|HG| = |H'G'|$.

5. In a translation, angles are taken to angles of the same measure. For translation $SNAKE \longrightarrow S'N'A'K'E'$, use proper geometric notation to indicate three pairs of angles that have same measure.

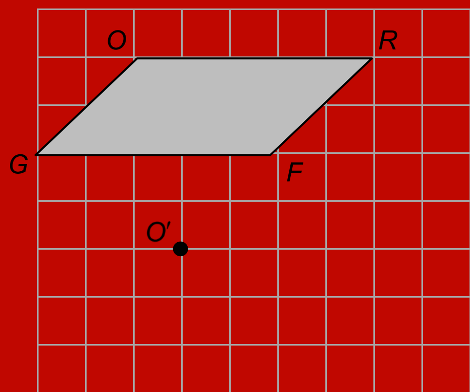
   $|\angle SNA| =$ __________ ________________ ________________

6. In a translation, parallel lines are taken to parallel lines. For translation $SNAKE \longrightarrow S'N'A'K'E'$, verify that $\overline{EK} \parallel \overline{NA}$ and $\overline{E'K'} \parallel \overline{N'A'}$.

7. Record the meanings of <u>translation</u> and <u>rigid motion</u> in **My Word Bank**.

PRACTICE 1

1. Start with the pre-image $\square FROG$.



a. Translate $\square FROG \rightarrow \square F'R'O'G'$.

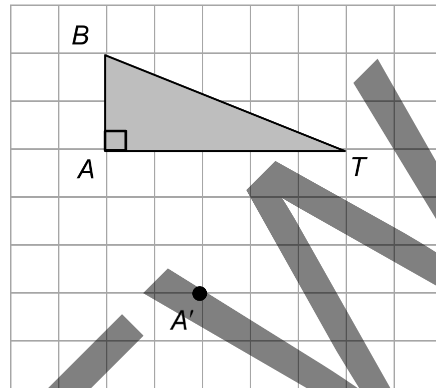
b. In the translation, where do $\overline{GO}$ and $\overline{FR}$ map in the image?

c. Demonstrate that lengths in the pre-image are taken to lengths in the image by demonstrating that $\left|\overline{OR}\right| = \left|\overline{O'R'}\right|$ and $\left|\overline{GO}\right| = \left|\overline{G'O'}\right|$.

d. Demonstrate that angle measures in the pre-image are taken to angle measures in the image by writing the measures of $\angle G$ and $\angle G'$.

e. Show that $\left|\overline{GO}\right| \parallel \left|\overline{FR}\right|$ in the pre-image, and that $\left|\overline{G'O'}\right| \parallel \left|\overline{F'R'}\right|$ in the image.

2. Start with the pre-image $\triangle BAT$.



a. Translate $\triangle BAT \rightarrow \triangle B'A'T'$.

b. $\overline{BA}$ maps to ____; $\overline{AT}$ maps to ____

$\left|\overline{BA}\right|$ = ____ units; $\left|\overline{B'A'}\right|$ = ____ units

$\left|\overline{AT}\right|$ = ____ units; $\left|\overline{A'T'}\right|$ = ____ units

c. If $\left|\angle T\right| = \left|\angle T'\right| = 22°$, then:

$\left|\angle A\right| = \left|\angle A'\right|$ = _____;

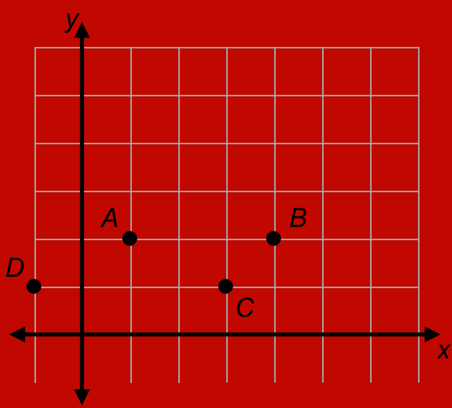
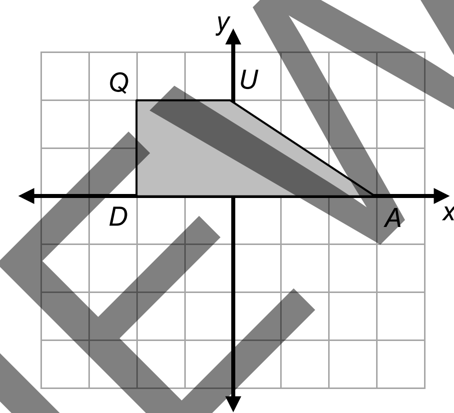
$\left|\angle B\right| = \left|\angle B'\right|$ = _____.

d. For this transformation, why is there no way to check if parallel lines are taken to parallel lines?

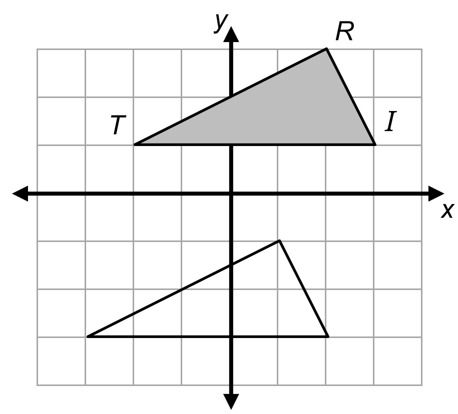
e. Does it appear that $B'A'T'$ will completely cover BAT?

TRANSLATIONS WITH COORDINATES

Follow your teacher's directions for (1) – (2). Small grid squares are one square unit of area.

(1) Transformation: $(x, y) \rightarrow$ (_____, _____)	(2) Transformation: $(x, y) \rightarrow$ (_____, _____)
	

$\triangle TRI$ is taken to $\triangle T'R'I'$ under a translation.



3. Label image points. Write a rule for the transformation.

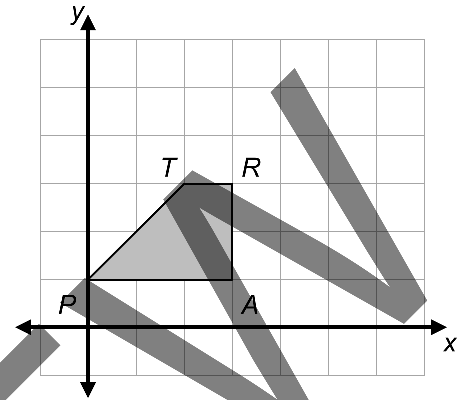
   $(x, y) \rightarrow$ (______, ______)

4. Since this is a rigid motion, angles are taken to angles. Name at least two pairs of corresponding pre-image/image angles that have equal measures.

5. Draw translation arrows $\overline{TT'}$, $\overline{RR'}$, $\overline{II'}$. Even though this is a rigid motion, why is there no way to check if parallel lines are taken to parallel lines?

TRANSLATIONS WITH COORDINATES
Continued

Here is a transformation (function) rule: $(x, y) \rightarrow (2x, y + 3)$.



6. Write image coordinates for this transformation.

   $T\,(2, 3) \rightarrow T'\,(\_\_\_, \_\_\_)$ $R\,(\_\_\_, \_\_\_) \rightarrow R'\,(\_\_\_, \_\_\_)$

   $A\,(\_\_\_, \_\_\_) \rightarrow A'\,(\_\_\_, \_\_\_)$ $P\,(\_\_\_, \_\_\_) \rightarrow P'\,(\_\_\_, \_\_\_)$

7. Draw and label the image for ⏢ *TRAP*.

8. Does it appear that ⏢ *TRAP* will exactly cover ⏢ *T'R'A'P'*?

9. Give an example where angles are NOT taken to angles of the same measure

10. Give an example where segments are NOT taken to segments of the same length.

11. There is a pair of parallel sides in the pre-image, and a cooresponding pair of parallel sides in the image. List them below.

    ______ || ______ and ______ || ______

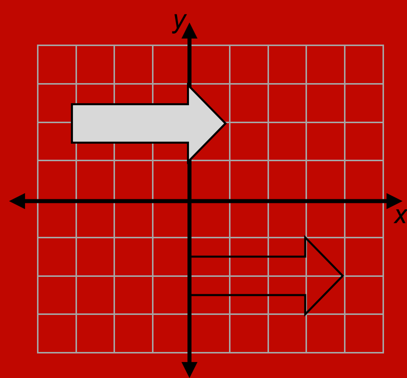
12. Why is this transformation NOT a rigid motion?

PRACTICE 2

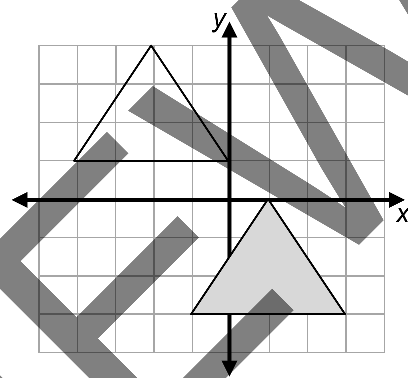
Each grid square is equal to one square unit of area.

For problems 1 – 2, a pre-image (shaded figure) is taken to its image under a transformation. Label one pre-image vertex A, and its corresponding image vertex A'. Draw a translation arrow from A to A' and write a translation rule using symbols.

1. $(x, y) \to$ (_____, _____)



2. $(x, y) \to$ (_____, _____)



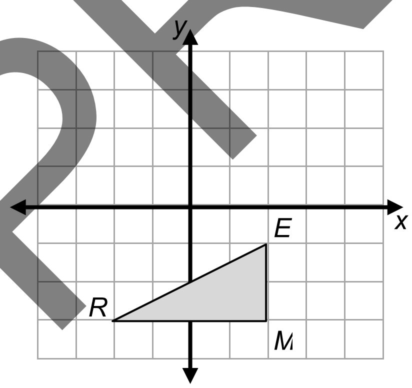
For problems 3 – 4, pre-image figures are shaded. For each transformation rule, write the pre-image and image coordinates, graph the image, and explain why the image represents a translation of the pre-image or not.

3. $(x, y) \to (x - 1, y + 5)$

R (___, ___) $\to$ R' (___, ___)

E (___, ___) $\to$ E' (___, ___)

M (___, ___) $\to$ M' (___, ___)



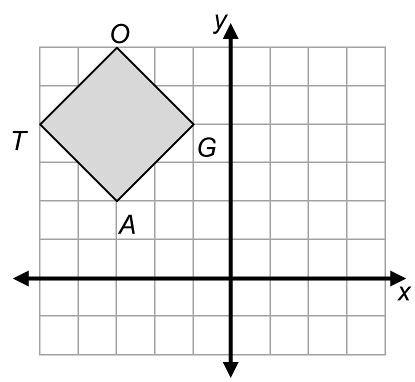
4. $(x, y) \to (x + 5, \frac{1}{2}y)$

T (___, ___) $\to$ T' (___, ___)

O (___, ___) $\to$ O' (___, ___)

G (___, ___) $\to$ G' (___, ___)

A (___, ___) $\to$ A' (___, ___)



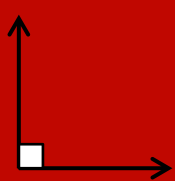
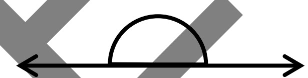
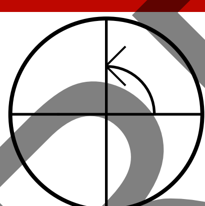
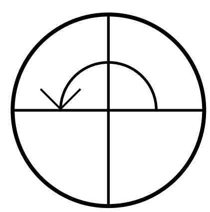
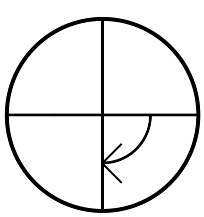
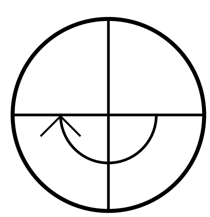
ROTATIONS

We will experiment with rotations and explore their properties. We will make conjectures about rotations of coordinate pairs.

[8.G.1a, 8.G.1b, 8.G.1c, 8.G.3; SMP1, 2, 3, 5, 6, 8]

GETTING STARTED

Fill in the blanks for each problem below.

1. This is a(n) ___________ angle. Its measure is _____ degrees. 	2. This is a(n) ___________ angle. Its measure is _____ degrees. 
3. There are _____ degrees in a quarter turn counterclockwise. 	4. There are _____ degrees in a half turn counterclockwise. 
5. There are _____ degrees in a quarter turn clockwise. 	6. There are _____ degrees in a half turn clockwise. 

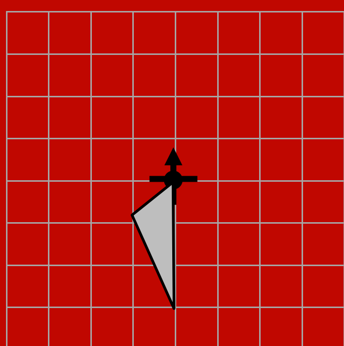
ABOUT ROTATIONS

Follow your teacher's directions for (1) – (3).

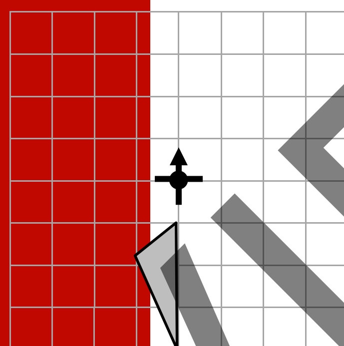
Patty Paper Rotations

- Trace the pre-image on patty paper with pencil markings on both sides of the paper.
- Make a small ↑ (axis arrow pointing up) to indicate the location of the center point and the initial orientation.
- Perform the rotation about the center using patty paper and trace the image.

(1)



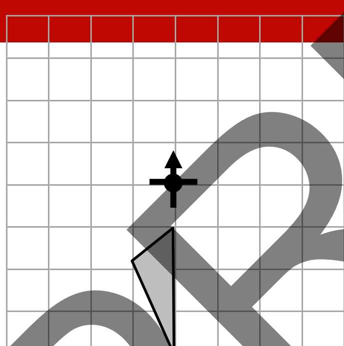
(2)



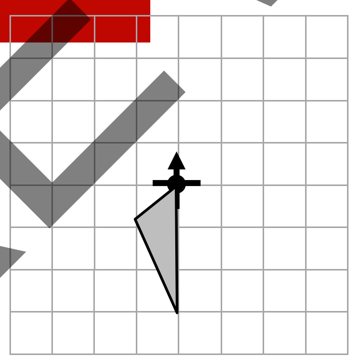
(3)

Rotate each triangle around the origin as indicated.

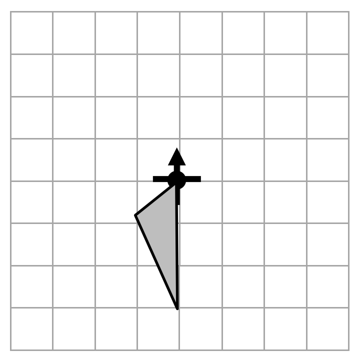
4. Counterclockwise 90°



5. Counterclockwise 90°



6. Clockwise 180°



7. In the five rotations above, choose any point in a pre-image figure and its corresponding image point. Are the distances from these two points to the center of rotation the same?

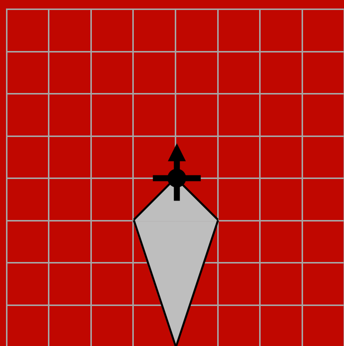
   What do you think about the other pairs of corresponding points?

8. Record the meaning of <u>rotation</u> in **My Word Bank**.

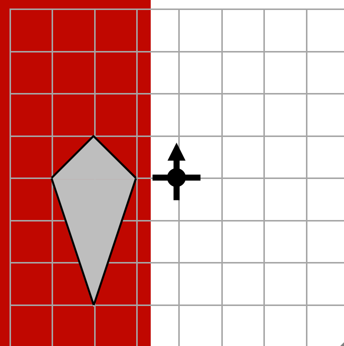
PRACTICE 3

Perform each rotations indicated below.

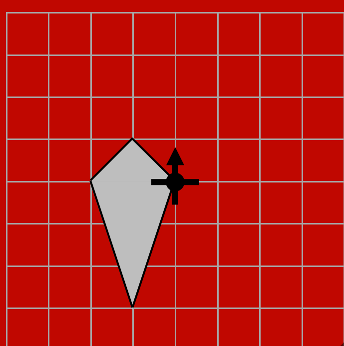
1. Counterclockwise 90°



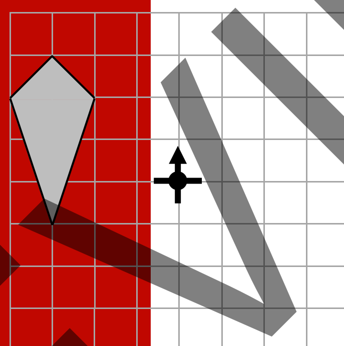
2. Counterclockwise 90°



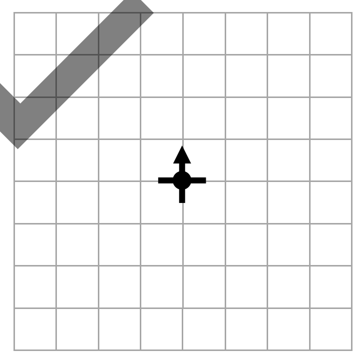
3. Counterclockwise 180°



4. Counterclockwise 180°



5. To show that every point in a plane maps to its image, draw the letter *F* somewhere on the grid below. Then rotate it counterclockwise 90° to find its image. Circle the image.



6. Does your conjecture about distances from problem 7 on **About Rotations** hold for the problems above? Restate and revise your conjecture as needed.

7. Give an example where line segments are taken to line segments of the same length.

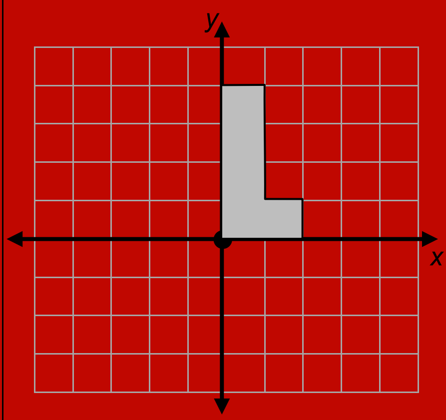
8. Give an example where angles are taken to angles of the same measure.

9. Does it appear that images will exactly cover corresponding pre-images?

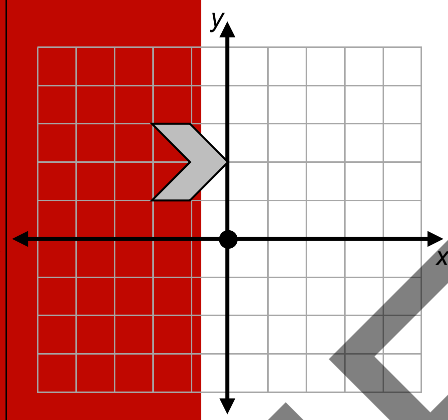
PRACTICE 4

For each pre-image below, show its image under a rotation about the origin. Label at least one corresponding point in each pre-image and image. Small grid squares are one square unit of area. Use patty paper as needed.

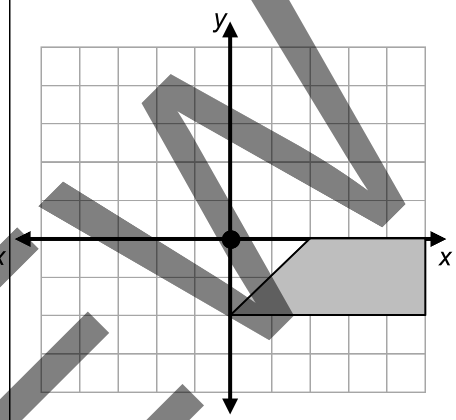
1. Counterclockwise 90°



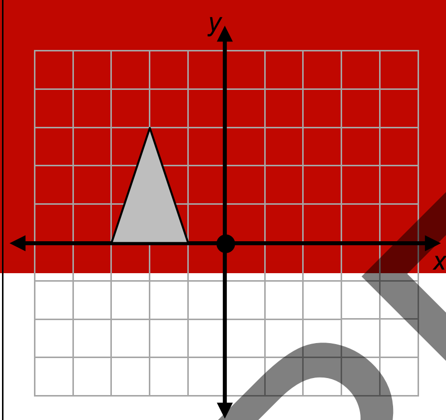
2. Clockwise 90°



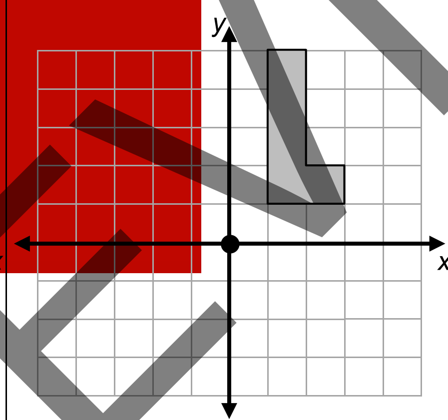
3. Counterclockwise 180°



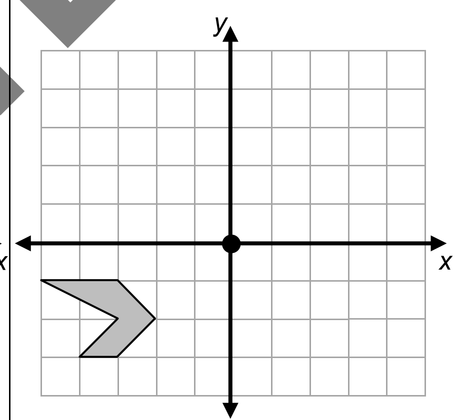
4. Counterclockwise 90°



5. Clockwise 90°



6. Counterclockwise 180°



Make conjectures about what happens to (x, y) when the plane is rotated as indicated.

7. Counterclockwise 90° (problems 1 and 4 above)	8. Clockwise 90° (problems 2 and 5 above)	9. Counterclockwise 180° (problems 3 and 6 above)
$x \rightarrow$ ______ $y \rightarrow$ ______ $(x, y) \rightarrow$ ________		

10. What happens to the center (0, 0) under a rotation?

REFLECTIONS

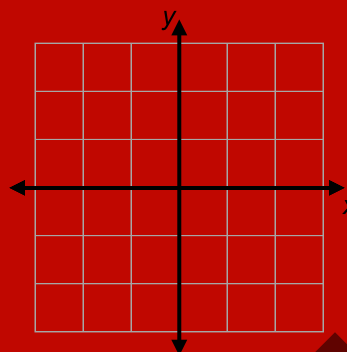
We will experiment with reflections and explore their properties. We will make conjectures about reflections of coordinate pairs. We will compare properties of translations, rotations, and reflections. Finally, we will define what it means for figures to be congruent.

[8.G.1a, 8.G.1b, 8.G.1c, 8.G.2, 8.G.3, 8.G.7; SMP1, 2, 3, 4, 5, 6, 7, 8]

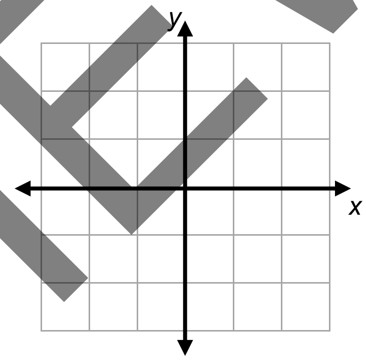
GETTING STARTED

Draw any letter from your first name in Quadrant III. Then perform each given transformation on that letter and circle the image. State whether it is a translation, rotation, or neither. Small grid squares are one square unit of area.

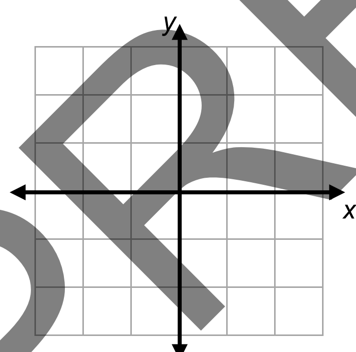
1. $(x, y) \rightarrow (x + 3, y + 2)$



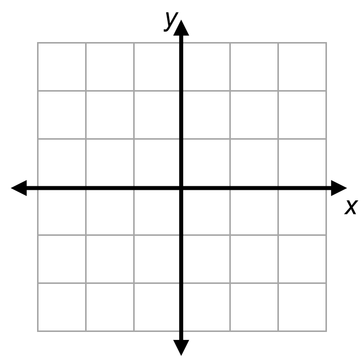
2. $(x, y) \rightarrow (y, -x)$



3. $(x, y) \rightarrow (-x, y)$



4. $(x, y) \rightarrow \left(\frac{1}{2}x, y + 4\right)$



5. Are any of the transformations above NOT rigid motions? Explain.

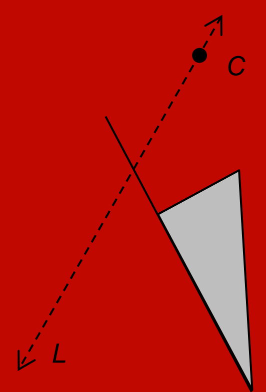
ABOUT REFLECTIONS

Patty Paper Reflections

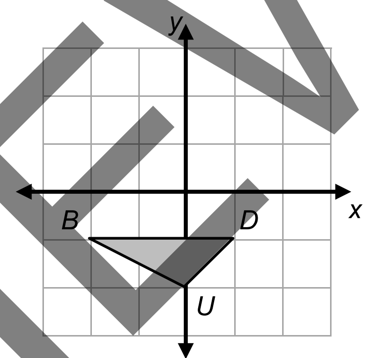
- Trace the pre-image on patty paper with pencil markings on both sides of the paper.
- Trace the line of reflection with a point on the line.
- Perform the reflection over the line, using the point to align the patty paper after the flip, and trace the image.

Follow your teacher's directions for (1) – (2). Small grid squares are one square unit of area.

(1)



(2)



3. Using problem 2, make a conjecture about the effect on coordinates under a reflection across the *x*-axis and give evidence.

   B → ____; (____, ____) → (____, ____)

   U → ____; (____, ____) → (____, ____)

   D → ____; (____, ____) → (____, ____)

4. Does a point on a line of reflection change location under a reflection (such as *C* in problem (1) or the origin in problem (2))?

5. Demonstrate for problem 2 that line segments are taken to line segments of equal length.

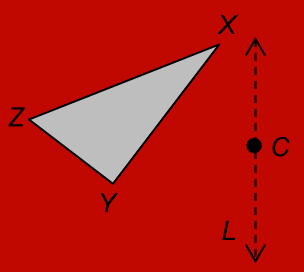
6. Why is it not relevant to consider parallel lines in problem (2) when determining that the transformation (reflection) is a rigid motion?

7. Record the meaning of <u>reflection</u> in **My Word Bank**.

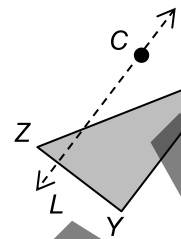
PRACTICE 5

For each problem below, reflect the pre-image across line *L*. Label the vertices in the images with X', Y', and Z'. Use patty paper as needed.

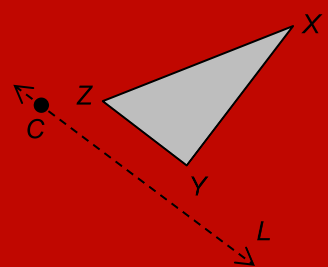
1.



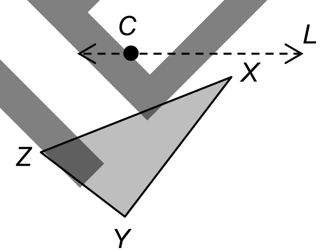
2.



3.



4.



5. Does it appear that each image will exactly cover each corresponding pre-image?

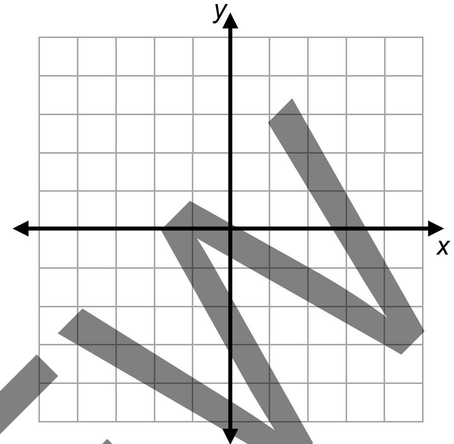
6. For each example, read the vertex letters on each pre-image in a clockwise direction, starting with *x*. Then read the vertex letters on each image in a clockwise direction, starting with *x*. Explain how they differ.

7. Which of the 3 rigid motions we have done requires flipping the patty paper over? How does that relate to problem 6?

8. For problems 1 – 4, lightly draw $\overline{ZZ'}$. What does the line of reflection appear to do to $\overline{ZZ'}$?

PRACTICE 6

Small grid squares are one square unit of area.



1. Draw pre-image ⏢ *FLIP* and shade it in.

   F (-5, 3) *L* (-3, 5)

   I (-1, 5) *P* (-1, 3)

2. Reflect ⏢ *FLIP* through the *y*-axis and record these image vertices.

   F' (____, ____) L' (____, ____)

   I' (____, ____) P' (____, ____)

   $(x, y) \rightarrow$ (____, ____)

How are the coordinates of the vertices of this image related to those of the pre-image?

3. Reflect ⏢ *FLIP* through the *x*-axis and record these image vertices. (Notice the “double prime” notation.)

   F'' (____, ____) L'' (____, ____) I'' (____, ____) P'' (____, ____)

   $(x, y) \rightarrow$ (____, ____)

How are the coordinates of the vertices of this image related to those of the pre-image?

4. Provide two examples of each description below.

a. Line segments are taken to line segments of the same length.	b. Angles are taken to angles of the same measure.	c. Parallel lines are taken to parallel lines.

ABOUT CONGRUENCE

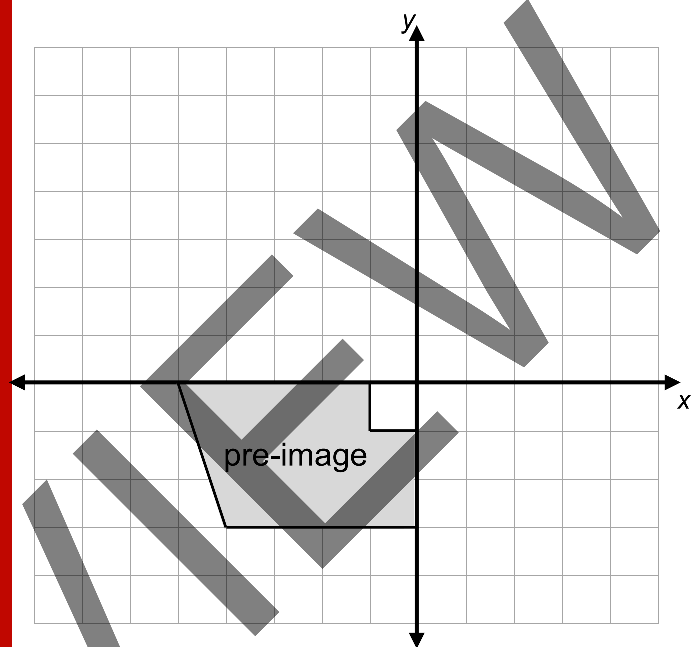
1. Record the meaning of congruent figures in **My Word Bank**.

Small grid squares are one square unit of area. Use patty paper if needed.

2. Using the pre-image, create Image *A* using the following two steps, and then label the new image.

   - Step 1: rotate the preimage clockwise 90° around the origin
   - Step 2: reflect the result about the *y*-axis

   Why is Image *A* congruent to the pre-image?



3. Using the pre-image, create Image *B* using the following two steps, and then label the new image.

   - Step 1: use the rule: $(x, y) \rightarrow (-x, y)$
   - Step 2: use the rule: $(x, y) \rightarrow (x - 8, y + 6)$

   Why is Image *B* congruent to the pre-image?

4. Use words or symbols to write a sequence of two steps to show that Image $A \cong$ Image B.

ABOUT CONGRUENCE
Continued

5. Using the pre-image, create and label Image *C* in quadrant IV. Use two steps and write them below.

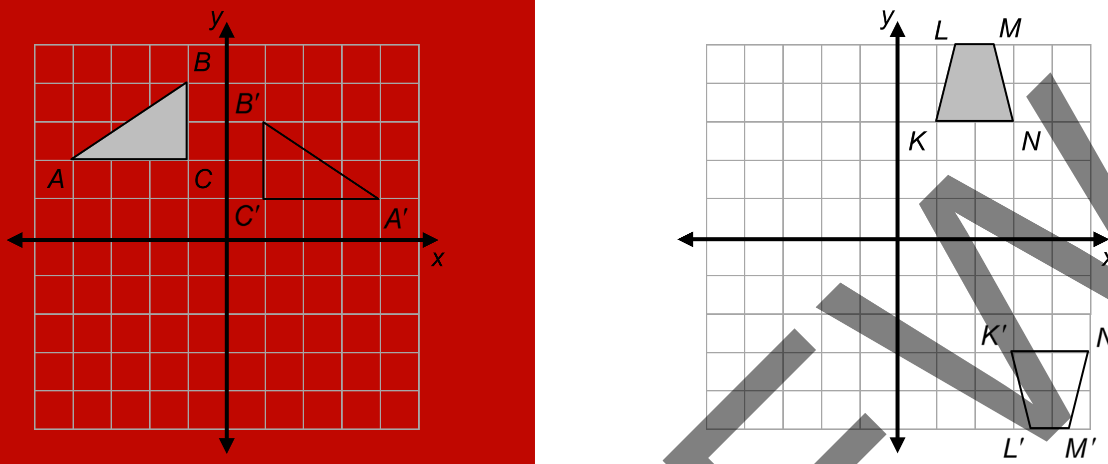
6. Use words or symbols to write a sequence of steps to show that Image $C \cong$ Image *A* OR Image $C \cong$ Image *B*.

7. Explain how rigid motions (translations, rotations, and reflections) are related to congruence.

8. Brigitte says that congruent figures have the same size and shape. Is Brigitte correct? Explain.

PRACTICE 7

Use the two graphs below for problems 1 – 4. Small grid squares are one square unit of area.



1. Describe a two-step sequence of transformations that maps each pre-image to its image. Use words or algebraic symbols.

a. For the triangles:	b. For the trapezoids:

2. When two figures are congruent, what do you know about their corresponding angles and corresponding sides?

3. List two pairs of congruent angles.

a. For the triangles:	b. For the trapezoids:

4. List two pairs of congruent sides.

a. For the triangles:	b. For the trapezoids:

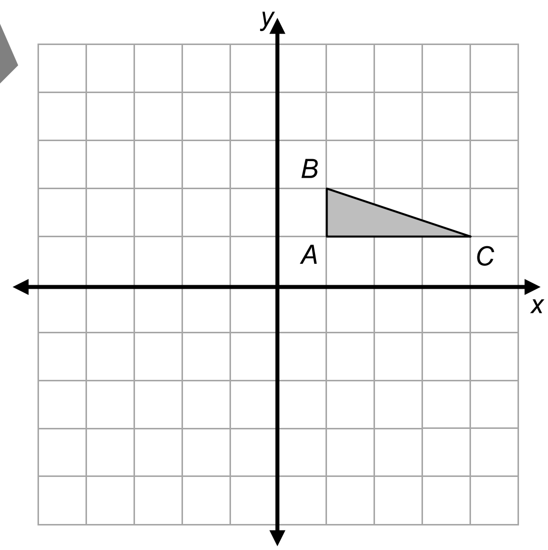
PRACTICE 7
Continued

5. Rashad says that if all the sides of one triangle have the same length as the corresponding sides in their image, then the triangles will be congruent. Is Rashad correct? Explain in writing or refute with a counter example.

6. Van says that if all the angles of one triangle have the same measure as the corresponding angles in their image, then the triangles will be congruent. Is Van correct? Explain in writing or refute with a counterexample.

Use the coordinate plane below with the given pre-image for problems 7 – 8.

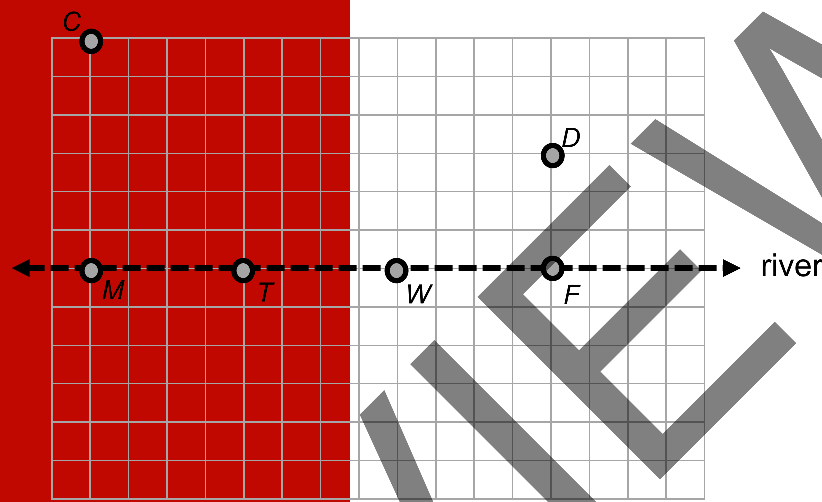
7. Use words or algebraic symbols to describe a two-step sequence of transformations for an image $\triangle DEF$ that is congruent to pre-image $\triangle ABC$ that is in in quadrant II. Be sure to draw and label the image.



8. Use words or algebraic symbols to describe a two-step sequence of transformations for an image $\triangle JKL$ that is NOT congruent to pre-image $\triangle ABC$ that is in in quadrant IV. Be sure to draw and label the image.

SWIMMING AT THE RIVER

Chris and David each live in a van down by the river. Chris (location *C*) frequently walks to the river for a short swim, and then continues walking to visit David (location *D*). His swimming stops for Monday, Tuesday, Wednesday, and Friday are marked (*M*, *T*, *W*, and *F*). There are no obstacles along the way, and he always walks in straight-line paths directly to that day's swimming spot (vertical, horizontal, or diagonal). Small grid squares are one square unit of area.



1. Find the total distance Chris walks each day. Swimming is not included in the distance calculations. Circle the day for which Chris walks the least.

Monday	Tuesday	Wednesday	Friday

SWIMMING AT THE RIVER
Continued

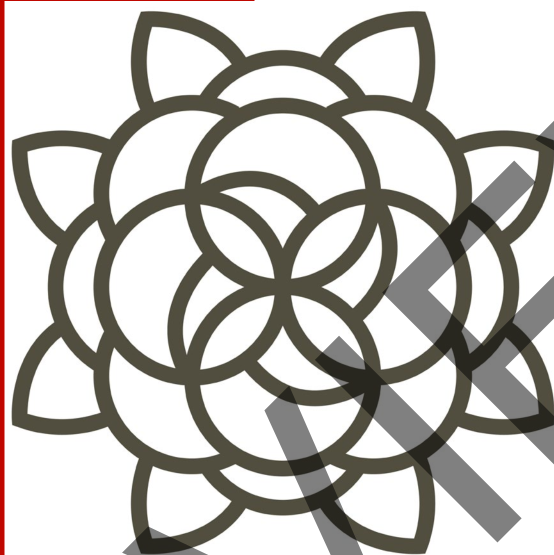
2. Choose two more locations at the river where Chris could stop to swim on Saturday (*S*) that you think might be a shorter distance than the one you circled in problem 1, and calculate those walking distances. How do these distances compare to the others?

3. Reflect pre-image *C* over the river at *M* and label it C'. Then draw segment $\overline{C'D}$. Explain how this line segment helps to support that the shortest walk circled in problem 1 may in fact be the shortest walk possible for Chris.

REVIEW

MANDALAS

A mandala is a symbol of the universe for many Buddhists and Hindus. Most mandalas have colorful, detailed geometric patterns or designs. Below is a simple mandala.



1. Use a colored pencil to shade at least two parts of the mandala that illustrate a translation.

   What color did you use? _______________ Why is this a translation?

2. Use a different colored pencil to shade at least two parts of the mandala that illustrate a reflection over a vertical or horizontal line. Indicate the line of reflection with a dashed line.

   What color did you use? _______________ Why is this a reflection?

3. Use a different colored pencil to shade at least two parts of the mandala that illustrate a rotation. Indicate the center point.

   What color did you use? _______________ Why is this a rotation?

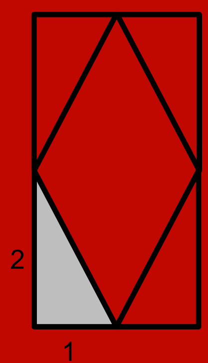
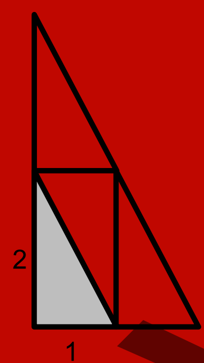
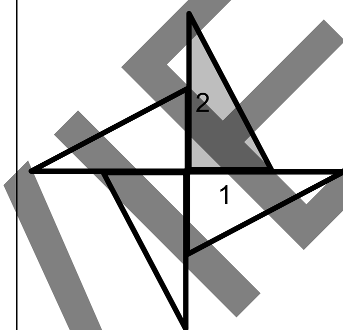
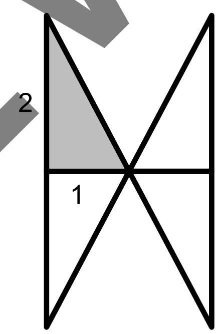
4. You may continue to color the mandala with other colors if you like.

POSTER PROBLEMS: CONGRUENCE

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _____.
- Each group will have a different colored marker. Our group marker is _____.

Part 2: Follow your teacher's directions. Create the diagrams below using rigid motions. Lengths are labeled in linear units on the shaded triangle. You may add labels and use other tools if useful.

Poster 1 (or 5)	**Poster 2 (or 6)**	**Poster 3 (or 7)**	**Poster 4 (or 8)**
			

A. Tape the four-quadrant graph paper to your poster. Let the shaded triangle represent the original pre-image. Draw it on the graph paper so that after some rigid motions are performed, the recreated finished diagram will fit.

B. Perform a sequence of one or two rigid motions on the pre-image for an image that continues to create your diagram. Your diagram should now have two triangles. Describe the rigid motion(s) you performed with words or algebraic symbols.

C. Perform another sequence of one or two rigid motions on the pre-image for an image that continues to create your diagram. Your diagram should now have three triangles. Describe the rigid motion(s) you performed with words or algebraic symbols.

D. Continue as in steps B and C above to complete your diagram.

Part 3: Return to your original poster. Try to follow the steps written by your classmates. Make final corrections and additions so that the directions are as clear as possible on the poster. Be prepared to share with the class.

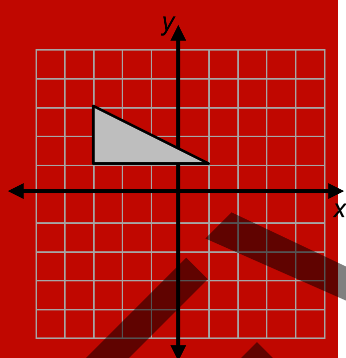
TALKING TRANSFORMATIONS: CONGRUENCE

Small grid squares are one square unit of area.

1. This is an activity for two players. Cut up the cards and place them face down.
   - For Round 1, Teammate A chooses a card, but does not show it to Teammate B.
   - A explains how to create the image through a sequence of rigid motions.
   - B creates the image on this page.
   - Teammates compare to see if directions are clear. Then BOTH teammates copy the image and write a sequence of steps to show the figures are congruent.
   - Teammates exchange roles for each round.

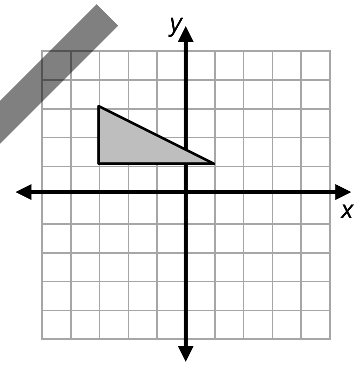
Round 1 (Card ___)

Steps:



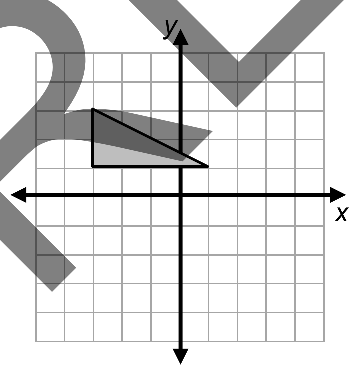
Round 2 (Card ___)

Steps:



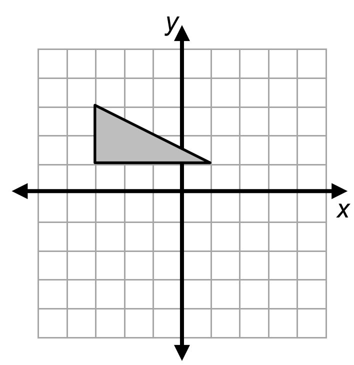
Round 3 (Card ___)

Steps:



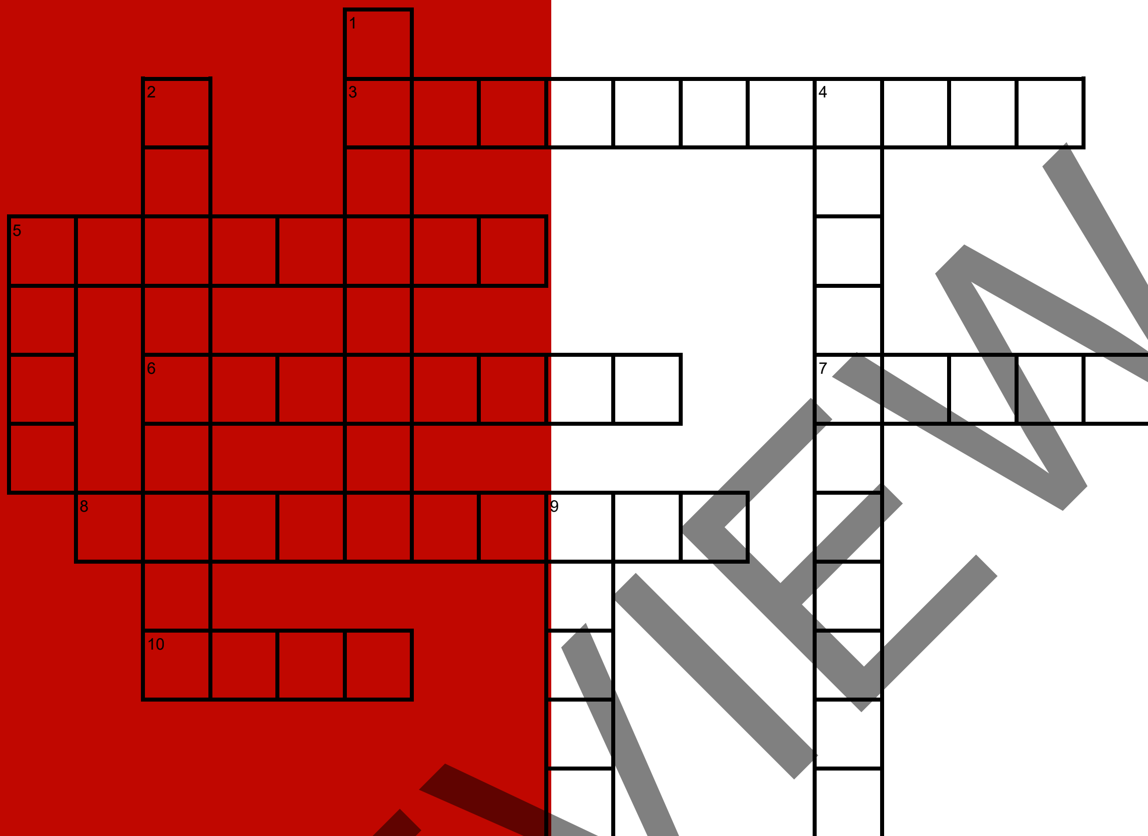
Round 4 (Card ___)

Steps:



2. How do you know that the figures and images in this activity are congruent?

FOCUS ON VOCABULARY



Across

3 a transformation that preserves distance and angle measure (2 words)

5 a transformation of the plane is one example of this; A linear equation in the form $y = mx + b$ is another

6 a "turn" at a given angle around a given point

7 informal name for a translation

8 a transformation where image points are mirror images of the pre-image

10 informal name for a rotation

Down

1 location of points in a plane prior to a transformation (usually hyphenated)

2 two shapes that exactly cover each other after a series of rigid motions

4 $(x, y) \rightarrow (x + 2, y - 3)$ is an example of a rule for this rigid motion

5 informal name for a reflection

9 location of points in a plane after a transformation

SPIRAL REVIEW

1. **READY-X.** Solve for the values of R, E, A, D, Y, X. Sums of rows and columns are indicated at the end of each row and column.

COLUMNS

		1	2	3	
ROWS	1	R	D	E	**22**
	2	R	X	A	**15**
	3	R	E	Y	**15**
	4	R	Y	Y	**11**
		20	**29**	**14**	

R = ______ E = ______ A = ______ D = ______ Y = ______ X = ______

2. Solve each system of equations.

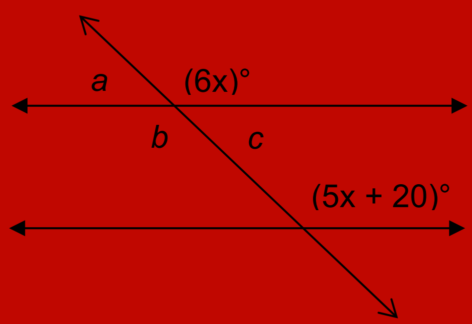
a.
$$\begin{cases} -y + 3x = 5 \\ 15 = 2x + y \end{cases}$$

b.
$$\begin{cases} -3y - 2x = -10 \\ 8 = 3x + y \end{cases}$$

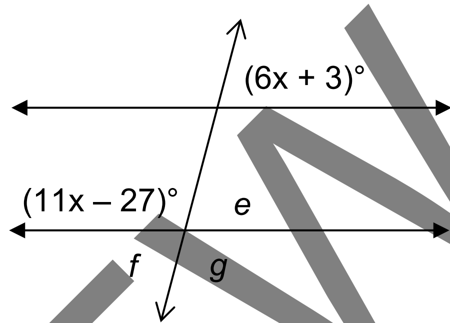
SPIRAL REVIEW
Continued

3. Find the value of x in each angle relationship below, then find the value of each labeled angle. Assume lines that look parallel are parallel.

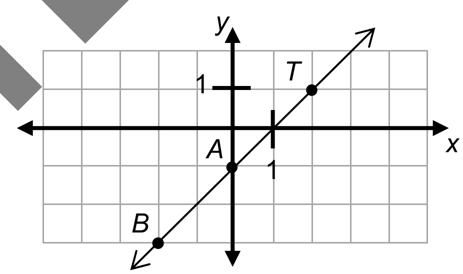
a.



b.



4. T.J. drew the line on the graph below and labeled three points B, A, and T.



   a. Which points could you use to find the slope of the line? Explain.

   b. What is the slope of the line? ______

   c. What is the y-intercept of the line? ______

   d. What is the equation, in slope-intercept form, for the slope of this line? ___________

   e. What is another point on this line? Use the equation.

   f. Find the distance from point B to point T using the Pythagorean Theorem. Answer as a radical and with an approximation rounded to the nearest hundredth.

5. Carson is holding a fundraiser for school. He is selling keychains at $2.50 each and stickers for $0.50 each. At the end of the week, he has raised $91.50 and sold 75 items combined. How many of each item were sold? Define your variables and show your equations.

SPIRAL REVIEW
Continued

Data was collected regarding students with siblings and pets. Responses are recorded and shown below.

	Students (A through J)									
	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
Sibling(s)	No	Yes	Yes	Yes	Yes	No	Yes	No	Yes	Yes
Pet(s)	Yes	No	Yes	No	Yes	No	No	Yes	No	Yes

6. Use the data above to complete the tables below.

Table I: Siblings and Pets Frequency Table

	Students with Pets	**Students with No Pets**	**Total**
Students with Siblings			
Students with No Siblings			
TOTAL			

Table II: Siblings and Pets Relative Frequency Table

(n = ____)	**Students with Pets**	**Students with No Pets**	**Total**
Students with Siblings			
Students with No Siblings			
TOTAL			

In your tables:

7. Draw a circle around the total number of students.

8. Draw a triangle around the total number of students with a pet.

9. Draw a square around the percent of students with no siblings.

10. Draw a trapezoid around the percent of students with no siblings and no pets.

REFLECTION

1. **Big Ideas**. Shade all circles that describe big ideas in this unit. Draw lines to show connections that you noticed.

- Use transformational geometry to investigate congruence and similarity
- Extend applications of volume to cylinders, cones, and spheres
- Explore bivariate data
- Complete the real number system
- Solve linear equations in one variable and linear systems in two variables
- Discover and apply properties of lines, angles, and triangles, including the Pythagorean Theorem
- Create, analyze, and use linear functions in problem solving
- Explore exponents and roots, and very large and very small quantities

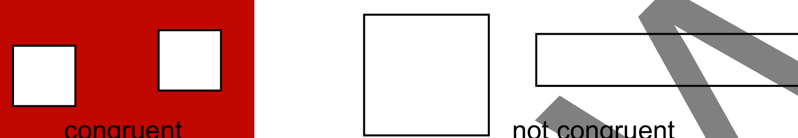
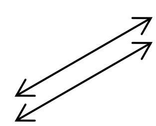
Give an example from this unit of one of the connections above.

2. **Unit Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before or something you would still like to work on.

3. **Mathematical Practice.** Explain how tools aided in your study of rigid motions [SMP5]. Then circle one more SMP on the back of this packet that you think was addressed in this unit and be prepared to share an example.

4. **More Connections.** We have been studying shapes and space. Where do you see transformations in the world around us?

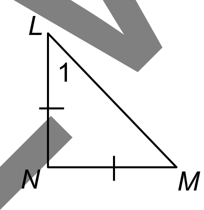
STUDENT RESOURCES

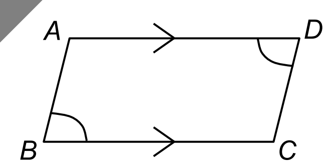
Word or Phrase	Definition
congruent figures	Two figures in the plane are <u>congruent figures</u> if the second can be obtained from the first by a sequence of one or more of translations, rotations, and reflections. Two squares are congruent if they have the same side-length.  congruent not congruent If $\triangle ABC$ is congruent to $\triangle DEF$, we write $\triangle ABC \cong \triangle DEF$.
function	A <u>function</u> is a rule that assigns to each input value exactly one output value. A function may also be referred to as a <u>transformation</u> or mapping. The collection of output values is the image of the function. Consider the function $y = 3x + 6$. For any input value, say $x = 10$, there is a unique output value, in this case $y = 36$. This output value is obtained by substituting the value of x into the equation. This function represents a straight line consisting of all ordered pairs of points (x, y) that satisfy the equation. Consider the transformation $(x, y) \rightarrow (-x, y)$. This transformation maps the x-coordinates in the plane to their opposites, while y-coordinates remain the same. In this case the image is a reflection about the y-axis.
image	The <u>image of a function or transformation</u> is the collection of its output values. The input values are then referred to as the pre-image. See <u>transformation</u>.
parallel	Two lines in a plane are <u>parallel</u> if they do not meet. Two line segments in a plane are <u>parallel</u> if the lines they lie on are parallel. 
reflection	A <u>reflection</u> of a plane through a line L is the transformation that maps each point to its mirror image on the other side of L. The line L is called the <u>line of reflection</u>. The transformation $(x, y) \rightarrow (x, -y)$ is a reflection of the plane through the x-axis.
rigid motion	A <u>rigid motion</u> is a transformation that preserves distances. Any rigid motion of the plane is a sequence of one or more translations, rotations, and reflections. Rigid motions also preserve lengths, angle measures, and parallel lines.
rotation	A <u>rotation</u> of a plane is a transformation that turns it through a given angle about a given point. The given angle is called the <u>angle of rotation</u>, and the given point is called the <u>center point of rotation</u>. The transformation $(x,y) \rightarrow (-y,x)$ is a rotation of the plane about the origin through angle 90°.

Word or Phrase	Definition
transformation	A transformation is a function that maps points in the plane (called the pre-image) to points in the plane (called the image). Rigid motion transformations include translations, rotations, and reflections.
translation	A translation of the plane is the transformation of the plane that maps pre-image points to image point in the same distance and direction. The transformation $(x, y) \rightarrow (x + 1, y + 2)$ slides all points 1 unit to the right and 2 units up.

Geometry Notation

These are examples of geometry diagrams and notations used in this program.





- A point is named using capital letters.

  Example: point M

- A polygon (e.g., triangle, parallelogram) is identified with a small symbol followed by its vertices.

  Examples: $\triangle LMN$, $\square ABCD$

- Line segment from point L to point N *is* named with the endpoints and a "bar" over them.

  Example: $\overline{LN}$

- The length (a measure) of a line segment from point L to point N is distinguished from the line segment (an object) by using absolute value symbols.

  Example: $\left|\overline{LN}\right|$

- An angle is named at its vertex and points on its rays, if needed.

  Example: The angle at L may be denoted $\angle L$, $\angle NLM$, $\angle MLN$, or $\angle 1$.

- The measure of an angle is distinguished from the angle (an object) using absolute value symbols.

  Example: The measure of $\angle L$ is written as $|\angle L|$.

- The symbol $\parallel$ indicates parallel lines. Arrows in a diagram indicate parallel segments as well.

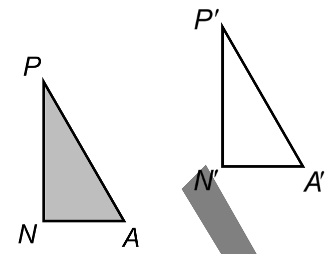
  Example: $\overline{AD} \parallel \overline{BC}$

- The symbol $\cong$ indicates congruence. Tick marks on the diagram above indicate congruent segments and arcs indicate congruent angles as well.

  Examples: Line segments $\overline{LN}$ and $\overline{NM}$ have the same length. Therefore, $\overline{LN} \cong \overline{NM}$.

  Angles at B and D have the same measure. Therefore, $\angle B \cong \angle D$.

Transformations of the Plane



A <u>transformation</u> is a function that maps points in the plane (called the pre-image) to points in the plane (called the <u>image</u>).

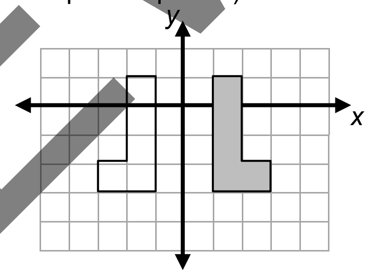
The input values (called the pre-image) are points in the plane. The output values (called the <u>image</u> of the transformation) are also points in the plane.

A transformation can be viewed as a <u>mapping</u> of pre-images (input values) to their corresponding images (output values).

In this figure, shaded triangle $\triangle PAN$ represents input values of a transformation and unshaded triangle $\triangle P'A'N'$ represents its image (output values).

The prime symbol (an apostrophe-like symbol) is often used to distinguish points in a pre-image (input values) from their images (output values).

We use the arrow notation $P \to P'$ (read "point *P* is taken to point *P* prime" or "*P* maps to *P* prime") to indicate that the image of the point P under the transformation is P'.

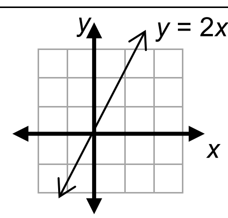
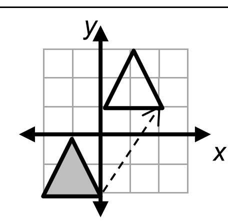


In a coordinate plane, we use the coordinates to describe the transformation as in the following example.

The reflection over the *y*-axis maps the shaded L-figure to a backwards L-figure. We use the arrow notation to describe this transformation. In this example, $(x, y) \to (-x, y)$.

Comparison of an Algebraic Function and a Geometric Function

Functions arise in many different contexts. The way we think of them and even the language we use to talk about them may be quite different for different areas of math. Here we compare a typical <u>function</u> we might meet in an algebra course and a typical function (we call it a <u>transformation</u>) that we might study in geometry.

Name of function	Linear function	Translation
Rule	Multiply by 2	Translate 2 units right and 3 units up
Description with symbols	x maps to $2x$ $x \to 2x$ $y = 2x$	(x, y) maps to $(x + 2, y + 3)$ $(x, y) \to (x + 2, y + 3)$
Graph		
Graph interpretation	The *x*-coordinates represent the inputs and the *y*-coordinates represent the outputs. The set of all input-output pairs is represented by the line.	A figure (shaded triangle - input) and its image (unshaded triangle - output) illustrate what happens to a typical figure in the plane. The translation arrow shows the direction and distance each point is moved.

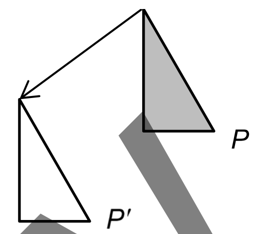
Translations, Rotations, and Reflections

Translations, rotations, and reflections are transformations of the plane that preserve distance between points.

A translation is a transformation that shifts all points the same distance and in the same direction.

This translation maps P to P' ($P \longrightarrow P'$).
(read "P maps to P prime").

The translation arrow shows the shift

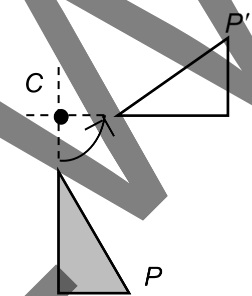


A rotation of a plane is a transformation that turns it through a given angle about a given point. The given point is called the center point of rotation. The given angle is called the angle of rotation.

This rotation maps P to P' ($P \longrightarrow P'$).

Point C is the center point of the rotation.

The angle of rotation is 90° (or a quarter counterclockwise).

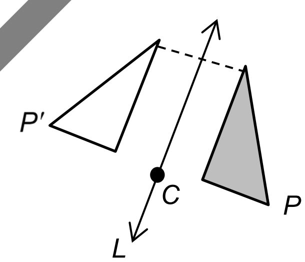


The reflection of a plane through a line L is the transformation that takes each point to its mirror image on the other side of L.

This reflection maps P to P' ($P \longrightarrow P'$).

Line L is the line of reflection.

Line L is the perpendicular bisector of $\overline{PP'}$.



Translations, rotations, and reflections preserve distances between points. Further, translations, rotations, and reflections

- map lines to lines,
- map line segments to line segments of the same length,
- map parallel lines to parallel lines, and
- map angles to angles of the same measure.

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
8.F.A	**Define, evaluate, and compare functions.**
8.F.1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
8.G.A	**Understand congruence and similarity using physical models, transparencies, or geometry software.**
8.G.1	Verify experimentally the properties of rotations, reflections, and translations:
a.	Lines are taken to lines, and line segments to line segments of the same length.
b.	Angles are taken to angles of the same measure.
c.	Parallel lines are taken to parallel lines.
8.G.2	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
8.G.3	Describe the effect of ~~dilations~~, translations, rotations, and reflections on two-dimensional figures using coordinates.
8.G.B	**Understand and apply the Pythagorean Theorem.**
8.G.7	Apply the Pythagorean Theorem to determine the unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

STANDARDS FOR MATHEMATICAL PRACTICE	
SMP1	Make sense of problems and persevere in solving them.
SMP2	Reason abstractly and quantitatively.
SMP3	Construct viable arguments and critique the reasoning of others.
SMP4	Model with mathematics.
SMP5	Use appropriate tools strategically.
SMP6	Attend to precision.
SMP7	Look for and make use of structure.
SMP8	Look for and express regularity in repeated reasoning.

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