Name ______________________________ Period __________ Date ______________

UNIT 7
STUDENT PACKET

Math Links
GRADE 8

LINEAR EQUATIONS AND SYSTEMS 1

Parent (or Guardian) signature ______________________________

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

point of intersection	slope-intercept form
solution to an equation solve an equation	substitution
system of linear equations	zero pair

CAN YOU SOLVE THIS EQUATION?

Solve this equation for *x* using any strategy. If you want to use an organized guess-and-check strategy, use the table below.

$$-4(x - 2) + 6 = 20 - 2(x - 1) - 7$$

Choose a value for *x*	**Substitute to get a value of the expression for the left side of the equation**	**Substitute to get a value of the expression for the right side of the equation**	**Difference between left and right sides**	**Are both sides equal?**

SOLVING SYSTEMS BY GRAPHING

We will solve systems of equations by graphing; and understand that a system can have exactly one, zero, or infinitely many solutions. We will use substitution to rewrite systems of equations as a single equation and verify solutions.

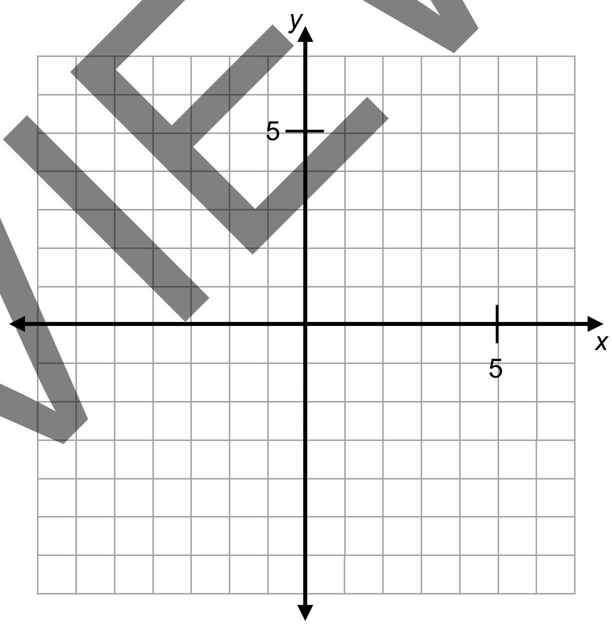
[8.EE.8a, 8.EE.8b, 8.EE.8c, 8.F.2, 8.F.3, 8.F.4; SMP1, 2, 3, 4]

GETTING STARTED

Write each equation below in slope-intercept form, complete the tables, and graph the equations on the coordinate plane to the right.

1. $-2x + y = 4$	***x***	***y***
	3	
	2	
	1	
	0	
	-1	

2. $y - 3 = 3x$	***x***	***y***
	3	
	2	
	1	
	0	
	-1	



3. In the tables above, circle the ordered pair in problem 1 and the ordered pair in problem 2 that are the same.

4. At what point do the two lines intersect on the graph?

5. Record the meanings of slope-intercept form and point of intersection in **My Word Bank**.

WHAT IS A SYSTEM OF EQUATIONS?

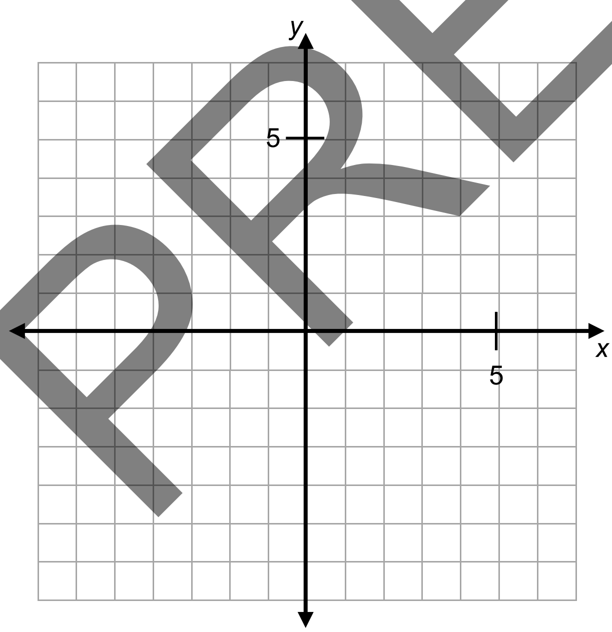
1. When a linear equation in two variables is graphed, what does the line represent?

2. When two linear equations are graphed in the same coordinate plane, and they intersect in one point, what does that point represent?

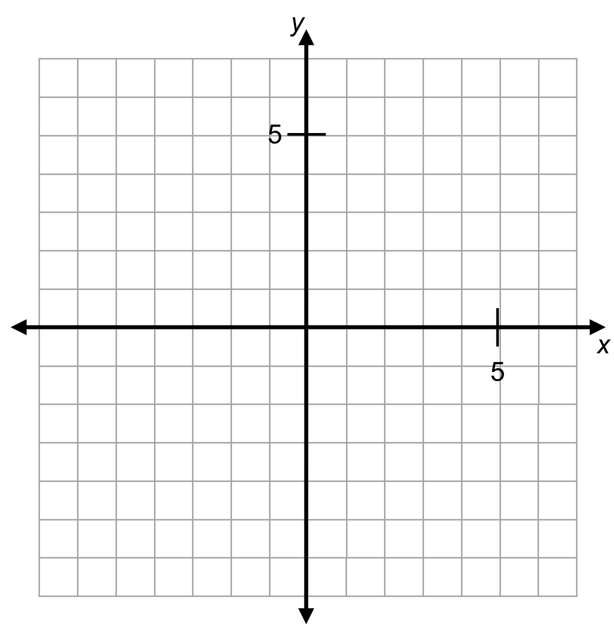
3. Record the meaning of a <u>system of linear equations</u> in **My Word Bank**.

For each system below, graph the lines on the same coordinate plane, and then write the solution as an ordered pair. Recall that equations in slope-intercept form tend to be easier to graph.

4.
$$\begin{cases} y = -2x + 4 \\ y = 2x \end{cases}$$



5.
$$\begin{cases} y + 1 = 4x \\ 6x - 3y = -3 \end{cases}$$



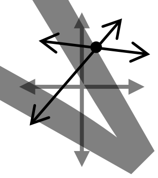
WHAT IS A SYSTEM OF EQUATIONS?
Continued

A system of two linear equations in two variables has **one solution**, **no solution**, or **infinitely many solutions**.

6. The system of linear equations to the right has **exactly one solution**.

   a. Circle the solution.

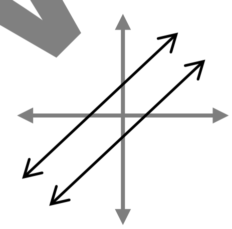
   b. Describe the slopes of these lines.



7. The system to the right has **no solutions**.

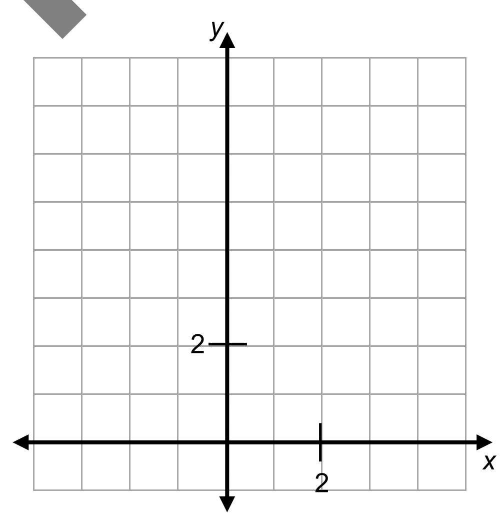
   a. Describe the lines and how their slopes relate to one another.

   b. Why do you think the graph of the system has no solutions?



8. Below is a system of equations.

$$\begin{cases} y = 2x + 3 \\ 2y = 4x + 6 \rightarrow \_\_\_\_\_\_\_\_\_\_ \end{cases}$$



   a. Change the second equation to slope-intercept form.

   b. What do you notice about these two equations?

   c. Graph both lines and describe what this graph looks like.

   d. Why do you think we say that this system has **infinitely many solutions**?

9. Consider the pair of equations in **Getting Started** a system of linear equations. Does this system have one solution, no solutions, or infinitely many solutions?

PRACTICE 1

1. A system of equations represented by two parallel lines has __________ solution(s).

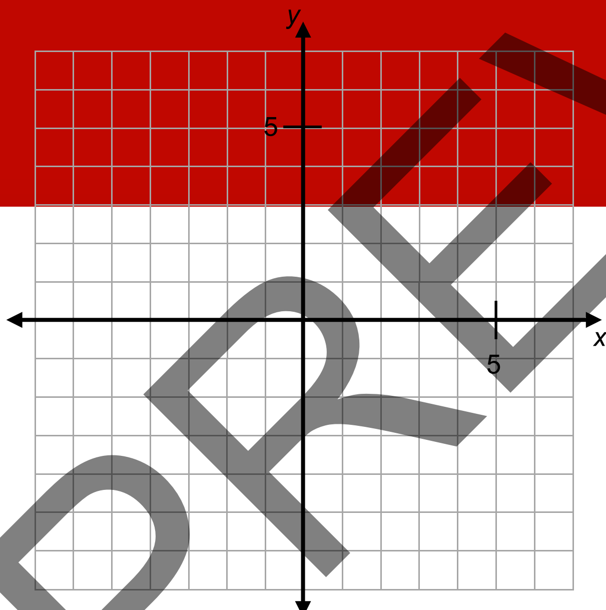
2. A system of equations represented by lines that coincide (equivalent equations) has _______________ solution(s).

3. A system of equations represented by lines intersecting in one point has __________ solution(s).

For the systems of equations below, first make sure equations are in slope-intercept form. Then graph the lines, determine the number of solutions, and write the solution(s), if any.

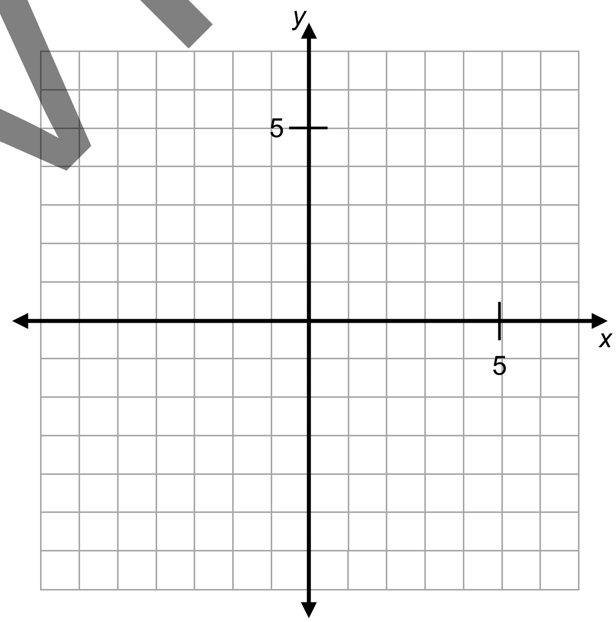
4.

$$\begin{cases} 3y = 9x + 3 \\ y = 3x - 5 \end{cases}$$

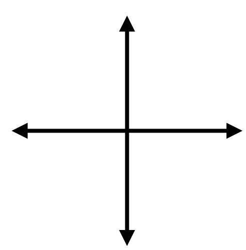


5.

$$\begin{cases} y - 5 = x \\ 2x + y = -4 \end{cases}$$



6. Sketch the graph of a system of linear equations with exactly one solution such that both lines have negative slopes.



PRACTICE 2

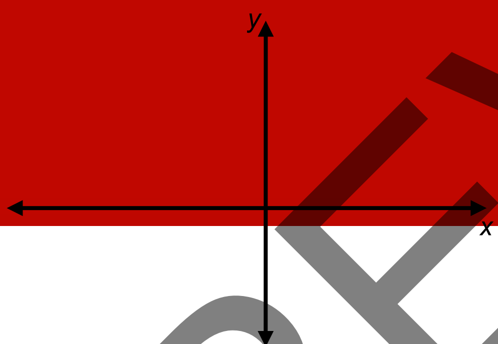
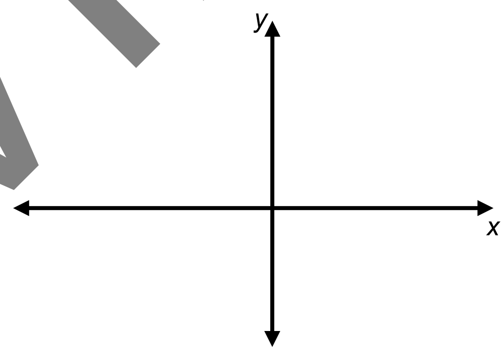
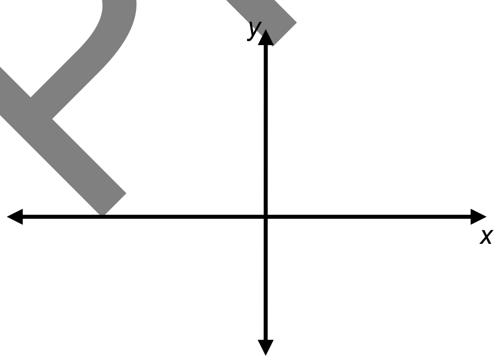
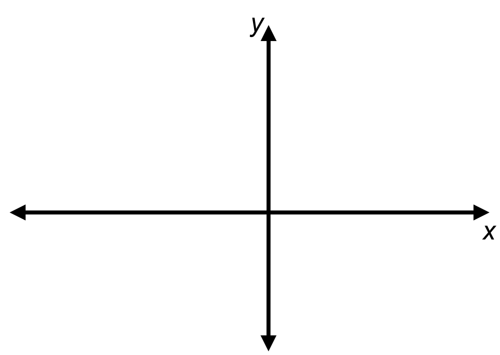
1. Coop looked at the system below and said, "I don't need to graph these lines. I know that there's no solution." Explain how Coop knows this.

$$\begin{cases} y = x + 2 \\ y = x + 1 \end{cases}$$

2. Smitty looked at the system below and said, "I don't need to graph these lines. I know that there are infinitely many solutions." Explain how Smitty knows this.

$$\begin{cases} y = 3x + 6 \\ y = 3(x + 2) \end{cases}$$

Sketch the graphs of lines that fit each description below. Then describe the type of solution(s) for each system.

3. Two lines with different slopes and different *y*-intercepts. 	4. Two lines with different slopes and the same *y*-intercepts. 
5. Two lines with the same slopes and different *y*-intercepts. 	6. Two lines with the same slopes and the same *y*-intercepts. 

USING SUBSTITUTION

Follow your teacher's directions for (1) – (6).

(1)	
(2)	(3)

Kim and her friend Jordan meet for lunch. Kim tells Jordan about the 100 Mile Walking Challenge she's been doing for a while. "I already have 40 miles, and starting tomorrow, I'm going to walk 8 miles per day," says Kim. "You should join the challenge." Jordan accepts and says, "Okay, you're way ahead of me, so I'm going to walk more miles per day to try to catch up."

(4)

Days	0	1									

USING SUBSTITUTION
Continued

(5)

(6)

7. Why do the solutions to problem 6 support your answer to problem 5?

8. If Kim and Jordan each continue to walk at their same pace, on what day have they walked the same number of miles?

   How many miles is this?

9. Use substitution and the two equations from problem 6 to write one equation in *x*. Then check to see if the *x*-value (day number from problem 8) is a solution to this equation.

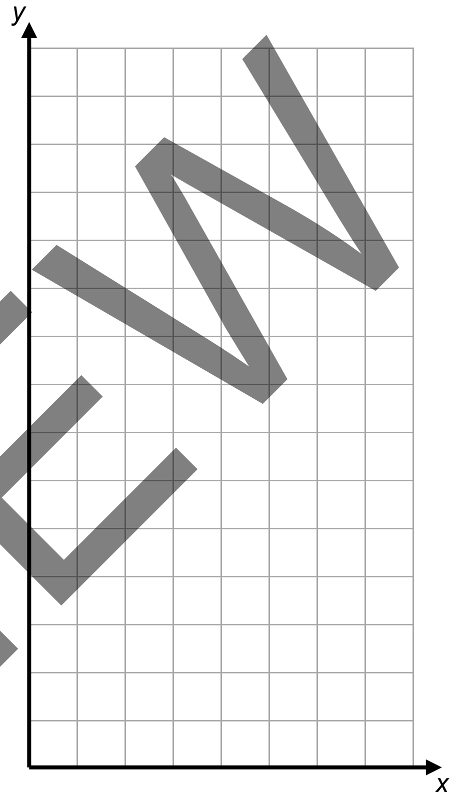
10. Record the meaning of <u>substitution</u> in **My Word Bank**.

PRACTICE 3

Naomi and Karolina are saving for a skateboard. Naomi has \$100 in the bank and will save \$30 each month. Karolina has \$40 in the bank and will save \$45 each month.

1. Complete the table below, graph the data, and write the input-output equations.

Naomi		**Karolina**	
Month # (*x*)	**Total saved in \$ (*y*)**	**Month # (*x*)**	**Total saved in \$ (*y*)**
0		0	
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	
7		7	
$y =$		$y =$	



2. Who is saving at a faster rate? Justify your answer by referring to some problem 1 representations.

3. During which month(s)…
   a. does Naomi have more money?
   b. does Karolina have more money?
   c. do they have the same amount of money?
   What do you notice about the table entries at this month?
   What do you notice about the graphs at this month?

4. Use substitution to write one equation in x equating Naomi's and Karolina's savings. Use this equation to verify the month at which they have the same amount of money. State your answer in a short sentence.

ESTIMATING SOLUTIONS TO SYSTEMS

Follow your teacher's directions.

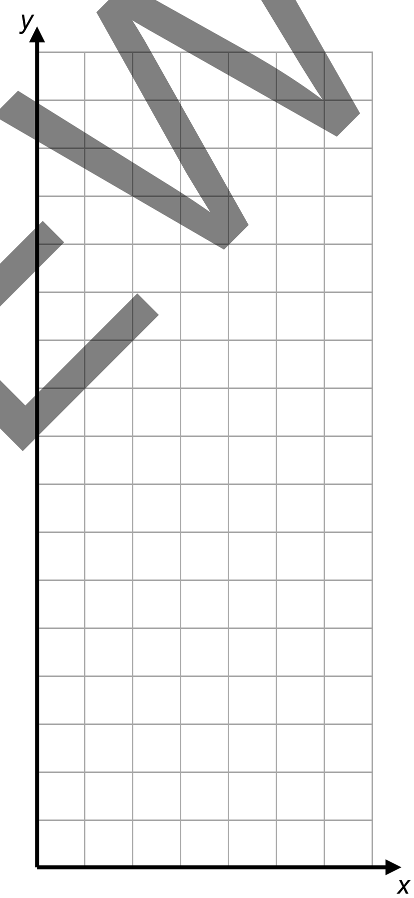
- Emmett and Gerry are traveling down the same street to the park near their school.
- They leave at the same time from different locations.
- Someone is taking pictures from a window above street level every 6 seconds so that they can keep track of the distance they travel over time.

(1)

(2)

(3)

Graph



Tables

Equations

(4)

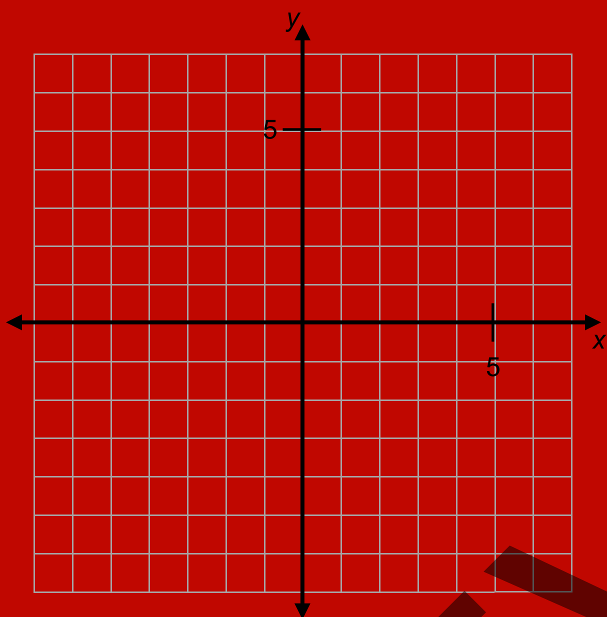
(5)

(6)

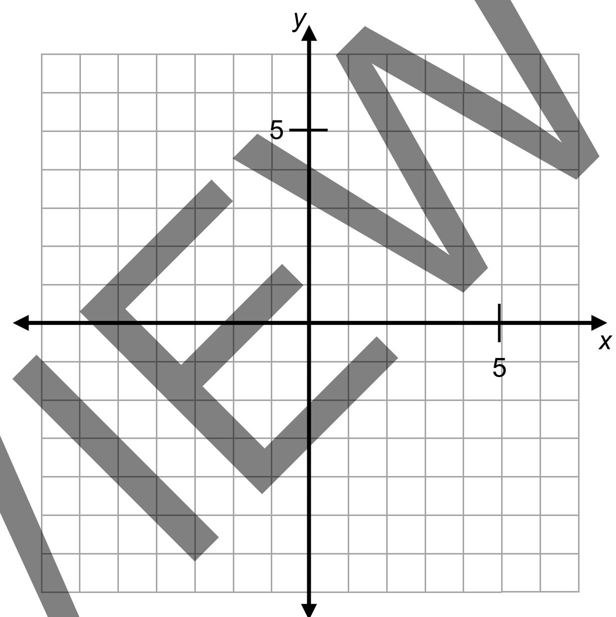
PRACTICE 4

Graph each system of equations. Make sure all equations are in slope-intercept form first.

1.
$$\begin{cases} y = -4x + 2 \\ y = 8x - 1 \end{cases}$$



2.
$$\begin{cases} y - 2 = x \\ 6x + y = -4 \end{cases}$$



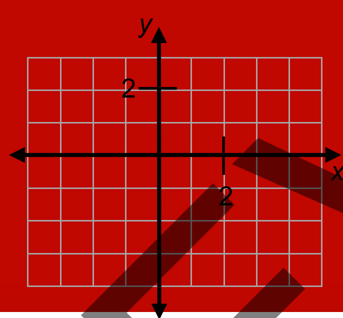
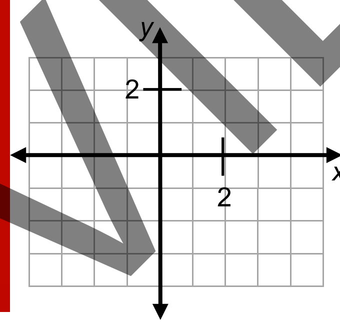
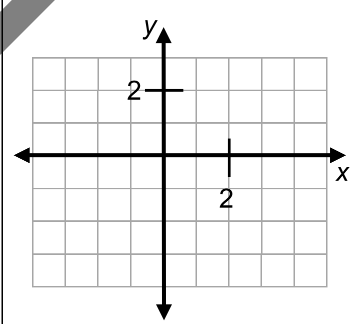
3. Ike looked at the graphs and said, “I can’t tell for sure what these solutions are.” Explain why you think Ike said this.

4. Use estimation and substitution to complete the table below.

	Problem 1	**Problem 2**
Estimate a solution from the graph.		
Write one equation in *x* using substitution.		
Substitute the estimated *x*-value into the equation and simplify both sides.		
What does the result from directly above tell you about your estimate?		

PRACTICE 5

Each pair of points below defines a line. Complete the table.

Lines… **through points…**	1. Lines *A* and *B* *A*: (0, -3) and (1, 0) *B*: (0, 1) and (-2, -1)	2. Lines *C* and *D* *C*: (0, -4) and (1, 0) *D*: (0, 2) and (-1, -2)	3. Lines *E* and *F* *E*: (0, 3) and (1, 1) *F*: (0, 0) and (-1, -2)
Find the slope of each line.			
Will the lines intersect? Explain.			
Graph the lines.			
State exact or estimated solution(s). (if any)			
Write equations for each line.			
Verify solution by substitution. (if possible)			

SOLVING EQUATIONS USING CUPS AND COUNTERS

We will solve equations using a model and balance techniques.

[8.EE.7a, 8.EE.7b; SMP6, 7]

GETTING STARTED

Write all equations below in slope intercept form if needed. Then use substitution to write one equation in x for each.

1. $\begin{cases} y - x = 4 \\ 2x - y = -1 \end{cases}$	2. $\begin{cases} 5 = 4x - y \\ 2x - y = 3 \end{cases}$

Solve using mental math. Substitute a value in for the variable to make each equation true.

3. $24 + x = 100$	4. $30 = 5x + 15$
5. $-39 = 13x$	6. $-3(x - 2) = -24$

7. Record the meanings of <u>solution to an equation</u> and <u>solve an equation</u> in **My Word bank**.

CUPS, COUNTERS, AND BALANCE

Follow your teacher's directions for (1) – (8).

(1)

Positive Counter	Negative Counter	Cup	Upside-down Cup

(2)	(3)
(4)	(5)

(6)

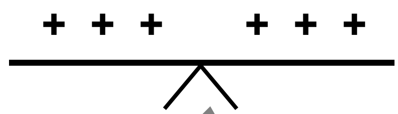
(7)

(8)

9. Record the meaning of <u>zero pair</u> in **My Word Bank**.

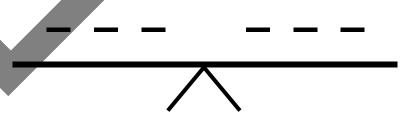
PRACTICE 6

To the right is a balanced scale with three positive counters on each side. For each action below, describe what happens to this original picture, and write a number sentence to represent the result. Make sketches as desired.



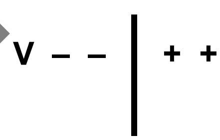
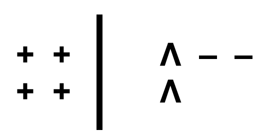
1. Four positive counters are added to each side.	2. Four positive counters are added to the left side.
3. Two negative counters are added to each side.	4. Six cups are added to each side.

To the right is a balanced scale with three negative counters on each side. For each action below, describe what happens to this original picture, and write a number sentence to represent the result. Make sketches as desired.



5. One positive counter is added to each side.	6. One positive counter is added to left side.
7. Two negative counters are removed from each side.	8. Five upside-down cups are added to each side.

For each cups and counters equation pictured below, write the equation as pictured, perform the indicated operation, and then write the new, resulting equation.

9. Equation pictured: ______________ Add counters to create zero pairs so that there are no counters on the left side.  Resulting equation: ______________	10. Equation pictured: ______________ Add cups to create zero pairs so that there are no upside down cups on the right side.  Resulting equation: ______________

SOLVING EQUATIONS WITH BALANCE 1

Follow your teacher's directions for (1) – (4).

When solving equations using cups and counters:

- Build each equation.
- Think: Can I do anything to either side (individually)?
- Think: Can I do anything to both sides (together)?
- Continue the process until the equation is solved.
- Write the solution and check it using substitution.
- Make a drawing of the process.

(1)	(2)
(3)	(4)

Build the equation below, find the solution, draw, and check..

5. $4(x - 1) = 2(x + 4)$	6. $3(2 + x) = x + 8$

SOLVING EQUATIONS WITH BALANCE 2

Follow your teacher's directions for (1) – (6).

(1)	
(2)	

(3)	(4)
(5)	(6)

Build each equation below, find the solution, draw, and check.

7. $5 - 3x - 2 = -(x - 3) + 4$	8. $-3(x - 1) = -2x - 3 - 2x$

PRACTICE 7

Solve by building each equation below. Record drawings. Check solutions.

1. $2x = -2x - 4$	2. $-x + 7 - x = -3 + 3x + 5$	3. $3(x - 2) = -2(x - 2)$
4. $-2(x + 1) = -3 - 4x - 3$	5. $-(3 + x) = -(-5 - x)$	6. $-2x + 4 = -3(2 + x) + 2x$

7. Try to solve the equation from **Can you Solve This?** (Opening Problem) using the model.

$$-4(x - 2) + 6 = 20 - 2(x - 1) - 7$$

Drawing:

Check:

SOLVING EQUATIONS ALGEBRAICALLY

We will solve equations algebraically, and understand than an equation can have exactly one, zero, or infinitely many solutions.

[8.EE.7a, 8.EE.7b; SMP3, 5, 7, 8]

GETTING STARTED

1. Consider the following expression: $-3x + 2(x + 1) - x - 6$

a. Make a cups and counters sketch of the expression.	b. Write the expression in its simplest form.
c. Consider the expression in part b. To the right, write an equation that sets this expression equal to 0. Then solve it using any method. What method did you choose?	

2. Solve the equation $-x - x = -8$ using any method.

   What method did you choose?

3. Solve the equation $-x = -8 + x$ using any method.

   What method did you choose?

4. How are the equations in problems 2 and 3 related?

FROM DRAWINGS TO PROCEDURES

Follow your teacher's directions.

Equation with work shown	Drawing
(1)	
(2)	
(3)	
(4)	

PRACTICE 8

Solve each equation below algebraically. Check by substitution. Build or draw as needed.

1. $x - 4 = 2x + 1$	2. $2(x - 1) = 2x + 1 + 3x$
3. $-3x - 6 = 3 - 6x$	4. $-(4x - 1) = -2x - 5$
5. $3(x + 1) = -5(x + 1)$	6. $-x - 4 - x = -2(2x - 2)$

7. Try to solve the equation from **Can you Solve This?** (Opening Problem) algebraically.

$-4(x - 2) + 6 = 20 - 2(x - 1) - 7$

DO ALL EQUATIONS HAVE EXACTLY ONE SOLUTION?

Draw a picture of each equation below and then explain what you think each person meant.

1. Sal looked at the equation $x + 3 = x + 1$ and said, "I don't think this can be solved!"	2. Yesenia looked at the equation $2x + 2 = 2(x + 1)$ and said, "Every value I substitute into this equation makes it true!"

3. Explain what it means for an equation to have **no solutions**. Refer to one of the problems above in your explanation.

4. Explain what it means for an equation to have **infinitely many solutions**. Refer to one of the problems above in your explanation.

> A linear equation in one variable has
> **one solution**, **no solution**, or **infinitely many solutions**.

Solve each equation below. Show your work and clearly state any solution(s).

5. $-2(x + 4) = 2(-4 - x)$	6. $5(x + 7) = 5x + 7$
7. $9 - x + 13x = 3(4x - 3)$	8. $-5x + 3 - 2x = -3(x - 1) - 4x$

PRACTICE 9

Solve algebraically. Check. Indicate if there are no solutions or infinitely many solutions.

1. $-(-x - 7) = 2x - 3x - 9$	2. $15x + 200 + 15x = 100 + 50x$
3. $-6(x + 2) = 3(-4 - 2x)$	4. $10(2x - 1) = 5(6x + 8)$
5. $4(x - 5) + 9 = -6 + 3x - 5$	6. $5(x + 20) = -30 - 10x + 100$
7. $-4x - x + 11 = -5(x - 3)$	8. $-2(10x + 4) = -4(6x - 8)$

REVIEW

POSTER PROBLEMS: LINEAR EQUATIONS AND SYSTEMS 1

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is ________.
- Each group will have a different colored marker. Our group marker is _________.

Part 2: Do the problems on the posters by following your teacher's directions.

Poster 1 (or 5)	Poster 2 (or 6)	Poster 3 (or 7)	Poster 4 (or 8)
$\begin{cases} y - 1 = 5x \\ y - 2x = 4 \end{cases}$	$\begin{cases} y - x = 6 \\ -4x + y = 0 \end{cases}$	$\begin{cases} -2x - y = -1 \\ 0 = -x - y \end{cases}$	$\begin{cases} -x = 4 - y \\ 4x + 2 = 2y \end{cases}$

A. Copy the problem. Write both equations in slope-intercept form.

B. Solve the system by graphing. Write the solution clearly.

C. Use substitution to write one equation in x.

D. Substitute the x-value from part B into the equation from part C as a check. After a successful check, explain why substituting this one x-value into the equation makes the ordered pair a true solution.

Part 3: Return to your seats. Work with your group.

For this system of equations, write both in slope-intercept form, and then explain how you know what the solution(s) is/are without graphing.

$$\begin{cases} x + y = 1 \\ 2x + 2y = 4 \end{cases}$$

BIG SQUARE PUZZLE: LINEAR EQUATIONS AND SYSTEMS 1

Your teacher will give you a puzzle to assemble.

Problems 1 and 2 each begin with descriptions of equations from the puzzle. Fill in the blanks with those equations, and finish the explanations.

1. The equation that has no solution is ________________________ because…

2. The equation that has infinitely many solutions is ________________________ because…

3. Choose any equation from the Big Square Puzzle other than the two above, copy it below, and solve it by drawing a cups and counters diagram.

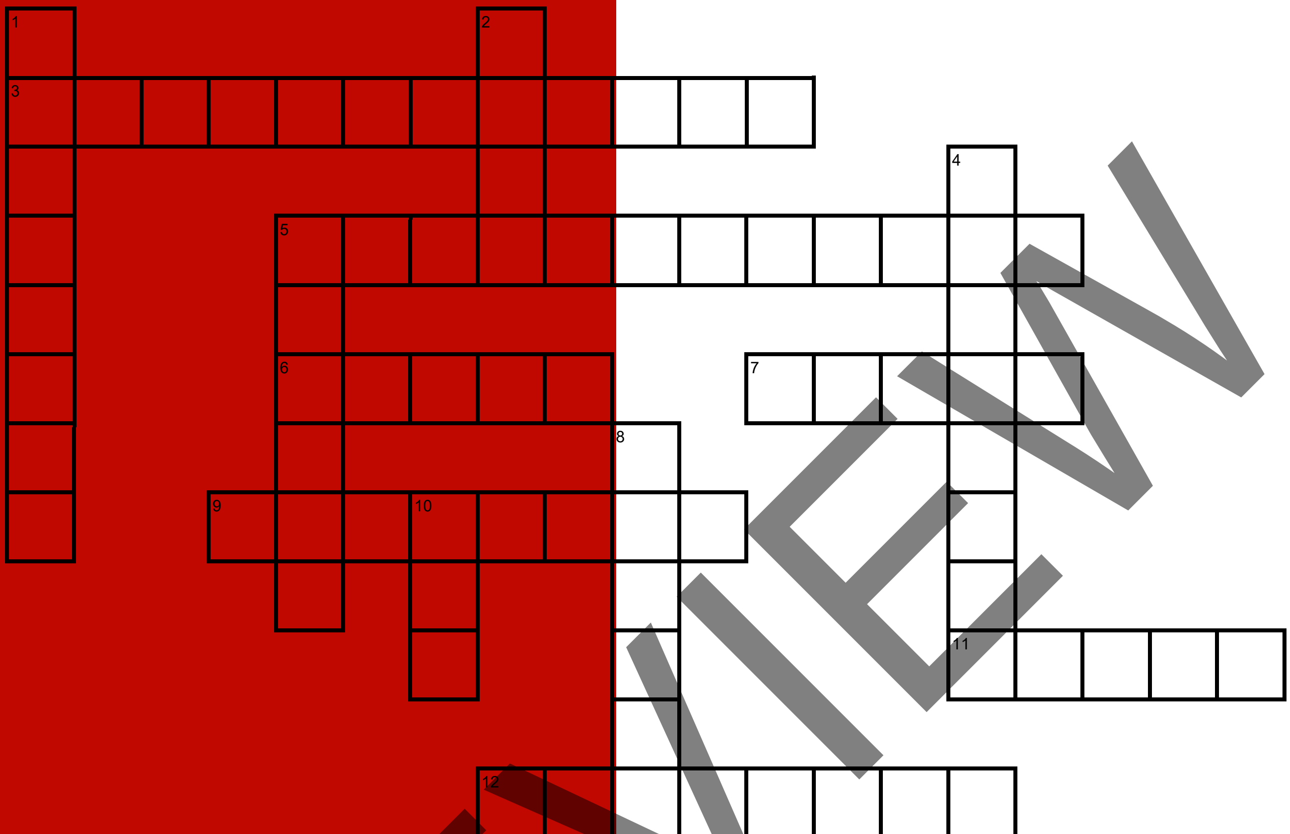
OPEN MIDDLE PROBLEMS: LINEAR EQUATIONS AND SYSTEMS 1

An equation has the following structure: $\boxed{}\,x + \boxed{} = \boxed{}\,x + \boxed{}$

Using the digits 1 to 9 at most one time each, place a digit in each box to create an equation:

1. with any solution	2. with the greatest solution
3. with the least solution	4. with a solution as close to 0 as possible

VOCABULARY REVIEW



Across

3 a property: $3(x - 4) = 3x - 12$

5 algebraic strategy for solving a system of equations

6 find a value to make an equation true

7 location where system of equations intersect to show a solution

9 one positive and one negative counter together (two words)

11 Systems of equations with no solutions have the same ______.

12 Zero is the additive ______.

Down

1 Property of equality: If $a = b$, then $a + c = b + c$

2 represents unknowns in our equation solving model

4 used with 2 Down

5 set of two or more equations in the same variables

8 multiplicative identity (two words)

10 number of solutions when two lines intersect

SPIRAL REVIEW

1. **READY-X.** Solve for the values of R, E, A, D, Y, X. Sums of rows and columns are indicated at the end of each row and column.

COLUMNS

ROWS	1	2	3	
1	R	D	E	**15**
2	A	A	A	**3**
3	Y	D	Y	**18**
4	X	D	A	**15**
	27	**13**	**11**	

R = _____ E = _____ A = _____ D = _____ Y = _____ X = _____

2. Solve each equation.

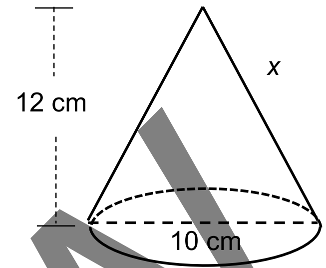
a. $4(m - 7) = -12$	b. $0 = 35(0.75 + n)$

3. Solve for y in terms of x.

a. $2x + y = 14$	b. $2y - x = 14$

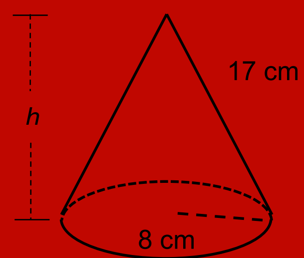
SPIRAL REVIEW
Continued

4. Find the slant height (x) and the volume of the cone to the right if the diameter is 10 cm and the height is 12 cm. Round to the nearest tenth. Hint: Use the Pythagorean Theorem to find the slant height.

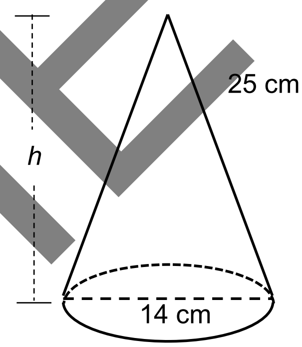


5. Find the height and the volume of the cones below. Round to the nearest tenth. (Cones not drawn to scale.)

a.

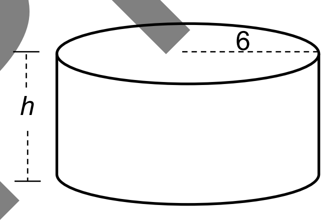


b.

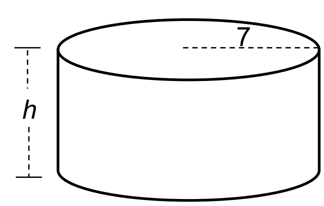


6. Find the height of each cylinder below. (Cylinders not drawn to scale.)

a. The cylinder below has a volume of 791.28 ft^3. Use $\pi = 3.14$.



b. The cylinder below has a volume of 1,386 cm^3. Use $\pi = \frac{22}{7}$

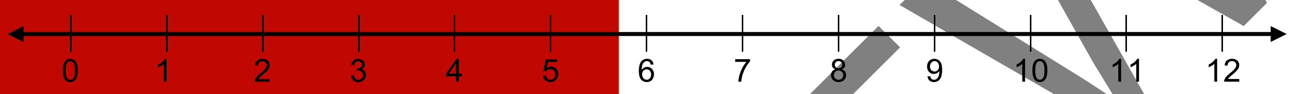


SPIRAL REVIEW
Continued

7. Graph each set below.

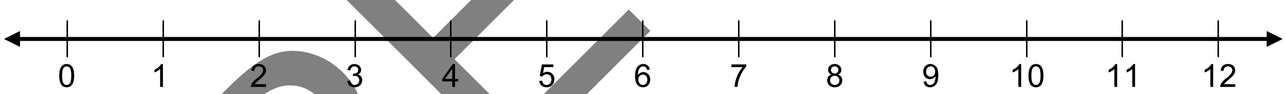
Set A			Set B			Set C		
2.75π	1.5π	π	$\sqrt{29}$	$\sqrt{6}$	$\sqrt{14}$	3^2	$\sqrt{9}$	$(\sqrt{9})^2$

Set A



Set B

Set C



8. Allie and Nico agree that the value of $\sqrt{45}$ is between 6 and 7. Allie thinks that $\sqrt{45}$ is closer to 6. Nico thinks that $\sqrt{45}$ is closer to 7.

   a. Which student do you think is more accurate? Explain.

   b. What would you say to the other student to help understand the error?

REFLECTION

1. **Big Ideas**. Shade all circles that describe big ideas in this unit. Draw lines to show connections that you noticed.

- Use transformational geometry to investigate congruence and similarity
- Extend applications of volume to cylinders, cones, and spheres
- Explore bivariate data
- Complete the real number system
- Solve linear equations in one variable and linear systems in two variables
- Discover and apply properties of lines, angles, and triangles, including the Pythagorean Theorem
- Create, analyze, and use linear functions in problem solving
- Explore exponents and roots, and very large and very small quantities

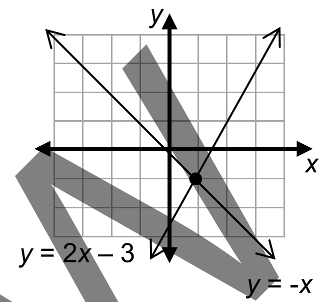
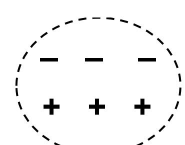
Give an example from this unit of one of the connections above.

2. **Unit Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before.

3. **Mathematical Practice.** Reflect on using cups and counters to solve equations. If they were helpful, how so? If not, why not [SMP5]? Then circle one more SMP on the back of this packet that you think was addressed in this unit and be prepared to share an example.

4. **Making Connections.** How are the number of solutions to an equation in one variable similar to the number of solutions to a system of equations in two variables?

STUDENT RESOURCES

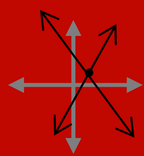
Word or Phrase	Definition
point of intersection	A point of intersection of two lines is a point where the lines meet. The two straight lines in the plane with equations $y = -x$ and $y = 2x - 3$ have point of intersection (1, -1). 
slope-intercept form	The slope-intercept form of the equation of a line is the equation $y = mx + b$, where m is the slope of the line, and b is the y-intercept of the line. The equation $y = 2x + 3$ determines a line with slope 2 and y-intercept 3.
solution to an equation	A solution to an equation involving variables consists of values for the variables which, when substituted, make the equation true. The value $x = 8$ is a solution to the equation $10 + x = 18$. If we substitute 8 for x in the equation, the equation becomes true: $10 + 8 = 18$.
solve an equation	To solve an equation refers to finding all values for the variables in the equation that, when substituted, make the equation true. Values that make an equation true are called solutions to the equation. To solve the equation $2x = 6$, one might think "two times what number is equal to 6?" Since $2(3) = 6$, the only value for x that satisfies this condition is 3. Therefore 3 is the solution.
substitution	Substitution refers to replacing a value or quantity with an equivalent value or quantity. If $y = x + 5$, and we know that $x = 3$, then we may use substitution to rewrite the first equation to get $y = 3 + 5$. If $y = x + 10$, and we know also that $y = 2x + 4$, then we may use substitution to write one equation in x to get $x + 10 = 2x + 4$.
system of linear equations	A system of linear equations is a set of two or more linear equations in the same variables. An example of a system of linear equations in x and y: $\begin{cases} x + y = 1 \\ x + 2y = 4 \end{cases}$
zero pair	In the signed counters model, a positive and a negative counter together form a zero pair. Let "+" represent a positive counter, and let "−" represent a negative counter. Then the following is an example of a collection of (three) zero pairs. 

Systems of Linear Equations

A system of equations is a set of two or more equations in the same variables. A solution to the system of equations consists of values for the variables which, when substituted, make all equations simultaneously true.

A system of linear equations has exactly one solution, infinitely many solutions, or no solution.

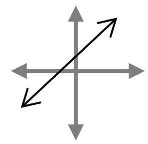
$$\begin{cases} y = -x + 3 \\ y = 2x - 3 \end{cases}$$



One solution at (2, 1)

These two lines intersect in aqexactly one point.This is the only pair of *x*- and *y*-values that statisfies these two equations simultaneously.

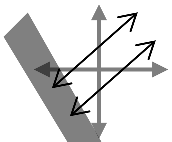
$$\begin{cases} y = x + 1 \\ 2y = 2x + 2 \end{cases}$$



Infinitely many solutions

Since the equations are equivalent, the two lines coincide. Every point on the line represents a solution.

$$\begin{cases} y = x - 2 \\ y = x + 2 \end{cases}$$



No solution

Since these two lines are parallel, they do not intersect. Thus these two equations have no solution in common.

Solving a System of Linear Equations by Graphing

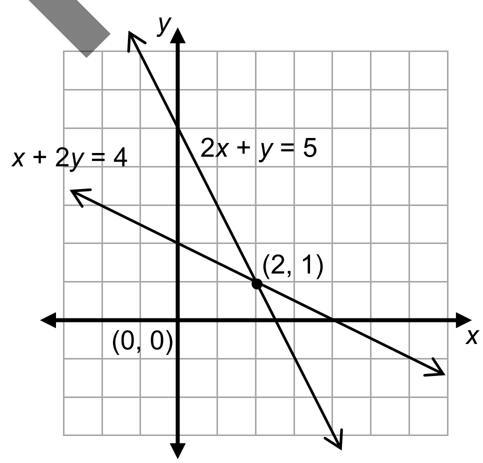
To solve a system of equations by graphing, graph both lines on the same set of axes and observe the point(s) of intersection, if any.

Solve by graphing: $\begin{cases} 2x + y = 5 \\ x + 2y = 4 \end{cases}$

1. Change each to slope-intercept form, $y = mx + b$.

   $2x + y = 5 \longrightarrow y = -2x + 5$

   $x + 2y = 4 \longrightarrow y = -\frac{1}{2}x + 2$



2. Graph each equation.

3. Observe the intersection of the lines, (2, 1). This represents the solution to the system. In other words, these are the x- and y- values that satisfy both equations. Remember that not every system of equations has exactly one solution.

4. Check by substituting solutions in the original equations to be sure they are correct.

   $2x + y = 5 \longrightarrow 2(2) + 1 = 5$ (true)

   $x + 2y = 4 \longrightarrow 2 + 2(1) = 4$ (true)

Properties of Arithmetic

Properties of arithmetic govern the manipulation of expressions (mathematical phrases).
For any three numbers *a*, *b*, and *c*:

- ✓ Associative property of addition
  $a + (b + c) = (a + b) + c$
- ✓ Commutative property of addition
  $a + b = b + a$
- ✓ Additive identity property
  (addition property of 0)
  $a + 0 = 0 + a = a$
- ✓ Additive inverse property
  $a + (-a) = -a + a = 0$
- ✓ Associative property of multiplication
  $a \bullet (b \bullet c) = (a \bullet b) \bullet c$
- ✓ Commutative property of multiplication
  $a \bullet b = b \bullet a$
- ✓ Multiplicative identity property
  (multiplication property of 1)
  $a \bullet 1 = 1 \bullet a = a$
- ✓ Multiplicative inverse property
  $a \bullet \frac{1}{a} = \frac{1}{a} \bullet a = 1 \quad (a \neq 0)$
- ✓ Distributive property relating addition and multiplication
  $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for any three numbers *a*, *b*, and *c*.

Properties of Equality

Properties of equality govern the manipulation of equations (mathematical sentences).
For any three numbers *a*, *b*, and *c*:

- ✓ Addition property of equality
  (Subtraction property of equality)
  If $a = b$ and $c = d$, then $a + c = b + d$
- ✓ Multiplication property of equality
  (Division property of equality)
  If $a = b$ and $c = d$, then $ac = bd$
- ✓ Reflexive property of equality: $a = a$
- ✓ Symmetric property of equality: If $a = b$, then $b = a$
- ✓ Transitive property of equality: If $a = b$, and $b = c$, then $a = c$

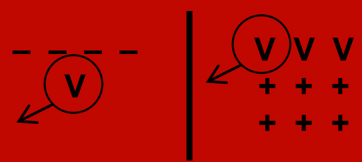
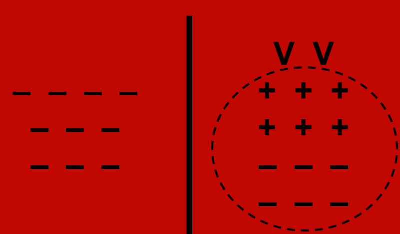
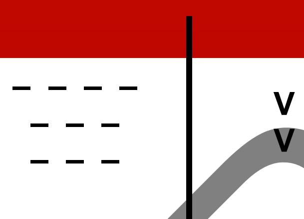
Solving Equations Using a Model 1

Let **+** represent 1 | Let **V** represent the unknown (like x)

Let **–** represent -1 | Let **Λ** represent the opposite of the unknown (like $-x$)

The following example illustrates one solution path. Other paths are possible to arrive at the same solution.

Solve: $-4 + x = 3(x + 2)$

Picture	Equation	What did you do?
	$-4 + x = 3x + 6$ $\underline{-x} \quad \underline{-x}$ $-4 = 2x + 6$	build the equation remove one x from each side
	$-4 = 2x + 6$ $\underline{+(-6)} \quad \underline{+(-6)}$ $-10 = 2x$	add -6 to each side remove zero pairs
	$\frac{-10}{2} = \boxed{\frac{2}{2}}x$ $-5 = x$	divide both sides by 2 put counters equally into cups (or do mentally) notice the use of the $\boxed{\text{big 1}}$

Check by substituting the solution into the original equation:

$-4 + x$	=	$3(x + 2)$	
$-4 + (-5)$	=	$3(-5 + 2)$	?
-9	=	$3(-3)$	?
-9	=	-9	true

Solving Equations Using a Model 2

Let **+** represent 1

Let **–** represent -1

Let **V** represent the unknown (like x)

Let **Λ** represent the opposite of the unknown (like $-x$)

The following example illustrates one solution path. Other paths are possible to arrive at the same solution.

Solve: $-2x - 1 = x - 4$

Picture	Equation	What did you do?
Λ Λ – | V – – – –	$-2x - 1 = x - 4$	build the equation
Λ Λ Λ – | V Λ – – – –	$\begin{aligned} -2x - 1 &= x - 4 \\ \underline{+(-x)} &= \underline{+(-x)} \\ -3x - 1 &= -4 \end{aligned}$	add the opposite of x to both sides remove the zero pair
Λ Λ Λ – | – – – –	$\begin{aligned} -3x - 1 &= -4 \\ \underline{-(-1)} &= \underline{-(-1)} \\ -3x &= -3 \end{aligned}$	remove -1 from both sides* *this gives the same result as adding 1 to each side
Λ Λ Λ V V V + + + | – – – + + + V V V	$\begin{aligned} -3x &= -3 \\ \underline{(+3x) + 3} &= \underline{+ 3 + (3x)} \\ 3 &= 3x \end{aligned}$	add $3x$ to both sides AND add 3 positives to both sides remove zero pairs
+ + + | V V V	$\begin{aligned} \frac{3}{3} &= \boxed{\frac{3}{3}}x \\ 1 &= x \end{aligned}$	divide both sides by 3 put counters equally into cups (or do mentally) notice the use of the big 1

Check by substituting the solution into the original equation:

$$\begin{aligned} -2x - 1 &= x - 4 & & \\ -2(1) - 1 &= 1 - 4 & & ? \\ -2 - 1 &= -3 & & ? \\ -3 &= -3 & & \text{true} \end{aligned}$$

Using Algebraic Techniques to Solve Equations

To solve equations using algebra:

- Use the properties of arithmetic to simplify each side of the equation (e.g., associative properties, commutative properties, inverse properties, distributive property).
- Use the properties of equality to isolate the variable (e.g., addition property of equality, multiplication property of equality).

Solve: $3 - x + 3 = 5x - 2x - 2$

Equation	What did you do?	Property
$3 - x + 3 = 5x - 2x - 2$ $6 - x = 3x - 2$	arithmetic collect like terms	distributive property $(5 - 2)x = 3x$
$6 - x = 3x - 2$ $+ 2 \qquad + 2$ $8 - x = 3x$	add 2 to both sides arithmetic	addition property of equality additive inverse/identity properties
$8 - x = 3x$ $+ x \quad + x$ $8 = 4x$	add x to both sides collect like terms	addition property of equality additive inverse/identity properties distributive property $3x + x = (3 + 1)x = 4x$
$\frac{8}{4} = \frac{4x}{4}$ $2 = x$	multiply both sides by $\frac{1}{4}$ (or divide both sides by 4) arithmetic	multiplication(division) property of equality multiplicative inverse/identity properties

Check by substituting the solution into the original equation:

$$\begin{aligned} 3 - x + 3 &= 5x - 2x - 2 & \\ 3 - 2 + 3 &= 5(2) - 2(2) - 2 & ? \\ 4 &= 10 - 4 - 2 & ? \\ 4 &= 4 & \text{true} \end{aligned}$$

COMMON CORE STATE STANDARDS

	STANDARDS FOR MATHEMATICAL CONTENT
8.EE.C	**Analyze and solve linear equations and pairs of simultaneous linear equations.**
8.EE.7	Solve linear equations in one variable.
a	a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
b	Solve linear equations ~~with rational number coefficients~~, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
8.EE.8	Analyze and solve pairs of simultaneous linear equations:
a	Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
b	~~Solve systems of two linear equations in two variables algebraically,~~ and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.
c	Solve real-world and mathematical problems leading to two linear equations in two variables.
8.F.A	**Define, evaluate, and compare functions.**
8.F.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
8.F.3	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; ~~give examples of functions that are not linear.~~ ~~*For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4) and (3, 9), which are not on a straight line.*~~
8.F.B	**Use functions to model relationships between quantities.**
8.F.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

	STANDARDS FOR MATHEMATICAL PRACTICE
SMP1	Make sense of problems and persevere in solving them.
SMP2	Reason abstractly and quantitatively.
SMP3	Construct viable arguments and critique the reasoning of others.
SMP4	Model with mathematics.
SMP5	Use appropriate tools strategically.
SMP6	Attend to precision.
SMP7	Look for and make use of structure.
SMP8	Look for and express regularity in repeated reasoning.

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