Name ______________________________ Period __________ Date ______________

UNIT 3
STUDENT PACKET

MathLinks
GRADE 8

THE ALGEBRA OF EXPONENTS AND ROOTS

Parent (or Guardian) signature ______________________________

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

conjecture	cube of a number
cube root	exponent notation
radical expression	scientific notation

FOLDING PAPER

Follow your teacher's directions for (1) – (3).

(1) – (2)

1		
2		
3		
4		
5		
6		
7		

(3) As the number of folds increases by 1, the number of sections created ____________.

From the third column, we see that the number of sections can be written as…

From the first row, it seems reasonable that $2^{\square} = \square$.

4. Continue the patterns below.

3^4 = ____ • ____ • ____ • ____ = ________	10^4 = ____ • ____ • ____ • ____ = ________
3^3	10^3
3^2	10^2
3^1	10^1
3^0	10^0

5. Record the meanings of <u>exponent notation</u> and <u>conjecture</u> in **My Word Bank**.

6. Make a conjecture about zero as an exponent for $x \neq 0$: x^0 = _____. Check this conjecture in the **Student Resources** and correct it if necessary.

EXPONENT FACTS AND RULES

We will work with expressions involving positive, negative, and zero exponents; and use patterns to make conjectures for rules that apply to expressions involving integer exponents.

[8.EE.1; SMP2, 3, 7, 8]

GETTING STARTED

Fill in the missing cells below. All bases and exponents are natural numbers.

	Base	Exponent	Exponent form	Factors (as a multiplication expression)	Value (simplified form)
1.	3	6			
2.			6^3		
3.				$10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10$	
4.					625
5.			1776^0	No factors	

Think about the properties of squares and cubes. Fill in the missing cells below. All side and edge lengths are in linear units, areas are in square units, and volumes are in cubic units.

	Squares ☐	
	Side length	Area
6.	5	
7.		100
8.	n	
9.		m^2

	Cubes	
	Edge length	Volume
10.	7	
11.		64
12.	p	
13.		k^3

14. x^3 may be read "*x* to the third power" or "*x* cubed." Write a numerical equation that relates to problem 10 using this notation.

INVESTIGATING TWO EXPONENT PATTERNS

Follow your teacher's directions for (1) – (12).

	Expression	**Factors (as a multiplication expression)**	**Exponent form**
(1)			
(2)			
(3)			
(4)			
(5)			
(6)			
(7)			
(8)			
(9)			
(10)			
(11)			
(12)			

Use the appropriate exponent rule if possible to rewrite each expression form.

	Expression	**A base to an exponent** (like a^b, if possible)	**Applicable rule** (circle one)
13.	$7^2 \bullet 7^3$		product power neither
14.	$(9^3)^2$		product power neither
15.	$3^6 \bullet 6^3$		product power neither
16.	$(x^5)^8$		product power neither
17.	$x^5 \bullet x^8$		product power neither

PRACTICE 1

1. The product rule for exponents is ____________________.

2. Circle all equations that correctly depict the product rule.

$2^5 \bullet 2^7 = 2^{35}$ $4^5 \bullet 6^5 = 10^5$ $9 \bullet 9^8 = 9^9$

$y^2 \bullet y^8 = y^{10}$ $w^5 \bullet w = w^6$ $m^3 \bullet n^3 = mn^3$

Apply the product rule.

3. $3^6 \bullet 3^6$	4. $6^2 \bullet 6^{12}$
5. $v^5 \bullet v^{10}$	6. $k^4 \bullet k$

7. Create a numerical expression such that after the product rule is applied, it is equal to 81.

8. The power rule for exponents is ____________________.

9. Circle all equations that correctly depict the power rule.

$(2^5)^7 = 2^{35}$ $(4^5)^5 = 4^{10}$ $9^2 \bullet 9^7 = 9^9$

$(y^2)^8 = y^{16}$ $(w^5)^6 = w^{30}$ $(m^3)^3 = m^6$

Apply the power rule.

10. $(3^6)^4$	11. $(9^2)^2$
12. $(v^5)^{10}$	13. $(k^3)^5$

14. Create a numerical expression such that after the power rule is applied, it is equal to 64.

15. Josue looked at problem 11 and said, “Even if I mixed up these two exponent rules, I could still get it right.” Explain what Josue meant.

A THIRD PATTERN: THE QUOTIENT RULE FOR EXPONENTS

Complete the table for powers of 10. Following patterns down each column may be helpful. Some side-by-side entries for columns III and IV will be exactly the same (like for problem 3).

	I **Expression**	II **Expanded Form**	III **Power of 10** **(fractions okay)**	IV **Power of 10** **(no fractions)**	V **Value** **(fractions okay)**
1.	$\frac{10^3}{10^0}$			10^3	
2.	$\frac{10^3}{10^1}$	$\frac{10 \bullet 10 \bullet 10}{10}$			
3.	$\frac{10^3}{10^2}$		10^1	10^1	
4.		$\frac{10 \bullet 10 \bullet 10}{10 \bullet 10 \bullet 10}$			
5.	$\frac{10^3}{10^4}$		$\frac{1}{10^1}$		
6.	$\frac{10^3}{10^5}$				

7. The expressions in column I are quotients with the same bases. The expressions in column IV are in exponent form. What fact about the exponents in column I determines when the exponents in column IV will be…
   a. Positive?

   b. Negative?

   c. Zero?

8. Do any of the expressions above with negative exponents result in a negative value?

9. Look at columns IV and V. How is 10^{-2} related to 10^2?

10. Look at columns I and IV: $\frac{10^3}{10^5} = 10^{\square}$, so $\frac{10^3}{10^5} = 10^{\boxed{-}}$

A THIRD PATTERN: THE QUOTIENT RULE FOR EXPONENTS
Continued

Complete the table just like the previous page, but for powers of 3.

	I **Expression**	II **Expanded Form**	III **Power of 3** **(fractions okay)**	IV **Power of 3** **(no fractions)**	V **Value** **(fractions okay)**
11.	$\frac{3^1}{3^0}$				
12.			3^0		
13.	$\frac{3^1}{3^2}$				$\frac{1}{3}$
14.	$\frac{3^1}{3^3}$				
15.		$\frac{3}{3 \bullet 3 \bullet 3 \bullet 3}$			
16.					

17. Look at columns IV and V. How is 3^{-1} related to 3^1?

18. Look at columns I and IV: $\frac{3^1}{3^5} = 3^{\square}$, so $\frac{3^1}{3^5} = 3^{\frac{\square}{\square}}$

19. Patti thinks that a base number to a negative power must result in a negative value. Is Patti correct? Explain.

20. Make four conjectures below. If $x \neq 0$, then:

$x^0 =$	$x^{-1} =$	$x^{-a} =$	$\frac{x^a}{x^b} =$

Check your conjectures in the **Student Resources**, and correct them if necessary.

PRACTICE 2

1. Rewrite each algebraic expression to illustrate a fact about exponents. Then create a numerical example. (0^0 and division by zero are not defined.)

Words	Symbols	Numerical Example
The value of a number to the zero power	$x^0 =$	
The meaning of -1 as an exponent	$x^{-1} =$	
The meaning of a negative exponent	$x^{-a} =$	
The product rule for exponents	$x^a \bullet x^b =$	
The power rule	$(x^a)^b =$	
The quotient rule	$\frac{x^a}{x^b} =$	

2. Why do none of the rules above apply to the expressions $5^2 \bullet 3^6$ and $\frac{7^2}{3^2}$?

Apply exponent rules to these expressions. Results with positive exponents in them are okay.

3. 8^{-2}	4. 8^{-6}	5. $8^{-2} \bullet 8^{-6}$
6. $9 \bullet 9^7$	7. $9 \bullet 9^{-7}$	8. $9^{-1} \bullet 9^7$
9. $(6^3)^2$	10. $(6^{-3})^2$	11. $(6^{-3})^{-2}$
12. $\frac{12^6}{12^6}$	13. $\frac{12^6}{12^4}$	14. $\frac{12^4}{12^6}$
15. $\frac{5^7}{5^0}$	16. $\frac{5^0}{5^7}$	17. $\frac{5^{-3}}{5^{-2}}$

PRACTICE 3

1. Write four different expressions equivalent to 2^8 that include exponents. Computing 2^8 is not necessary.

2. Using the quotient rule, write $\frac{5^4}{5^8}$ as 5 to a single power _____

   Now write this expression with a positive exponent. _____

Simplify by using positive exponents.

3. $\dfrac{(5^2)^2}{5^3}$	4. $\dfrac{5^3 \bullet 5^2}{(5^3)^2}$	5. $\dfrac{5^3 \bullet 5^2}{5^4 \bullet 5^2}$	6. $\dfrac{(5^2)^2}{5^6}$
7. $2^{-3} \bullet 2^5$	8. $2^3 \bullet 2^{-5}$	9. $\dfrac{2^2 \bullet 2^{-3}}{2^3}$	10. $\dfrac{2^{-3} \bullet 2^{-3}}{2^{-2}}$
11. $3^4 \bullet \dfrac{1}{3^4}$	12. $3^{-4} \bullet \dfrac{1}{3^4}$	13. $3^4 \bullet \dfrac{1}{3^{-4}}$	14. $\dfrac{2 \bullet 3^{20}}{6 \bullet 3^{17}}$

PRACTICE 4: EXTEND YOUR THINKING

Apply exponent rules and write as a simple fraction or decimal.

1.	$\left(\left(\frac{1}{2}\right)^2\right)^3$	→	→	
2.	$\left(\frac{1}{2}\right)^2 \bullet \left(\frac{1}{2}\right)^3$	→	→	
3.	$\left(\left(\frac{2}{3}\right)^2\right)^3$	→	→	
4.	$\left(\frac{2}{3}\right)^2 \bullet \left(\frac{2}{3}\right)^3$	→	→	
5.	$((0.2)^2)^3$	→	→	
6.	$(0.2)^2 \bullet (0.2)^3$	→	→	

7. If $\left(\frac{1}{4}\right)^3 = \left(\frac{1}{2}\right)^n$, what is the value of *n*?

Compute.

8. $(-2)^2$	9. $(-2)^3$	10. $(-2)^4$	11. $(-2)^5$
12. $(-2)^{-2}$	13. $(-2)^{-3}$	14. $(-2)^{-4}$	15. $(-2)^{-5}$

16. Karin thinks that a negative number to a negative power must be negative. Is she correct? Explain.

LARGE AND SMALL QUANTITIES

We will write very large and very small quantities in different ways, including scientific notation. We will solve problems involving large and small quantities.

[8.EE.3, 8.EE.4; SMP1, 3, 4, 5, 6, 8]

GETTING STARTED

1. Fill in the place value chart below and locate the decimal point.

trillions							Hundred-thousands			hundreds		ones		hundredths		

Write each number:

2. One trillion, twenty billion, three hundred million, four thousand, five hundred

3. Seventy-eight thousandths

4. Six ten-thousandths

Round each number below as directed.

Number	Nearest hundreds	Nearest tenths	One significant digit	Two significant digits
Example: 74,259	74,300		70,000	74,000
5. 9,587				
6. 20,345,678				
Example: 12.2319		12.2	10	12
7. 5.1309				
8. 0.561494				

WHAT IN THE WORLD?

Follow your teacher's directions for (1) – (4).

(1)	(2)
______ is ______ times greater than ______. ______ is ______ times greater than ______.	

(3) A 2021 estimate of the ______________________ was ________________.

a.	b.
c.	d.

(4)

a.	b.	c.	d.

5. Complete the table below.

Country	**Population**			**US population is how many times larger?** (estimate using simple, rounded values)
	rounded (nearest thousand)	**rounded** (one significant digit)	**a whole number times a power of 10**	
United States	331,003,000			
South Korea	51,269,000			
Australia	25,500,000			
Oman	5,107,000			
Iceland	341,000			

WHAT IN THE WORLD?
Continued

6. Use the given populations in problem 5 to write these numbers in scientific notation. Use two significant digits.

United States	Oman
South Korea	Iceland
Australia	

Write an estimate of each population in scientific notation.

7. Egypt's population is about 2 times that of South Korea.	8. Russia's population is about 3 times that of South Korea.
9. Mexico's population is about 2.5 times that of South Korea	10. Indonesia's population is about 10 times that of Australia.
11. Colombia's population is about 10 times that of Oman.	12. Compare the populations of Iceland and Lebanon, which is about 6.8×10^6 people.

13. The two most populous countries in the world are India (1,380,193,000) and China (1,439,324,000).

 a. Write population estimates for both countries in scientific notation, rounded to two significant digits.

 b. Estimate the fraction of the total world population in China.

 c. About what fraction of the total world population lives in these two countries combined?

14. Record the meaning of <u>scientific notation</u> in **My Word Bank.**

*All given population estimates are from worldometers.info, May, 2021, rounded to the nearest thousand.

PRACTICE 5

Without computing, fill in the blanks to make the expressions equivalent.

1. 0.2×400 and $2 \times$ ______.	2. 55×10^6 and $5.5 \times 10^{\square}$.

Determine whether each number below is in scientific notation.
If **not**, write it in scientific notation.

3. 346×10^5	4. 2.25×10^6	5. 89.7×10^7	6. 0.14×10^{10}

Use symbols <, =, or > to compare each pair of numbers below. Show work or explain.

7. 1.5×10^8 ______ 451×10^5	8. 0.57×10^7 ______ 32×10^6

9. Dr. Jerry Buss purchased the Los Angeles Lakers basketball team in 1979 for approximately $67.5 million. At his death in 2013, the team was reportedly worth about $1 billion.

   a. Write each dollar amount as a single digit times a power of 10.

   b. Write each value using scientific notation.

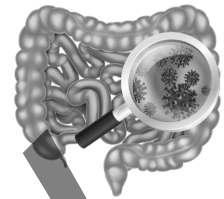
   c. By approximately what factor did the team value grow over that time period?

10. Abel multiplied two large quantities on his calculator and got: 8.72 E12
    Explain what his calculator is doing and the number this represents.

11. Frazier was trying to multiply $5,400,000,000,000,000 \times 75,000$ using his phone, but when he input the first number he got a message that 15 digits is the maximum allowed. Describe how he can multiply these numbers, and write the result using scientific notation.

A GUT FEELING

Follow your teacher's directions for (1) – (4).



(1)

Did you know that "gut health" is very important for things like your immunity, digestion, sleep, skin health, and even mental health? Your gut has trillions of living organisms in it (bacteria); some that are good, and some that are bad. More bacteria live in your gut than in the rest of your body!

(2) The average bacterium is between _____ and _____ micrometers .

1 micrometer (μm) ≈ _____ meters (m).

(3)

(4)

a.	b.	c.	d.

5. A 70 kg human has about 100 trillion bacteria in their gut that weigh a total of about 200g (the approximate weight of a medium sized mango).

   a. Write 1 trillion in scientific notation.

   b. Calculate this human weight if 1kilogram (kg) ≈ 2.2 pounds (lb).

   c. Gerry weighs about $\frac{2}{3}$ of this human weight. What is Gerry's weight (in lb)?

   d. About how many bacteria are in Gerry's gut? Write in scientific notation.

6. Compare the number of bacteria in your gut to the number of people on earth.

PRACTICE 6

Determine whether each number below is in scientific notation.
If **not**, write it in scientific notation.

1. 346×10^{-5}	2. 2.25×10^{-6}	3. 89.7×10^{-4}	4. 0.14×10^{-3}

Use symbols <, =, or > to compare each pair of numbers below. Show work or explain.

5. 0.123×10^{-4} _____ 3.2×10^{-6}	6. 0.571×10^{-5} _____ 32×10^{-7}

Complete the table below.

	Number pairs	**Write each as a single digit times a power of 10. Circle the greater number.**	**How many times greater is the circled quantity?**
7.	0.002×10^{-4} 200×10^{-6}		
8.	60×10^{-4} 0.3×10^{-3}		
9.	0.08×10^{-3} 400×10^{-5}		

10. Mazie multiplied two small quantities on her calculator and got: 4.61 E-9
Explain what her calculator is doing and what number this represents.

11. Andonia was trying to multiply 0.00000000007×0.5 using her phone, but when she input the first number, she got a message that she can't enter more than 10 digits after the decimal point. Show how to multiply these numbers, and write the result using scientific notation.

BIG AND SMALL JIGSAW

1. Discuss with your group: *about* how many times greater is 5.1×10^{7} than 1.9×10^{-5}?

Weight conversions	1,000 mg = 1 g	1,000 g = 1 kg	1,000 kg = 1 metric ton
Length conversions	10 mm = 1 cm	1,000 mm = 1 m	1,000 m = 1 km

2. Your teacher will give your group a card. Copy the information.

   We have the _______ _____________. Weight: ________________; Length: ___________.

3. With your group, convert measurements to at least one different unit and also to scientific notation.

   Weight:

   Length:

4. Partner with someone outside your group to compare a BIG and SMALL animal/object.

   We are comparing a BIG ________________ and a SMALL ____________________.

How many times heavier is the BIGGER one than the SMALLER one?	How many times longer is the BIGGER one than the SMALLER one?

5. Partner with someone else outside your group to compare BIG and SMALL.

   We are comparing a BIG ________________ and a SMALL ____________________.

How many times heavier is the BIGGER one than the SMALLER one?	How many times longer is the BIGGER one than the SMALLER one?

6. Return to your original group to share and compare. Make corrections as needed.

PRACTICE 7: EXTEND YOUR THINKING

Perform each operation below. Write final results in scientific notation.

1. $(3.4 \times 10^{-4}) \bullet (2 \times 10^{8})$	2. $(0.00000004) \bullet (21{,}000{,}000{,}000)$
3. $(9.1 \times 10^{-16}) \bullet (1{,}000{,}000{,}000)$	4. $(7.2 \times 10^{12}) \bullet (0.000000002)$
5. $\dfrac{2.6 \times 10^{24}}{1.3 \times 10^{14}}$	6. $\dfrac{1.3 \times 10^{-8}}{2.6 \times 10^{12}}$
7. $10^9 + 10^9 + 10^9 + 10^9 + 10^9 + 10^9 + 10^9$	8. $(5.4 \times 10^{6}) - (3.1 \times 10^{5})$

9. An internet search in 2021 showed that LeBron James earned $44.5 million. At the same time, the median salary for a middle school teacher in the US was $60,810 per year.

   a. Write both amounts as numbers rounded to two significant digits.

   b. Write both amounts in scientific notation.

   c. James makes approximately how many times more money than a middle school teacher in the US?

10. Go back to **What in the World?** The Earth's population was approximately ____________.

    The number of ants on Earth is approximately 10 trillion. Write both numbers rounded to a single digit times a power of 10. Then estimate the number of times the ant population is greater than the Earth's population.

EXPONENTS AND ROOTS

We will work with squares, square roots, cubes, and cube roots of rational numbers, and solve equations involving variables squared and cubed.

[8.EE.2; SMP1, 3, 4, 5, 6]

GETTING STARTED

1. Complete the table of squares and cubes below.

number (***n***)	4	0.4	$\frac{1}{4}$	-4	-0.4	$-\frac{1}{4}$
squared $(n)^2$						
cubed $(n)^3$						

Make conjectures by filling in each blank with *positive* or *negative*.

2. A positive number squared is ________________.

3. A positive number cubed is ________________.

4. A negative number squared is ________________.

5. A negative number cubed is ________________.

Compute.

6. $\sqrt{25}$	7. $\sqrt{0.25}$	8. $\sqrt{\frac{1}{25}}$

9. $\sqrt[3]{y}$ may be read the "third root of *y*" or the "cube root of *y*." It may be thought of as the answer to the question, "what number to the third power equals *y*?" Write a numerical equation that relates the fact that $(4)^3 = 64$ using this notation.

10. Record the meanings of <u>cube of a number</u>, <u>cube root</u>, and <u>radical expression</u> in **My Word Bank**.

MORE EXPLORING WITH EXPONENTS AND ROOTS

1. Complete the table of squares and cubes below.

number (***n***)	$\frac{5}{4}$	$-1\frac{3}{4}$	1.6	-2.5
squared $(n)^2$				
cubed $(n)^3$				

2. Why can the cube of a number be negative, while the square of a number cannot?

3. Why can't $\sqrt{-25}$ be calculated?

4. Use a calculator to find an approximate value for $3\sqrt{2}$.

Demonstrate whether each equation is true or false.

5. $\sqrt{25}+\sqrt{1} = 5 + 1$	6. $\sqrt{25}+\sqrt{1} = \sqrt{25+1}$	7. $2\sqrt{5} = \sqrt{10}$

Compute each cube root. Recall that $\sqrt[3]{64} = 4$ because 4 • 4 • 4 = 64.

8. $\sqrt[3]{8}$	9. $\sqrt[3]{-8}$	10. $\sqrt[3]{27}$	11. $\sqrt[3]{-27}$

12. Why can $\sqrt[3]{-1}$ be written as an integer, but $\sqrt{-1}$ cannot ?

PRACTICE 8

Compute.

1. $(\sqrt{7})^2$	2. $\sqrt{\frac{4}{9}}$	3. $\sqrt[3]{8}$	4. $(\sqrt[3]{7})^3$
5. $(\sqrt{25})(\sqrt{49})$	6. $\sqrt[3]{64}$	7. $\sqrt[3]{-8}$	8. $\sqrt[3]{-27}$
9. $(\sqrt{12})(\sqrt{12})$	10. $\sqrt[3]{1000}$	11. $\sqrt[3]{\frac{1}{27}}$	12. $\sqrt[3]{\frac{8}{27}}$
13. $2\sqrt{16}$	14. $\sqrt[3]{0.008}$	15. $\sqrt[3]{-1}$	16. $\left(\sqrt[3]{3\frac{3}{8}}\right)$

17. True or false? Explain. $\sqrt{4} + \sqrt{4} = \sqrt{8}$

18. Explain why the square root of a negative integer can never be an integer.

19. The cube root of a negative integer sometimes has an integer result and sometimes does not. Support this statement with numerical examples.

Explain each relationship with a sketch and numbers. Use squares, square roots, cubes, and cube roots.

20. The area of a square and its side lengths.	21. The volume of a cube and its edge lengths.

SOLVING EQUATIONS WITH EXPONENTS

Follow your teacher's directions for (1) – (4).

(1)
(2)
(3)
(4)

Solve each equation below. If rational number solutions do not exist for *x*, leave them in square root or cube root form.

5. $x^2 = 64$	6. $x^2 = \frac{1}{4}$	7. $x^2 = 0.04$	8. $x^2 = 5$
9. $x^3 = 27$	10. $x^3 = \frac{1}{8}$	11. $x^3 = 0.001$	12. $x^3 = 3$

13. Consider the equation $x^2 = 36$. Substitute the following numerical expressions into this equation to prove that they all make the equation true.

value	work	value	work
6	$(\_\_\_\_)^2$ = _____ • _____ = 36	$\sqrt{36}$	$(\sqrt{\_\_\_\_})^2$ = _____ • _____ = _____

PRACTICE 9

Solve each equation below. If rational number solutions don't exist for *x*, leave in square root or cube root form.

1. $x^2 = 100$	2. $x^2 = \frac{1}{64}$	3. $x^2 = 0.09$	4. $x^2 = 7$
5. $x^2 = 144$	6. $x^2 = \frac{49}{81}$	7. $x^2 = 0.36$	8. $x^2 = 11$
9. $x^3 = 64$	10. $x^3 = \frac{1}{64}$	11. $x^3 = 0.008$	12. $x^3 = 7$
13. $x^3 = 125$	14. $x^3 = \frac{8}{27}$	15. $x^3 = 0.064$	16. $x^3 = 11$

17. Consider the equation $x^3 = 216$. Substitute the following numerical expressions into this equation to prove that they all make the equation true.

value	work	value	work
6	$(\_\_\_\_)^3 = \_\_\_\_ \bullet \_\_\_\_ \bullet \_\_\_\_ = 216$	$\sqrt[3]{216}$	$\_\_\_\_ = \_\_\_\_ \bullet \_\_\_\_ \bullet \_\_\_\_ = \_\_\_\_$

18. Write equations for which $x = 2$ is a solution.

a. In the form $ax = b$, where *a* and *b* are integers.	b. In the form $x^2 = b$, where *b* is an integer.	c. In the form $x^3 = b$, where *b* is an integer.

19. Explain why $x^3 = -1$ has a solution while $x^2 = -1$ does not.

REVIEW

WHY DOESN'T IT BELONG?: THE ALGEBRA OF EXPONENTS AND ROOTS

For each set below, choose at least two of the four entries and explain why each doesn't belong with the others.

Set 1 Simplify each expression.

A $(x^2)^2$	**B** $x^8 \bullet x^{-4}$	
C $\frac{x^6}{x^8}$	**D** $\frac{x^{-4}}{x^{-8}}$	

Set 2 Write each expression with a base to a non-negative exponent.

A $x^4 \bullet x^{-4}$	**B** $\frac{1}{x^{-4}}$	
C $(x^2)^{-1}$	**D** x^{-4}	

Set 3 Simplify the two expressions and solve the two equations.

A $\sqrt{9}$	**B** $\sqrt[3]{-27}$	
C $x^2 = 9$	**D** $x^3 = 27$	

POSTER PROBLEMS: THE ALGEBRA OF EXPONENTS AND ROOTS

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is ____.
- Each group will have a different colored marker. Our group marker is _____.

Part 2: Follow your teacher's directions. Do the problems on posters. Use R3 for reference.

Poster 1 (or 5)	**Poster 2 (or 6)**	**Poster 3 (or 7)**	**Poster 4 (or 8)**
The Shake	The Jiffy	The Outhouse	The Attogram

A. Using the info provided by your teacher, read about your group's unit of measure. To what common measurement is this related? Write this number in decimals, using its common measurement.

B. Read about this group's unit of measure. Write this number in scientific notation, using its common measurement.

C. Write a problem using this structure: "How many times longer/bigger is a [something measured in time, area, or weight] than a(n) [corresponding teeny measurement]?"

D. Solve the problem that was written in above in problem C.

Part 3: Return to your seats. Work with your group, and show all your work.

1. Read about the quasihemidemisemiquaver. Say it, and rewrite it showing important syllable breaks.

2. A quaver is an eighth note. Each "prefix" is a signal to successively halve the note. Show why quasihemidemisemiquaver means 128^{th} note.

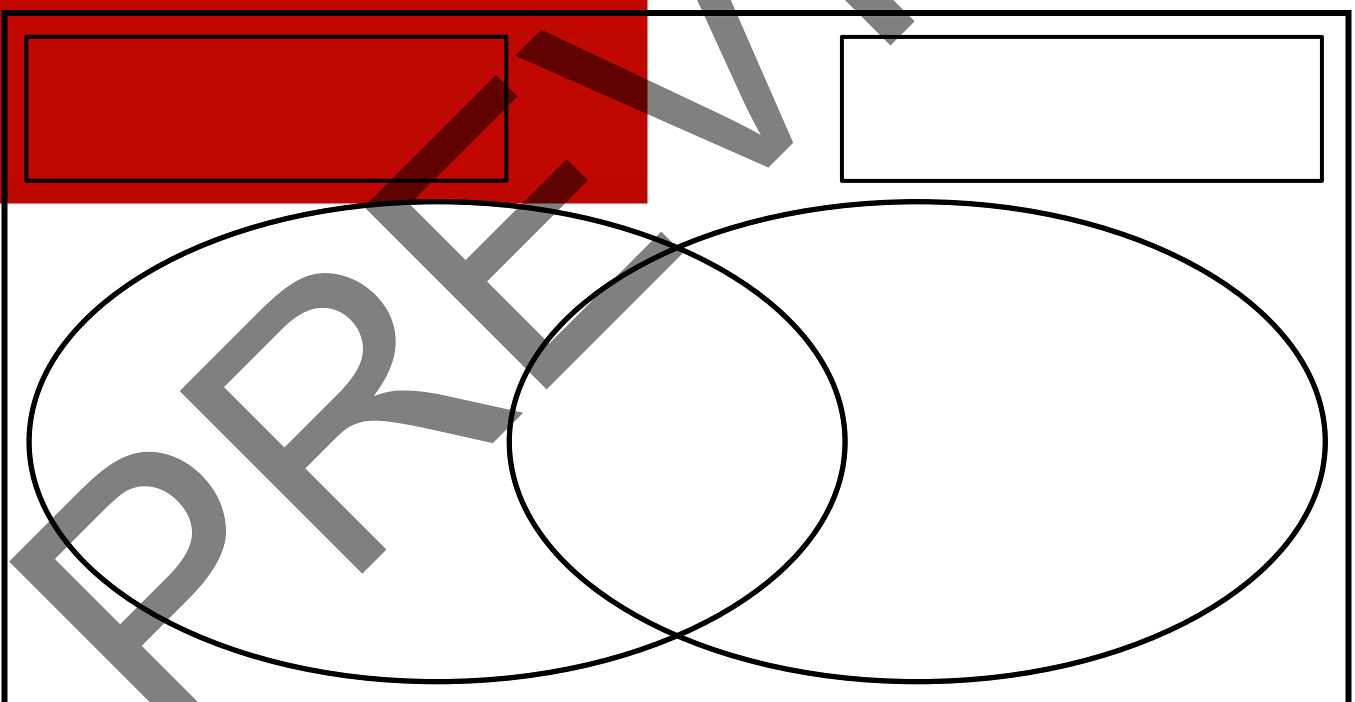
3. How is a quasihemidemisemiquaver related to the opening problem about folding paper?

MATCH AND COMPARE SORT: THE ALGEBRA OF EXPONENTS AND ROOTS

1. Individually, match words with descriptions. Record results.

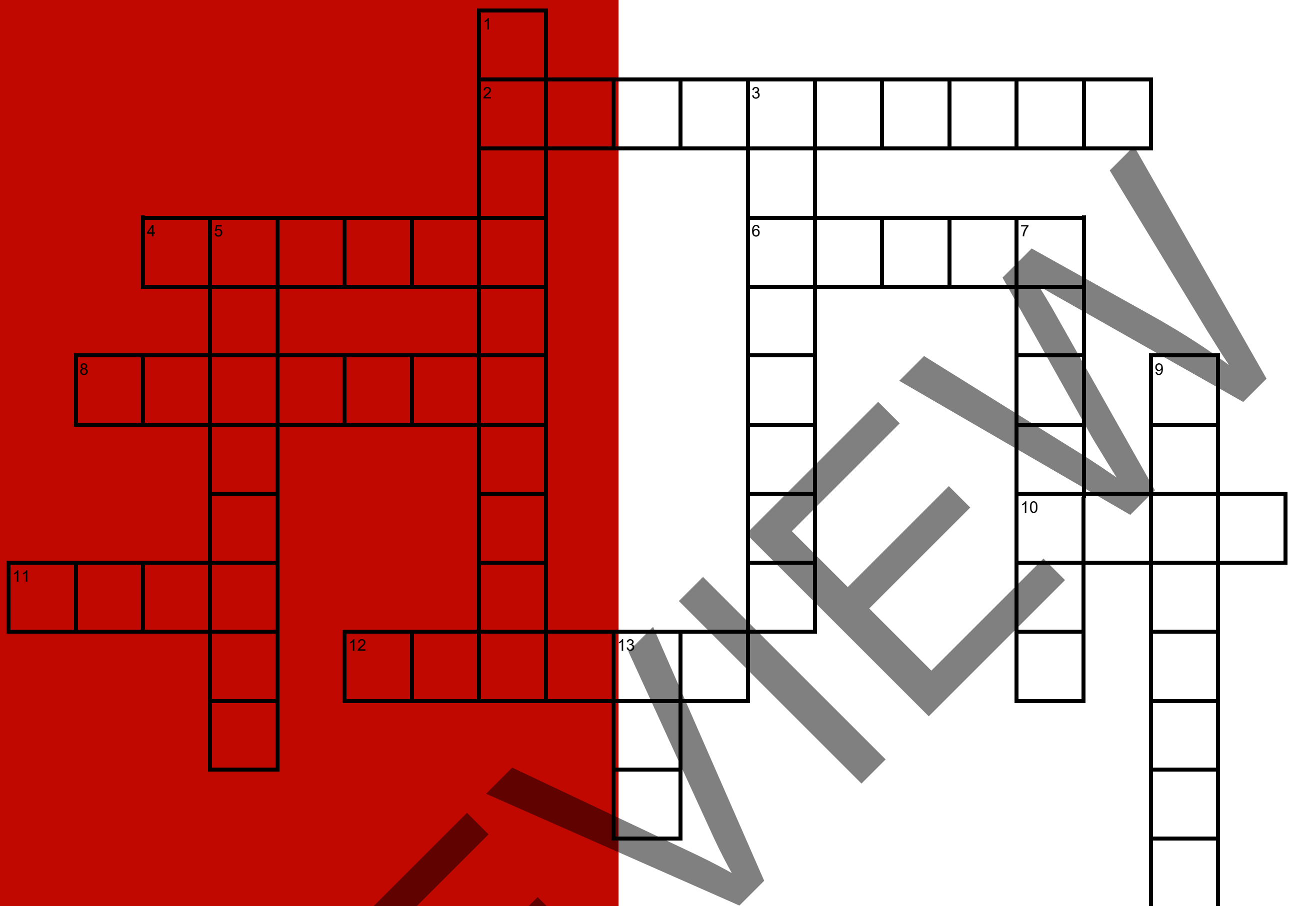
Card set △			Card set ○		
Card number	**word**	**Card letter**	**Card number**	**word**	**Card letter**
I			**I**		
II			**II**		
III			**III**		
IV			**IV**		

2. Partners, choose a pair of numbered matched cards and record the attributes that are the same and those that are different.



3. Partners, choose another pair of numbered matched cards and discuss the attributes that are the same and those that are different.

FOCUS ON VOCABULARY



Across

2 hypothesis, educated guess

4 A number to the second power

6 Rule for exponents where bases are same and exponents are multiplied

8 Rule for exponents where bases are the same and exponents are added

10 27 is example of perfect ____

11 In 5^{-2}, 5 is the _____

12 In $9 \bullet 9 \bullet 9$, 9 is a _____

Down

1 The product of a number between 1 and 10 and power of 10 is in _____ notation

3 In 5^{-2}, -2 is the _____

5 Rule for exponents where bases are the same and exponents are subtracted

7 $\sqrt[3]{8}$, $\sqrt{x}$, and $2\sqrt{16}$ are ____ expressions

9 The ___ ___ of 27 is 3 (two words)

13 A number to the zero power

SPIRAL REVIEW

1. **READY-X.** Solve for the values of R, E, A, D, Y, X. Sums of rows and columns are indicated at the end of each row and column.

COLUMNS

ROWS

R	Y	R	**13**
E	Y	E	**19**
A	Y	A	**29**
D	Y	X	**17**
18	**36**	**24**	

R = _____ E = _____ A = _____ D = _____ Y = _____ X = _____

2. Solve each equation below.

a. $4(m - 7) = -12$	b. $0 = 35(0.75 + n)$

SPIRAL REVIEW
Continued

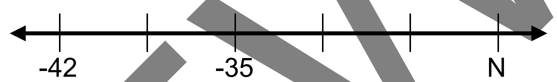
3. Solve for y in terms of x.

a. $2x + y = 14$	b. $2y - x = 14$

4. Find the value of N.

a.

b.

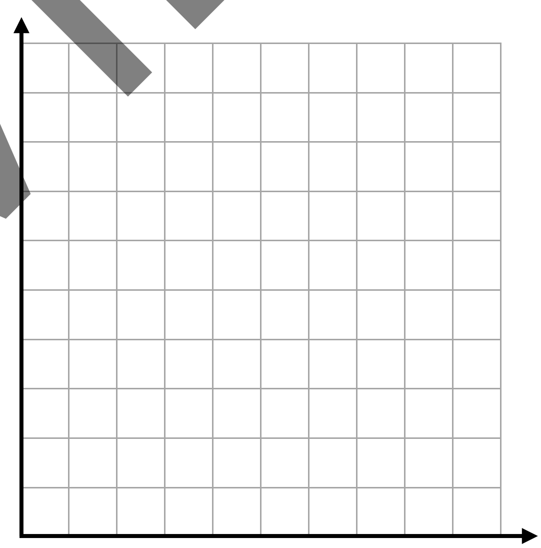


5. The table below has some costs and quantities for purchasing ride tickets at the fair.

a. Complete the table.

number of tickets (x)	cost in $ (y)	$\frac{\text{cost (\$)}}{\text{ticket}}$
1	1.5	
5	7.5	
10		
15		
20		

b. Graph with appropriate labels.



c. Write an equation to represent the situation.

d. Use evidence from these representations to explain whether this situation represents a proportional relationship.

REFLECTION

1. **Big Ideas**. Shade all circles that describe big ideas in this unit. Draw lines to show connections that you noticed.

- Use transformational geometry to investigate congruence and similarity
- Extend applications of volume to cylinders, cones, and spheres
- Explore bivariate data
- Complete the real number system
- Solve linear equations in one variable and linear systems in two variables
- Discover and apply properties of lines, angles, and triangles, including the Pythagorean Theorem
- Create, analyze, and use linear functions in problem solving
- Explore exponents and roots, and very large and very small quantities

Give an example from this unit of one of the connections above.

2. **Unit Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before or something you would still like to work on.

3. **Mathematical Practice.** Explain how repeated calculations revealed a pattern that could be generalized to a rule [SMP 8]. Then circle one more SMP on the back of this packet that you think was addressed in this unit and be prepared to share an example.

4. **More Connections.** Describe interesting idea (story) that was revealed by analyzing large or small numbers in this unit.

STUDENT RESOURCES

Word or Phrase	Definition
conjecture	A conjecture is a statement that is proposed to be true, but has not been proven to be true nor to be false. After creating a table of sums of odd numbers such as 1 + 3 = 4, 1 + 5 = 6, 5 + 7 = 12, 3 + 9 = 12, etc., we may make a conjecture that the sum of any two odd numbers is an even number. This conjecture can be proven to be true.
cube of a number	The cube of a number n is the number $n^3 = n \bullet n \bullet n$. The cube of -5 is $(-5)^3 = (-5)(-5)(-5) = -125$.
cube root	The cube root of a number n is the number whose cube is equal to n. That is, the cube root of n is the value of x such that $x^3 = n$. The cube root of n is written $\sqrt[3]{n}$. The cube root of -125 is $\sqrt[3]{-125} = -5$, because $(-5)^3 = (-5)(-5)(-5) = -125$.
exponent notation	The exponent notation b^n (read as " b to the power n ") is used to express n factors of b. The number b is the base, and the natural number n is the exponent. Exponent notation is extended to arbitrary integer exponents by setting $b^0 = 1$ and $b^{-n} = \frac{1}{b^n}$. $2^3 = 2 \bullet 2 \bullet 2 = 8$ (the base is 2 and the exponent is 3) $3^2 \bullet 5^3 = 3 \bullet 3 \bullet 5 \bullet 5 \bullet 5 = 1{,}125$ (the bases are 3 and 5) $2^0 = 1$ $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
radical expression	A radical expression is an expression involving a root, such as a square root. $\sqrt{20}$ and $5\sqrt{3}$ are radical expressions.
scientific notation	Scientific notation for a positive number represents the number as a product of a decimal between 1 and 10 and a power of 10. It is typically used to write either very large numbers or very small numbers. In scientific notation, the number 245,000 is written as 2.45×10^5. In scientific notation, the number 0.0063 is written as 6.3×10^{-3}.

Numbers Squared and Cubed

Why do we say that a number raised to the second power is "squared"? The reason has to do with the area formula for squares. The area of a square of side length s is given by

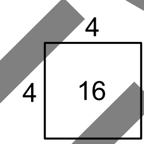
$$\text{area} = s \bullet s = s^2 .$$

A square with side length 4 units has area "4 squared" = 4^2 = 16 square units.

What about "square root" – where does that term come from?

Here the reason is that a "root" can also refer to the solution of an equation. A "square root" has to do with finding the side length of a square of a given area; that is, of solving the equation $s^2 = A$. For a given area A, the side length s of the square with area A is side length = $s = \sqrt{A}$ = "square root of A."

A square with area 16 square units has side length $\sqrt{16}$ = 4 units.



$\rightarrow 4^2 = 16$ and $\sqrt{16} = 4$

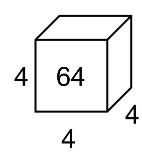
Why do we say that a number raised to the third power is "cubed"? In this case, the answer has to do with the volume formula for cubes. The volume of a cube with side length s is given by

$$\text{volume} = s \bullet s \bullet s = s^3 .$$

A cube with side length 4 units has volume "4 cubed" = 4^3 = 64 cubic units.

In turn, a "cube root" has to do with finding the side length of a cube of a given volume, that is, of solving the equation $s^3 = V$. For a given volume V, the side length s of the cube with volume V is side length = $s = \sqrt[3]{V}$ = "cube root of V."

A cube with volume 64 cubic units has side length $\sqrt[3]{64}$ = 4 units.



$\rightarrow 4^3 = 64$ and $\sqrt[3]{64} = 4$

Although we assume here that V is positive, the cube root of a negative number can be found by solving the equation, $s^3 = V$. The square root of a negative number is not a real number.

Squaring a number and finding the square root of a number are inverse operations. Similarly, cubing a number and finding the cube root of a number are inverse operations.

Three Facts and Three Rules for Exponents

Definitions and Rules		Example
Meaning of positive exponent:	$x^m = x \bullet x \bullet \ldots \bullet x$ (*m* factors)	$3^4 = 3 \bullet 3 \bullet 3 \bullet 3$ (4 factors of 3)
Fact about zero as an exponent:	$x^0 = 1,\ x \neq 0$	$3^0 = 1$, (0^0 is not defined)
Fact about a negative exponent:	$x^{-a} = \frac{1}{x^a},\ x \neq 0$	$3^{-2} = \frac{1}{3^2}$, (0 cannot be in the denominator because division by 0 is not defined)
Product rule for exponents:	$x^a \bullet x^b = x^{a+b}$	$3^2 \bullet 3^1 = 3 \bullet 3 \bullet 3 = 3^{2+1} = 3^3$
Power rule for exponents:	$(x^a)^b = x^{a \bullet b}$	$(3^2)^3 = 3^2 \bullet 3^2 \bullet 3^2 = 3^{3 \bullet 2} = 3^6$
Quotient rule for exponents:	$\frac{x^a}{x^b} = x^{a-b},\ x \neq 0$	$\frac{3^4}{3^6} = \frac{3 \bullet 3 \bullet 3 \bullet 3}{3 \bullet 3 \bullet 3 \bullet 3 \bullet 3 \bullet 3} = 3^{4-6} = 3^{-2}$, (0 cannot be in the denominator because division by 0 is not defined)

The three rules above apply to expressions with the same base numbers. For example:
$5^3 \bullet 4^2 = (5 \bullet 5 \bullet 5) \bullet (4 \bullet 4)$, and the product rule does not apply.

Making Sense of Zero and Negative Exponents

These patterns show that the definitions for zero and negative exponents are reasonable.

Pattern: Divide by 2	Result of the division	Pattern as a product	Pattern in exponent form
	Start with 8	$2 \bullet 2 \bullet 2$	2^3
$8 \div 2$	4	$2 \bullet 2$	2^2
$4 \div 2$	2	2	2^1
$2 \div 2$	1	1	2^0
$1 \div 2$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2^1}$ or 2^{-1}
$\frac{1}{2} \div 2$	$\frac{1}{4}$	$\frac{1}{2 \bullet 2}$	$\frac{1}{2^2}$ or 2^{-2}
$\frac{1}{4} \div 2$	$\frac{1}{8}$	$\frac{1}{2 \bullet 2 \bullet 2}$	$\frac{1}{2^3}$ or 2^{-3}

Scientific Notation				
Given number	**Related decimal between 1 and 10**	**Power of 10**	**Number in scientific notation**	**Reasoning**
120,000,000	1.2	10^8	1.2×10^8	The given number is 10^8 times 1.2; adjust place values by multiplication.
0.0000345	3.45	10^{-5}	3.45×10^{-5}	3.45 is 10^5 times the given number; adjust place values by multiplication.

Some of the benefits of scientific notation:

(1) Scientific notation is useful for writing numbers with very large or very small values in a compact way.

(2) The power of 10 gives an immediate clue to the relative size of the number.

Error alert!

When comparing numbers in scientific notation such as 2.5×10^{12} and 8.76×10^8, a common mistake is to focus on the fact that 8.76 > 2.5. **Focus on the exponent!**

$$2.5 \times 10^{12} = 2{,}500{,}000{,}000{,}000$$
$$8.76 \times 10^{8} = 876{,}000{,}000$$

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
8.EE.A	**Work with radicals and integer exponents.**
8.EE.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
8.EE.2	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. ~~Know that $\sqrt{2}$ is irrational.~~
8.EE.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*
8.EE.4	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

STANDARDS FOR MATHEMATICAL PRACTICE	
SMP1	Make sense of problems and persevere in solving them.
SMP2	Reason abstractly and quantitatively.
SMP3	Construct viable arguments and critique the reasoning of others.
SMP4	Model with mathematics.
SMP5	Use appropriate tools strategically.
SMP6	Attend to precision.
SMP7	Look for and make use of structure.
SMP8	Look for and express regularity in repeated reasoning.

9 781614 450542